



SCHOLAR COACH

Boutique Academic Coaching & Tutoring

AP PHYSICS 1: ALGEBRA-BASED · COURSE REVIEW AND PRACTICE

AP Physics 1: Algebra-Based **Mastery.**

Every unit on the exam: concise concept review, worked examples, exam-style writing, practice problems with full explanations, a mixed practice test, and a reference section.

INSIDE

Words in Context · Inferences · Command of Evidence · Central Ideas and Details · Function of a Sentence · Algebra · Advanced Math · Problem-Solving and Data Analysis · Geometry and Trigonometry · Three Full-Length Practice Exams with Explanations



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Course review, practice, and a mixed test.

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Start Here: The Exam, Your Plan, and Your Tools

How the AP Physics 1: Algebra-Based exam is built, how points are earned, and how to use this book.

F Start Here: The Exam, Your Plan, and Your Tools

How AP Physics 1 is structured and scored, what each free-response type rewards, how to plan your weeks, and the math habits that protect points.

Why This Matters

Knowing the shape of the exam before you study saves weeks. This chapter explains how AP Physics 1 is built, which units carry the most weight, what each free-response type rewards, how to pace your study, and the math habits that protect points. This book teaches Units 1 to 3 in full; Units 4 to 8 follow the same structure.

The 2027 Exam

The exam is on **Wednesday, May 5, 2027 (Session 2)**. It is a hybrid digital exam of three hours. Section I has 42 multiple-choice questions in 85 minutes and counts 50%. Section II has 4 free-response questions in 95 minutes and counts 50%; you view the questions in the testing app and handwrite answers in a paper booklet. Calculators are allowed on both sections and a reference sheet is provided. The multiple-choice section for 2027 grew from 40 questions and 80 minutes, and the free-response time shrank from 100 minutes. Treat questions as single-correct. No calculus is required: algebra, trigonometry, and proportional reasoning are enough.

Unit	Topic	Multiple-choice weight
1	Kinematics	10-15%
2	Force and Translational Dynamics	18-23%
3	Work, Energy, and Power	18-23%
4	Linear Momentum	10-15%
5	Torque and Rotational Dynamics	10-15%
6	Energy and Momentum of Rotating Systems	5-8%
7	Oscillations	5-8%
8	Fluids	10-15%

A note on g

The exam's numerical problems default to $g = 10 \text{ m/s}^2$, and answers using 9.8 or 9.81 are not penalized. This book uses 9.8 m/s^2 throughout so you can check every answer exactly; expect your results to differ by about 2% from a 10 m/s^2 solution. Always say which value you used.

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Practices and Skills

Practice	Skills	What it looks like
1 Creating Representations	1.A diagrams; 1.B quantitative graphs; 1.C qualitative graph sketches	Free-body diagrams, energy bar charts, labeled axes with scales
2 Mathematical Routines	2.A derive; 2.B calculate or estimate; 2.C compare; 2.D predict factors of change	Symbolic derivations, answers with units, ratios such as "quadruples"
3 Scientific Questioning and Argumentation	3.A design procedures; 3.B apply a law to make a claim; 3.C justify with evidence	Experiment plans, claim-evidence-reasoning, justification from data

Use the exam's task verbs the way they are used in this book: **derive** means start from a principle and show the algebra in the given symbols; **calculate** means a numerical answer with units; **justify** means a claim backed by a principle or data; **sketch** means qualitative shape with labeled features.

The Four Free-Response Types

Type	What you do	Where points come from
Mathematical Routines	Derive and calculate	Naming the starting principle, symbolic substitution, units
Translation Between Representations	Move among graphs, diagrams, equations, words	Correct shapes and labels, bars that sum correctly, consistency with the story
Experimental Design and Analysis	Plan a procedure and analyze data	Variables, measurable quantities, linear graph, slope tied to the target quantity
Qualitative/Quantitative Translation	Connect a qualitative claim to an equation	Claim, equation, and reasoning that agree with one another

In the 2025 exam these four carried about 10, 12, 10, and 8 points in that order; point values can change, so read the directions. Across all four, graders reward a named principle, visible substitutions, labeled axes with quantity and unit, one best-fit line drawn with a straightedge, and justifications that tie to the specific scenario. Common losses: connect-the-dots graphs, plotting $1/x$ when the slope should be the target, writing "gravity" or "it" without a referent, and a claim that contradicts the reasoning.

Study Plans

Week	4-week plan	8-week plan
1	Units 1 and 2: read, do examples, practice set	Unit 1 and its practice set
2	Unit 3 and Units 4 to 5 core ideas	Unit 2 and Unit 2 free response
3	Units 6 to 8; mixed practice test	Unit 3 and energy bar-chart drills
4	Free-response clinic; review traps	Unit 4 momentum
5		Unit 5 torque
6		Units 6 and 7
7		Unit 8 and mixed test
8		Free-response clinic and weak spots

How to Use This Book

- ◆ Read Why This Matters, then Core Ideas with a pencil: redraw each figure from memory.
- ◆ Cover each worked example's solution and try it first; compare your units and signs.
- ◆ Write the Model Response in your own words before reading ours; check it against the grader list.
- ◆ Do the practice sets with a timer: about 1.5 minutes per multiple-choice item.
- ◆ Keep a mistake log naming the trap, not just the topic.

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Math Habits That Protect Points

Four habits

Solve symbolically first, substitute numbers last, and carry units through every line.

Use factor-of-change reasoning: if $K = \frac{1}{2}mv^2$ and v doubles, K quadruples.

Decide the sign convention (positive direction, origin, system) before you write an equation.

Linearize: choose axes so the slope or intercept equals the quantity you want, such as d versus t^2 with slope $a/2$.

Practice

CHECK Front Check

Answer each item. Show your reasoning.

- Which statement about the 2027 exam is correct?
 - Section I has 42 multiple-choice questions in 85 minutes and counts 50%.
 - Section I has 40 multiple-choice questions in 80 minutes and counts 50%.
 - Section II has 6 free-response questions in 95 minutes.
 - Calculators are allowed only in Section II.
- A student finds that a car's speed doubles. By what factor does its kinetic energy change?
 - 2
 - 4
 - 1/2
 - 8
- A block's position is measured at several times as it accelerates uniformly from rest. A student plans to plot position versus time.
 - Identify the graph that gives a straight line and explain what its slope represents.
 - State two features that make the graph score well.
- A student writes "the ball has no energy at the top of its path because gravity stops it." Identify two errors and write a correct statement. Take the ball and Earth as the system and neglect air resistance.

Answers Answers and Explanations

F Start Here: The Exam, Your Plan, and Your Tools

CHECK · FRONT CHECK

- A. The 2027 format is 42 questions in 85 minutes for Section I. Choice B is the old format.
- B. $K \propto v^2$, so doubling the speed multiplies K by $2^2 = 4$. Choice A treats K as proportional to speed.
- (a) Plot position (vertical) against time squared (horizontal): $x = \frac{1}{2}at^2$ from rest gives a straight line with slope $a/2$. (b) Axes labeled with quantity and unit, and one best-fit line drawn with a straightedge through the data rather than connecting the dots.
- Error 1: speed is zero at the top but the system still has energy: the ball-Earth system has gravitational potential energy mgh there. Error 2: gravity does not "stop" the ball; the gravitational force from Earth reduces its upward velocity by changing its momentum and kinetic energy continuously. Correct: at the top the ball's kinetic energy is zero, but the system's total energy is unchanged, all stored as gravitational potential energy.

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Unit 1: Kinematics

Exam weighting: about 10-15% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

1 Kinematics

Describing motion with scalars, vectors, graphs, and motion maps; constant acceleration, relative velocity, and projectiles, all with algebra and clear sign conventions.

Why This Matters

Kinematics carries 10-15% of multiple choice, but its tools run through every later unit: graphs, signs, vectors, and the habit of defining a coordinate system. Students lose points by mixing up distance with displacement and by reading a graph's height when its slope is needed. Numerical work in this book uses $g = 9.8 \text{ m/s}^2$; the exam's own default is 10 m/s^2 and a 9.8 or 9.81 answer is accepted, so your results may differ from a 10 m/s^2 solution by about 2%.

Core Ideas

1.1 Scalars and Vectors in One Dimension

A **scalar** has only magnitude (distance, speed, time, mass). A **vector** has magnitude and direction (displacement, velocity, acceleration). In one dimension, direction is carried by a sign, so you must first choose an origin and a positive direction and say so. Displacement $\Delta x = x_f - x_i$ depends only on endpoints; distance adds every leg and is never negative. Speed is distance over time; average velocity is displacement over time, so a runner who finishes where she started has zero average velocity but nonzero average speed.

1.2 Displacement, Velocity, and Acceleration

Average velocity is $v_{avg} = \Delta x / \Delta t$ and average acceleration is $a_{avg} = \Delta v / \Delta t$. Instantaneous values are slopes at a point on a graph. An object can have zero velocity and nonzero acceleration (the top of a vertical toss). An object speeds up when velocity and acceleration have the same sign and slows down when the signs are opposite.

Constant-acceleration model (one dimension)

$$v = v_0 + a t$$

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a \Delta x$$

Use only when a is constant. For free fall near Earth with up positive, $a = -9.8 \text{ m/s}^2$ on the way up, at the top, and on the way down (air resistance negligible).

1.3 Representing Motion

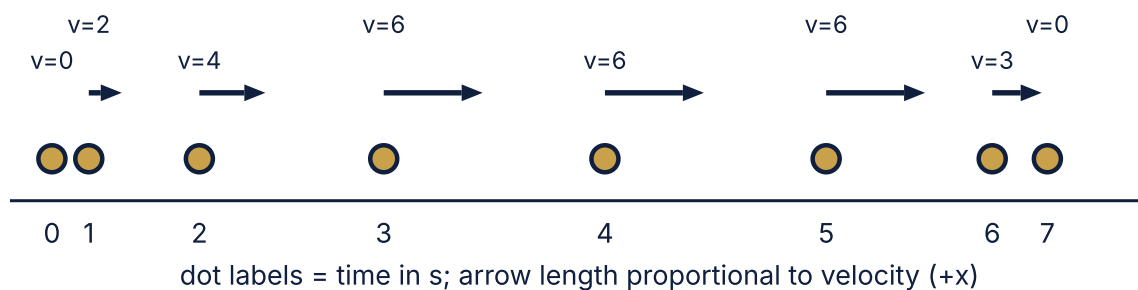
One motion can be shown four ways: words, a **motion map** (dots at equal time intervals with velocity arrows), graphs, and equations. The cart below starts at rest, accelerates at $+2 \text{ m/s}^2$ for 3 s, coasts at 6 m/s for 2 s, then brakes at -3 m/s^2 for 2 s.

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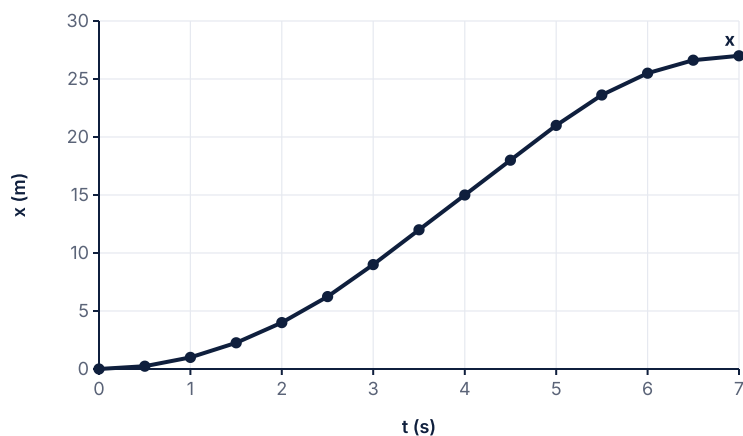
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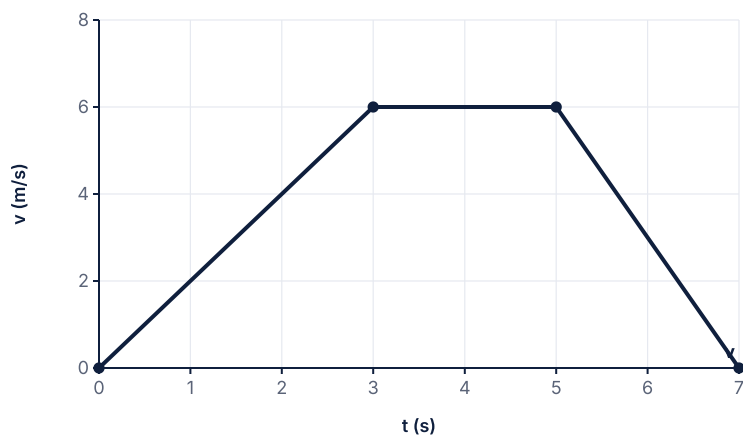
Motion map of the cart (dots every 1.0 s)



Dots spread out while speeding up, are evenly spaced at constant velocity, and bunch up while slowing.



Position: curved while accelerating, straight at constant velocity.

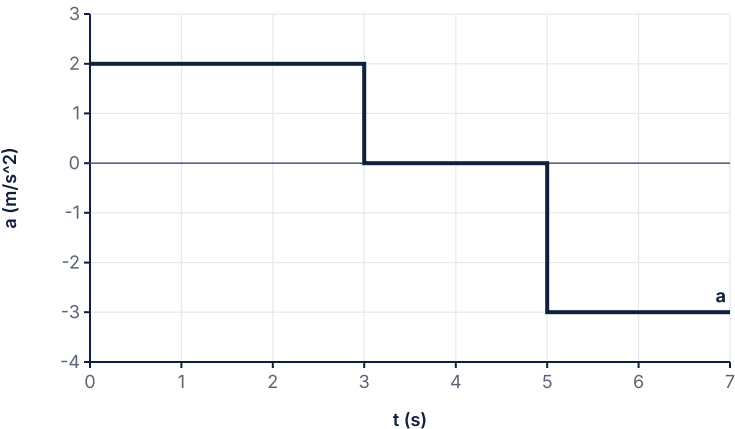


Velocity: slopes are the accelerations $+2$, 0 , -3 m/s^2 .

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Acceleration: constant in each stage, jumps between stages.

Graph feature	Meaning	Example from the cart
Slope of x-t	velocity	slope of x-t at 4 s is 6 m/s
Slope of v-t	acceleration	-3 m/s ² from 5 to 7 s
Area under v-t	displacement	9 + 12 + 6 = 27 m
Area under a-t	change in velocity	+6 m/s from 0 to 3 s
Curved x-t	changing velocity	parabola from 0 to 3 s

Label each axis with quantity and unit, and remember that area below the time axis counts as negative displacement.

1.4 Reference Frames and Relative Motion

Every velocity is measured relative to some frame. For objects A, B, and C, velocities add as vectors: $\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$, read as "A relative to C equals A relative to B plus B relative to C." A passenger walking 1.5 m/s toward the front of a train moving 20 m/s has a ground velocity of 21.5 m/s; toward the back, 18.5 m/s. Unless told otherwise, the frame is inertial and measurements are relative to the ground.

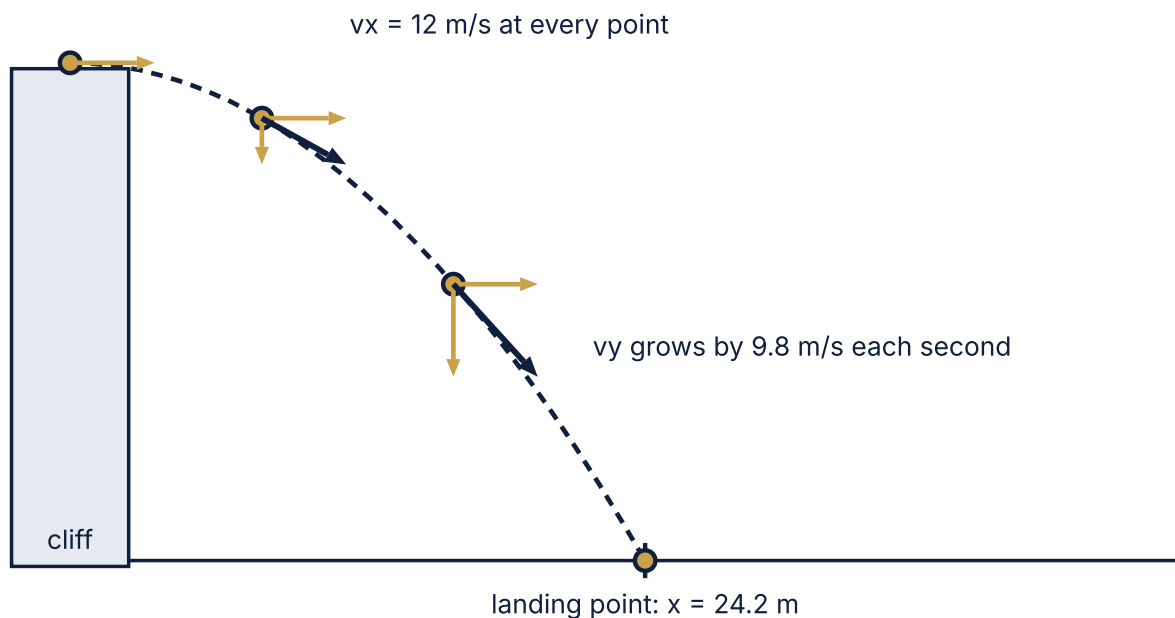
1.5 Vectors and Motion in Two Dimensions

A vector in a plane is described by components along chosen axes: $A_x = A \cos \theta$ and $A_y = A \sin \theta$ for angle θ from the x axis, and $A = \sqrt{A_x^2 + A_y^2}$, $\tan \theta = A_y/A_x$. The central idea of projectile motion is **independence of axes**: the horizontal and vertical motions share only the time. With air resistance negligible, $a_x = 0$ and $a_y = -g$, so v_x stays constant while v_y changes by 9.8 m/s every second.

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Horizontal launch: the horizontal arrow never changes, the vertical arrow grows, and the velocity (navy) tilts steeper.

Projectile procedure

1. Draw axes and resolve the launch velocity: $v_{0x} = v_0 \cos \theta$, $v_{0y} = v_0 \sin \theta$.
2. Write horizontal equations with $a_x = 0$ and vertical equations with $a_y = -g$ separately.
3. Link them only through the common time t .
4. At the highest point $v_y = 0$ but v_x is unchanged, so the speed is $v_0 \cos \theta$, not zero.

Worked Examples

EXAMPLE 1: DISTANCE, DISPLACEMENT, SPEED, VELOCITY

A jogger runs 40 m east, then 15 m west, taking 10 s in all. Take east as positive. Find the distance, displacement, average speed, and average velocity.

SOLUTION

Distance = $40 + 15 = 55 \text{ m}$.

Displacement $\Delta x = +40 + (-15) = +25 \text{ m}$, so 25 m east.

Average speed = $55/10 = 5.5 \text{ m/s}$ (a scalar).

Average velocity = $+25/10 = +2.5 \text{ m/s}$, i.e., 2.5 m/s east. The two averages differ because the path doubled back.

EXAMPLE 2: READING A V-T GRAPH

For the cart whose v-t graph appears above, find the total displacement, the average velocity over the 7 s, and the acceleration from 5 s to 7 s.

SOLUTION

Displacement is the area under the curve: triangle $\frac{1}{2}(3)(6) = 9 \text{ m}$, rectangle $2 \times 6 = 12 \text{ m}$, triangle $\frac{1}{2}(2)(6) = 6 \text{ m}$. Total = 27 m.

Average velocity = $27/7 = 3.9 \text{ m/s}$ in the +x direction.

Slope from 5 s to 7 s: $a = (0 - 6)/(7 - 5) = -3.0 \text{ m/s}^2$. The negative sign with positive velocity means the cart slows down.

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EXAMPLE 3: HORIZONTAL LAUNCH

Using the projectile figure, a ball rolls off a 20 m high cliff at 12 m/s horizontally. Find the fall time, horizontal range, and the speed and direction of the velocity just before landing.

SOLUTION

Vertical: $\Delta y = \frac{1}{2}gt^2$ so $t = \sqrt{2(20)/9.8} = 2.02$ s.

Horizontal: $x = v_x t = 12(2.02) = 24.2$ m.

Landing components: $v_x = 12$ m/s, $v_y = gt = 19.8$ m/s downward.

Speed = $\sqrt{12^2 + 19.8^2} = 23.1$ m/s at $\tan^{-1}(19.8/12) = 58.8$ deg below the horizontal.

EXAMPLE 4: RELATIVE MOTION ON A RIVER

A boat moves at 4.0 m/s relative to the water across a river 120 m wide with a 1.5 m/s current. (a) Heading straight across, find the crossing time, downstream drift, and speed relative to the ground. (b) At what angle upstream must it head to land directly opposite, and how long does that take?

SOLUTION

(a) Across: $t = 120/4.0 = 30$ s. Drift = $1.5(30) = 45$ m. Ground speed = $\sqrt{4.0^2 + 1.5^2} = 4.27$ m/s.

(b) Cancel the current: $4.0 \sin \theta = 1.5$, so $\theta = 22.0$ deg upstream of the straight-across line.

Remaining crossing component = $4.0 \cos \theta = \sqrt{16 - 2.25} = 3.71$ m/s, so $t = 120/3.71 = 32.4$ s. The straight route is longer in time.

Write It Like a 5**Model Response**

Prompt: A student tosses a ball straight up, releasing it at 14.7 m/s from hand height. Sketch v-t and a-t graphs for the flight until the ball returns to the hand, and justify the acceleration at the highest point. Take up as positive. The velocity graph is a straight line starting at +14.7 m/s at $t = 0$, crossing zero at $t = 1.5$ s, and ending at -14.7 m/s at $t = 3.0$ s; its slope is -9.8 m/s^2 . The acceleration graph is a horizontal line at -9.8 m/s^2 for the whole flight. At the top, the velocity is momentarily zero but the acceleration is not: the only force on the ball is the gravitational force from Earth, so by Newton's second law the acceleration is the gravitational field strength, 9.8 m/s^2 downward, at every instant.

What the grader looks for

1. A stated sign convention.
2. v-t line with correct intercepts and one straight slope.
3. a-t constant and negative, not zero at the top.
4. A justification that names the cause (net force or gravitational field) and links it to the instant asked.
5. Axes labeled with quantity and unit.

Coach's Playbook

- 1 Sketch, then choose the origin and positive direction.
- 2 List knowns and unknowns with signs; pick the equation that omits the quantity you neither know nor need.
- 3 For graphs, decide whether the question asks for a slope, an area, or a value.
- 4 Split 2D motion into axes and carry only t between them.
- 5 Check units and sign against the story.

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Common traps

1. Saying acceleration is zero at the top of a toss; only velocity is zero. 2. Reading the height of a v-t graph as acceleration or the slope of an x-t graph as position. 3. Using $\Delta x = vt$ when velocity changes, or the constant-acceleration equations across stages with different accelerations. 4. Labeling a graph axis with only a unit or only a symbol, or joining scattered data points instead of drawing one best-fit line with a straightedge.

Quick Check

Q1. A car's v-t graph is a horizontal line at -5 m/s. What are its acceleration and direction? A: Zero acceleration; it moves in the negative direction at constant speed. Q2. A ball is dropped from rest. How far has it fallen after 2.0 s? A: $\frac{1}{2}(9.8)(2.0)^2 = 19.6$ m. Q3. A swimmer swims 2.0 m/s relative to still water across a 1.0 m/s current. What is her speed relative to the shore? A: $\sqrt{4+1} = 2.2$ m/s.

Challenge Corner

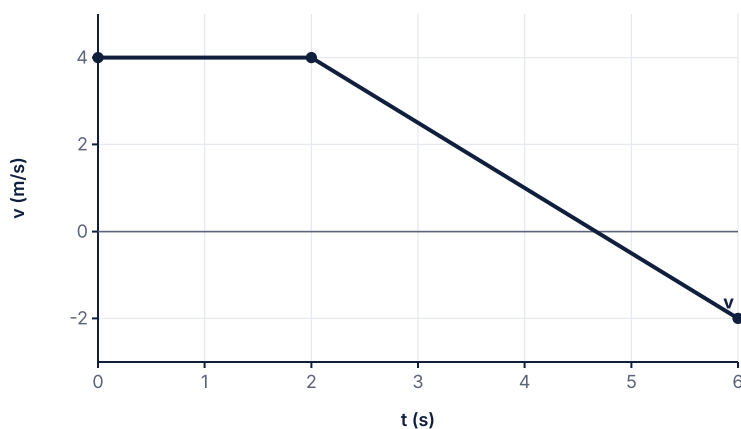
A car passes a stopped police cruiser at a constant 25 m/s. At that instant the cruiser accelerates from rest at 4.0 m/s^2 . When and where does it catch the car, and how fast is it going? Set positions equal: $25t = \frac{1}{2}(4.0)t^2 = 2t^2$, so $t = 12.5$ s (the other root $t = 0$ is the starting pass). Position = $25(12.5) = 312.5$ m. Cruiser speed = $4.0(12.5) = 50$ m/s, twice the car's speed, because catching up means making up the area difference on a v-t graph.

Practice

PRACTICE Unit 1 Practice

Show your work. A calculator is allowed. Justify conclusions.

1. The velocity of an object versus time is shown. What is the object's displacement from $t = 0$ to $t = 6$ s?



Velocity of an object.

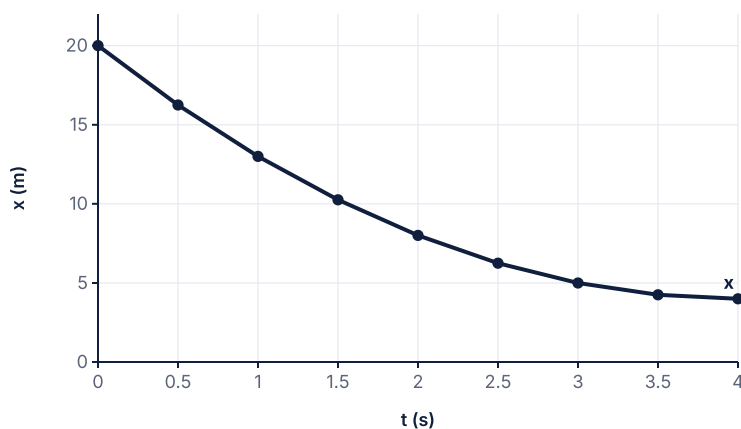
- (A) 8 m (B) 12 m (C) 14.7 m (D) 24 m
2. A ball is launched at 20 m/s at 30 degrees above the horizontal. Air resistance is negligible. What is its speed at the highest point?
(A) 0 (B) 10 m/s (C) 17.3 m/s (D) 20 m/s
3. A ball dropped from rest falls 20 m in a time t . How far does it fall from rest in a time $2t$?
(A) 20 m (B) 40 m (C) 60 m (D) 80 m
4. A moving walkway carries riders east at 2.0 m/s relative to the floor. A rider walks west at 1.5 m/s relative to the walkway. What is the rider's velocity relative to the floor?
(A) 0.5 m/s east (B) 3.5 m/s east (C) 1.5 m/s west
(D) 2.0 m/s east

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5. Ball A is dropped from a height H . Ball B is launched horizontally from the same height at the same instant. Air resistance is negligible. Which statement is correct?
- (A) A hits first, and both hit with the same speed.
 (B) Both hit the ground at the same time, and B hits with the greater speed.
 (C) Both hit at the same time with the same speed.
 (D) B hits later because it travels farther.
6. The graph shows the position of a cart. Which description fits the motion from 0 to 4 s?



Position of a cart.

- (A) The cart moves in the negative direction and slows down, so its acceleration is positive.
 (B) The cart moves in the positive direction and speeds up.
 (C) The cart moves in the negative direction and speeds up.
 (D) The cart moves in the positive direction and slows down.
7. A car travelling at 30 m/s brakes with a constant acceleration of magnitude 6.0 m/s^2 until it stops.
- (a) Derive $v^2 = v_0^2 + 2a\Delta x$ starting from $v = v_0 + at$ and $\Delta x = v_0t + \frac{1}{2}at^2$.
 (b) Calculate the braking distance.
 (c) A driver's reaction time is 0.50 s at the initial speed. Calculate the total distance from the moment the hazard appears.
8. A ball is tossed vertically upward from hand height at 14.7 m/s and caught at the same height. Take up as positive.
- (a) Sketch the velocity-time graph and the acceleration-time graph, labeling the intercepts and values.
 (b) Draw a motion map for the rising half of the flight.
 (c) Calculate the maximum height above the hand.
9. A cart on an inclined track starts from rest and rolls down. Available equipment: a meterstick, a stopwatch, a ramp with an adjustable support, and a cart. Design an experiment to determine the cart's acceleration.
- (a) Describe the procedure, including what you vary, what you measure, and how you reduce timing error.
 (b) State which two quantities you will graph so that the result is a straight line, and how the slope gives the acceleration.
 (c) Identify one systematic source of error and its effect.
10. Ball A is dropped from rest from a height of 19.6 m. Ball B is launched horizontally from the same height at 15 m/s. Air resistance is negligible.
- (a) Compare the times of flight qualitatively, then calculate them.
 (b) Calculate how far from the base B lands.
 (c) Compare the speeds of A and B just before landing and justify with components.

Answers Unit 1: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

1 Kinematics

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PRACTICE · UNIT 1 PRACTICE

1. B. Area under v - t : 8 m for the first 2 s, then a positive triangle (+5.33 m) and a negative triangle (-1.33 m), total +12 m. Choice C (14.7 m) is the distance, which ignores the sign of the second triangle.
2. C. Only the vertical component is zero at the top; the horizontal component stays $20 \cos 30^\circ = 17.3$ m/s. Choice A is the common error of treating the top as a stop.
3. D. Fall distance grows as t^2 , so doubling the time multiplies the distance by 4. Choice B treats distance as proportional to time.
4. A. Add vectors with east positive: $+2.0 + (-1.5) = +0.5$ m/s. Choice B adds magnitudes and ignores the opposite directions.
5. B. The vertical motions are identical, so the times are equal. B also keeps its horizontal velocity, so its landing speed is larger. Choice C forgets the horizontal component.
6. A. The slope is negative (velocity negative) and its magnitude shrinks to zero at 4 s, so velocity is increasing toward zero: acceleration is positive ($+2$ m/s²). Choice B confuses a falling position with speeding up.
7. (a) 1 point: Solve the first for $t = (v - v_0)/a$; substituting gives $\Delta x = v_0(v - v_0)/a + \frac{(v - v_0)^2}{2a} = \frac{v^2 - v_0^2}{2a}$, so $v^2 = v_0^2 + 2a\Delta x$.
 (b) 1 point setup, 1 point answer: $0 = 30^2 + 2(-6.0)\Delta x$, so $\Delta x = 900/12 = 75$ m.
 (c) 1 point: During reaction the speed is constant: $30 \times 0.50 = 15$ m. Total = $15 + 75 = 90$ m.
8. (a) Velocity: straight line from (0, +14.7 m/s) through (1.5 s, 0) to (3.0 s, -14.7 m/s), slope -9.8 m/s². Acceleration: horizontal line at -9.8 m/s² from 0 to 3.0 s. Axes labeled with quantity and unit.
 (b) Dots in order, evenly timed, spacing decreasing as they rise, velocity arrows pointing up and getting shorter, ending at zero length at the top; acceleration arrow downward.
 (c) $h = v_0^2/(2g) = 14.7^2/(2 \times 9.8) = 11.0$ m (check: average velocity 7.35 m/s times 1.5 s = 11.0 m).
9. (a) Set a fixed incline. Release the cart from rest at several measured distances d (for example 0.20, 0.40, 0.60, 0.80, 1.00 m) from a fixed finish line; time each run at least three times and average to reduce reaction-time error; mark the release point each time.
 (b) Graph d (vertical) versus t^2 (horizontal). Because $d = \frac{1}{2}at^2$ from rest, the best-fit straight line (drawn with a straightedge, not connect-the-dots) has slope $a/2$, so $a = 2 \times$ slope. Axes labeled with quantity and unit.
 (c) Any one: the release is not truly from rest or the stopwatch is started late, which makes every measured time shorter or longer by about a constant amount and shifts the intercept; friction or a nonuniform track would make the line curve.
10. (a) Qualitative: vertical motion is independent of horizontal motion, and both start with zero vertical velocity from the same height, so the times are equal. $t = \sqrt{2(19.6)/9.8} = 2.0$ s for both.
 (b) $x = v_x t = 15 \times 2.0 = 30$ m.
 (c) A: $v = gt = 19.6$ m/s downward. B: $v_y = 19.6$ m/s and $v_x = 15$ m/s, so $v = \sqrt{15^2 + 19.6^2} = 24.7$ m/s. B is faster because it has the same vertical component plus a horizontal one.

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Unit 2: Force and Translational Dynamics

Exam weighting: about 18-23% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

2 Force and Translational Dynamics

Systems, free-body diagrams, Newton's three laws, gravitation, friction, springs, and circular motion, taught with force diagrams and one consistent method.

Why This Matters

Force and translational dynamics is the heaviest unit (18-23% of multiple choice) and feeds every later unit. Points are lost for missing or mislabeled force arrows, for putting a third-law pair on one object, and for writing "gravity" or "it" without naming which object exerts the force on which.

Core Ideas

2.1 Systems and Center of Mass

A **system** is the object or group of objects you choose to analyze. Forces between members of the system are **internal**; forces from anything outside are **external**. Only external forces can change the motion of the system's center of mass. Friction between a block and the cart carrying it is internal if both are in the system, but friction between the cart and the floor is external. The center of mass of point masses on a line is $x_{cm} = (m_1x_1 + m_2x_2 + \dots)/(m_1 + m_2 + \dots)$.

Newton's second law for a system

$$\vec{a}_{cm} = \frac{\sum \vec{F}_{ext}}{M}, \text{ where } M \text{ is the total mass.}$$

Internal forces occur in third-law pairs and cancel in the sum, so they cannot accelerate the center of mass.

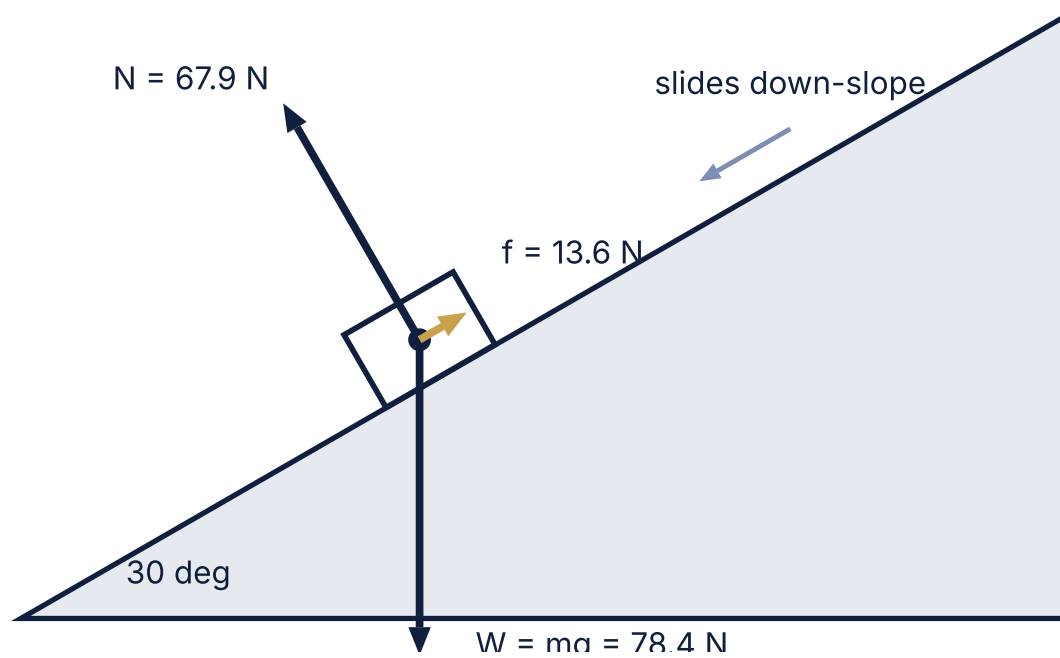
2.2 Forces and Free-Body Diagrams

A force is an interaction between two objects. A **system diagram** shows the objects and the boundary; a **free-body diagram** (FBD) shows one object as a dot and only the forces exerted on it, drawn from the dot, arrow lengths reflecting magnitudes, and each arrow labeled by source and object. Never draw acceleration, velocity, or components as forces.

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Free-body diagram of the 8.0 kg block sliding down a rough 30 degree incline; the block slides down the slope, so kinetic friction points up the slope.

Force	Exerted by	Direction	Model
Gravitational	Earth (or planet)	toward its center	$F_g = mg$ near the surface
Normal	surface	perpendicular to surface, away from it	adjusts; $N \neq mg$ in general
Tension	rope or string	along the rope, pulling the object	same magnitude along an ideal rope
Friction	surface	parallel to surface	static or kinetic (see 2.7)
Spring	spring	toward equilibrium length	$F_s = kx$ (see 2.8)

2.3 Newton's Third Law

If object A exerts a force on B, then B exerts a force on A of equal magnitude, opposite direction, and the same type. The pair acts on different objects, so the two members never appear on the same FBD and never cancel. In a truck-car collision the forces are equal in magnitude; the car accelerates more because its mass is smaller. A book's weight and the table's normal force on it are *not* a pair (both act on the book); the partner of the weight is the book's pull on Earth.

2.4 Newton's First Law

If the net external force on an object is zero, its velocity does not change: it stays at rest or moves in a straight line at constant speed. Equilibrium means $\sum \vec{F} = 0$ along each axis, not that nothing acts on the object; a crate pushed at constant speed has friction equal and opposite to the push.

2.5 Newton's Second Law

Newton's second law

$\sum \vec{F} = m\vec{a}$, applied along each axis: $\sum F_x = ma_x$, $\sum F_y = ma_y$.

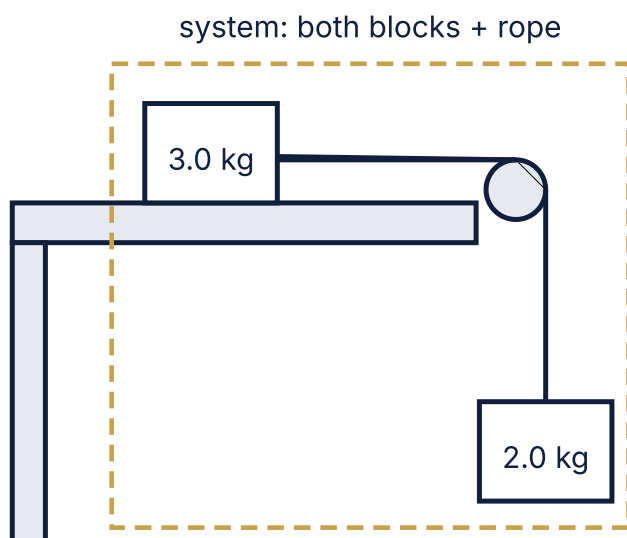
Procedure: choose the system, draw its FBD, pick axes (one along the acceleration), write one equation per axis with signs, solve.

For connected objects, an ideal rope and pulley give the same tension at both ends, and that tension generally equals neither weight.

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Free-body diagrams



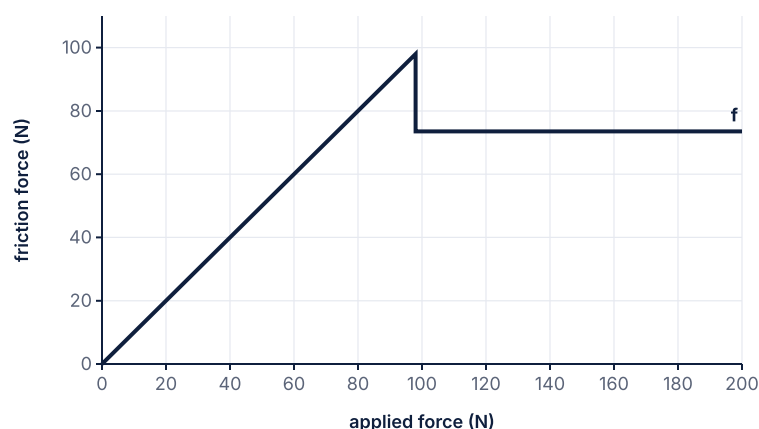
Two blocks joined over a pulley; W_2 is longer than T , so the hanging block accelerates downward.

2.6 Gravitational Force

Any two masses attract with $F_g = Gm_1m_2/r^2$, where r is the center-to-center distance and $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$. The field strength at distance r from a mass M is $g = GM/r^2$, so near a planet's surface $F_g = mg$. Because $F_g \propto 1/r^2$, doubling the distance divides the force by 4: at a height equal to Earth's radius above the surface ($r = 2R$) the field is $9.8/4 = 2.45 \text{ m/s}^2$. Inertial mass (in $F = ma$) and gravitational mass (in F_g) are numerically equal, so all objects in free fall share one acceleration.

2.7 Kinetic and Static Friction

Friction acts parallel to the surfaces. **Kinetic friction** (sliding) has magnitude $f_k = \mu_k N$ and opposes the sliding. **Static friction** (no sliding) adjusts to whatever is needed up to a maximum: $f_s \leq \mu_s N$. Find N from the force balance perpendicular to the surface, not by assuming mg . Typically $\mu_s > \mu_k$; neither depends on contact area or speed in this model.



Friction on the 25 kg crate: static friction matches the push up to 98 N, then kinetic friction stays at 73.5 N.

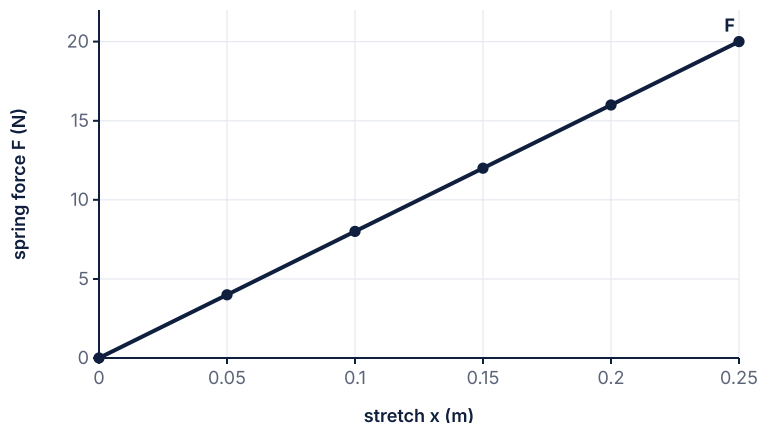
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2.8 Spring Forces

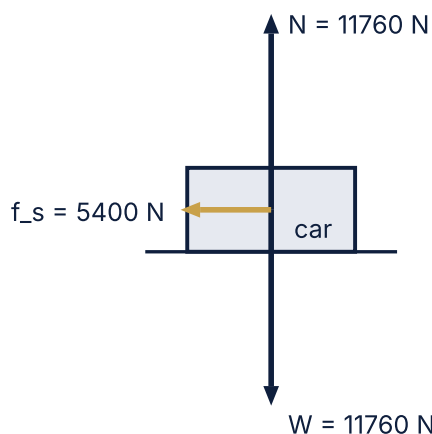
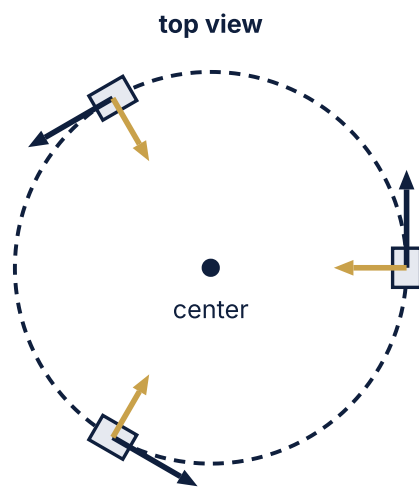
An ideal spring exerts $F_s = -kx$, where x is the stretch or compression from the relaxed length and k (N/m) is the spring constant. The minus sign says the force opposes the displacement; magnitudes are $|F| = kx$. On an F - x graph, k is the slope. A mass hanging at rest satisfies $kx = mg$.



Hooke's law: a straight line through the origin; slope = $k = 80$ N/m.

2.9 Circular Motion

An object moving in a circle of radius r at constant speed v is accelerating because its direction changes. The acceleration points toward the center with magnitude $a_c = v^2/r$, and the period is $T = 2\pi r/v$. "Centripetal force" is a role, not a new force: it is the net force toward the center, supplied by tension, gravity, friction, or the normal force. Draw only real forces, then write $\sum F_{\text{toward center}} = mv^2/r$.



rear view; center of curve to the left

navy: velocity (tangent); gold: acceleration and net force (toward center)

Car on a flat curve at 15 m/s: static friction toward the center supplies the net force; N and W cancel vertically.

Circular motion checklist

Net force toward the center = mv^2/r ; no outward force acts on the object.

Doubling v at fixed r quadruples the required force; doubling r at fixed v halves it.

At the top of a vertical circle, both T (or N) and mg point toward the center: $T + mg = mv^2/r$.

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Worked Examples

EXAMPLE 1: FRICTION ON A LEVEL FLOOR

A 25 kg crate is pushed with a horizontal force of 120 N. The coefficients are $\mu_s = 0.40$ and $\mu_k = 0.30$. Does it move, and what is its acceleration?

SOLUTION

$N = mg = 25(9.8) = 245$ N, so $f_{s,max} = 0.40(245) = 98$ N. Since $120 > 98$, the crate slides.

Then $f_k = 0.30(245) = 73.5$ N.

$a = (120 - 73.5)/25 = 1.86$ m/s² in the direction of the push. (A 90 N push would not move it; friction would be 90 N.)

EXAMPLE 2: BLOCK ON AN INCLINE

The 8.0 kg block in the diagram slides down a 30 degree incline with $\mu_k = 0.20$. Find N , the friction force, and the acceleration. What minimum μ_s would keep it at rest?

SOLUTION

Perpendicular to the slope: $N = mg \cos 30^\circ = 78.4(0.866) = 67.9$ N.

$f_k = 0.20(67.9) = 13.6$ N up the slope.

Along the slope (down positive): $mg \sin 30^\circ - f_k = ma$, so $a = (39.2 - 13.6)/8.0 = 3.2$ m/s² down the slope.

At rest, $mg \sin \theta \leq \mu_s mg \cos \theta$, so $\mu_s \geq \tan 30^\circ = 0.58$.

EXAMPLE 3: TWO BLOCKS AND A PULLEY

A 3.0 kg block on a frictionless table is tied to a 2.0 kg hanging block over an ideal pulley (diagram above). Find the acceleration and the tension.

SOLUTION

Hanging block (down positive): $m_2 g - T = m_2 a$. Table block: $T = m_1 a$.

Add: $m_2 g = (m_1 + m_2) a$, so $a = 19.6/5.0 = 3.92$ m/s².

$T = m_1 a = 3.0(3.92) = 11.8$ N, less than $m_2 g = 19.6$ N, which is why the hanging block accelerates.

EXAMPLE 4: CAR ON A FLAT CURVE

A 1200 kg car rounds a flat curve of radius 50 m. (a) What is the greatest speed with $\mu_s = 0.60$? (b) What friction force is needed at 15 m/s, and is it available? (c) What is the greatest speed on wet pavement with $\mu_s = 0.30$?

SOLUTION

Friction supplies the center-directed force: $\mu_s mg = mv^2/r$, so $v_{max} = \sqrt{\mu_s g r}$ and mass cancels.

(a) $v_{max} = \sqrt{0.60(9.8)(50)} = 17.1$ m/s.

(b) $f = mv^2/r = 1200(225)/50 = 5400$ N; the maximum is $0.60(1200)(9.8) = 7056$ N, so yes.

(c) $v_{max} = \sqrt{0.30(9.8)(50)} = 12.1$ m/s, a factor of $1/\sqrt{2}$ smaller.

Write It Like a 5

Model Response

Prompt: A book rests on a level table. A student says, "The normal force and the weight are equal and opposite, so by Newton's third law they are a third-law pair." Evaluate the claim. The forces are equal and opposite, but they are not a third-law pair: partners act on different objects and are the same type. The normal force (table on book) and the weight (Earth on book) both act on the book, and one is contact while the other is gravitational. They are equal only because the book's acceleration is zero, so $N - mg = 0$. The partners are the book's force on the table and the book's gravitational force on Earth.

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What the grader looks for

1. States that the claim is false. 2. Names the objects acting in each force (table on book, Earth on book). 3. Says third-law partners act on different objects and share a type. 4. Explains the equality using the second law with zero acceleration. 5. Identifies the correct partners.

Coach's Playbook

- 1 Name the system; list every touching object and long-range force on it.
- 2 Draw the FBD from a dot; label each force "A on B".
- 3 Pick axes along the acceleration.
- 4 Write $\sum F = ma$ per axis with signs; solve for N before friction.
- 5 Check limits: does friction exceed $\mu_s N$? Is T between the weights?

Common traps

1. Drawing the normal force as always upward on a tilted or looped surface; it is perpendicular to the surface. 2. Putting both members of a third-law pair on one diagram or calling friction internal when the ground is outside the system. 3. Writing $N = mg$ on an incline or with an angled push, and using μN for static friction as if it were always at its maximum. 4. Drawing a "centripetal" or outward "centrifugal" force in addition to the real forces.

Quick Check

Q1. Where is the center of mass of 2.0 kg at $x = 0$ and 6.0 kg at $x = 4.0$ m? A: $24/8.0 = 3.0$ m. Q2. A satellite's distance triples. Gravitational force changes by? A: $1/9$. Q3. A 0.50 kg mass hangs at rest and stretches a spring 0.098 m. Find k . A: $k = mg/x = 4.9/0.098 = 50$ N/m.

Challenge Corner

A 2.0 kg block rests on a frictionless 30 degree incline, held by a spring of constant 80 N/m parallel to the slope. How far is the spring stretched, and what is the normal force? Along the slope the spring balances the weight component: $kx = mg \sin 30^\circ = 2.0(9.8)(0.50) = 9.8$ N, so $x = 9.8/80 = 0.1225$ m = 0.12 m. Perpendicular: $N = mg \cos 30^\circ = 19.6(0.866) = 17.0$ N, less than the weight.

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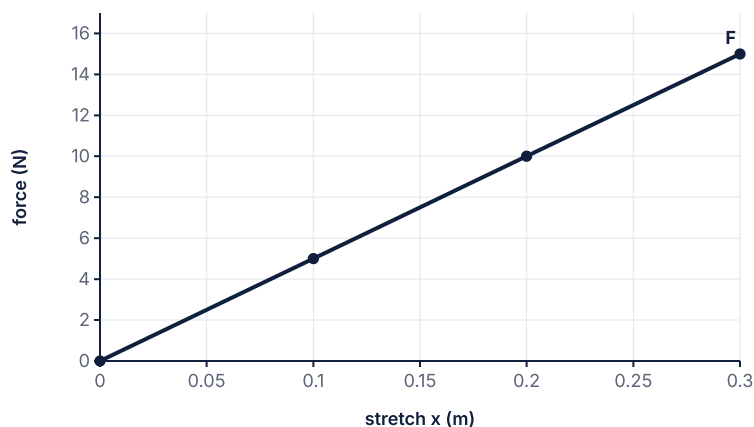
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Practice

PRACTICE Unit 2 Practice

Show your work. A calculator is allowed. Justify conclusions.

1. A 5000 kg truck collides head-on with a 1000 kg car. How does the magnitude of the force of the truck on the car compare with the force of the car on the truck?
 (A) The force of the truck on the car is 5 times larger.
 (B) The force of the car on the truck is 5 times larger.
 (C) They are equal.
 (D) It depends on which vehicle is moving faster.
2. A 4.0 kg block on a frictionless floor is pushed right by a 20 N force and left by an 8 N force. What is its acceleration?
 (A) 3.0 m/s^2 to the right (B) 7.0 m/s^2 to the right (C) 5.0 m/s^2 to the right
 (D) 2.0 m/s^2 to the left
3. At a height above Earth's surface equal to Earth's radius R , the gravitational field strength is closest to
 (A) 4.9 m/s^2 (B) 2.45 m/s^2 (C) 1.2 m/s^2
 (D) 9.8 m/s^2
4. The graph shows the force of a spring versus its stretch. What force is required to stretch the spring 0.25 m?



Spring force versus stretch.

- (A) 2.5 N (B) 7.5 N (C) 15 N (D) 12.5 N
5. A ball on a string moves in a horizontal circle of fixed radius at speed v with string tension T . If the speed doubles at the same radius, the tension becomes
 (A) $4T$ (B) $2T$ (C) $T/2$ (D) T
6. A 5.0 kg block rests on a rough 25 degree incline. What is the friction force on it?
 (A) 44.4 N up the incline (B) 20.7 N down the incline (C) 20.7 N up the incline
 (D) 49 N up the incline
7. Masses $m_1 = 5.0 \text{ kg}$ and $m_2 = 3.0 \text{ kg}$ hang from the two ends of a light string over an ideal pulley and are released from rest.
 (a) Derive an expression for the acceleration in terms of m_1 , m_2 , and g , and one for the tension.
 (b) Calculate both.
 (c) Justify, without calculation, why the tension is between the two weights.
8. A 10 kg box is pulled across a rough floor at constant velocity by a rope with a 40 N tension directed 30 degrees above the horizontal.
 (a) Draw a free-body diagram of the box, labeling each force by its source.
 (b) Calculate the normal force and the kinetic friction force.
 (c) Calculate μ_k .
 (d) A student claims $N = mg$ here. Evaluate the claim.

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9. A spring's constant is unknown. Available: the spring, a stand, a meterstick, and a set of 50 g masses. No force sensors are allowed.
- Describe a procedure that gives data to find k .
 - State what to plot, how to use the graph to get k , and why a best-fit line is better than one pair of values.
 - Identify one limitation of treating the spring as ideal.
10. A 1000 kg car travels at constant speed on a flat circular track of radius 40 m, with $\mu_s = 0.50$.
- State the direction of the net force on the car and which force supplies it.
 - Calculate the maximum speed without slipping.
 - Predict, with justification, how the maximum speed changes if the radius is doubled, and if the car carries a heavy load.

Answers Unit 2: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

2 Force and Translational Dynamics

PRACTICE · UNIT 2 PRACTICE

- C. Third-law partners always have equal magnitude. Choice A is tempting because the car's acceleration is larger, but that comes from its smaller mass, not a larger force.
- A. The net force is $20 - 8 = 12$ N right, so $a = 12/4.0 = 3.0$ m/s². Choice B adds the forces instead of subtracting them.
- B. The distance from Earth's center is $2R$, and the field varies as $1/r^2$, so $g = 9.8/4 = 2.45$ m/s². Choice A halves the field, treating it as proportional to $1/r$.
- D. The slope is 5 N/ 0.10 m = 50 N/m, so $F = 50(0.25) = 12.5$ N. Choice C reads the highest plotted point, which is at 0.30 m.
- A. Tension supplies the center-directed force mv^2/r , which grows as v^2 , so doubling v multiplies it by 4. Choice B treats tension as proportional to speed.
- C. The block is in equilibrium, so static friction equals the weight component along the slope, $5.0(9.8) \sin 25^\circ = 20.7$ N, directed up the slope. Choice A uses the perpendicular component $mg \cos \theta$.
- (a) For m_1 (down positive): $m_1 g - T = m_1 a$. For m_2 (up positive): $T - m_2 g = m_2 a$. Adding: $a = (m_1 - m_2)g / (m_1 + m_2)$. Substituting back: $T = 2m_1 m_2 g / (m_1 + m_2)$.
 (b) $a = 2.0(9.8)/8.0 = 2.45$ m/s²; $T = 2(5.0)(3.0)(9.8)/8.0 = 36.75$ N, about 37 N.
 (c) m_1 accelerates down, so its weight (49 N) exceeds T ; m_2 accelerates up, so T exceeds its weight (29.4 N). Thus $29.4 < T < 49$ N.
- (a) Four forces from a dot: weight (Earth on box) down, normal (floor on box) up, tension (rope on box) 30 degrees above horizontal, kinetic friction (floor on box) horizontal opposing motion.
 (b) Constant velocity: horizontal $f = 40 \cos 30^\circ = 34.6$ N; vertical $N + 40 \sin 30^\circ = mg$, so $N = 98 - 20 = 78$ N.
 (c) $\mu_k = f/N = 34.6/78 = 0.44$.
 (d) False: the vertical component of the tension helps support the box, so N (78 N) is less than mg (98 N).
- (a) Hang the spring from the stand with the meterstick beside it; record the relaxed length; add masses one at a time (50 g, 100 g, ...) and record the length each time after the mass is at rest; stretch x = length minus relaxed length; force $F = mg$ at equilibrium.
 (b) Plot F (vertical, with unit N) against x (horizontal, m) and draw one best-fit straight line with a straightedge; the slope is k in N/m. A fit uses all points and averages out reading errors, while a single pair could carry one large error.
 (c) Any one: the spring's own mass adds to the stretch; the spring may exceed its elastic limit at large loads so the plot curves; reading error with the meterstick.
- (a) The net force points toward the center of the circle; static friction (road on tires) supplies it.
 (b) $\mu_s mg = mv^2/r$, so $v = \sqrt{0.50(9.8)(40)} = 14.0$ m/s.
 (c) Since $v_{max} = \sqrt{\mu_s g r}$, doubling r increases it by $\sqrt{2}$ to 19.8 m/s. The mass cancels, so extra load does not change the maximum speed: friction's maximum and the required force both scale with m .

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Unit 3: Work, Energy, and Power

Exam weighting: about 18-23% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

3 Work, Energy, and Power

Kinetic and potential energy, work, conservation of energy with energy bar charts, and power, applied to tracks, springs, and falling objects with algebra only.

Why This Matters

Work, energy, and power carry 18-23% of multiple choice, and energy bar charts and energy graphs are a favorite free-response representation. Students lose points by drawing kinetic energy for an object that is momentarily at rest, treating gravity as both a potential energy and a work, and writing $\Delta K = \frac{1}{2}m(\Delta v)^2$. Energy is a scalar, so there are no components to resolve, but you must say which system you chose.

Core Ideas

3.1 Translational Kinetic Energy

A moving object has **translational kinetic energy** $K = \frac{1}{2}mv^2$, measured in joules (1 J = 1 kg m²/s²). It is a scalar and never negative. Because $K \propto v^2$, tripling the speed multiplies K by 9. The change is $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$; a car going from 15 to 25 m/s has K rise by a factor $(25/15)^2 = 2.78$, not 1.67. A system of several objects has total kinetic energy equal to the sum of each object's K .

3.2 Work

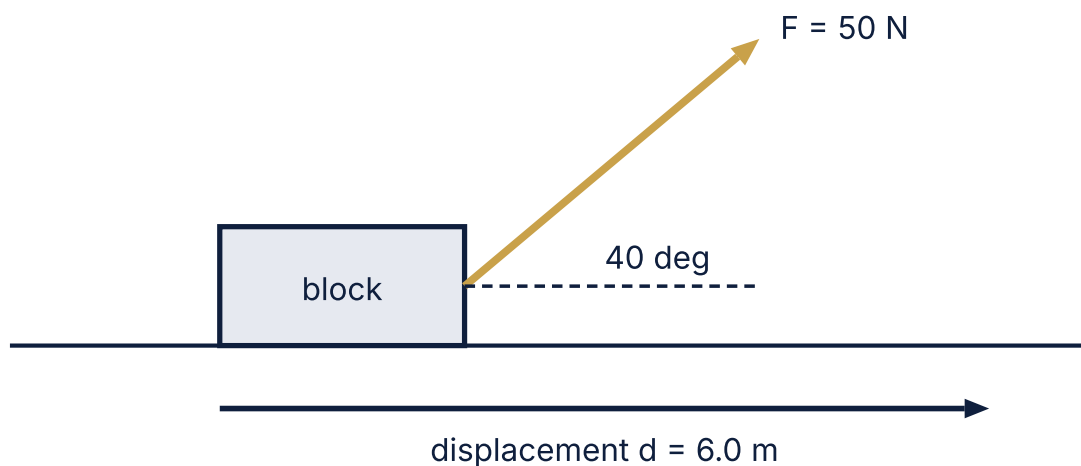
Work is energy transferred to or from a system by an external force acting through a displacement. For a constant force, $W = Fd \cos \theta$, where θ is the angle between the force and the displacement. The reference conventions treat positive work as work done *on* the system; check the live sheet for the exact wording. If $\theta < 90^\circ$ the work is positive (energy added); at 90° it is zero (the normal force on a sliding block, the tension on an object moving in a circle); beyond 90° it is negative (kinetic friction, a person lowering a weight).

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$$W = F d \cos(\theta) = 50(6.0)\cos 40 = 230 \text{ J}$$



Only the component of the force along the displacement does work.

Work and the work-energy theorem

$W = Fd \cos \theta$ for a constant force; for a varying force, W is the area under the graph of $F \cos \theta$ versus position.

$W_{\text{net}} = \Delta K$: the net work done on an object equals its change in kinetic energy.

Gravity near Earth's surface does $W = -mg \Delta h$; kinetic friction does $W = -f_k d$ and the energy becomes thermal energy.

3.3 Potential Energy

Potential energy belongs to a system of interacting objects. For a ball and Earth near the surface, $U_g = mgh$, where h is the height above a reference level you choose; only changes in U_g have physical meaning, so moving the zero does not change any prediction. A spring stores $U_s = \frac{1}{2}kx^2$, where x is the stretch or compression, which equals the area under the spring's F - x graph ($\frac{1}{2} \cdot x \cdot kx$). Doubling the stretch quadruples the stored energy. These forms hold when the force is conservative: the energy can be fully recovered as kinetic energy.

3.4 Conservation of Energy

For a **closed system** with no energy transferred in or out, the total energy is constant. Choose the system first. If the system includes Earth, gravity is internal and appears as U_g ; if it does not, gravity is an external force doing work. Never count it both ways. When an external force does work or friction produces thermal energy E_{th} :

Energy bookkeeping

$K_i + U_i + W_{ext} = K_f + U_f + E_{th}$ with $E_{th} = f_k d$ for sliding friction.

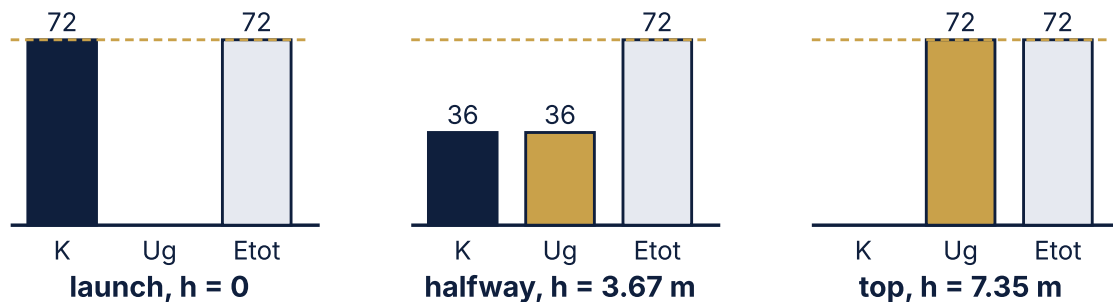
Energy bar charts show K , U_g , U_s , E_{th} at chosen positions; the sum of the bars must match in a closed system, and each bar must be zero or positive.

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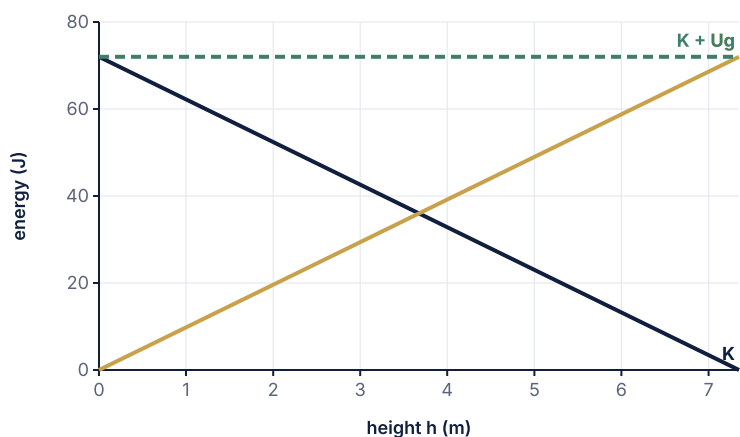
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Energy bar charts for a 1.0 kg ball tossed up at 12 m/s (energies in J)



Total energy (dashed gold line) stays 72 J; K is zero at the top because the ball is momentarily at rest.



U_g is linear in height and the total is a horizontal line when only gravity does work.

On graphs, U_g is a straight line against height, K is the mirror image, and the total is horizontal; plotting U_g against time gives a curve. Speed does not always drop when a spring is touched: it falls only once the spring force exceeds the weight.

3.5 Power

Power is the rate of energy transfer: $P_{avg} = W/\Delta t = \Delta E/\Delta t$, in watts (1 W = 1 J/s). For a constant force at angle θ to the velocity, $P = Fv \cos \theta$. A motor lifting a load at constant speed has $P = mgv$. Efficiency is useful output power divided by input power, which is always less than 1. Doubling the time for the same work halves the power; it does not change the work.

Worked Examples

EXAMPLE 1: KINETIC ENERGY AND THE WORK-ENERGY THEOREM

A 1200 kg car speeds up uniformly from 15 m/s to 25 m/s while travelling 200 m. Find the change in kinetic energy, the net force, and the average power of the net force.

SOLUTION

$$\Delta K = \frac{1}{2}(1200)(25^2 - 15^2) = 600(400) = 2.4 \times 10^5 \text{ J.}$$

$$W_{net} = F_{net}d = \Delta K, \text{ so } F_{net} = 2.4 \times 10^5 / 200 = 1200 \text{ N.}$$

$$\text{Time: average speed } 20 \text{ m/s gives } t = 200/20 = 10 \text{ s, so } P_{avg} = 2.4 \times 10^5 / 10 = 2.4 \times 10^4 \text{ W (check: } Fv_{avg} = 1200 \times 20).$$

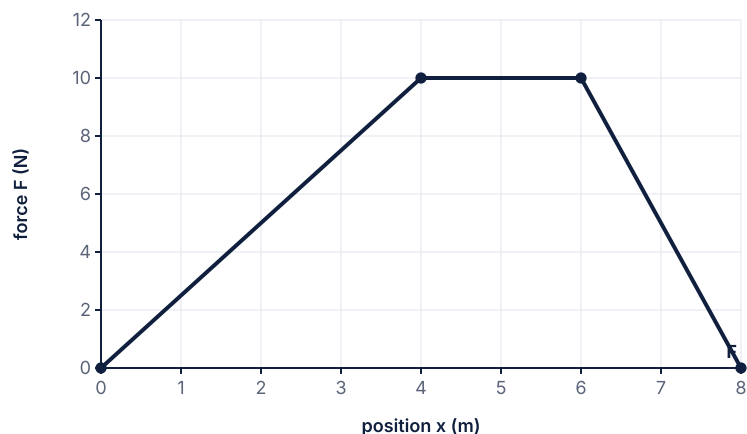
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EXAMPLE 2: WORK FROM A FORCE-POSITION GRAPH

The force on a 2.0 kg object, starting from rest, varies as shown. Find the work done from 0 to 8 m and the final speed.



Force along the displacement for a 2.0 kg object that starts at rest.

SOLUTION

Area: triangle $\frac{1}{2}(4)(10) = 20$ J; rectangle $2 \times 10 = 20$ J; triangle $\frac{1}{2}(2)(10) = 10$ J. $W = 50$ J.

$W_{\text{net}} = \Delta K = \frac{1}{2}mv^2$, so $v = \sqrt{2(50)/2.0} = 7.07$ m/s.

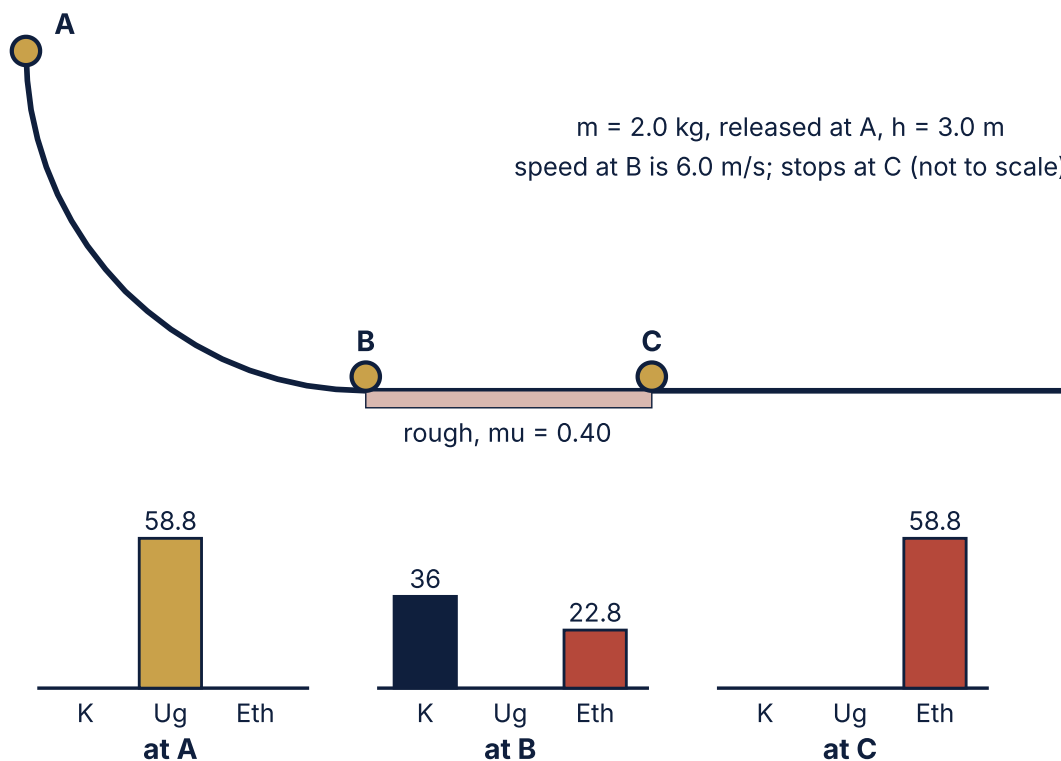
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EXAMPLE 3: ENERGY WITH FRICTION

A 2.0 kg block released from rest at A slides down a curved track 3.0 m high and reaches B at 6.0 m/s. A rough floor with $\mu_k = 0.40$ then stops it at C. Find the thermal energy produced on the curve, the speed at B without friction, and the distance BC.



Energy bars for the block: all Ug at A, 36 J of K plus 22.8 J of thermal energy at B, all thermal energy at C.

SOLUTION

Take block + Earth + track as the system. At A: $U_g = mgh = 2.0(9.8)(3.0) = 58.8 \text{ J}$, $K = 0$.

At B: $K = \frac{1}{2}(2.0)(6.0)^2 = 36 \text{ J}$, so $E_{th} = 58.8 - 36 = 22.8 \text{ J}$ on the curve.

With no friction: $v = \sqrt{2gh} = 7.67 \text{ m/s}$.

On the floor: $K_B = f_k d = \mu_k mg d$, so $d = 36 / (0.40 \times 2.0 \times 9.8) = 4.6 \text{ m}$.

EXAMPLE 4: SPRING LAUNCH

A 0.50 kg block compresses a spring ($k = 400 \text{ N/m}$) by 0.10 m on a frictionless table, then is released and slides up a frictionless 30 degree incline. Find its speed leaving the spring and how far up the incline it travels.

SOLUTION

$U_s = \frac{1}{2}(400)(0.10)^2 = 2.0 \text{ J}$ becomes kinetic energy: $v = \sqrt{2(2.0)/0.50} = 2.83 \text{ m/s}$.

At the highest point $K = 0$ and $U_g = 2.0 \text{ J}$, so $h = 2.0 / (0.50 \times 9.8) = 0.41 \text{ m}$.

Distance along the slope: $h / \sin 30^\circ = 0.82 \text{ m}$.

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Write It Like a 5

Model Response

Prompt: A ball is released from rest above a vertical spring, falls onto it, and compresses it to the lowest point. Friction and air resistance are negligible. Take the ball, spring, and Earth as the system, and describe energy bar charts at release, at maximum speed, and at the lowest point. At release the ball is at rest, so $K = 0$; all energy is gravitational potential and the spring bar is zero. At maximum speed the spring force equals the weight, so the net force is zero and the speed is largest: K is at its maximum, U_g has decreased, and U_s is part of the total. At the lowest point the ball is momentarily at rest, so $K = 0$ again; U_g is smallest and U_s is largest, equal to the original U_g difference. The sum of the bars is the same in all three, because with no friction the system is closed and the total energy is constant. The ball keeps speeding up after touching the spring until the spring force exceeds the weight.

What the grader looks for

1. States the system. 2. $K = 0$ at the two turning points, with a reason. 3. Maximum K tied to zero net force. 4. Equal total bars with the conservation reasoning. 5. Correct relative sizes of U_g and U_s at each position.

Coach's Playbook

- 1 State the system and whether energy crosses its boundary.
- 2 Pick initial and final states; list K , U_g , U_s , E_{th} at each.
- 3 Write the energy equation first; substitute numbers last.
- 4 For work, name the force and angle; for graphs, use the area.
- 5 Check signs: friction removes mechanical energy; spring energy needs the stretch from relaxed length.

Common traps

1. Showing kinetic energy at the top of a toss or the lowest point of a spring compression, where the speed is momentarily zero. 2. Using $\Delta K = \frac{1}{2}m(\Delta v)^2$ instead of $\frac{1}{2}m(v_f^2 - v_i^2)$. 3. Using energy conservation for a collision without checking for lost energy, or counting gravity as both U_g and a work. 4. Drawing total energy sloped or U_g as a curve against height.

Quick Check

Q1. A 3.0 kg cart moves at 4.0 m/s. Find K . A: $\frac{1}{2}(3.0)(16) = 24$ J. Q2. A force of 10 N pushes a box 5.0 m at 90 degrees to the force. What is the work? A: Zero, since $\cos 90^\circ = 0$. Q3. A motor does 4800 J of work in 12 s. Find its power. A: $4800/12 = 400$ W.

Challenge Corner

A 1500 kg car climbs a 5.0 degree slope at a constant 20 m/s against a 600 N resistive force. What power must the engine deliver to the wheels? Constant velocity means balanced forces along the slope: $F_{engine} = 600 + mg \sin 5^\circ = 600 + 1500(9.8)(0.0872) = 1881$ N. Then $P = Fv = 1881 \times 20 = 3.8 \times 10^4$ W, about 38 kW. Without the slope, only 12 kW would be needed.

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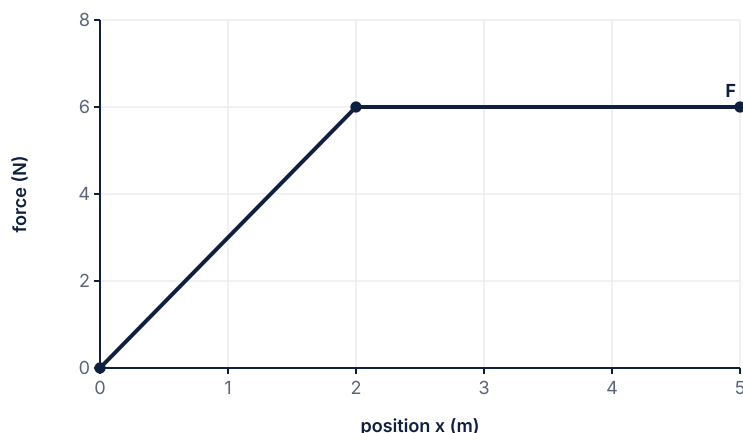
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Practice

PRACTICE Unit 3 Practice

Show your work. A calculator is allowed. Justify conclusions.

1. A car's speed increases from 10 m/s to 30 m/s. By what factor does its kinetic energy increase?
(A) 3 (B) 6 (C) 9 (D) 12
2. A person lowers a box at constant speed by a rope. Which statement describes the work done on the box?
(A) The person does positive work and gravity does negative work of equal size.
(B) The person does negative work and gravity does positive work of equal size.
(C) Both do zero work because the speed is constant.
(D) The person does negative work and gravity does zero work.
3. A ball is dropped from rest at a height of 20 m. Air resistance is negligible. What is its speed when it is 5.0 m above the ground?
(A) 9.9 m/s (B) 14.0 m/s (C) 19.8 m/s (D) 17.1 m/s
4. The force on an object along its displacement increases linearly from 0 to 6 N over the first 2.0 m, then stays at 6 N for another 3.0 m. How much work is done in the 5.0 m?



Force versus position.

- (A) 24 J (B) 30 J (C) 18 J (D) 12 J
5. A motor lifts a 20 kg crate 3.0 m at constant speed in 4.0 s. What is the motor's average power output?
(A) 588 W (B) 147 W (C) 196 W (D) 49 W
6. A spring stores 2.0 J when stretched 0.10 m. How much energy does it store when stretched 0.30 m?
(A) 6.0 J (B) 4.0 J (C) 8.0 J (D) 18 J
7. A 4.0 kg block slides from rest down a rough ramp from a height of 2.5 m. Friction does -15 J of work on it.
(a) Starting from conservation of energy, derive an expression for the speed at the bottom in terms of m , g , h , and the work done by friction.
(b) Calculate the speed.
(c) Calculate the fraction of the initial gravitational potential energy converted to thermal energy.
8. A 0.40 kg block is pressed against a spring ($k = 250$ N/m) until the spring is compressed 0.20 m, then released from rest. The block slides up a rough incline; friction does a total of -1.0 J of work during the slide. Take the block, spring, incline, and Earth as the system.
(a) Draw energy bar charts (K , U_g , U_s , E_{th}) for the compressed position and for the highest point reached, with the reference level at the compressed position.
(b) Calculate the height gained.
(c) Sketch the mechanical energy ($K + U_g$) after leaving the spring versus distance along the incline, and say what its slope represents.

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9. A student wants to find how the launch speed of a ball from a spring launcher depends on the spring compression x . Available: the launcher fixed horizontally at the edge of a table, a meterstick, carbon paper and white paper, and a ball. No timers are allowed.
- Describe the procedure and what is measured.
 - Explain how the horizontal range gives the launch speed, and what to graph to get a straight line.
 - State how the slope relates to the spring constant and the ball's mass, and identify one source of error.
10. Two identical 2.0 kg blocks start from rest at the top of two frictionless ramps of the same height, 1.5 m. Ramp A is inclined at 30 degrees; ramp B at 60 degrees.
- Compare the speeds at the bottom, qualitatively and then by calculation.
 - Compare the times to reach the bottom and justify.
 - Compare the average power delivered by gravity on the two ramps and calculate both.

Answers Unit 3: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

3 Work, Energy, and Power

PRACTICE · UNIT 3 PRACTICE

- C. $K \propto v^2$ and $(30/10)^2 = 9$. Choice A would be right only if K were proportional to speed.
- B. The rope pulls up while the box moves down, so the person's work is negative; gravity pulls down along the motion, so its work is positive. Net work is zero because $\Delta K = 0$. Choice C confuses zero net work with zero individual works.
- D. It has fallen 15 m: $v = \sqrt{2(9.8)(15)} = 17.1$ m/s. Choice D is the speed at the ground (20 m fall); choice B averages the heights instead.
- A. Area: triangle $\frac{1}{2}(2)(6) = 6$ J plus rectangle $3 \times 6 = 18$ J gives 24 J. Choice B uses 6 N times 5 m and ignores the ramp.
- B. Work = $mgh = 20(9.8)(3.0) = 588$ J, and $P = 588/4.0 = 147$ W. Choice A is the energy in joules, not divided by time.
- D. $U_s \propto x^2$, so tripling the stretch gives $9 \times 2.0 = 18$ J. Choice A treats the energy as proportional to the stretch.
- (a) Block + Earth: $mgh + W_f = \frac{1}{2}mv^2$ with W_f negative, so $v = \sqrt{2(gh + W_f/m)}$.
(b) $mgh = 98$ J, $K = 98 - 15 = 83$ J, so $v = \sqrt{2(83)/4.0} = 6.4$ m/s.
(c) Thermal energy is 15 J out of 98 J, so $15/98 = 0.15$ (15%).
- (a) Compressed: $U_s = 5.0$ J, all other bars 0 ($K = 0$ at rest). Highest point: $K = 0$, $U_g = 4.0$ J, $U_s = 0$, $E_{th} = 1.0$ J; the total is 5.0 J in both charts.
(b) $U_s = \frac{1}{2}(250)(0.20)^2 = 5.0$ J. $5.0 - 1.0 = 4.0 = mgh$, so $h = 4.0/(0.40 \times 9.8) = 1.02$ m.
(c) A straight line sloping downward: each meter along the slope, mechanical energy decreases by the kinetic friction force, so the slope's magnitude equals f_k . Axes labeled with quantity and unit.
- (a) Clamp the launcher horizontally so the ball leaves the table edge level. For each of at least five compressions x (measured with the meterstick), fire three times and mark where the ball lands on the carbon paper; measure the table height H once and the range R from the table edge to the average landing mark.
(b) Fall time $t = \sqrt{2H/g}$ is the same for all shots, so $v = R/t$ and $v \propto R$. Energy conservation gives $\frac{1}{2}mv^2 = \frac{1}{2}kx^2$, so $v^2 = (k/m)x^2$ and $R^2 = (2Hk/(mg))x^2$. Graph R^2 versus x^2 (axes labeled with quantity and unit) and draw one best-fit line with a straightedge.
(c) Slope = $2Hk/(mg)$, so $k = \text{slope} \times mg/(2H)$. Errors: friction inside the launcher means less energy reaches the ball; the ball's rotation or landing-mark spread adds scatter.
- (a) Energy conservation: $mgh = \frac{1}{2}mv^2$, with the same h , mass, and no friction, so the speeds are equal: $v = \sqrt{2(9.8)(1.5)} = 5.4$ m/s. The normal force does no work and gravity does the same work on both.
(b) The steeper ramp has the larger acceleration ($g \sin \theta$) and the shorter length, so B takes less time. $a_A = 4.9$, $L_A = 3.0$ m, $t_A = \sqrt{2(3.0)/4.9} = 1.11$ s; $a_B = 8.49$, $L_B = 1.73$ m, $t_B = 0.64$ s.
(c) The work is the same, $mgh = 29.4$ J, but the time differs: $P_A = 29.4/1.11 = 26.6$ W and $P_B = 29.4/0.639 = 46.0$ W, so B has the greater average power.

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Unit 4: Linear Momentum

Exam weighting: about 10-15% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

4 Linear Momentum

Momentum, impulse, conservation, and collisions: choose the system, draw the graph correctly, and keep energy conservation away from collisions where objects stick.

Why This Matters

Unit 4 carries about 10 to 15 percent of the multiple-choice section, and momentum was the Mathematical Routines free-response question on the 2025 exam. Points vanish when students pick the wrong system, use energy conservation on a collision where objects stick, write impulse as Ft with no change symbol, or draw a momentum graph with a gap at the collision. This book uses $g = 9.8 \text{ m/s}^2$; the exam's stated default is 10 m/s^2 , and either earns full credit.

Core Ideas

4.1 Linear Momentum

Linear momentum is a vector pointing along the velocity: $\vec{p} = m\vec{v}$, in kg m/s . A system's momentum is the vector sum of its parts. In one dimension choose a positive direction and let signs carry direction: a 2.0 kg cart moving left at 3.0 m/s has $p = -6.0 \text{ kg m/s}$. A system can have zero total momentum while its parts move fast, like two skaters pushing apart.

Momentum is not energy. Since $K = p^2/2m$, equal momenta mean the lighter object has more kinetic energy. Momentum is a vector that can cancel; kinetic energy is a nonnegative scalar.

Momentum facts

$\vec{p} = m\vec{v}$, units kg m/s (the same as N s).

System: $\vec{p}_{\text{sys}} = \sum m_i \vec{v}_i$, added with signs.

$K = \frac{p^2}{2m}$: at equal p , the smaller mass has the larger K .

4.2 Change in Momentum and Impulse

Newton's second law says the net force equals the rate of change of momentum, $\vec{F}_{\text{net}} = \Delta\vec{p}/\Delta t$. Rearranged, the **impulse** of the net force equals the change in momentum, with Δ meaning final minus initial. On a force-time graph, impulse is the signed area under the curve, and the average force is that area divided by the contact time. A ball that bounces back receives more impulse than one that merely stops, because $\Delta p = mv_f - mv_i$ reverses sign. Writing $p = Ft$ is a classic error: force changes momentum, it does not equal it.

Impulse-momentum theorem

$\vec{J} = \vec{F}_{\text{net}} \Delta t = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$

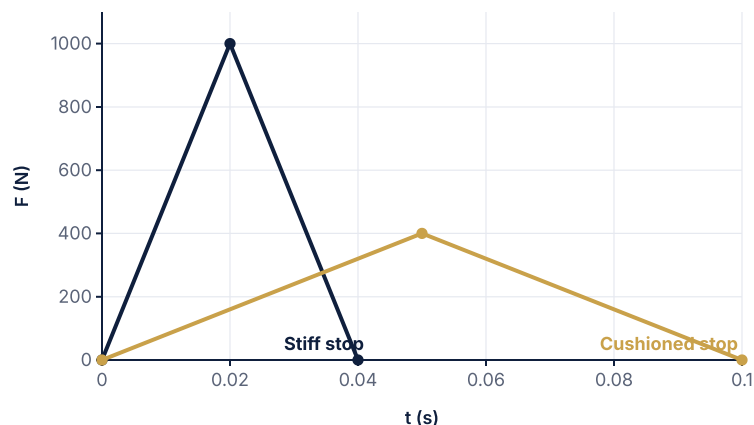
Impulse = area under the net-force vs time graph; $F_{\text{avg}} = J/\Delta t$.

For a fixed change in momentum, longer contact time means smaller average force (airbags, bent knees).

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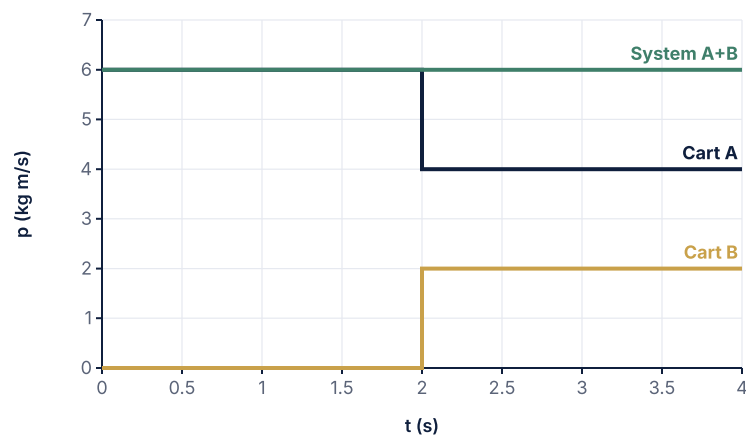
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Two stops with the same impulse: each triangle has area 20 N s. The cushioned stop has a far smaller peak force.

4.3 Conservation of Linear Momentum

If the net external force on a system is zero, its momentum does not change: $\Delta \vec{p}_{sys} = 0$. Internal forces cancel by Newton's third law: equal, opposite, same duration. So everything depends on the system you choose. Include both colliding objects and their mutual forces are internal. Friction from a track or table is external, and so is a pull from something outside the system. In a brief collision the external impulse is tiny next to the internal one, so momentum just before equals momentum just after. Credit requires naming the scenario: \"the net external force on the cart-block system is zero because the track is frictionless and gravity is balanced by the normal force.\" Each component conserves separately, so horizontal momentum can be conserved while vertical is not.



Cart A (2.0 kg, 3.0 m/s) sticks to cart B (1.0 kg, at rest) at $t = 2$ s. The parts trade momentum; the system's line is one continuous horizontal line.

Conservation of momentum, step by step

1. Choose the system and say why the net external force is zero (or negligible during the collision).
2. Choose a positive direction and write $\sum p_{before} = \sum p_{after}$ with signs.
3. Solve for the unknown; check the sign and size.
4. Only then use energy, if the question asks.

4.4 Elastic and Inelastic Collisions

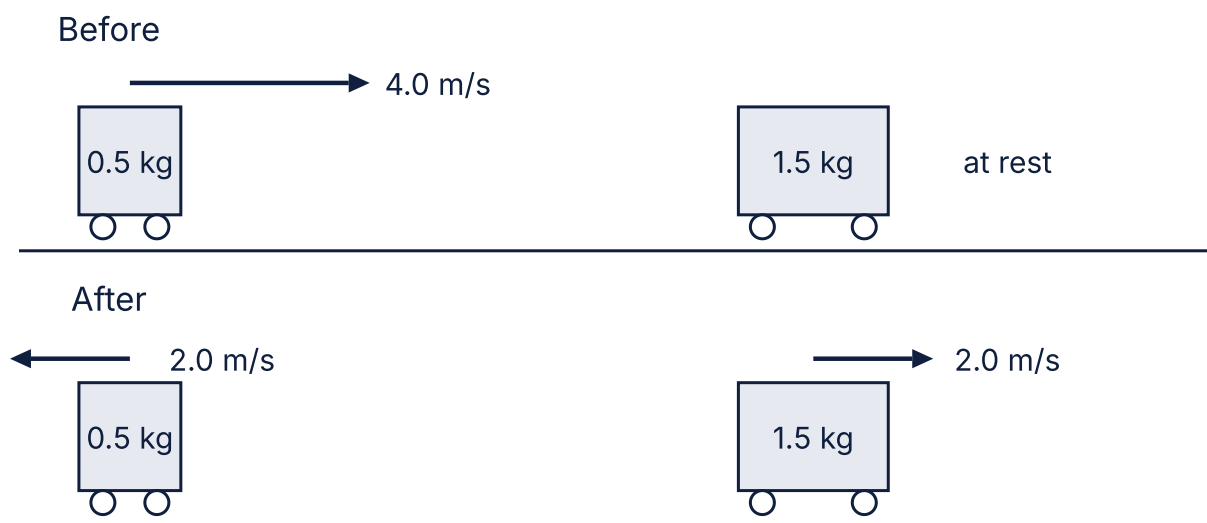
Collision type	Momentum of system	Kinetic energy	Example
Elastic	Conserved	Conserved	Hard spheres, spring bumpers
Inelastic	Conserved	Decreases (thermal, sound, deformation)	Crumpling cars
Perfectly inelastic	Conserved	Largest loss; objects move together	Sticking carts, bullet in block
Explosion/recoil	Conserved	Increases (stored energy released)	Skaters pushing apart

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Momentum is conserved in all of these when the external force is negligible; kinetic energy is conserved only in the elastic case. In a head-on elastic collision in one dimension, the speed of approach equals the speed of separation. A light object hitting a heavy one rebounds and barely moves it. Energy conservation can **never** give the speed after objects stick together, because that collision loses energy.

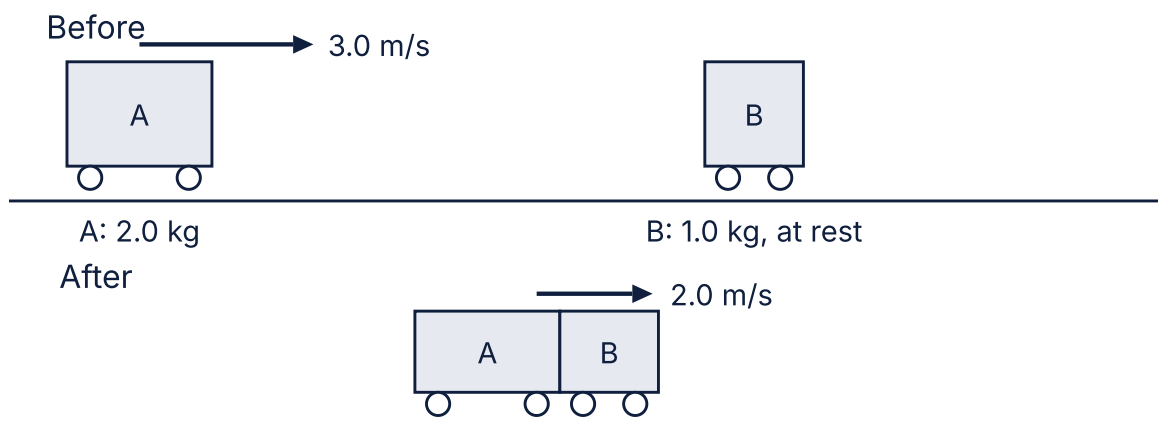


Elastic collision, right is positive. Momentum: $2.0 = -1.0 + 3.0$ kg m/s. Kinetic energy: $4.0 \text{ J} = 1.0 \text{ J} + 3.0 \text{ J}$.

Worked Examples

EXAMPLE 1: CARTS THAT STICK

Cart A (2.0 kg) moves right at 3.0 m/s and couples to cart B (1.0 kg), initially at rest, on a frictionless track. Find the common speed and the kinetic energy lost.



Perfectly inelastic collision, before and after.

SOLUTION

System A + B: the track is frictionless and gravity cancels the normal force, so horizontal $F_{ext} = 0$ and momentum is conserved.

$m_A v_A = (m_A + m_B) v_f$, so $v_f = (2.0)(3.0)/3.0 = 2.0$ m/s right.

$K_i = \frac{1}{2}(2.0)(3.0)^2 = 9.0 \text{ J}$; $K_f = \frac{1}{2}(3.0)(2.0)^2 = 6.0 \text{ J}$. The 3.0 J lost became thermal energy and sound.

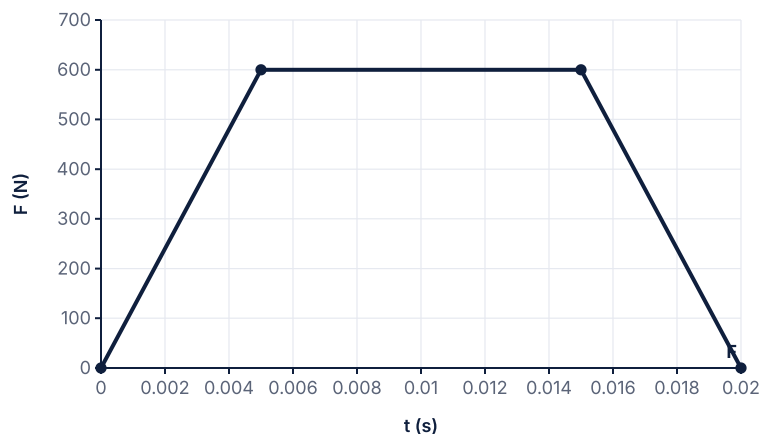
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EXAMPLE 2: IMPULSE FROM A FORCE-TIME GRAPH

A 0.45 kg ball at rest is kicked; the force follows the graph. Find the impulse, the ball's speed, and the average force.



Force of foot on ball.

SOLUTION

Impulse = trapezoid area: $J = \frac{1}{2}(0.020 + 0.010)(600) = 9.0 \text{ N}\cdot\text{s}$.

$J = \Delta p = mv_f - 0$, so $v_f = 9.0/0.45 = 20 \text{ m/s}$.

$F_{avg} = J/\Delta t = 9.0/0.020 = 450 \text{ N}$, below the 600 N peak.

EXAMPLE 3: A BOUNCE

A 0.20 kg ball hits a wall at 12 m/s and rebounds at 8.0 m/s. Contact lasts 0.015 s. Find the average force of the wall on the ball.

SOLUTION

Toward the wall is positive: $p_i = +2.4$ and $p_f = -1.6 \text{ kg m/s}$.

$\Delta p = -1.6 - 2.4 = -4.0 \text{ kg m/s}$; the reversal makes the change larger than the initial momentum.

$F_{avg} = -4.0/0.015 = -267 \text{ N}$: about 270 N away from the wall. The ball's 2 N weight is negligible.

EXAMPLE 4: BALLISTIC PENDULUM

A 0.020 kg bullet embeds in a 1.98 kg block hanging from light strings; the block rises 0.080 m. Find the bullet's speed.

SOLUTION

Stage 1 (collision): momentum of bullet + block is conserved, $mv = (m + M)V$. Kinetic energy is NOT conserved here.

Stage 2 (swing): only gravity does work, so $\frac{1}{2}V^2 = gh$ and $V = \sqrt{2(9.8)(0.080)} = 1.25 \text{ m/s}$.

$v = (m + M)V/m = (2.00)(1.25)/0.020 = 125 \text{ m/s}$.

Write It Like a 5**Model Response**

Prompt: Cart A (0.80 kg) moves at 1.5 m/s on a low-friction track and sticks to cart B (0.40 kg), initially at rest. (a) Sketch the A+B system's momentum versus time. (b) Justify it. (c) Find the final speed. (d) A student uses conservation of kinetic energy for (c); explain the error. Response: (a) One horizontal line at $p = 1.2 \text{ kg m/s}$ through the whole interval, no jump or slope at the collision. (b) The net external force on the A+B system is approximately zero because the track is low-friction and the normal force balances gravity; the forces between the carts are internal and equal and opposite, so the system's momentum cannot change. (c) $m_A v_A = (m_A + m_B) v_f$ gives $v_f = (0.80)(1.5)/1.20 = 1.0 \text{ m/s}$. (d) Kinetic energy is not conserved when objects stick: $K_i = 0.90 \text{ J}$ but $K_f = 0.60 \text{ J}$. Setting $K_i = K_f$ would wrongly give 1.2 m/s.

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What the grader looks for

One continuous horizontal line; a justification tied to this scenario (net external force on the chosen system is zero), not just a law's name; a starting equation before numbers; correct substitution with units; and the statement that energy is lost when objects stick.

Coach's Playbook

- 1 Name the system and list its external forces; friction is external unless the surface is in the system.
- 2 Sketch before and after with a positive direction and velocity arrows.
- 3 Write the principle first, then substitute.
- 4 Check units ($\text{N s} = \text{kg m/s}$) and signs (a rebound reverses p).
- 5 Test the answer: $K_f \leq K_i$ unless something exploded.

Common traps

(1) A momentum graph that slopes or drops at the collision instead of staying one horizontal line. (2) Using energy conservation to find the speed after objects stick. (3) Calling friction internal when the surface is outside the chosen system. (4) Writing $p = Ft$, or forgetting that Δp is final minus initial so a bounce is a bigger change than a stop.

Quick Check

Q1: Equal momentum, different masses: which has more kinetic energy? A: The lighter one, since $K = p^2/2m$. Q2: What does the area under a net force-time graph give? A: Impulse, equal to Δp . Q3: In a perfectly inelastic collision, what is conserved? A: Momentum only.

Challenge Corner

Cart A (2.0 kg) moves right at 5.0 m/s and strikes cart B (3.0 kg, at rest) fitted with a spring bumper, $k = 2000 \text{ N/m}$. Find the maximum compression. Solution: at maximum compression the carts share one velocity, $V = (2.0)(5.0)/5.0 = 2.0 \text{ m/s}$. The kinetic energy missing, $25 - 10 = 15 \text{ J}$, is stored in the spring: $\frac{1}{2}kx^2 = 15$, so $x = \sqrt{30/2000} = 0.12 \text{ m}$.

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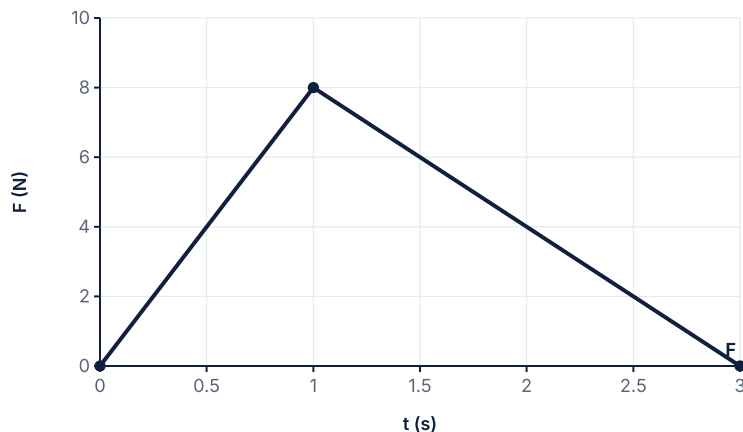
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Practice

PRACTICE Unit 4 Practice

Show your work. A calculator is allowed. Justify conclusions.

- Object X has mass m and object Y has mass $4m$. They have the same momentum. What is K_Y/K_X ?
 (A) $1/4$ (B) $1/2$ (C) 1 (D) 4
- The graph shows the net force on a 3.0 kg object initially at rest. What is its speed at $t = 3$ s?



Net force on the object.

- 12 m/s (B) 2.7 m/s (C) 1.3 m/s (D) 4.0 m/s
- A crash-test dummy is stopped from the same speed with and without an airbag; the airbag lengthens the stopping time. Compared with no airbag, the airbag makes which quantity smaller?
 (A) The impulse on the dummy (B) The change in momentum of the dummy
 (C) The average force on the dummy (D) The mass of the dummy
 - A 3.0 kg cart moving right at 4.0 m/s collides with a 1.0 kg cart moving left at 2.0 m/s on a frictionless track, and they stick. What is their velocity?
 (A) 3.5 m/s right (B) 1.0 m/s right (C) 2.5 m/s right
 (D) 3.0 m/s right
 - Skater A (50 kg) and skater B (75 kg) at rest on ice push apart; A moves away at 3.0 m/s. What is the ratio of A's kinetic energy to B's?
 (A) 1.0 (B) 0.67 (C) 1.5 (D) 2.25
 - A 1.0 kg lump of clay falls straight down onto a 2.0 kg cart rolling horizontally at 3.0 m/s on a frictionless track and sticks. Immediately after, the cart's speed is
 (A) 2.0 m/s (B) 3.0 m/s (C) 2.5 m/s (D) 1.0 m/s
 - Design an experiment to test whether the impulse delivered to a low-friction cart equals its change in momentum. Available: a cart of known mass, a track, a force sensor mounted on the cart, a motion detector, a string, pulley and hanging masses, and a computer.
 (a) Describe the procedure, including what you measure and how you vary the independent variable.
 (b) Explain how you would analyze the data, naming the quantities plotted so that the slope tests the hypothesis.
 (c) Name one source of error and how to reduce it.
 - Cart X (0.50 kg) moving at 2.0 m/s hits stationary cart Y (0.50 kg). In trial 1 they stick together. In trial 2 an elastic collision occurs through spring bumpers.
 (a) Predict whether the speed of Y just after the collision is greater in trial 1 or trial 2, and justify qualitatively using momentum.
 (b) Calculate the kinetic energy lost in trial 1 as a fraction of the initial kinetic energy.
 (c) Describe what happens to the lost energy.

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9. A 0.058 kg tennis ball moving at 20 m/s toward a racket is returned in the opposite direction. The force by the racket was measured: (ms): 0, 2, 4, 6, 8
 F (N): 0, 400, 800, 400, 0
 (a) Estimate the impulse using trapezoids.
 (b) Determine the speed of the ball leaving the racket.
 (c) Calculate the average force during the 8 ms contact and compare it with the peak force.
10. A 0.040 kg ball moving horizontally at 20 m/s strikes a 0.96 kg block resting on a rough table ($\mu_k = 0.30$) and embeds in it.
 (a) Is momentum of the ball-block system conserved during the very short collision? Justify.
 (b) Calculate the speed of the combination just after the collision.
 (c) Is momentum of the system conserved while the combination slides to a stop? Justify.
 (d) Calculate how far it slides.

Answers Unit 4: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

4 Linear Momentum

PRACTICE · UNIT 4 PRACTICE

1. A. Equal p gives $K = p^2/2m$, so $K_Y/K_X = m/(4m) = 1/4$. Choosing 4 assumes the heavier object has more energy; at equal momentum it has less.
2. D. Impulse is the triangle's area, $(1/2)(3)(8) = 12$ N s, and $v = J/m = 12/3.0 = 4.0$ m/s. Choosing 12 stops at the impulse; 2.7 divides the peak force by the mass and ignores time.
3. C. The same speed to zero means the same Δp and the same impulse, but $F_{avg} = \Delta p/\Delta t$, so a longer time gives a smaller average force. The tempting 'smaller impulse' is wrong because the impulse is fixed by Δp .
4. C. Right is positive: $(3.0)(4.0) + (1.0)(-2.0) = 10$ kg m/s $= (4.0)v$, so $v = 2.5$ m/s right. Dropping the sign of the left-moving cart gives 3.5 m/s.
5. C. Starting from rest, $50(3.0) = 75 v_B$ so $v_B = 2.0$ m/s. $K_A = 225$ J and $K_B = 150$ J, ratio 1.5. The momenta are equal in size (ratio 1.0), but energies are not.
6. A. Only horizontal momentum is conserved because the track's normal force changes vertical momentum: $(2.0)(3.0) = (3.0)v$ gives 2.0 m/s. Keeping 3.0 m/s ignores the added mass.
7. (a) 3 points. Level the track. Mount the force sensor on the cart and the motion detector behind it. Pull the cart with a string over a pulley and change the hanging mass for each trial. Each trial, record force vs time and velocity vs time over the same interval.
 (b) 3 points. For each trial, find impulse as the area under the F - t graph (software integration or trapezoids) and $\Delta p = m(v_f - v_i)$ from the motion detector. Plot Δp (kg m/s) on the vertical axis against J (N s) on the horizontal axis with labeled axes and units, and draw one best-fit line with a straightedge. If impulse equals change in momentum, the line passes through the origin with slope 1.0.
 (c) 1 point. Friction or a tilted track adds an external force that the sensor may not measure; reduce it by leveling, using low-friction wheels, and zeroing the sensor before each run.
8. (a) 2 points. Trial 2 gives the greater speed for Y. In trial 1 the carts move together, so momentum gives $v = (0.50)(2.0)/1.0 = 1.0$ m/s. In trial 2 the equal-mass elastic collision leaves X stopped and Y at 2.0 m/s, which carries all the momentum; this is consistent with both momentum and energy being conserved.
 (b) 2 points. $K_i = \frac{1}{2}(0.50)(2.0)^2 = 1.0$ J; $K_f = \frac{1}{2}(1.0)(1.0)^2 = 0.50$ J. Lost fraction $= 0.50/1.0 = 50\%$.
 (c) 1 point. The lost energy becomes thermal energy and sound and deforms the objects.
9. (a) 2 points. Areas: $0.4 + 1.2 + 1.2 + 0.4 = 3.2$ N s.
 (b) 3 points. Taking the racket's direction as positive, $p_i = (0.058)(-20) = -1.16$ kg m/s. $J = \Delta p$ gives $3.2 = 0.058v_f + 1.16$, so $v_f = 35$ m/s.
 (c) 2 points. $F_{avg} = 3.2/0.008 = 400$ N, half the 800 N peak, because the force is zero at both ends of the contact.
10. (a) 2 points. Yes, approximately. The ball-block forces are internal; the friction and normal forces are external but their impulse over the very short collision is negligible next to the internal impulse.
 (b) 2 points. $mv = (m + M)V$ gives $V = (0.040)(20)/1.00 = 0.80$ m/s.
 (c) 2 points. No. Kinetic friction from the table is an external force on the system, so its momentum decreases to zero.
 (d) 2 points. $\frac{1}{2}V^2 = \mu_k g d$ gives $d = (0.80)^2/[2(0.30)(9.8)] = 0.11$ m.

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Unit 5: Torque and Rotational Dynamics

Exam weighting: about 10-15% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

5 Torque and Rotational Dynamics

Rotational kinematics, torque and lever arms, rotational inertia, equilibrium, and the rotational form of Newton's second law, with the lab-design habits that earn credit.

Why This Matters

Unit 5 is worth about 10 to 15 percent of the multiple-choice section, and the 2025 experimental-design question was a torque and meterstick lab that averaged under 5 out of 10. Students lost points assuming a spring scale reads the hanging weight, mixing up similar distances, plotting $1/x$ for a straight line, and drawing hinge forces in the wrong direction.

Core Ideas

5.1 Rotational Kinematics

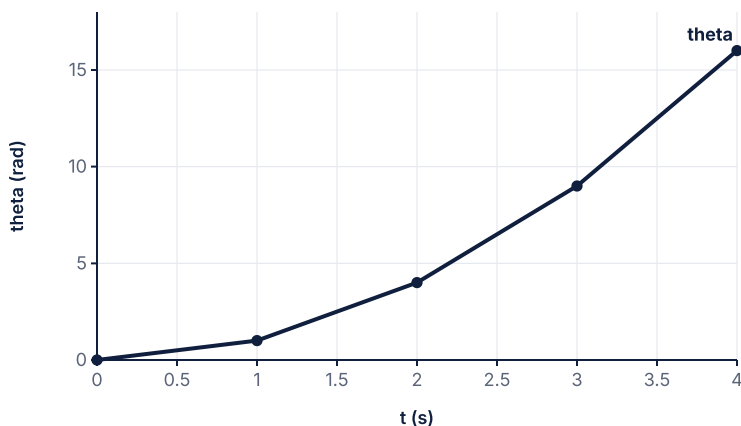
Angular position θ is measured in radians, with one revolution equal to 2π rad. Angular velocity is $\omega = \Delta\theta/\Delta t$ and angular acceleration is $\alpha = \Delta\omega/\Delta t$. Choose counterclockwise as positive and let signs carry direction. For constant α the equations are exact copies of linear kinematics with $x \rightarrow \theta$, $v \rightarrow \omega$, $a \rightarrow \alpha$. On an ω - t graph, the slope is α and the area is $\Delta\theta$. A spinning object can be momentarily at rest ($\omega = 0$) while $\alpha \neq 0$.

Constant angular acceleration

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \text{ (radians only; convert revolutions and rpm first).}$$



A wheel starting from rest with $\alpha = 2.0 \text{ rad/s}^2$ turns through $\theta = t^2$: a parabola, like constant-acceleration position.

5.2 Connecting Linear and Rotational Motion

Every point on a rigid rotating object shares the same ω and α , but points farther from the axis move faster. A point at distance r has arc length $s = r\theta$, tangential speed $v = r\omega$, tangential acceleration $a_t = r\alpha$, and centripetal acceleration $a_c = v^2/r = \omega^2 r$ directed toward the axis. The tangential part changes speed, the centripetal part changes direction. These relations hold only with θ in radians.

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5.3 Torque

Torque measures how effectively a force produces rotation about a chosen axis: $\tau = rF \sin \theta$, where θ is the angle between the position vector from the axis to the point of application and the force. Equivalently, $\tau = r_{\perp} F$, with $r_{\perp} = r \sin \theta$ the **lever arm**, the perpendicular distance from the axis to the line of action. The unit is $\text{N}\cdot\text{m}$ (not joules). A force whose line of action passes through the axis, or one parallel to r , has zero torque. Torque depends on the axis chosen. Counterclockwise torque is positive.

Torque

$$\tau = rF \sin \theta = r_{\perp} F = rF_{\perp}$$

Sign: counterclockwise positive, clockwise negative. Net torque $\tau_{\text{net}} = \sum \tau$ about the same axis.

5.4 Rotational Inertia

Rotational inertia I is an object's resistance to changes in angular velocity. For point masses $I = \sum m_i r_i^2$. It depends on the mass, how it is distributed, and the axis chosen. Mass farther from the axis counts as the square of its distance. For extended objects the exam supplies I or lets you compare shapes (for the same M and R , a hoop has $I = MR^2$ and a solid disk $\frac{1}{2}MR^2$).

$r = 0.20 \text{ m}$ each: $I = 0.080$



axis at gold dot

$r = 0.50 \text{ m}$ each: $I = 0.50$



Two 1.0 kg masses on a light rod. Moving the masses outward multiplies I by 6.25, since I is proportional to r squared.

5.5 Rotational Equilibrium and Newton's First Law in Rotational Form

An object in rotational equilibrium has $\alpha = 0$, so $\sum \tau = 0$ about any axis. Static equilibrium needs both $\sum \vec{F} = 0$ and $\sum \tau = 0$. Because the torque equation holds for any axis, pick the pivot where the unknown forces act: they then have zero lever arm and drop out. Each uniform object's weight acts at its center. A hinge exerts a force with both horizontal and vertical components, usually not along the beam. A spring scale reports a force, not a torque, and its reading is not automatically the hanging weight: use the torque balance to find it.

5.6 Newton's Second Law in Rotational Form

Rotational second law

$$\alpha = \frac{\tau_{\text{net}}}{I}, \text{ so } \tau_{\text{net}} = I\alpha.$$

For a pulley or wheel combined with hanging masses: write $F = ma$ for each mass, $\tau = I\alpha$ for the wheel, and connect them with $a = \alpha R$ (no slipping).

Tensions on the two sides of a pulley with rotational inertia differ; their difference supplies the net torque.

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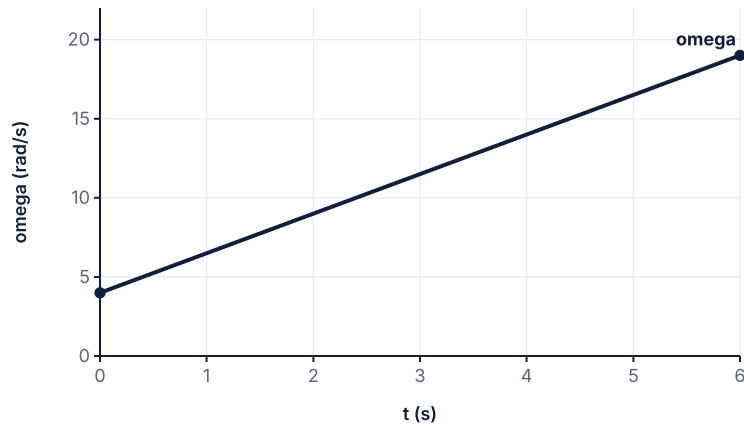
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Worked Examples

EXAMPLE 1: WHEEL KINEMATICS

A wheel spinning at 4.0 rad/s speeds up at a constant 2.5 rad/s² for 6.0 s. Find its final angular velocity and how many revolutions it makes.



Slope = 2.5 rad/s²; area = 69 rad.

SOLUTION

$$\omega = \omega_0 + \alpha t = 4.0 + (2.5)(6.0) = 19 \text{ rad/s.}$$

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 24 + 45 = 69 \text{ rad, the area under the graph.}$$

$$\text{Revolutions} = 69/2\pi = 11 \text{ rev.}$$

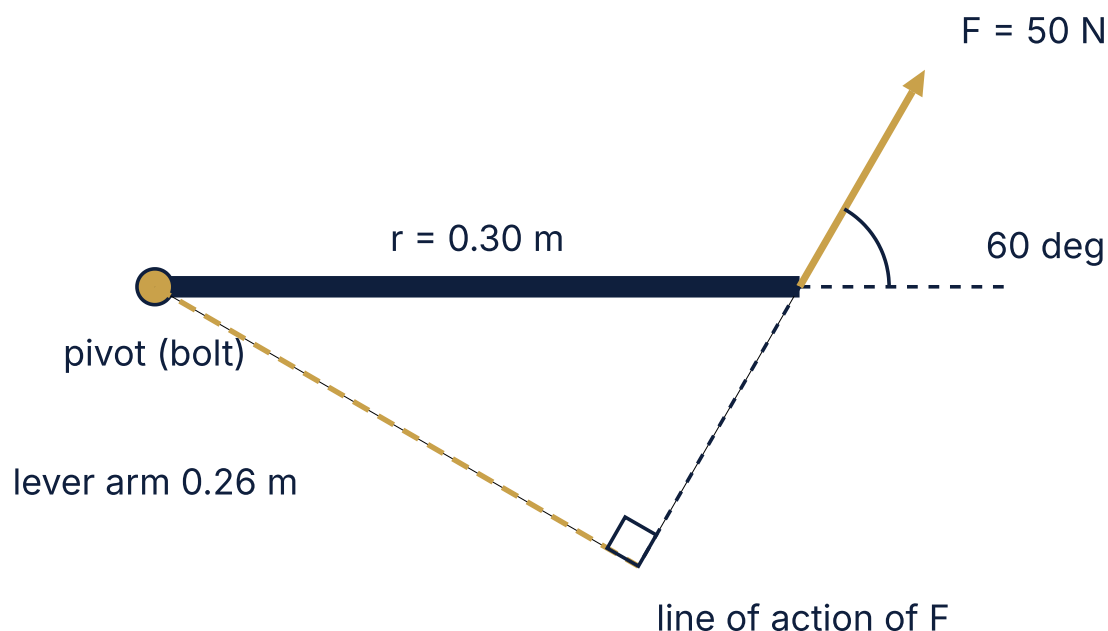
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EXAMPLE 2: TORQUE WITH AN ANGLED FORCE

A 0.30 m wrench is pulled with 50 N at 60 degrees to the wrench. Find the torque on the bolt.



The lever arm is the perpendicular distance from the pivot to the line of action: $0.30 \sin 60^\circ = 0.26 \text{ m}$, so torque = $0.26 \text{ m} \times 50 \text{ N} = 13 \text{ N m}$.

SOLUTION

$$\tau = rF \sin \theta = (0.30)(50) \sin 60^\circ = 13 \text{ N m}.$$

Check with the lever arm: $r_\perp = 0.30 \sin 60^\circ = 0.26 \text{ m}$ and $0.26 \times 50 = 13 \text{ N m}$.

Only the component of the force perpendicular to the wrench, 43 N, turns the bolt.

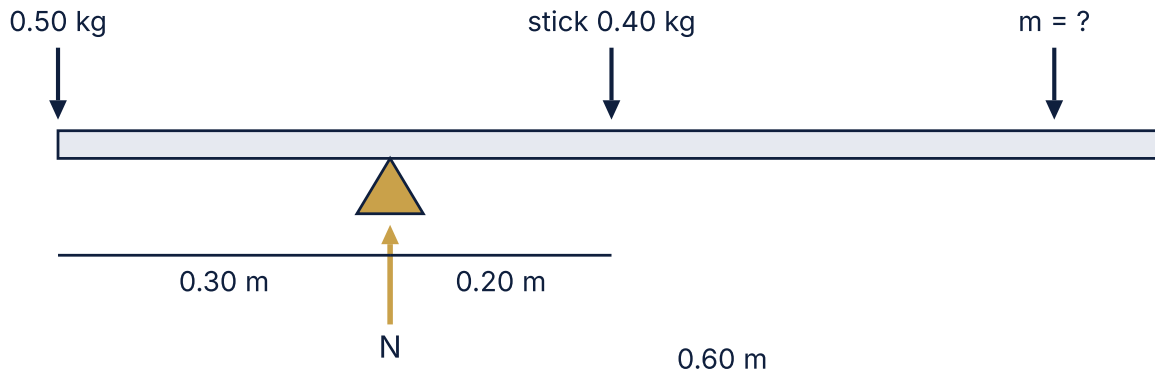
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EXAMPLE 3: BALANCING A STICK

A uniform 0.40 kg stick (1.0 m) rests on a pivot at the 0.30 m mark. A 0.50 kg mass hangs at the 0 mark and an unknown mass m hangs at the 0.90 m mark, so that the stick is balanced. Find m and the pivot force.



Uniform stick on a pivot at the 0.30 m mark. The stick's weight acts at its center (0.50 m mark).

SOLUTION

Torques about the pivot: left (counterclockwise) = $(0.50)g(0.30)$; right = $(0.40)g(0.20) + mg(0.60)$. The pivot force has zero lever arm. $0.15 = 0.080 + 0.60m$, so $m = 0.12$ kg (0.117).

$\sum F = 0$: $N = (0.50 + 0.40 + 0.117)(9.8) = 9.96$ N upward.

EXAMPLE 4: HANGING MASS ON A DISK

A 1.0 kg mass hangs from a string wrapped on a solid disk pulley ($M = 2.0$ kg, $R = 0.20$ m, $I = \frac{1}{2}MR^2$) free to spin. Find the mass's acceleration and the tension.

SOLUTION

Mass: $mg - T = ma$. Disk: $TR = I\alpha = \frac{1}{2}MR^2(a/R)$, so $T = \frac{1}{2}Ma$.

Combine: $mg = (m + M/2)a$, so $a = 9.8/2.0 = 4.9$ m/s² (half of g).

$T = \frac{1}{2}(2.0)(4.9) = 4.9$ N (less than $mg = 9.8$ N), $\alpha = a/R = 24.5$ rad/s². Check: $TR = 0.98 = I\alpha = (0.040)(24.5)$.

Write It Like a 5**Model Response**

Prompt: Two wheels of equal mass and radius, a hoop and a solid disk, start from rest. The same constant net torque acts on each for 2.0 s. (a) Which has the greater angular velocity at 2.0 s? Justify. (b) By what factor? Response: (a) The solid disk. The rotational form of Newton's second law gives $\alpha = \tau_{\text{net}}/I$. Torque, mass, and radius are equal, but the hoop's mass is all at radius R , so $I_{\text{hoop}} = MR^2$ is larger than $I_{\text{disk}} = \frac{1}{2}MR^2$. The disk has the larger α , and since $\omega = \alpha t$ with the same t and $\omega_0 = 0$, the disk has the larger ω . (b) $\omega_{\text{disk}}/\omega_{\text{hoop}} = I_{\text{hoop}}/I_{\text{disk}} = MR^2/(\frac{1}{2}MR^2) = 2$.

What the grader looks for

The named principle ($\tau_{\text{net}} = I\alpha$) first; what is the same (torque, time) and what differs (I); the link from α to ω ; a factor derived symbolically; claim and justification that agree.

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Coach's Playbook

- 1 Draw the object, mark the axis, and label each force at its point of application.
- 2 For torque, find the angle between r and F , or the lever arm, and write the sign.
- 3 In equilibrium, choose the pivot to eliminate the unknown forces, then also write $\sum F = 0$.
- 4 For rolling-wheel or pulley problems, write $F = ma$ and $\tau = I\alpha$ separately and tie them by $a = \alpha R$.
- 5 In a lab, put the numerator quantity against the denominator quantity so the slope is the target.

Common traps

(1) Treating the hanging weight as the spring-scale reading, or mixing up the distance to the pivot with the distance to the center of mass. (2) Plotting $1/x$ to force linearity instead of plotting the numerator against the denominator. (3) Drawing a hinge force horizontal or along the beam and using only $\sum F = 0$ without torque. (4) Calling angular acceleration zero when $\omega = 0$ momentarily, or constant when the torque changes.

Quick Check

Q1: What is the torque when F is parallel to r ? A: Zero, since $\sin 0 = 0$. Q2: If I doubles and τ_{net} stays fixed, what happens to α ? A: It halves. Q3: How do the speeds of two points on a spinning disk at 0.10 m and 0.20 m compare? A: The outer point is twice as fast; both have the same ω .

Challenge Corner

A uniform horizontal beam (10 kg, 4.0 m) is hinged to a wall and held by a cable attached to its far end at 30 degrees above horizontal; a 20 kg mass hangs from that end. Find the tension and the hinge force. Solution: torques about the hinge: $T \sin 30^\circ (4.0) = (10)(9.8)(2.0) + (20)(9.8)(4.0)$, so $2T = 980$ and $T = 490$ N. Horizontal: the cable pulls toward the wall, so the hinge pushes away from it: $H_x = T \cos 30^\circ = 424$ N. Vertical: $H_y = 294 - 490 \sin 30^\circ = 49$ N up. Magnitude = $\sqrt{424^2 + 49^2} = 427$ N, tilted slightly above horizontal, not along the beam.

Practice

PRACTICE Unit 5 Practice

Show your work. A calculator is allowed. Justify conclusions.

1. A wheel starts from rest and has constant angular acceleration 3.0 rad/s^2 . Through what angle does it turn in 4.0 s?
 (A) 12 rad (B) 24 rad (C) 36 rad (D) 48 rad
2. A 20 N force acts at the end of a 0.50 m lever. The force makes an angle of 30 degrees with the lever. What is the torque about the pivot?
 (A) 10 N m (B) 8.7 N m (C) 5.0 N m (D) 2.5 N m
3. Two 1.0 kg point masses are 1.0 m apart on a light rod. What is the rotational inertia about an axis perpendicular to the rod through one of the masses?
 (A) 1.0 kg m^2 (B) 2.0 kg m^2 (C) 0.25 kg m^2
 (D) 0.50 kg m^2
4. A light plank balances on a central pivot with a 4.0 kg mass 0.60 m to the left. Where must a 3.0 kg mass be placed on the right?
 (A) 0.45 m (B) 0.60 m (C) 0.80 m (D) 1.2 m
5. A wheel's net torque doubles and, at the same time, its rotational inertia doubles. Its angular acceleration
 (A) doubles (B) halves (C) quadruples
 (D) stays the same

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6. A wheel of radius 0.40 m spins at a constant 5.0 rad/s. What is the centripetal acceleration of a point on its rim?
 (A) 2.0 m/s² (B) 10 m/s² (C) 25 m/s² (D) 5.0 m/s²
7. A student has a meterstick of unknown mass, a knife-edge pivot, a set of known masses with string loops, and a ruler. Design an experiment to determine the stick's mass M .
 (a) Describe the setup and procedure, including what quantities are measured and how many trials.
 (b) State the equation relating the measured quantities and explain how the data should be graphed so that M comes from the slope.
 (c) Identify one assumption and how you would check it.
8. A door is 0.90 m wide and hinged along one edge. A person applies a 20 N force in each of three cases: (1) perpendicular to the door at the far edge; (2) perpendicular to the door 0.45 m from the hinge; (3) at the far edge, but with the force at 30 degrees to the door's surface (the line from the hinge).
 (a) Rank the torques about the hinge from greatest to least, justifying with the torque equation.
 (b) Calculate the torque in each case.
 (c) What perpendicular force at 0.45 m produces the torque of case 1?
9. A 2.0 kg block and a 1.0 kg block hang on opposite sides of a pulley, a solid disk of mass 1.0 kg and radius 0.10 m ($I = \frac{1}{2}MR^2$). The string does not slip.
 (a) Derive an expression for the acceleration in terms of m_1 , m_2 , and M .
 (b) Calculate the acceleration and both tensions.
 (c) Explain why the two tensions differ.
10. The graph shows the angular velocity of a flywheel ($I = 0.50 \text{ kg m}^2$, radius 0.25 m) slowed by friction at its axle.
 (a) Determine the angular acceleration.
 (b) Calculate the angle turned before stopping, in revolutions.
 (c) Determine the net torque.
 (d) A student says, 'At $t = 4 \text{ s}$ the angular acceleration is zero because the wheel has stopped.' Evaluate this claim.

Answers Unit 5: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

5 Torque and Rotational Dynamics

PRACTICE · UNIT 5 PRACTICE

- B. $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(3.0)(4.0)^2 = 24 \text{ rad}$. Choosing 12 uses αt , which is the angular velocity, not the angle.
- C. $\tau = rF \sin \theta = (0.50)(20) \sin 30^\circ = 5.0 \text{ N m}$. The 10 N m answer leaves out the sine; 8.7 N m uses the cosine.
- A. Only the other mass contributes: $I = (1.0)(1.0)^2 = 1.0 \text{ kg m}^2$, because the mass on the axis has $r = 0$. The center-axis value of 0.50 kg m^2 is for a different axis.
- C. Balance of torques about the pivot: $(4.0)(0.60) = (3.0)x$, so $x = 0.80 \text{ m}$. The answer 0.45 m comes from inverting the mass ratio.
- D. $\alpha = \tau_{\text{net}}/I$, so doubling both leaves α unchanged. Choosing 'doubles' tracks only the torque and forgets that I also changed.
- B. $a_c = \omega^2 r = (5.0)^2(0.40) = 10 \text{ m/s}^2$. The 2.0 value is the speed $v = \omega r$ in m/s, not an acceleration; 25 is ω^2 without the radius.
- (a) 3 points. Balance the empty meterstick on the pivot to confirm its center is at the 50.0 cm mark. Hang a known mass m from a loop at the 0 cm end and slide the pivot along the stick until it balances; read the pivot position p with the ruler. Distances: $d = p$ (pivot to mass) and $e = 0.500 - p$ (pivot to stick center). Repeat for at least five different masses.
 (b) 3 points. Torque balance about the pivot gives $mgd = Mge$, so $m = M(e/d)$. Plot m (kg) on the vertical axis against e/d (unitless) on the horizontal axis, label axes with quantity and unit, and draw one straight best-fit line. The slope is M .
 (c) 1 point. Assumption: the stick is uniform so its weight acts at 50.0 cm; checked by balancing the unloaded stick at the center, or the line's intercept should be near zero.
- (a) 2 points. Ranking: case 1 > case 2 = case 3. $\tau = rF \sin \theta$; case 1 has the largest r and $\sin 90^\circ = 1$. Case 2 halves r . Case 3 has the same r as case 1 but $\sin 30^\circ = 0.5$, a lever arm of 0.45 m, equal to case 2.
 (b) 3 points. (1) $0.90 \times 20 \times 1 = 18 \text{ N m}$. (2) $0.45 \times 20 = 9.0 \text{ N m}$. (3) $0.90 \times 20 \times \sin 30^\circ = 9.0 \text{ N m}$.
 (c) 2 points. $F = 18/0.45 = 40 \text{ N}$.
- (a) 3 points. $m_1g - T_1 = m_1a$; $T_2 - m_2g = m_2a$; for the pulley, $(T_1 - T_2)R = \frac{1}{2}MR^2(a/R)$, so $T_1 - T_2 = \frac{1}{2}Ma$. Adding gives $a = (m_1 - m_2)g/(m_1 + m_2 + M/2)$.
 (b) 3 points. $a = (1.0)(9.8)/(3.0 + 0.5) = 2.8 \text{ m/s}^2$. $T_1 = 2.0(9.8 - 2.8) = 14 \text{ N}$; $T_2 = 1.0(9.8 + 2.8) = 12.6 \text{ N}$. Check: $(14 - 12.6)(0.10) = 0.14 \text{ N m} = I\alpha = (0.0050)(28)$.
 (c) 1 point. The pulley has rotational inertia, so a net torque $(T_1 - T_2)R$ is required to give it angular acceleration; the tensions must differ.

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10. (a) 2 points. Slope: $\alpha = (0 - 12)/4.0 = -3.0 \text{ rad/s}^2$.
(b) 2 points. Area = $\frac{1}{2}(12)(4.0) = 24 \text{ rad}$, which is $24/2\pi = 3.8 \text{ rev}$.
(c) 2 points. $\tau_{\text{net}} = I\alpha = (0.50)(-3.0) = -1.5 \text{ N m}$ (opposing the rotation).
(d) 2 points. The claim is incorrect. The graph is a straight line with constant slope, so $\alpha = -3.0 \text{ rad/s}^2$ at every instant including $t = 4 \text{ s}$; $\omega = 0$ is its value there, not the slope.

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Unit 6: Energy and Momentum of Rotating Systems

Exam weighting: about 5-8% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

6 Energy and Momentum of Rotating Systems

Rotational kinetic energy, work, angular momentum and its conservation, rolling, and satellite motion: the same two conservation laws, now with I and ω .

Why This Matters

Unit 6 is the smallest in the course, about 5 to 8 percent of the multiple-choice section, but its ideas show up in energy and experimental free-response questions. Students lose points by assuming angular momentum is always conserved, dropping the rotational term in rolling energy, and using the wrong radius for satellites. The exam gives moments of inertia when needed; you supply the physics.

Core Ideas

6.1 Rotational Kinetic Energy

A rotating rigid body has kinetic energy $K_{rot} = \frac{1}{2}I\omega^2$, in joules. Doubling ω quadruples it. An object that both translates and rotates has total kinetic energy $K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2$, and both terms belong in the energy conservation equation.

6.2 Torque and Work

A torque that acts through an angle does work: $W = \tau\Delta\theta$ for a constant torque along the rotation, with $\Delta\theta$ in radians. On a τ - θ graph, work is the area. The work-energy theorem holds: $W_{net} = \Delta K_{rot}$. Power is $P = \tau\omega$ (or $W/\Delta t$ on average). A torque opposing the rotation, like friction in a bearing, does negative work.

Energy of rotation

$$K_{rot} = \frac{1}{2}I\omega^2 \quad W = \tau \Delta\theta \quad P = \tau\omega$$

$$W_{net} = \Delta K_{rot} \quad (\text{angles in radians})$$

6.3 Angular Momentum and Angular Impulse

For a rigid body spinning about a fixed axis, $L = I\omega$, in $\text{kg m}^2/\text{s}$. A point mass moving with velocity v has angular momentum about a chosen point $L = r_{\perp}mv = rmv \sin \theta$, where θ is the angle between $\vec{r}$ and $\vec{v}$; a particle moving in a straight line has nonzero L about any point not on its path. The rotational form of Newton's second law is $\tau_{net} = \Delta L/\Delta t$, so the **angular impulse** $\tau_{net}\Delta t$ equals ΔL .

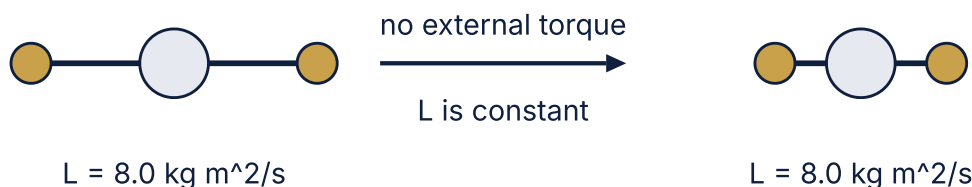
6.4 Conservation of Angular Momentum

If the net external torque on a system about an axis is zero, its angular momentum about that axis is constant: $I_i\omega_i = I_f\omega_f$. Internal torques cancel. A pivot force can be external yet exert no torque about the pivot, so L about the pivot can be conserved even when linear momentum is not. Changing I changes ω inversely, and K changes too: pulling weights in does positive work, so K rises. When two rotating objects lock together, L is conserved but K decreases.

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Before: $I = 4.0$, $\omega = 2.0$ rad/sAfter: $I = 2.5$, $\omega = 3.2$ rad/s

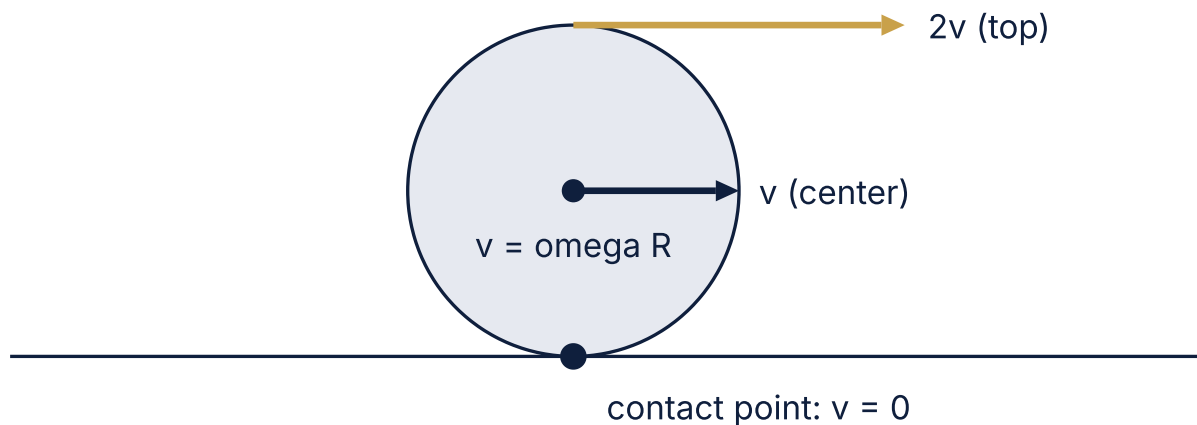
Top view of a person on a low-friction stool holding weights. Pulling the weights in lowers I , so ω rises and L stays the same.

Angular momentum procedure

1. Choose the system and axis; state why the net external torque about that axis is zero.
2. Write $\sum L_i = \sum L_f$ using $I\omega$ for spinning bodies and $r_{\perp}mv$ for moving particles.
3. Solve for the unknown; then use energy only if asked.

6.5 Rolling

An object rolls **without slipping** when the contact point is at rest relative to the surface: $v_{cm} = \omega R$. Kinetic energy is $\frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2$. With $I = cMR^2$ this is $\frac{1}{2}Mv_{cm}^2(1 + c)$. Static friction at the contact point does no work, because that point does not move, so mechanical energy is conserved while rolling. Released from rest at height h , a rolling object reaches $v = \sqrt{2gh/(1 + c)}$, which does not depend on mass or radius, and its acceleration down an incline is $a = g \sin \theta / (1 + c)$.



A wheel rolling without slipping at center speed v . Relative to the ground the top moves at $2v$ and the contact point does not move.

Rolling object (no slipping)	I	Rotational share of KE	a down incline
Hoop	MR^2	1/2	$0.50 g \sin \theta$
Solid disk or cylinder	$\frac{1}{2}MR^2$	1/3	$0.67 g \sin \theta$
Solid sphere	$\frac{2}{5}MR^2$	2/7	$0.71 g \sin \theta$
Frictionless slide (no rotation)	-	0	$1.00 g \sin \theta$

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6.6 Motion of Orbiting Satellites

For a circular orbit, gravity supplies the centripetal force: $GMm/r^2 = mv^2/r$, so $v = \sqrt{GM/r}$ and $T = 2\pi r/v$, which gives $T^2 = 4\pi^2 r^3/GM$. The satellite's mass cancels. Kinetic energy is $K = GMm/2r$, potential energy is $U = -GMm/r$, and total energy $E = -GMm/2r$ is negative for a bound orbit. A larger orbit means slower speed, longer period, and a higher (less negative) total energy. For an elliptical orbit, gravity always points toward the planet, so the torque about the planet is zero and $L = mrv \sin \theta$ is constant: at the closest and farthest points $r_p v_p = r_a v_a$, and the satellite is fastest at perigee. Mechanical energy $K + U$ is also constant. The line to the planet sweeps equal areas in equal times (Kepler's second law).

Satellite relations

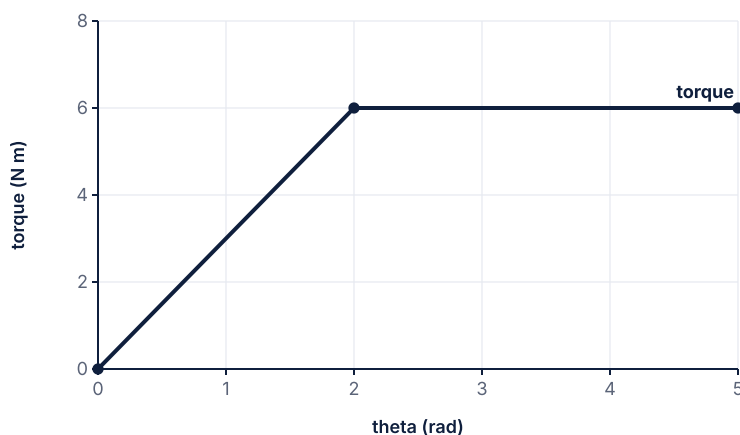
$$v = \sqrt{GM/r} \quad T = 2\pi r/v \quad E = -\frac{GMm}{2r} \quad (\text{circular})$$

$$\text{Elliptical: } r_p v_p = r_a v_a \text{ and } \frac{1}{2}mv^2 - GMm/r = \text{constant.}$$

Worked Examples

EXAMPLE 1: TORQUE DOING WORK

A flywheel ($I = 2.0 \text{ kg m}^2$) starts at rest. A torque acts through 5.0 rad as shown. Find the work done and the final angular velocity.



Torque on the flywheel.

SOLUTION

Work = area: ramp $\frac{1}{2}(2.0)(6.0) = 6.0 \text{ J}$ plus rectangle $(3.0)(6.0) = 18 \text{ J}$, total 24 J.

$$W_{\text{net}} = \Delta K_{\text{rot}} = \frac{1}{2}I\omega^2, \text{ so } \omega = \sqrt{2(24)/2.0} = 4.9 \text{ rad/s.}$$

EXAMPLE 2: A PERSON ON A STOOL

A student on a low-friction stool holds weights, with $I = 4.0 \text{ kg m}^2$ and $\omega = 2.0 \text{ rad/s}$. She pulls the weights in so $I = 2.5 \text{ kg m}^2$. Find the new ω and the change in kinetic energy.

SOLUTION

No external torque about the vertical axis (friction is low), so L is conserved: $\omega_f = 4.0(2.0)/2.5 = 3.2 \text{ rad/s}$.

$$K_i = \frac{1}{2}(4.0)(2.0)^2 = 8.0 \text{ J}, \quad K_f = \frac{1}{2}(2.5)(3.2)^2 = 12.8 \text{ J.}$$

The 4.8 J increase came from the work her muscles did pulling the weights inward.

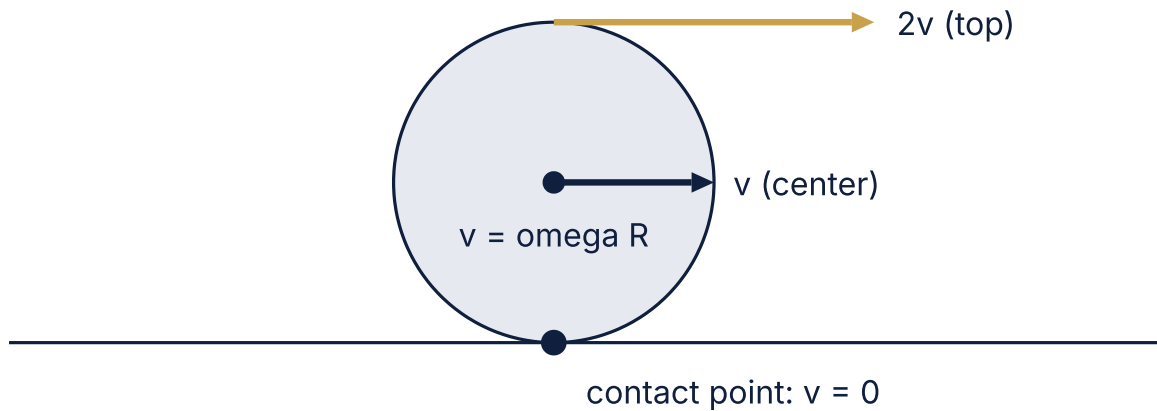
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EXAMPLE 3: ROLLING VERSUS SLIDING

A solid sphere ($I = \frac{2}{5}MR^2$, $R = 0.10$ m) rolls without slipping from rest down a ramp, dropping 0.50 m. Find its speed and angular velocity at the bottom, and compare with a frictionless slide.



A wheel rolling without slipping at center speed v . Relative to the ground the top moves at $2v$ and the contact point does not move.

SOLUTION

Energy: $Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}(\frac{2}{5}MR^2)(v/R)^2 = \frac{7}{10}Mv^2$, so $v = \sqrt{10gh/7} = \sqrt{10(9.8)(0.50)/7} = 2.6$ m/s.

$\omega = v/R = 26$ rad/s.

A frictionless slide gives $\sqrt{2gh} = 3.1$ m/s. The sphere is slower because $2/7$ of its energy is in rotation.

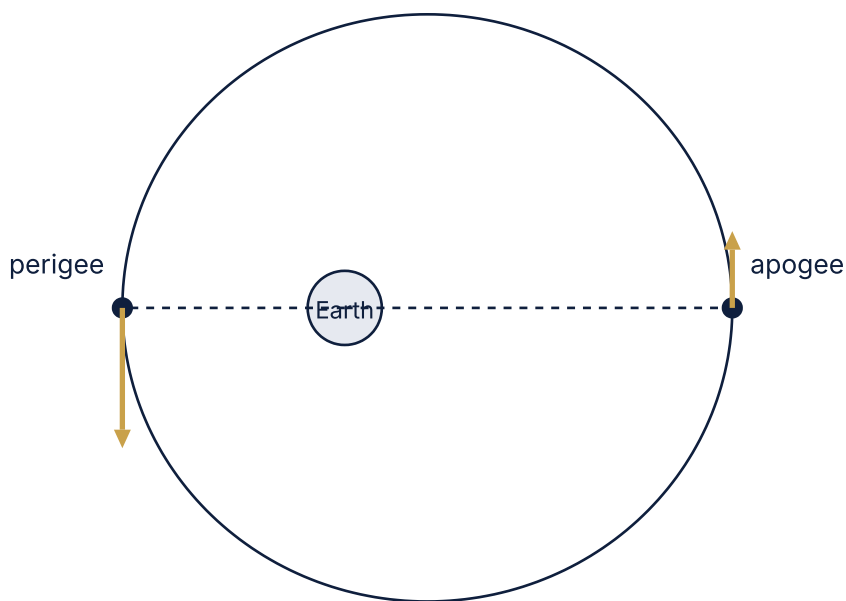
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EXAMPLE 4: AN ELLIPTICAL ORBIT

A satellite's closest distance to Earth's center is 7.00×10^6 m, where its speed is 8.50×10^3 m/s. Its farthest distance is 1.22×10^7 m. Find its speed there, and check energy conservation ($GM_E = 3.98 \times 10^{14}$ N m²/kg).



perigee: $r = 7.00\text{e}6$ m, $v = 8.50\text{e}3$ m/s

apogee: $r = 1.22\text{e}7$ m, $v = 4.88\text{e}3$ m/s

gold arrows show velocity

An elliptical orbit. The gravitational force points to Earth, so it exerts no torque about Earth; angular momentum is constant and the satellite is fastest at perigee.

SOLUTION

Gravity exerts no torque about Earth, so $r_p v_p = r_a v_a$: $v_a = (7.00 \times 10^6)(8.50 \times 10^3)/(1.22 \times 10^7) = 4.88 \times 10^3$ m/s.

Energy per unit mass at perigee: $\frac{1}{2}v_p^2 - GM/r_p = 3.61 \times 10^7 - 5.69 \times 10^7 = -2.08 \times 10^7$.

At apogee: $1.19 \times 10^7 - 3.26 \times 10^7 = -2.07 \times 10^7$ J/kg. The totals agree to rounding. The satellite is slowest where it is farthest.

Write It Like a 5**Model Response**

Prompt: Disk 1 ($I_1 = 0.50$ kg m²) spins at 12 rad/s on a frictionless axle. A non-rotating disk 2 ($I_2 = 0.25$ kg m²) is dropped onto it and they stick. (a) Is angular momentum conserved? Justify. (b) Find the final ω . (c) Is kinetic energy conserved? Response: (a) Yes. The net external torque on the two-disk system about the axle is zero: the axle is frictionless, gravity and the support force act along the axis, and the friction between the disks is internal. (b) $I_1\omega_1 = (I_1 + I_2)\omega_f$ gives $\omega_f = (0.50)(12)/0.75 = 8.0$ rad/s. (c) No. $K_i = \frac{1}{2}(0.50)(12)^2 = 36$ J and $K_f = \frac{1}{2}(0.75)(8.0)^2 = 24$ J; 12 J became thermal energy.

What the grader looks for

A conservation claim tied to the system and axis (external torque is zero); the equation written before numbers; correct use of the summed rotational inertia; and the statement that L is conserved but K is not, with the energy accounted for.

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Coach's Playbook

- 1 Decide first whether the question calls for energy, angular momentum, or both.
- 2 For rolling, list translational and rotational terms and use $\omega = v/R$ to put them in one unknown.
- 3 Use $r_{\perp}$ for particle angular momentum: look for the perpendicular distance from the point to the line of motion.
- 4 Orbit problems: pick circular (use $v = \sqrt{GM/r}$) or elliptical (use rv constant and energy).
- 5 Sanity check: pulling in raises both ω and K ; sticking lowers K .

Common traps

(1) Conserving K in a collision where disks or wheels lock together. (2) Dropping the rotational term in rolling, or giving friction credit for doing work on a rolling wheel. (3) Using the planet's radius instead of the distance from its center, or calling a satellite's speed independent of orbit radius. (4) Forgetting that a pivot force prevents conservation of linear momentum but not angular momentum about the pivot.

Quick Check

Q1: If ω triples, how does K_{rot} change? A: It becomes 9 times larger. Q2: A satellite moves to a larger circular orbit; what happens to its speed? A: It decreases, since $v = \sqrt{GM/r}$. Q3: Does static friction do work on a wheel rolling without slipping? A: No; its point of application is instantaneously at rest.

Challenge Corner

A uniform rod (mass 2.0 kg, length 1.2 m, $I = \frac{1}{3}ML^2$ about one end) is released from rest from a horizontal position, pivoted at one end. Find the angular velocity and the speed of the free end at the bottom. Solution: the center of mass falls $L/2$, so $Mg(L/2) = \frac{1}{2}(\frac{1}{3}ML^2)\omega^2$ and $\omega = \sqrt{3g/L} = \sqrt{3(9.8)/1.2} = 4.9 \text{ rad/s}$. The free end has $v = \omega L = 5.9 \text{ m/s}$.

Practice

PRACTICE Unit 6 Practice

Show your work. A calculator is allowed. Justify conclusions.

1. A wheel with rotational inertia 0.40 kg m^2 spins at 10 rad/s . What is its rotational kinetic energy?
 (A) 20 J (B) 40 J (C) 4.0 J (D) 2.0 J
2. A constant torque of 3.0 N m acts on a wheel through 2.0 revolutions in the direction of rotation. How much work does it do?
 (A) 18.8 J (B) 37.7 J (C) 6.0 J (D) 75.4 J
3. A person on a low-friction stool pulls two weights toward the body while spinning. Which statement is correct?
 (A) The angular momentum and angular velocity both increase.
 (B) The angular momentum is unchanged and the kinetic energy is unchanged.
 (C) The angular momentum decreases because the moment of inertia decreases.
 (D) The angular momentum is unchanged, and both the angular velocity and kinetic energy increase.
4. A solid disk and a hoop of equal mass and radius roll without slipping from rest down the same ramp. Compared with the hoop, the speed of the disk at the bottom is
 (A) the same, because mass and radius are equal (B) less, because it has less rotational inertia
 (C) about 1.15 times as great (D) twice as great
5. A satellite moves from a circular orbit of radius r to a circular orbit of radius $2r$. Its speed changes from v to
 (A) $v/\sqrt{2}$ (B) $v/2$ (C) $2v$ (D) $\sqrt{2}v$

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6. A net torque of 4.0 N m acts on a body of rotational inertia 2.0 kg m^2 , initially at rest, for 3.0 s. What is its angular velocity at the end?
- (A) 2.0 rad/s (B) 12 rad/s (C) 24 rad/s (D) 6.0 rad/s
7. A student has a solid cylinder, a ramp, a meterstick, and a stopwatch only. Design an experiment to determine the cylinder's ratio $c = I/(MR^2)$ from the motion of the cylinder rolling without slipping from rest.
- (a) Describe the procedure, including which quantities you measure and how you obtain the final speed.
- (b) Explain how the data are graphed so that the slope gives c , and give the relationship you use.
- (c) State what value you expect for a solid cylinder.
8. Satellite A orbits Earth in a circle of radius r . Satellite B, of equal mass, orbits in a circle of radius $2r$.
- (a) Which has the greater speed? Justify using Newton's second law and gravity.
- (b) Derive the ratio of their periods T_B/T_A and evaluate it.
- (c) Which has the greater total mechanical energy? Justify with an expression.
9. A solid cylinder ($M = 3.0 \text{ kg}$, $R = 0.20 \text{ m}$, $I = \frac{1}{2}MR^2$) rolls without slipping from rest down a 2.0 m long incline at 30 degrees to the horizontal.
- (a) Derive the acceleration in terms of g and the angle.
- (b) Calculate the speed at the bottom.
- (c) Calculate the minimum coefficient of static friction required.
- (d) The friction is not zero, yet mechanical energy is conserved. Explain.
10. A 0.50 kg ball of clay moving at 4.0 m/s strikes the rim of a stationary disk (mass 2.0 kg, radius 0.30 m, $I = \frac{1}{2}MR^2$) tangentially and sticks. The disk can rotate freely about a fixed vertical axle through its center.
- (a) Justify why angular momentum of the clay-disk system about the axle is conserved.
- (b) Calculate the angular velocity just after the collision.
- (c) Calculate the kinetic energy lost.
- (d) Explain why the linear momentum of the system is not conserved.

Answers**Unit 6: Answers and Explanations**

Read every explanation, including the ones for problems you answered correctly.

6 Energy and Momentum of Rotating Systems**PRACTICE · UNIT 6 PRACTICE**

- A. $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.40)(10)^2 = 20 \text{ J}$. 40 J forgets the one-half; 4.0 is $I\omega$ (the angular momentum), a different quantity.
- B. $\Delta\theta = 2.0(2\pi) = 12.6 \text{ rad}$, and $W = \tau\Delta\theta = 3.0(12.6) = 37.7 \text{ J}$. 6.0 J uses 2.0 revolutions as if it were radians.
- D. No external torque, so $L = I\omega$ is constant. I falls, so ω rises, and $K = L^2/2I$ rises because the person does work pulling the weights inward. Choice B fails because K is not conserved.
- C. $v_{\text{disk}} = \sqrt{4gh/3}$ and $v_{\text{hoop}} = \sqrt{gh}$, ratio $\sqrt{4/3} = 1.15$. Equal mass and radius does not give equal speed: the hoop's larger I takes more of the energy into rotation.
- A. $v = \sqrt{GM/r}$, so doubling r divides v by $\sqrt{2}$. Choosing $v/2$ forgets the square root.
- D. Angular impulse equals ΔL : $\tau t = I\omega$, so $\omega = (4.0)(3.0)/2.0 = 6.0 \text{ rad/s}$. 12 is ΔL (angular momentum), not ω ; 2.0 is α with no time.
- (a) 3 points. Set the ramp at a small angle; measure ramp length L along the slope and the vertical drop h with the meterstick. Release from rest and time the roll over distance L with the stopwatch (average several trials). For constant acceleration from rest the average speed is L/t , so final speed $v = 2L/t$. Repeat for at least five different release heights (different h).
- (b) 3 points. Energy conservation gives $Mgh = \frac{1}{2}Mv^2(1+c)$, so $v^2 = [2g/(1+c)]h$. Plot v^2 (m^2/s^2) on the vertical axis against h (m) on the horizontal axis, label axes with quantity and unit, and draw one best-fit line through the data with a straightedge. The slope $= 2g/(1+c)$, so $c = 2g/\text{slope} - 1$.
- (c) 1 point. For a solid cylinder $c = 1/2$, slope $= 4g/3 = 13 \text{ m/s}^2$ (using $g = 9.8$).
- (a) 3 points. B is slower. Gravity supplies the centripetal force: $GMm/r^2 = mv^2/r$, so $v = \sqrt{GM/r}$, decreasing with r . Specifically $v_B = v_A/\sqrt{2}$.
- (b) 3 points. $T = 2\pi r/v = 2\pi r\sqrt{r/GM}$, so $T \propto r^{3/2}$ and $T_B/T_A = 2^{3/2} = 2.8$.
- (c) 2 points. B. $E = -GMm/2r$; for $2r$ it equals half of A's value, i.e. less negative, so B has the greater total energy.
- (a) 3 points. Along the incline: $Mg\sin\theta - f = Ma$. About the center: $fR = I\alpha = \frac{1}{2}MR^2(a/R)$, so $f = \frac{1}{2}Ma$. Then $Mg\sin\theta = \frac{3}{2}Ma$ and $a = \frac{2}{3}g\sin\theta$.
- (b) 2 points. $a = \frac{2}{3}(9.8)(0.50) = 3.27 \text{ m/s}^2$; $v = \sqrt{2aL} = \sqrt{2(3.27)(2.0)} = 3.6 \text{ m/s}$.
- (c) 2 points. $f = \frac{1}{2}Ma = 4.9 \text{ N}$ and $N = Mg\cos\theta = 25.5 \text{ N}$, so $\mu_s \geq 0.19$.
- (d) 1 point. Static friction acts at the contact point, which is instantaneously at rest, so it does no work and no energy is converted to thermal energy.

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10. (a) 2 points. The external forces on the system are gravity, the normal force, and the axle force; each acts through (or parallel to) the axle and has zero torque about it, so net external torque is zero and L is constant.
- (b) 3 points. $mvR = (I + mR^2)\omega$. $I = \frac{1}{2}(2.0)(0.30)^2 = 0.090$; $mR^2 = 0.045$. $(0.50)(4.0)(0.30) = 0.60 = 0.135\omega$, so $\omega = 4.4$ rad/s.
- (c) 2 points. $K_i = \frac{1}{2}(0.50)(4.0)^2 = 4.0$ J; $K_f = \frac{1}{2}(0.135)(4.44)^2 = 1.33$ J; lost 2.7 J.
- (d) 1 point. The axle exerts an external force on the disk, so the system's linear momentum is not constant even though the torque about the axle is zero.

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Unit 7: Oscillations

Exam weighting: about 5-8% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

7 Oscillations

Simple harmonic motion, the periods of spring-mass systems and pendulums, how to read and build oscillation graphs, and how energy moves between kinetic and potential forms.

Why This Matters

Oscillations carry about 5 to 8 percent of the multiple-choice questions and show up in free response as a spring, a timed pendulum, or an energy-graph translation. Students lose points by using the spring-mass formula for a pendulum, by saying period depends on amplitude, and by drawing energy graphs that ignore where the object is at rest. This book uses $g = 10 \text{ m/s}^2$, the exam default.

Core Ideas

7.1 Defining Simple Harmonic Motion (SHM)

An object oscillates when a **restoring force** pulls or pushes it back toward an **equilibrium position**, the place where the net force is zero. The motion is **simple harmonic** only when that restoring force is proportional to the displacement and points opposite to it. A mass on a spring obeying Hooke's law is the model case; the largest displacement from equilibrium is the **amplitude** A .

Simple harmonic motion

Condition: net force along the motion is $F_x = -kx$, where x is displacement from equilibrium and k is a positive constant.

Newton's second law then gives $a_x = -\frac{k}{m}x$: acceleration is proportional to displacement and opposite to it.

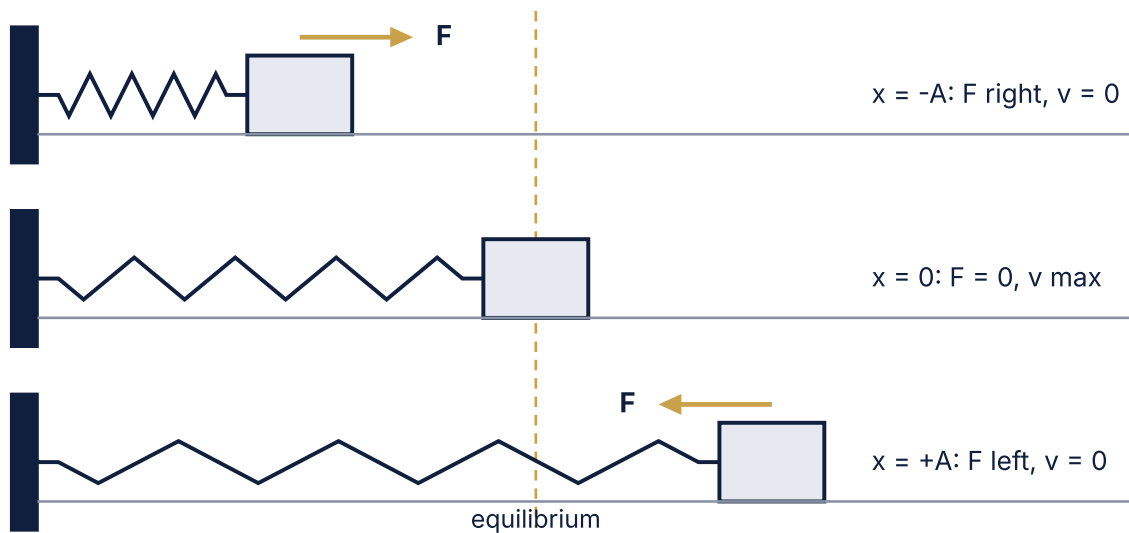
The minus sign is a direction statement: with x positive to the right, the force and acceleration point left when the object is right of equilibrium.

At the turning points ($x = \pm A$) the object is momentarily at rest while force and acceleration are largest. At equilibrium the net force is zero, yet the speed is greatest.

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Horizontal spring-mass oscillator on a frictionless surface. Positive x is to the right. The spring force always points toward equilibrium.

Position	Speed	Net force and acceleration	Energy picture
$x = +A$ or $-A$	0	maximum magnitude, toward equilibrium	all spring potential energy
$x = 0$	maximum	zero	all kinetic energy
between	between	proportional to x	shared

7.2 Frequency and Period of SHM

The **period** T is the time for one complete cycle (seconds). The **frequency** f is cycles per second, in hertz, and $f = 1/T$. Neither the spring-mass nor the pendulum period depends on amplitude.

Periods you must know

Spring-mass: $T_s = 2\pi\sqrt{\frac{m}{k}}$. Larger mass means a longer period; stiffer spring means a shorter period.

Simple pendulum (small angles, about 15 degrees or less): $T_p = 2\pi\sqrt{\frac{L}{g}}$. Depends on length and gravitational field strength only; the bob's mass cancels.

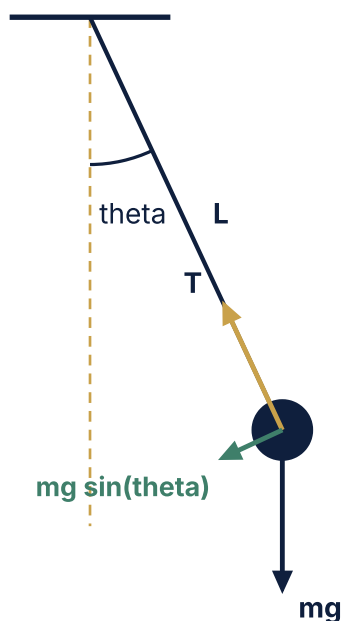
Never swap the formulas: a pendulum has no spring constant, and a spring-mass system does not depend on g .

For a pendulum the restoring force is $mg \sin \theta \approx mg x/L$ for small angles, so $k_{\text{eff}} = mg/L$ and the mass cancels. A vertical spring is still a spring-mass system: gravity only shifts equilibrium to a stretch mg/k . Use proportional reasoning: $T \propto \sqrt{m}$, $T \propto 1/\sqrt{k}$, $T \propto \sqrt{L}$, and $T \propto 1/\sqrt{g}$.

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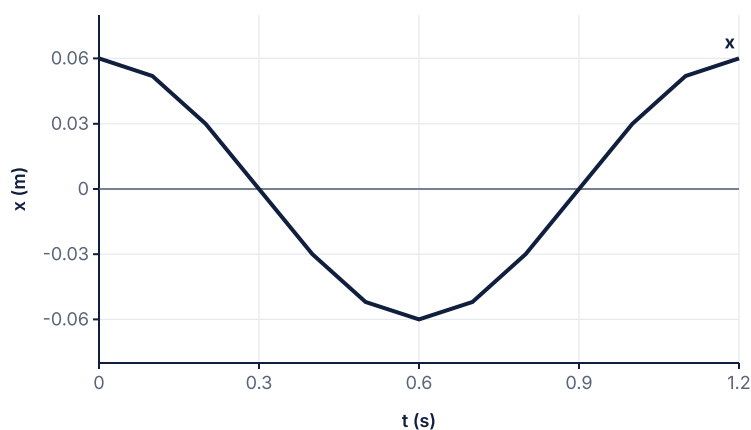
Along the arc, the net force is the component $mg \sin(\theta)$ toward equilibrium.

$T - mg \cos(\theta)$ supplies the centripetal force.

Pendulum bob displaced by angle θ . Only the tangential part of gravity acts as the restoring force.

7.3 Representing and Analyzing SHM

The position of an object released from rest at $x = +A$ is $x = A \cos(2\pi t/T)$; if it starts at equilibrium and moves in the positive direction the graph is a sine. The **angular frequency** $\omega = 2\pi/T = 2\pi f$ (rad/s) equals $\sqrt{k/m}$ for a spring. The slope of a position-time graph is velocity, so it is steepest at equilibrium crossings and zero at peaks; the slope of a velocity-time graph is acceleration, and the acceleration graph is a flipped copy of the position graph because $a = -\omega^2 x$.



Position of an oscillator with $A = 0.060$ m and $T = 1.2$ s, released from rest at $+A$.

Maximum values and speed

$$v_{\max} = A\omega = \frac{2\pi A}{T} \text{ at } x = 0, \text{ and } a_{\max} = \omega^2 A \text{ at } x = \pm A.$$

$$\text{At any position, } v = \omega \sqrt{A^2 - x^2}.$$

Doubling the amplitude doubles $v_{\max}$ and $a_{\max}$ but leaves T unchanged.

Experiments. Time at least 10 full oscillations and divide; timing a partial cycle loses points. To find k , plot T^2 against m (slope $4\pi^2/k$). To find g , plot T^2 against L (slope $4\pi^2/g$). Draw one straightedge best-fit line, label axes with quantity and unit, and take the slope from two points on the line.

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7.4 Energy of Simple Harmonic Oscillators

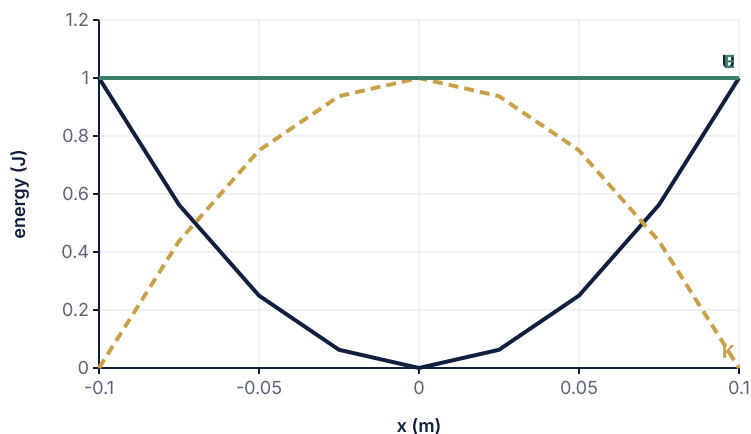
With no friction, mechanical energy is conserved. Spring energy is $U_s = \frac{1}{2}kx^2$, a parabola against x ; kinetic energy is the inverted parabola and the total is a horizontal line.

Energy relations

$$E = \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \text{ and } v_{\max} = A\sqrt{k/m}.$$

Pendulum: $E = mgh$ at the turning point (height $h = L(1 - \cos \theta)$ above the lowest point) and $v_{\max} = \sqrt{2gh}$.

Total energy scales with A^2 ; doubling the amplitude quadruples E .



$k = 200 \text{ N/m}$, $A = 0.10 \text{ m}$: spring energy U , kinetic energy K and total E against position.

For a hanging mass, include both spring and gravitational energy. Friction or drag removes energy and the amplitude decays, while the period of a lightly damped oscillator barely changes.

Worked Examples

EXAMPLE 1: SPRING-MASS PERIOD

A 0.50 kg block on a frictionless table is attached to a spring with $k = 200 \text{ N/m}$. Find the period and frequency. Then find the period if the block is replaced by a 2.0 kg block.

SOLUTION

$$T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.50/200} = 2\pi(0.050) = 0.314 \text{ s}.$$

$$f = 1/T = 3.18 \text{ Hz}.$$

With 2.0 kg the mass is 4 times larger, so T is $\sqrt{4} = 2$ times larger: $T = 0.628 \text{ s}$. The amplitude does not matter.

EXAMPLE 2: PENDULUM ON TWO WORLDS

A pendulum of length 0.80 m is released from a small angle. Find its period where $g = 10 \text{ m/s}^2$ and where $g = 1.6 \text{ m/s}^2$ (the Moon). What length gives a 2.0 s period on Earth?

SOLUTION

$$\text{Earth: } T = 2\pi\sqrt{0.80/10} = 1.78 \text{ s}.$$

$$\text{Moon: } T = 2\pi\sqrt{0.80/1.6} = 4.44 \text{ s}. \text{ The ratio is } \sqrt{10/1.6} = 2.5, \text{ so the pendulum is slower on the Moon.}$$

$$\text{Solve for length: } L = gT^2/(4\pi^2) = 10(4.0)/39.48 = 1.01 \text{ m}. \text{ The mass of the bob never enters.}$$

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EXAMPLE 3: READING A POSITION GRAPH

Use the table for an oscillator with $A = 0.060$ m and $T = 1.2$ s. Find f , ω , $v_{\max}$, $a_{\max}$, and the speed at $x = 0.030$ m.

t (s)	0	0.3	0.6	0.9	1.2
x (m)	0.060	0	-0.060	0	0.060

SOLUTION

$$f = 1/1.2 = 0.83 \text{ Hz and } \omega = 2\pi/1.2 = 5.24 \text{ rad/s.}$$

$$v_{\max} = A\omega = 0.060(5.24) = 0.314 \text{ m/s, reached at } t = 0.3 \text{ s and } 0.9 \text{ s when the graph crosses zero.}$$

$$a_{\max} = \omega^2 A = (5.24)^2(0.060) = 1.64 \text{ m/s}^2 \text{ at } t = 0 \text{ and } 0.6 \text{ s.}$$

$$v = \omega\sqrt{A^2 - x^2} = 5.24\sqrt{0.0036 - 0.0009} = 0.272 \text{ m/s.}$$

EXAMPLE 4: ENERGY METHOD

A 0.50 kg block on a spring ($k = 200$ N/m) is released from rest at $x = 0.10$ m on a frictionless surface. Find its speed at $x = 0.060$ m.

x (m)	U (J)	K (J)	E (J)
0.10	1.00	0	1.0
0.06	0.36	0.64	1.0
0	0	1.00	1.0

SOLUTION

$$\text{Total energy: } E = \frac{1}{2}kA^2 = \frac{1}{2}(200)(0.10)^2 = 1.0 \text{ J.}$$

$$\text{Spring energy at } 0.060 \text{ m: } \frac{1}{2}(200)(0.060)^2 = 0.36 \text{ J, so } K = 1.0 - 0.36 = 0.64 \text{ J.}$$

$$v = \sqrt{2K/m} = \sqrt{2(0.64)/0.50} = 1.6 \text{ m/s. Check with } v_{\max} = A\sqrt{k/m} = 0.10(20) = 2.0 \text{ m/s; } 1.6 \text{ is smaller, as it must be away from equilibrium.}$$

Write It Like a 5**Model Response**

Prompt: A block on a frictionless horizontal surface is attached to a spring and released from rest at displacement A . Justify that the motion is simple harmonic, and explain how the period changes if the block is released from $2A$ instead. Response: The spring exerts a force $F = -kx$ on the block, proportional to its displacement from equilibrium and directed toward equilibrium. Gravity and the normal force cancel, so this is the net force and $a = -(k/m)x$. The period $T = 2\pi\sqrt{m/k}$ contains only m and k , so releasing from $2A$ leaves it unchanged: the block travels twice as far but its maximum speed $A\sqrt{k/m}$ also doubles.

What the grader looks for

Named law ($F = -kx$); net force identified; the period formula with amplitude absent; a clear claim of no change; and a physical reason (larger speeds).

Coach's Playbook

- 1 Name the system: spring-mass or pendulum, then choose that formula.
- 2 Write the energy at the turning point first; it fixes the total.
- 3 Use factors of change: write the ratio T_2/T_1 and cancel everything that does not change.
- 4 For graphs, mark where speed is zero (peaks) and where acceleration is zero (crossings).

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- 5 In lab questions, say what is held constant and linearize with T^2 .

Common traps

- 1) Using $T = 2\pi\sqrt{m/k}$ for a pendulum, or saying a pendulum's period depends on the bob's mass. 2) Saying a larger amplitude gives a longer period. 3) Drawing kinetic energy as nonzero at the turning point, or total energy as sloped. 4) Timing only part of a cycle, or measuring from an arbitrary starting point, then calling the result the period.

Quick Check

Q: What is the period if frequency is 5 Hz? A: 0.20 s. Q: What is zero at the equilibrium position? A: Net force and acceleration (speed is not zero). Q: A pendulum's length is quadrupled; what happens to T ? A: It doubles.

Challenge Corner

A 0.40 kg block hangs from a spring with $k = 100$ N/m. It is held at the position where the spring is unstretched and released from rest. Find the equilibrium stretch, the amplitude, the maximum speed and the lowest point. Solution: equilibrium stretch $= mg/k = 0.40(10)/100 = 0.040$ m. The block starts 0.040 m above equilibrium, so $A = 0.040$ m. $v_{\max} = A\sqrt{k/m} = 0.040\sqrt{2500} = 0.632$ m/s. Energy check: $mgh = 0.16$ J minus spring energy 0.080 J leaves $K = 0.080$ J, so $v = 0.632$ m/s. The lowest point is 0.040 m below equilibrium, a stretch of 0.080 m.

Practice

PRACTICE Unit 7 Practice

Show your work. A calculator is allowed. Justify conclusions. Use $g = 10$ m/s².

- Which statement defines simple harmonic motion?
 - The speed of the object is constant as it moves back and forth.
 - The net force on the object is proportional to its displacement from equilibrium and points toward equilibrium.
 - The acceleration of the object is constant and points toward equilibrium.
 - The object repeats the same motion at regular time intervals.
- A block of mass m on a spring of constant k has period T . The block is replaced by one of mass $4m$ and the spring by one of constant $2k$. The new period is

(A) $T/2$	(B) T	(C) $2T$	(D) $\sqrt{2} T$
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- A simple pendulum is moved to a planet where the gravitational field strength is four times as large as on Earth. Its length is unchanged. The ratio of the new period to the old period is

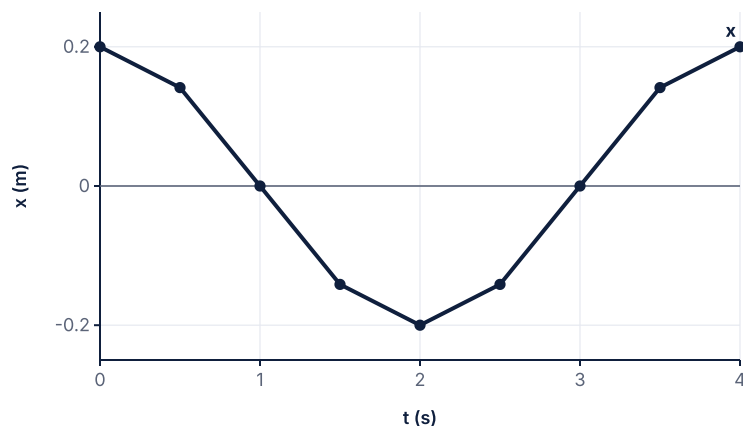
(A) $1/\sqrt{2}$	(B) $1/2$	(C) 2	(D) $1/4$
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4. An object oscillates with $x = 0.20 \cos(2\pi t/4.0)$ in meters, shown in the graph. At $t = 1.0$ s the object



- (A) has zero speed and maximum acceleration
(B) has maximum speed and maximum acceleration
(C) has maximum speed and zero net force
(D) has zero speed and zero net force
5. An oscillating block on a spring has total energy E . When its displacement is half the amplitude, its kinetic energy is
(A) $3E/4$ (B) $E/2$ (C) $E/4$ (D) $E/8$
6. Students measure the period T of a mass m on a spring for several masses. Which graph is a straight line whose slope can be used to determine the spring constant k ?
(A) T against m (B) T against $1/m$ (C) T^2 against $1/m$
(D) T^2 against m
7. A group times a pendulum. For each length L they time 10 full oscillations (t_{10}) with a stopwatch.
L (m): 0.25, 0.50, 0.75, 1.00, 1.25
 t_{10} (s): 10.1, 14.2, 17.5, 20.1, 22.6
(a) Describe how to time the oscillations to reduce error. (b) Compute T and T^2 for each length and state what to plot so that the slope gives g . (c) The best-fit slope is $4.08 \text{ s}^2/\text{m}$. Determine g . (d) Identify one reason the measurement of g could differ from 10 m/s^2 .
8. A 0.25 kg block hangs from a vertical spring with $k = 100 \text{ N/m}$ and is at rest in equilibrium.
(a) Find the spring stretch at equilibrium. (b) The block is pulled 0.040 m farther down and released. Find the period and the maximum speed. (c) Use energy to verify the maximum speed. Use $g = 10 \text{ m/s}^2$.
9. A student states: 'A heavier pendulum bob swings with a longer period because gravity pulls on it harder.'
(a) Use the derivation of the pendulum period to evaluate the claim. (b) State what happens to the period when the same pendulum is taken to the Moon and justify. (c) The pendulum amplitude is doubled from 5 degrees to 10 degrees. Predict the change in the period.
10. A pendulum has a 0.30 kg bob on a string of length 1.2 m. It is released from rest at an angle of 20 degrees from vertical. Use $g = 10 \text{ m/s}^2$.
(a) Calculate the height of the bob above its lowest point at release. (b) Calculate the bob's speed at the lowest point. (c) Calculate the period. (d) Calculate the tension in the string at the lowest point.

Answers Unit 7: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

7 Oscillations

PRACTICE · UNIT 7 PRACTICE

- B. SHM requires a restoring force proportional to displacement. Choice D describes constant acceleration, which is projectile-like; choice C is true of any periodic motion but not specific to SHM.
- D. $T \propto \sqrt{m/k}$, so $T' = T\sqrt{4/2} = \sqrt{2}T$. Choice C comes from changing only the mass or ignoring the square root.
- B. $T \propto 1/\sqrt{g}$, so four times g gives half the period. Choice A uses a factor of 2 instead of 4 inside the root.
- C. At $t = 1.0 \text{ s}$, $x = 0.20 \cos(\pi/2) = 0$, the equilibrium position, so speed is maximum and net force and acceleration are zero. Choice B confuses speed with acceleration.

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5. A. $U = \frac{1}{2}k(A/2)^2 = E/4$, so $K = 3E/4$. Choice B assumes energy splits evenly at half the amplitude, but energy depends on the square of the displacement.
6. D. $T^2 = (4\pi^2/k)m$ is linear in m with slope $4\pi^2/k$. Choice A is a curve, since $T \propto \sqrt{m}$; choice C is inverse, not linear.
7. (a) 2 points: time at least 10 full oscillations and divide by 10; start and stop at the same point in the cycle (preferably at equilibrium); use a small angle (under 15 degrees); repeat and average.
 (b) 3 points: $T = t_{10}/10$ gives 1.01, 1.42, 1.75, 2.01, 2.26 s and $T^2 = 1.02, 2.02, 3.06, 4.04, 5.11 \text{ s}^2$. Plot T^2 (vertical) against L (horizontal); $T^2 = (4\pi^2/g)L$ so slope $= 4\pi^2/g$.
 (c) 2 points: $g = 4\pi^2/\text{slope} = 39.48/4.08 = 9.7 \text{ m/s}^2$.
 (d) 1 point: for example reaction time in stopping the watch, air drag, string stretch, measuring L to the string's top rather than the bob's center, or using a large angle.
8. (a) 1 point: $\Delta y = mg/k = 0.25(10)/100 = 0.025 \text{ m}$.
 (b) 3 points: $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.0025} = 0.31 \text{ s}$ (independent of gravity and of the 0.040 m amplitude). $\omega = \sqrt{k/m} = 20 \text{ rad/s}$ and $v_{\text{max}} = A\omega = 0.040(20) = 0.80 \text{ m/s}$.
 (c) 3 points: start (lowest point): spring stretch $0.025 + 0.040 = 0.065 \text{ m}$. Spring energy difference to equilibrium: $\frac{1}{2}(100)(0.065^2 - 0.025^2) = 0.18 \text{ J}$. Gravity energy released: $mg(0.040) = 2.5(0.040) = 0.10 \text{ J}$. Then $\frac{1}{2}mv^2 = 0.18 - 0.10 = 0.08 \text{ J}$, so $v = \sqrt{2(0.08)/0.25} = 0.80 \text{ m/s}$, matching (b).
9. (a) 2 points: the restoring force is $mg \sin \theta \approx mg x/L$ and $F = ma$ gives $a = (g/L)x$ after m cancels, so $T = 2\pi\sqrt{L/g}$ does not contain the mass. The claim is incorrect: the heavier bob also has more inertia.
 (b) 2 points: on the Moon g is smaller, so $T \propto 1/\sqrt{g}$ increases (about 2.5 times larger for $g = 1.6 \text{ m/s}^2$ versus 10).
 (c) 1 point: no change; in the small-angle range the period is independent of amplitude (the maximum speed doubles, so the larger arc takes the same time).
10. (a) 2 points: $h = L(1 - \cos 20^\circ) = 1.2(1 - 0.940) = 0.072 \text{ m}$.
 (b) 2 points: energy conservation $mgh = \frac{1}{2}mv^2$ so $v = \sqrt{2gh} = \sqrt{2(10)(0.0724)} = 1.2 \text{ m/s}$.
 (c) 2 points: $T = 2\pi\sqrt{L/g} = 2\pi\sqrt{0.12} = 2.18 \text{ s}$ (20 degrees is slightly above the small-angle range, so this is approximate).
 (d) 2 points: at the bottom the net force is centripetal: $T_{\text{string}} - mg = mv^2/L$, so $T_{\text{string}} = 0.30(10) + 0.30(1.447)/1.2 = 3.0 + 0.36 = 3.4 \text{ N}$.

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Unit 8: Fluids

Exam weighting: about 10-15% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

8 Fluids

Density, pressure in static fluids, buoyancy as a Newton's second law problem, and the continuity and Bernoulli equations for moving fluids.

Why This Matters

Fluids carry 10 to 15 percent of the multiple-choice questions. The unit moved into Physics 1 only recently, so the free-response writers like it: a recent exam asked students to compare buoyant force and tension for the same object in two liquids. Points are lost by treating pressure as a force, by writing the density symbol and the pressure symbol interchangeably, by leaving weight or buoyancy off a free-body diagram, and by reaching for Bernoulli when the prompt asks for Newton's second law. This book uses $g = 10 \text{ m/s}^2$, water density 1000 kg/m^3 and atmospheric pressure $P_0 = 1.0 \times 10^5 \text{ Pa}$.

Core Ideas

8.1 Internal Structure and Density of Substances

A **fluid** is a substance that flows and takes the shape of its container: liquids and gases. Its particles are free to move past one another, which is why a fluid can exert force on any surface it touches. **Density** is the mass per unit volume, a property of the material, not of the sample: cutting a block in half halves its mass and its volume but leaves ρ unchanged. Liquids are treated as incompressible, so ρ stays constant; a gas is much less dense (air is about 1.2 kg/m^3).

Density

$\rho = \frac{m}{V}$ in kg/m^3 . Convert carefully: $1 \text{ g/cm}^3 = 1000 \text{ kg/m}^3$ and $1 \text{ L} = 10^{-3} \text{ m}^3$.

For a mixture or a system made of several parts, $\rho_{\text{avg}} = m_{\text{total}}/V_{\text{total}}$; add masses and volumes, never densities.

Material	Approx. density (kg/m^3)
Air	1.2
Oil (cooking)	800 to 920
Water	1000
Aluminum	2700
Iron	7900

8.2 Pressure

Pressure is force per unit area, $P = F/A$, where F is the force perpendicular to the surface. It is a scalar measured in pascals ($1 \text{ Pa} = 1 \text{ N/m}^2$), and a fluid at rest pushes perpendicular to any surface in contact with it. In a fluid at rest, pressure increases with depth because the layers above must be supported. At depth h below the surface of a uniform fluid, the pressure is the surface pressure plus the weight per unit area of the column above.

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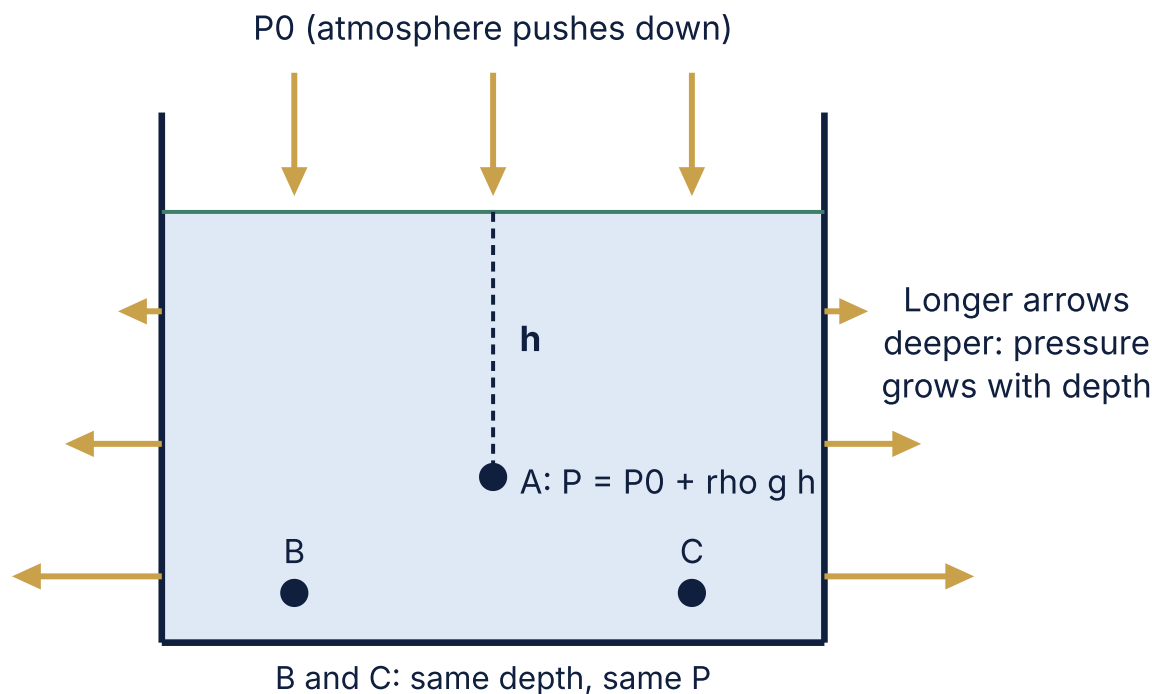
Pressure in a static fluid

Absolute pressure: $P = P_0 + \rho gh$, where h is the depth below the surface and P_0 is the pressure at the surface.

Gauge pressure is ρgh , the part above P_0 .

Two points at the same depth in the same connected fluid have the same pressure, whatever the shape of the container.

Force on a surface: $F = PA$. Pressure differences push, not pressure alone.



Open tank of water. Pressure at depth h is $P_0 + \rho gh$ and acts perpendicular to every surface; points at the same depth share one pressure.

Pascal's principle: a pressure change applied to an enclosed fluid is transmitted undiminished to every part of it. A hydraulic lift uses it: $P = F_1/A_1 = F_2/A_2$, so $F_2 = F_1(A_2/A_1)$. The large piston gets the larger force but moves a shorter distance, because the volume moved is equal ($A_1 d_1 = A_2 d_2$) and the work $F_1 d_1 = F_2 d_2$ is conserved.

8.3 Fluids and Newton's Laws

A fluid pushes up harder on the bottom of a submerged object than down on its top, since the bottom is deeper. The net upward force is the **buoyant force**. It equals the weight of the fluid that the object displaces.

Buoyant force

$F_b = \rho_{\text{fluid}} V_{\text{submerged}} g$, directed upward. The density is the fluid's, not the object's.

Fully submerged: $V_{\text{submerged}} = V_{\text{object}}$, and F_b does not depend on depth.

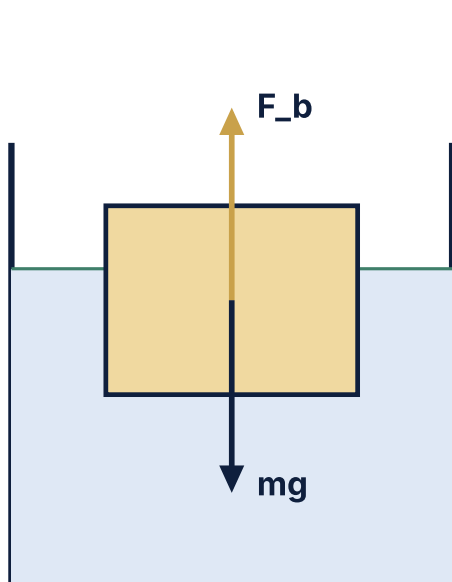
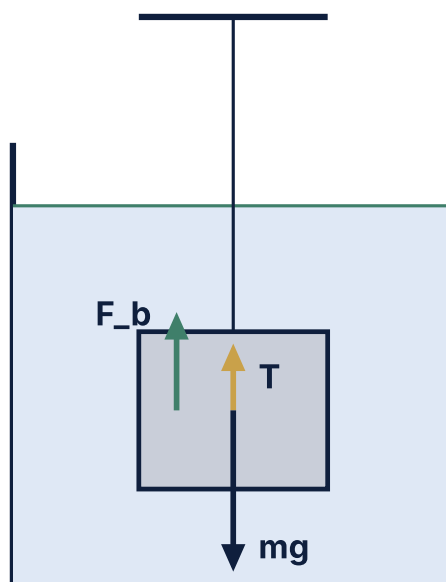
Floating at rest: $F_b = mg$, so $\frac{V_{\text{sub}}}{V} = \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}}$.

Always apply Newton's second law to the object: $\sum F = ma$ with F_b up and mg down.

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Floating: $F_b = mg$ Held by string: $T + F_b = mg$

Free-body forces on a floating block (left) and on a fully submerged block held by a string (right). Arrow lengths show the force balance.

Compare cases by the object's weight: the weight mg is the same in any fluid, but F_b changes with ρ_{fluid} . A denser fluid gives more buoyant force, so a floating object rides higher and a hanging object feels less tension. A released, fully submerged object accelerates upward if $\rho_{\text{fluid}} > \rho_{\text{object}}$ and downward otherwise, with $a = g(\rho_{\text{fluid}}/\rho_{\text{object}} - 1)$ taken upward. By Newton's third law the object pushes down on the fluid with F_{bf} , so a beaker on a scale reads higher when an object hangs in it.

8.4 Fluids and Conservation Laws

Conservation of mass for a steady, incompressible fluid gives the **continuity equation**: what flows into a section of pipe flows out. The volume flow rate $Q = Av$ is the same everywhere along the pipe, so a narrow section means a faster fluid. Conservation of energy per unit volume gives **Bernoulli's equation** for an ideal fluid (steady, incompressible, nonviscous) along a streamline.

Continuity and Bernoulli

$A_1v_1 = A_2v_2$ (flow rate $Q = Av$ in m^3/s).

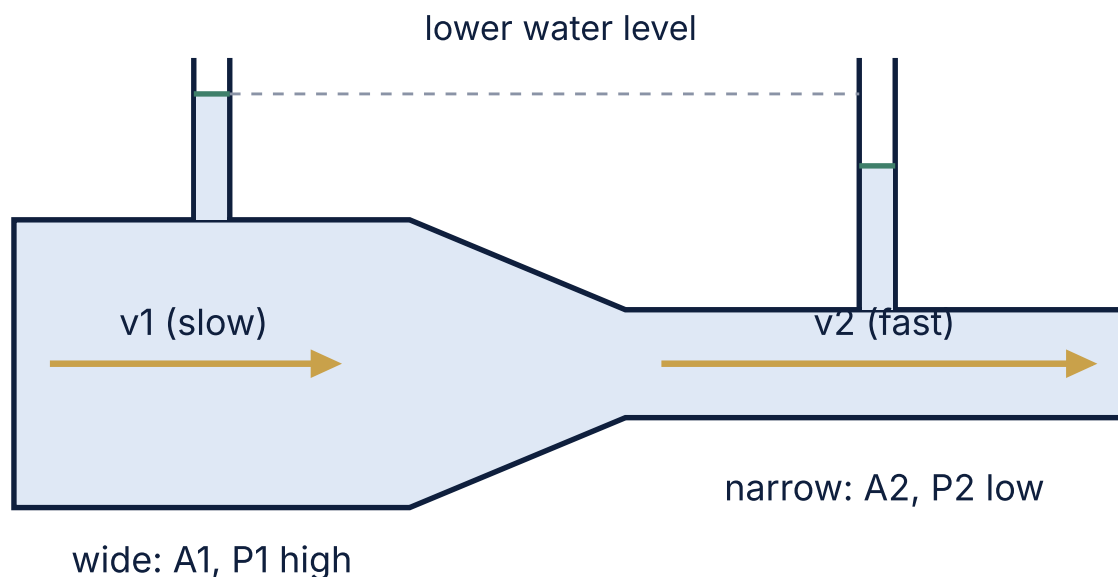
$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$, where y is height.

Special cases: with $v = 0$ it becomes $P = P_0 + \rho g h$; for a small hole at depth h in an open tank, $v = \sqrt{2gh}$ (Torricelli).

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Horizontal pipe that narrows. Continuity makes the fluid faster in the narrow section, and Bernoulli's equation says the pressure there is lower, shown by the lower water level in the narrow section's tube.

Why is the pressure lower where the fluid is faster? The fluid speeds up entering the narrow section, so the net force on it must point forward: the pressure behind it is greater than the pressure ahead. Faster flow comes with lower pressure only when height is unchanged. Climbing a pipe costs pressure through the ρgy term as well. Bernoulli's equation also explains lift (faster air over a curved wing gives a lower pressure above) qualitatively.

Worked Examples

EXAMPLE 1: DENSITY OF A MIXTURE

A tank holds 200 cm^3 of water ($\rho = 1000 \text{ kg/m}^3$) and 300 cm^3 of oil ($\rho = 800 \text{ kg/m}^3$). Assume the volumes add. Find the average density.

Liquid	V (cm ³)	rho (g/cm ³)	m (g)
Water	200	1.00	200
Oil	300	0.80	240
Total	500		440

SOLUTION

Water mass: $1.0 \text{ g/cm}^3 \times 200 = 200 \text{ g} = 0.200 \text{ kg}$. Oil mass: $0.80 \times 300 = 240 \text{ g} = 0.240 \text{ kg}$.

Total mass = 0.440 kg ; total volume = $500 \text{ cm}^3 = 5.0 \times 10^{-4} \text{ m}^3$.

$\rho_{\text{avg}} = 0.440 / (5.0 \times 10^{-4}) = 880 \text{ kg/m}^3$. The result lies between 800 and 1000, closer to the oil because there is more of it.

EXAMPLE 2: PRESSURE AND FORCE ON A HATCH

A submarine hatch of area 0.50 m^2 is 40 m below the surface of seawater ($\rho = 1030 \text{ kg/m}^3$). The cabin air is at $1.0 \times 10^5 \text{ Pa}$. Find the absolute pressure outside and the net inward force on the hatch.

SOLUTION

Gauge pressure: $\rho gh = 1030(10)(40) = 4.12 \times 10^5 \text{ Pa}$.

Absolute pressure outside: $P_0 + \rho gh = 1.0 \times 10^5 + 4.12 \times 10^5 = 5.12 \times 10^5 \text{ Pa}$.

The inside pushes back with $1.0 \times 10^5 \text{ Pa}$, so the net pressure difference is the gauge pressure: $F = (\rho gh)A = 4.12 \times 10^5(0.50) = 2.06 \times 10^5 \text{ N}$, directed inward.

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EXAMPLE 3: FLOATING AND BEING PUSHED UNDER

A wooden block of volume $2.0 \times 10^{-3} \text{ m}^3$ and density 600 kg/m^3 floats in water. Find the fraction submerged. Then find the downward force a hand must apply to hold it just fully under.

Case	F_b (N)	mg (N)
Floating	12	12
Held under	20	12 (plus 8 N hand)

SOLUTION

Floating: $V_{\text{sub}}/V = \rho_{\text{block}}/\rho_{\text{water}} = 0.60$, so $1.2 \times 10^{-3} \text{ m}^3$ is submerged.

Weight: $mg = 600(2.0 \times 10^{-3})(10) = 12 \text{ N}$. Fully submerged, $F_b = 1000(2.0 \times 10^{-3})(10) = 20 \text{ N}$.

Newton's second law with zero acceleration: $F_b = mg + F_{\text{hand}}$, so $F_{\text{hand}} = 20 - 12 = 8 \text{ N}$ downward.

EXAMPLE 4: CONTINUITY THEN BERNOULLI

Water flows through a horizontal pipe whose diameter narrows from 8.0 cm to 4.0 cm. In the wide section the speed is 2.0 m/s and the pressure is $2.0 \times 10^5 \text{ Pa}$. Find the speed and pressure in the narrow section and the flow rate.

Section	d (cm)	v (m/s)	P (Pa)
Wide	8.0	2.0	2.0e5
Narrow	4.0	8.0	1.7e5

SOLUTION

Area scales with diameter squared: $v_2 = v_1(d_1/d_2)^2 = 2.0(2)^2 = 8.0 \text{ m/s}$.

Horizontal, so $P_2 = P_1 - \frac{1}{2}\rho(v_2^2 - v_1^2) = 2.0 \times 10^5 - 500(64 - 4) = 1.7 \times 10^5 \text{ Pa}$.

$Q = A_1 v_1 = \pi(0.040)^2(2.0) = 1.0 \times 10^{-2} \text{ m}^3/\text{s}$.

Write It Like a 5**Model Response**

Prompt: A block hangs from a string, fully submerged first in water and then in a denser salt solution. Compare the tension in the string in the two cases. Response: The block is in equilibrium in both cases, so the net force is zero: $T + F_b = mg$. The weight mg is the same in both, because the block's mass does not change. The buoyant force is $F_b = \rho_{\text{fluid}} V g$, and the block is fully submerged in both, so V is the same; the salt solution has greater density, so F_b is larger there. Since $T = mg - F_b$, the tension is smaller in the salt solution.

What the grader looks for

Newton's second law (not Bernoulli) named as the starting principle; all three forces identified with directions; a statement that the weight is the same; $F_b = \rho V g$ with the fluid density and equal volume; a conclusion consistent with the chosen comparison.

Coach's Playbook

- 1 Underline the principle the prompt names (Newton's second law, conservation, Bernoulli) and start there.
- 2 Draw the free-body diagram for the object, not the fluid: weight down, buoyant force up, plus tension or contact forces.
- 3 Write rho for density and P for pressure; never swap them.
- 4 For pipes, apply continuity first to get speeds, then Bernoulli for pressure.

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- 5 Identify what is the same in both situations (weight, volume, flow rate) before comparing.

Common traps

1) Using the object's density in $F_b = \rho V g$ instead of the fluid's. 2) Drawing buoyant force and weight pointing the same way or omitting one of them. 3) Treating pressure as a force, or writing ρ and P interchangeably. 4) Saying pressure is higher where the fluid is faster, or skipping continuity and applying Bernoulli with unknown speeds.

Quick Check

Q: What is the gauge pressure 5.0 m under fresh water? A: $1000(10)(5.0) = 5.0 \times 10^4$ Pa. Q: A pipe's radius is halved; speed becomes? A: 4 times. Q: A block floats with 25% of its volume above the water. Its density? A: 750 kg/m^3 .

Challenge Corner

An open tank holds water 3.0 m deep. A hole of area 2.0 cm^2 is 1.8 m below the surface, 1.2 m above the ground. Find the exit speed, the flow rate, the time to fill a 60 L bucket, and the horizontal range of the stream. Solution: $v = \sqrt{2gh} = \sqrt{2(10)(1.8)} = 6.0 \text{ m/s}$. $Q = Av = 2.0 \times 10^{-4}(6.0) = 1.2 \times 10^{-3} \text{ m}^3/\text{s} = 1.2 \text{ L/s}$, so a 60 L bucket takes 50 s. The stream is a horizontal projectile: fall time $t = \sqrt{2(1.2)/10} = 0.49 \text{ s}$, so range = $6.0(0.49) = 2.9 \text{ m}$.

Practice

PRACTICE Unit 8 Practice

Show your work. A calculator is allowed. Justify conclusions. Use $g = 10 \text{ m/s}^2$.

- A solid brass block is cut into two equal halves. The density of each half compared with that of the original block is
 (A) half as great (B) twice as great
 (C) the same (D) dependent on the shape of each half
- Three open containers with different shapes are filled with the same liquid to the same height. The pressure at the bottom of each is
 (A) the same in all three
 (B) greatest in the container with the largest volume
 (C) greatest in the container with the widest base
 (D) least in the container that is widest at the top
- An aluminum block and a lead block have the same volume and are both fully submerged in water. The buoyant force is
 (A) greater on the lead block because it is heavier
 (B) greater on the aluminum block because it is less dense
 (C) zero on both because they are at rest
 (D) the same on both blocks
- Water flows steadily through a pipe whose radius decreases to half its original value. The speed of the water in the narrow part compared with the wide part is
 (A) twice as great (B) four times as great (C) the same
 (D) half as great
- A block floats in water with three-quarters of its volume below the surface. The density of the block is
 (A) 250 kg/m^3 (B) 1330 kg/m^3 (C) 750 kg/m^3
 (D) 1000 kg/m^3
- Water flows through a horizontal pipe that narrows. In the narrow section, compared with the wide section, the water has
 (A) greater speed and greater pressure (B) greater speed and lower pressure
 (C) lower speed and lower pressure (D) lower speed and greater pressure

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7. A student wants to determine the density of an irregular metal sample using a spring scale, a beaker of water ($\rho_w = 1000 \text{ kg/m}^3$) and string. Five samples of the same metal are weighed in air (W) and while fully submerged (W').
- W (N): 1.35, 2.70, 4.05, 5.40, 6.75
 W' (N): 0.87, 1.68, 2.58, 3.37, 4.26
- (a) Derive an expression for W in terms of the buoyant force $F_b = W - W'$ that has slope equal to a ratio of densities. (b) Describe what to plot and compute F_b for each sample. (c) The best-fit slope is 2.68. Determine the density of the metal. (d) Explain one way the setup could make the result inaccurate, and how it affects the answer.
8. A hydraulic lift has a small piston of area $4.0 \times 10^{-3} \text{ m}^2$ and a large piston of area 0.20 m^2 . It raises a car of mass 1200 kg at constant speed. Use $g = 10 \text{ m/s}^2$.
- (a) Calculate the force on the small piston. (b) Calculate the gauge pressure in the fluid. (c) The car is raised 0.10 m . Calculate how far the small piston moves and justify using conservation of energy.
9. A 3.0 kg block of volume $1.5 \times 10^{-3} \text{ m}^3$ hangs from a string while fully submerged in water (1000 kg/m^3), then in salt water (1200 kg/m^3). Use $g = 10 \text{ m/s}^2$.
- (a) Draw free-body diagrams for the block in each fluid. (b) Using Newton's second law, calculate the tension in each case. (c) Justify the comparison of the tensions in a short paragraph.
10. Water flows in a pipe of diameter 10 cm at ground level with speed 1.0 m/s and pressure $3.0 \times 10^5 \text{ Pa}$. The pipe rises 4.0 m and narrows to a diameter of 5.0 cm . Use $\rho = 1000 \text{ kg/m}^3$ and $g = 10 \text{ m/s}^2$.
- (a) Calculate the speed in the narrow section. (b) Calculate the pressure at the upper section. (c) Explain which change, speed or height, causes the larger pressure drop.

Answers Unit 8: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

8 Fluids

PRACTICE • UNIT 8 PRACTICE

- C. Density is a property of the material, so each half has the same mass-to-volume ratio. Choice A confuses density with mass.
- A. Pressure depends only on depth, density and g ($P = P_0 + \rho gh$), not on shape or volume. Choices B and C confuse pressure with total weight of liquid.
- D. $F_b = \rho_w V g$ uses the volume displaced, equal for both. The block's own density does not enter, so choice A (heavier means more buoyancy) fails.
- B. $A_1 v_1 = A_2 v_2$ and $A \propto r^2$, so halving r quarters A and quadruples v . Choice A uses r instead of r^2 .
- C. Floating: $\rho_{\text{block}}/\rho_w = V_{\text{sub}}/V = 0.75$, so 750 kg/m^3 . Choice A mistakes the fraction above the surface for the density ratio.
- B. Continuity makes the narrow section faster; Bernoulli with no height change makes the faster section lower pressure. Choice A says faster means more pressure, a common error.
- (a) 2 points: the block is in equilibrium: $F_b = W - W'$, and $F_b = \rho_w V g$ while $W = \rho_m V g$. Dividing, $W = (\rho_m/\rho_w)F_b$, so the slope of W against F_b is ρ_m/ρ_w .
 (b) 2 points: $F_b = 0.48, 1.02, 1.47, 2.03, 2.49 \text{ N}$. Plot W (vertical) against F_b (horizontal), labeled with units, and draw one straight best-fit line.
 (c) 2 points: $\rho_m = \text{slope} \times \rho_w = 2.68(1000) = 2.7 \times 10^3 \text{ kg/m}^3$.
 (d) 2 points: for example, air bubbles clinging to the sample or the sample touching the beaker would make W' read low or high; air bubbles add buoyancy, lowering W' and raising F_b , so the slope is too small and the density too low.
- (a) 1 point: constant speed means net force zero: $F_2 = mg = 1200(10) = 1.2 \times 10^4 \text{ N}$. $F_1 = F_2(A_1/A_2) = 1.2 \times 10^4(0.020) = 240 \text{ N}$.
 (b) 2 points: $P = F_1/A_1 = 240/(4.0 \times 10^{-3}) = 6.0 \times 10^4 \text{ Pa}$ (check: $F_2/A_2 = 1.2 \times 10^4/0.20 = 6.0 \times 10^4 \text{ Pa}$).
 (c) 3 points: the fluid volume moved is equal: $A_1 d_1 = A_2 d_2$, so $d_1 = 0.10(0.20)/(4.0 \times 10^{-3}) = 5.0 \text{ m}$. Check with work: $F_1 d_1 = 240(5.0) = 1.2 \times 10^3 \text{ J} = F_2 d_2 = 1.2 \times 10^4(0.10) \text{ J}$. The work input equals the work output for an ideal lift, with no energy created.
- (a) 2 points: both diagrams have $mg = 30 \text{ N}$ downward, F_b upward, and T upward.
 (b) 3 points: $T = mg - F_b$. Water: $F_b = 1000(1.5 \times 10^{-3})(10) = 15 \text{ N}$, $T = 30 - 15 = 15 \text{ N}$. Salt water: $F_b = 1200(1.5 \times 10^{-3})(10) = 18 \text{ N}$, $T = 30 - 18 = 12 \text{ N}$.
 (c) 2 points: the weight is the same in both, since the mass is unchanged; the block is fully submerged, so the displaced volume is the same, but the salt water has greater density, so F_b is greater. Since $T + F_b = mg$ for zero acceleration, a larger F_b means a smaller tension.
- (a) 2 points: continuity: $v_2 = v_1(d_1/d_2)^2 = 1.0(2)^2 = 4.0 \text{ m/s}$.
 (b) 3 points: $P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) - \rho g \Delta y = 3.0 \times 10^5 + 500(1 - 16) - 1000(10)(4.0) = 3.0 \times 10^5 - 7.5 \times 10^3 - 4.0 \times 10^4 = 2.5 \times 10^5 \text{ Pa}$ (2.525e5 unrounded).
 (c) 1 point: the height change gives a drop of $4.0 \times 10^4 \text{ Pa}$, much larger than the $7.5 \times 10^3 \text{ Pa}$ from the increase in speed.

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Mixed Practice Test

A mixed set of multiple-choice and free-response questions across all units, with answers, explanations, and scoring guidance.

T Mixed Practice Test

A cumulative test over all eight units, weighted like the exam, with scoring guides.

Instructions and timing (about 50 minutes): Part A is 12 multiple-choice questions (about 20 minutes) and Part B is three free-response questions (about 30 minutes). A calculator is allowed. Use $g = 10 \text{ m/s}^2$. Score with the guide that follows each answer. The weights below come from the exam's multiple-choice ranges.

Unit	Topic	Exam weight	Items in this test
1	Kinematics	10-15%	MC 1; FR 1(d)
2	Force and Translational Dynamics	18-23%	MC 2, 3
3	Work, Energy, and Power	18-23%	MC 4, 5; FR 1
4	Linear Momentum	10-15%	MC 6, 7
5	Torque and Rotational Dynamics	10-15%	MC 8; FR 2
6	Energy and Momentum of Rotating Systems	5-8%	MC 9; FR 2
7	Oscillations	5-8%	MC 10; FR 3
8	Fluids	10-15%	MC 11, 12; FR 3

PRACTICE TEST Mixed Practice Test

Part A: choose the best answer for items 1 to 12. Part B: items 13 to 15 are free response; show your work with units and justify claims.

- A ball is thrown horizontally at 12 m/s from the edge of a cliff 20 m above level ground. Ignore air resistance and use $g = 10 \text{ m/s}^2$. The ball lands a horizontal distance from the base of the cliff of
(A) 17 m (B) 24 m (C) 48 m (D) 240 m
- A 60 kg person stands on a scale in an elevator that is accelerating upward at 2.0 m/s^2 . Use $g = 10 \text{ m/s}^2$. The scale reads
(A) 480 N (B) 600 N (C) 720 N (D) 120 N
- A car travels at 15 m/s around a flat circular curve of radius 50 m. The minimum coefficient of static friction needed to keep it on the road is
(A) 0.23 (B) 0.30 (C) 0.90 (D) 0.45
- A frictionless roller-coaster cart starts from rest at a height of 20 m. Its speed when it passes a point 5.0 m above the ground is closest to
(A) 10 m/s (B) 17 m/s (C) 20 m/s (D) 30 m/s
- A motor lifts a 50 kg crate 6.0 m at constant speed in 10 s. Use $g = 10 \text{ m/s}^2$. The average power the motor delivers to the crate is
(A) 30 W (B) 500 W (C) 300 W (D) 3000 W
- A 0.20 kg ball strikes a wall at 10 m/s and rebounds along the same line at 6.0 m/s. The contact time is 0.010 s. The magnitude of the average force on the ball is
(A) 320 N (B) 80 N (C) 200 N (D) 32 N
- A 3.0 kg cart moving at 4.0 m/s collides with and sticks to a 1.0 kg cart at rest. The kinetic energy lost in the collision is
(A) 0 J (B) 6.0 J (C) 18 J (D) 24 J

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8. A uniform beam of length 3.0 m and mass 10 kg is attached to a wall by a hinge and held horizontal by a vertical rope at its far end. A 15 kg load hangs 2.0 m from the wall. Use $g = 10 \text{ m/s}^2$. The tension in the rope is
 (A) 125 N (B) 250 N (C) 100 N (D) 150 N
9. A skater spinning at 2.0 rad/s has rotational inertia 4.0 kg m^2 . She pulls in her arms and her rotational inertia becomes 1.6 kg m^2 . Her new angular speed is
 (A) 0.8 rad/s (B) 2.0 rad/s (C) 5.0 rad/s (D) 12.5 rad/s
10. Which single change would double the period of a mass oscillating on an ideal spring?
 (A) Quadrupling the mass (B) Doubling the mass
 (C) Doubling the amplitude (D) Quadrupling the spring constant
11. A diver is 30 m below the surface of a freshwater lake ($\rho = 1000 \text{ kg/m}^3$). The pressure at the surface is $1.0 \times 10^5 \text{ Pa}$. Use $g = 10 \text{ m/s}^2$. The absolute pressure at the diver is
 (A) $3.0 \times 10^5 \text{ Pa}$ (B) $1.3 \times 10^5 \text{ Pa}$ (C) $3.0 \times 10^6 \text{ Pa}$ (D) $4.0 \times 10^5 \text{ Pa}$
12. A block floats in water with 40 percent of its volume submerged. It is moved to a liquid of density 800 kg/m^3 and floats there. The fraction of its volume submerged in that liquid is
 (A) 0.20 (B) 0.32 (C) 0.50 (D) 0.40
13. [10 points] A 2.0 kg block slides from rest down a frictionless curved ramp from a height of 1.8 m onto a horizontal surface. The first 2.0 m of the surface are rough, with $\mu_k = 0.30$; beyond it the surface is frictionless and ends at a spring of constant $k = 300 \text{ N/m}$. Use $g = 10 \text{ m/s}^2$.
 (a) [2 points] Calculate the speed of the block at the bottom of the ramp.
 (b) [3 points] Calculate the energy dissipated on the rough section and the speed of the block when it leaves the rough section.
 (c) [2 points] Calculate the maximum compression of the spring.
 (d) [3 points] Calculate the time the block takes to cross the rough section.
14. [10 points] A horizontal turntable (a disk) with rotational inertia 0.50 kg m^2 about its axle spins at 4.0 rad/s. A 1.0 kg lump of clay is dropped onto the turntable 0.50 m from the axle and sticks there. Treat the clay as a point mass and ignore axle friction during the landing.
 (a) [3 points] Calculate the angular speed of the turntable immediately after the clay sticks.
 (b) [2 points] Calculate the mechanical energy lost and state where it goes.
 (c) [2 points] Explain why angular momentum is conserved here even though the linear momentum of the system is not.
 (d) [3 points] A constant frictional torque of 0.40 N m then acts on the axle. Calculate the time for the turntable to stop.
15. [10 points] A 0.40 kg block hangs at rest from a vertical spring of constant 80 N/m . Use $g = 10 \text{ m/s}^2$.
 (a) [2 points] Calculate the stretch of the spring at equilibrium.
 (b) [3 points] The block is pulled 0.030 m farther down and released. Calculate the period and the maximum speed of the resulting oscillation.
 (c) [3 points] The block (volume $2.0 \times 10^{-4} \text{ m}^3$) is now suspended at rest, fully submerged in water of density 1000 kg/m^3 . Calculate the new stretch of the spring.
 (d) [2 points] The original setup (in air) is moved to a planet where g is one-sixth as large. State and justify how the period and the equilibrium stretch change.

Key Mixed Practice Test: Answers, Explanations, and Scoring

T Mixed Practice Test

PRACTICE TEST · MIXED PRACTICE TEST

1. B. Unit 1. Fall time $t = \sqrt{2h/g} = 2.0 \text{ s}$, so range $= 12(2.0) = 24 \text{ m}$. Choice A uses a time of about 1.4 s (wrong formula); choice D multiplies by the height.
2. C. Unit 2. $N - mg = ma$ gives $N = 60(10 + 2.0) = 720 \text{ N}$. Choice A subtracts the acceleration, which is the downward-accelerating case.
3. D. Unit 2. Static friction supplies the centripetal force: $\mu mg = mv^2/r$, so $\mu = v^2/(rg) = 225/500 = 0.45$. Choice A forgets the factor in front, using half the answer.
4. B. Unit 3. $\frac{1}{2}mv^2 = mg(20 - 5)$ so $v = \sqrt{2(10)(15)} = 17 \text{ m/s}$. Choices A and C use the height 5 m or 20 m instead of the drop of 15 m.
5. C. Unit 3. At constant speed the work done is $mgh = 50(10)(6.0) = 3000 \text{ J}$, and $P = W/t = 300 \text{ W}$. Choice D forgets to divide by the time.
6. A. Unit 4. $\Delta p = 0.20(6.0 - (-10)) = 3.2 \text{ kg m/s}$ since the direction reverses, and $F = \Delta p/\Delta t = 320 \text{ N}$. Choice B uses the difference of speeds (4.0) and ignores the reversal.

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7. B. Unit 4. Momentum: $v = 12/4 = 3.0$ m/s. Initial KE is 24 J and final is $\frac{1}{2}(4.0)(9.0) = 18$ J, so 6.0 J is lost. Choice C is the final KE, and choice D is the initial KE.
8. D. Unit 5. Torque about the hinge: $T(3.0) = 100(1.5) + 150(2.0) = 450$ N m, so $T = 150$ N. Choice B is the total weight, which would be right only if the rope were at the hinge.
9. C. Unit 6. Angular momentum is conserved: $\omega_2 = I_1\omega_1/I_2 = 4.0(2.0)/1.6 = 5.0$ rad/s. Choice D squares the inertia ratio, as for the kinetic energy.
10. A. Unit 7. $T \propto \sqrt{m}$, so quadrupling the mass doubles the period. Choice B doubles the period only if T were proportional to mass, and the amplitude does not matter.
11. D. Unit 8. $P = P_0 + \rho gh = 1.0 \times 10^5 + 1000(10)(30) = 4.0 \times 10^5$ Pa. Choice A gives only the gauge pressure.
12. C. Unit 8. The block's density is $0.40(1000) = 400$ kg/m³. Floating in the oil, the fraction submerged is $400/800 = 0.50$. Choice D keeps the fraction from the water case, ignoring the denser fluid.
13. Scoring guide: (a) 2 points: $mgh = \frac{1}{2}mv^2$ gives $v = \sqrt{2gh} = 6.0$ m/s (1 for method, 1 for answer).
 (b) 3 points: friction force $\mu mg = 0.30(2.0)(10) = 6.0$ N, energy dissipated $= 6.0(2.0) = 12$ J (1 point); KE after $= 36 - 12 = 24$ J (1 point); $v = \sqrt{2(24)/2.0} = 4.9$ m/s (1 point).
 (c) 2 points: no friction beyond, so $24 = \frac{1}{2}kx^2$ (1 point), $x = \sqrt{48/300} = 0.40$ m (1 point).
 (d) 3 points: $a = -\mu g = -3.0$ m/s² (1 point); $v = v_0 + at$ (1 point); $t = (6.0 - 4.9)/3.0 = 0.37$ s (1 point); check: distance $= (36 - 24)/(2 \times 3.0) = 2.0$ m. Model response (b): The only non-conservative force is friction: $W_f = -\mu_k mgd = -12$ J. The block started with $\frac{1}{2}mv^2 = 36$ J, so it leaves the rough section with 24 J, and $v = 4.9$ m/s.
14. Scoring guide: (a) 3 points: L conserved: $I_i\omega_i = I_f\omega_f$ (1); $I_f = 0.50 + 1.0(0.50)^2 = 0.75$ kg m² (1); $\omega_f = 0.50(4.0)/0.75 = 2.7$ rad/s (1).
 (b) 2 points: $K_i = \frac{1}{2}(0.50)(16) = 4.0$ J, $K_f = \frac{1}{2}(0.75)(2.67)^2 = 2.67$ J; loss $= 1.3$ J (1); goes into thermal energy and deformation as the clay slips/sticks (1).
 (c) 2 points: the net external torque on the system about the axle is zero (the clay's weight and the axle force exert no torque about the axle) (1); the axle exerts a horizontal external force on the turntable while it spins, so the linear momentum is not conserved (1).
 (d) 3 points: $\alpha = \tau/I = 0.40/0.75 = 0.53$ rad/s² (1); $\omega = \omega_0 - \alpha t$ (1); $t = 2.67/0.533 = 5.0$ s (1). Model response (c): Angular momentum about the axle is conserved because no external torque acts about the axle during the landing. Linear momentum is not conserved because the axle exerts external forces on the system.
15. Scoring guide: (a) 2 points: $kx = mg$ (1); $x = 0.40(10)/80 = 0.050$ m (1).
 (b) 3 points: $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.40/80} = 0.44$ s (1 point, with the amplitude not mattering); amplitude is 0.030 m (1); $v_{\max} = A\sqrt{k/m} = 0.030(14.1) = 0.42$ m/s (1).
 (c) 3 points: $F_b = \rho Vg = 1000(2.0 \times 10^{-4})(10) = 2.0$ N (1); equilibrium: $kx + F_b = mg$ (1); $x = (4.0 - 2.0)/80 = 0.025$ m (1).
 (d) 2 points: the period is unchanged, since $T = 2\pi\sqrt{m/k}$ has no g (1); the stretch mg/k is one-sixth as large, about 0.0083 m (1). Model response (d): The period depends only on m and k , so it stays the same. The equilibrium stretch is mg/k , which scales with g , so it becomes one-sixth of 0.050 m.

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Free-Response Clinic

Exam-style free-response questions with point-by-point scoring guides and full-credit model responses.

X Free-Response Clinic

Four full free-response practice tasks in the exam's four styles, plus a short extra, with point-by-point scoring guides.

Five free-response tasks in the exam's four styles. The point values are Scholar Coach practice values. The College Board does not publish per-question point values on its course page, so use these only to practice scoring yourself, and check your teacher's current guidance. Give yourself about 95 minutes for the four main questions and 8 minutes for the short extra. Use $g = 10 \text{ m/s}^2$. Write derivations from a named principle, label every axis with a quantity and a unit, and state what stays the same.

Question	Style	Practice points	Unit focus
1	Mathematical Routines	10	Energy, momentum, friction
2	Translation Between Representations	12	Oscillations, energy
3	Experimental Design and Analysis	10	Oscillations, linearization
4	Qualitative/Quantitative Translation	8	Fluids, Newton's second law
Extra	Short answer	4	Fluids, Bernoulli

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CLINIC Free-Response Clinic

Answer each part in order; scoring guides at the end give practice point values.

1. Question 1: Mathematical Routines [10 points]

A block of mass m is released from rest at height h on a frictionless curved ramp. At the bottom it collides with a block of mass $3m$ at rest on a horizontal surface and sticks to it. The combined blocks then slide a distance d across a rough part of the surface with coefficient of kinetic friction μ before stopping. Use symbols m , h , μ and g .

(a) [2 points] Derive an expression for the speed v_1 of the first block just before the collision.

(b) [3 points] Derive an expression for the speed v_2 of the combined blocks just after the collision. State the principle you use and justify why it applies to the collision but energy conservation does not.

(c) [3 points] Derive an expression for d in terms of h and μ .

(d) [2 points] Calculate d for $h = 0.80$ m and $\mu = 0.50$. Use $g = 10$ m/s².

2. Question 2: Translation Between Representations [12 points]

A 0.50 kg block on a frictionless horizontal surface is attached to a spring of constant 200 N/m. The block is pulled 0.10 m from equilibrium and released from rest at $t = 0$. Take the pull direction as positive.

(a) [3 points] Sketch graphs of position, velocity and acceleration against time for one full period, labeling the period and the maximum value of each quantity.

(b) [3 points] Draw bar charts of kinetic energy, spring energy and total energy at $x = +0.10$ m, $x = +0.050$ m and $x = 0$, with numerical values.

(c) [3 points] Write an equation for $x(t)$ and use it to calculate the position and velocity at $t = 0.050$ s.

(d) [3 points] The amplitude is doubled. Sketch the spring-energy and total-energy curves against position for both amplitudes on one set of axes and state how the period changes.

3. Question 3: Experimental Design and Analysis [10 points]

A student has a spring, a stand, a set of masses (each known to 0.001 kg), a stopwatch and a meterstick. The student does not have a force sensor. The task is to determine the spring constant k .

Data: m (kg): 0.100, 0.200, 0.300, 0.400, 0.500

Time for 10 full oscillations t_{10} (s): 2.89, 3.98, 4.91, 5.67, 6.29

(a) [3 points] Describe a procedure that uses only the listed equipment, including how to reduce the timing error.

(b) [2 points] Identify the quantities to plot so that the slope gives k and write the equation relating the slope to k .

(c) [3 points] Complete a table of T and T^2 and, using the best-fit slope of 0.787 s²/kg, determine k .

(d) [2 points] The best-fit line has a small positive vertical intercept. Give a physical reason and describe how to present the graph so that the slope is valid.

4. Question 4: Qualitative/Quantitative Translation [8 points]

Two identical solid cubes of side 0.10 m and density 600 kg/m³ are released in two beakers. Cube X floats in water (1000 kg/m³) and cube Y floats in oil (800 kg/m³).

(a) [2 points] In a short paragraph compare the buoyant forces on the cubes. Begin from Newton's second law.

(b) [2 points] Compare the volumes submerged and justify.

(c) [2 points] Calculate the depth of cube X below the water surface.

(d) [2 points] A third identical cube is held fully submerged in a liquid of density 500 kg/m³ and released. Calculate its initial acceleration, including direction. Use $g = 10$ m/s².

5. Short extra [4 points]

Water ($\rho = 1000$ kg/m³) flows steadily through a horizontal pipe. Its cross-sectional area narrows from 4.0×10^{-3} m² to 1.0×10^{-3} m², and the speed in the wide section is 1.5 m/s.

(a) [2 points] Calculate the speed in the narrow section.

(b) [2 points] Calculate how much lower the pressure is in the narrow section.

Key Free-Response Clinic: Scoring Guides and Model Responses**X Free-Response Clinic**

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CLINIC · FREE-RESPONSE CLINIC

1. Scoring guide (practice values): (a) 2 points: energy conservation $mgh = \frac{1}{2}mv_1^2$ (1); $v_1 = \sqrt{2gh}$ (1).
 (b) 3 points: momentum is conserved because the external forces (gravity, normal) are vertical and cancel, and friction is negligible during the short collision (1); $mv_1 = (m + 3m)v_2$ (1); $v_2 = v_1/4 = \frac{1}{4}\sqrt{2gh}$ (1). Energy is not used because the collision is inelastic and mechanical energy is lost to thermal energy and deformation.
 (c) 3 points: work-energy or kinematics for the slide: $\frac{1}{2}(4m)v_2^2 = \mu(4m)gd$ (1); $d = v_2^2/(2\mu g)$ (1); $d = (2gh/16)/(2\mu g) = h/(16\mu)$ (1).
 (d) 2 points: $d = 0.80/(16 \times 0.50) = 0.10$ m (2; one point for substituting correctly). Check: $v_1 = 4.0$ m/s, $v_2 = 1.0$ m/s, $d = 1.0/(2 \times 0.5 \times 10) = 0.10$ m.
 Model response (b): the blocks form a system; the only horizontal external force is negligible friction during the collision, so p is conserved: $mv_1 = 4mv_2$ and $v_2 = \frac{1}{4}\sqrt{2gh}$. Energy is not conserved because the blocks stick together.
2. Scoring guide (practice values): (a) 3 points: $T = 2\pi\sqrt{m/k} = 0.31$ s; $\omega = 20$ rad/s. Position: cosine starting at +0.10 m, zero at $T/4$, minimum -0.10 m at $T/2$ (1). Velocity: negative sine, starting at zero, minimum -2.0 m/s at $T/4$ (1). Acceleration: $-40 \cos$ shape, starting at -40 m/s², peaking at $+40$ m/s² at $T/2$ (1). Axes labeled with units and period marked.
 (b) 3 points: total $E = \frac{1}{2}kA^2 = 1.0$ J at every position (1). At $x = 0.10$: $U = 1.0$ J, $K = 0$ (1). At $x = 0.050$: $U = 0.25$ J, $K = 0.75$ J; at $x = 0$: $U = 0$, $K = 1.0$ J (1).
 (c) 3 points: $x = 0.10 \cos(20t)$ in meters, with t in seconds and the angle in radians (1). At 0.050 s: $x = 0.10 \cos(1.0) = 0.054$ m (1). $v = -A\omega \sin(\omega t) = -2.0 \sin(1.0) = -1.7$ m/s, toward equilibrium (1).
 (d) 3 points: parabola $U = \frac{1}{2}kx^2$ is the same curve for both amplitudes (1); the total-energy line for $2A$ is at 4.0 J versus 1.0 J, and the parabola is extended to ± 0.20 m (1); the period is unchanged, 0.31 s, because $T = 2\pi\sqrt{m/k}$ has no amplitude (1).
 Model response (d): the amplitude does not change m or k , so the period stays 0.31 s. The energy is four times larger because $E \propto A^2$, but each cycle covers twice the distance at twice the maximum speed.
3. Scoring guide (practice values): (a) 3 points: hang a mass from the spring and displace it a small vertical distance (1); release and time 10 full oscillations with the stopwatch, starting at equilibrium, then divide by 10 (1); repeat for each mass and average repeated timings (1).
 (b) 2 points: plot T^2 (vertical) against m (horizontal) (1); $T^2 = (4\pi^2/k)m$ so slope $= 4\pi^2/k$ and $k = 4\pi^2/\text{slope}$ (1).
 (c) 3 points: $T = 0.289, 0.398, 0.491, 0.567, 0.629$ s and $T^2 = 0.0835, 0.158, 0.241, 0.321, 0.396$ s² (1 for table); $k = 4\pi^2/0.787$ (1); $k = 50$ N/m (1).
 (d) 2 points: the spring itself has mass and part of it moves, so $T^2 = 4\pi^2(m + m_s/3)/k$ and the line meets the axis above zero; the slope is unaffected (1). Draw one straight best-fit line with a straightedge, axes labeled with quantity and unit, and take the slope from two points on the line, not from the data (1).
 Model response (a): Hang a known mass, pull it down a small distance and release it. Start the stopwatch as the mass crosses equilibrium and stop at the 10th crossing in the same direction. Divide by 10 to get T . Repeat for each mass.
4. Scoring guide (practice values): (a) 2 points: each cube floats at rest, so by Newton's second law $F_b = mg$ (1); the cubes are identical, so the weights are equal and the buoyant forces are equal even though the fluids differ (1).
 (b) 2 points: $F_b = \rho_f V_{\text{sub}} g$ is the same, so the denser water gives the smaller submerged volume: cube Y (oil) sinks deeper (1); V_{sub} in water is 0.60 of the cube, in oil 0.75 (1).
 (c) 2 points: $0.60 = \rho_c/\rho_w$ so depth $= 0.60(0.10) = 0.060$ m (2; 1 for the ratio method). Cube Y: 0.075 m.
 (d) 2 points: $m = 600(1.0 \times 10^{-3}) = 0.60$ kg, $mg = 6.0$ N; $F_b = 500(1.0 \times 10^{-3})(10) = 5.0$ N (1); $a = (6.0 - 5.0)/0.60 = 1.7$ m/s² downward (1).
 Model response (a): Each cube is at rest, so Newton's second law gives zero net force: $F_b = mg$. Both cubes have the same mass, so the buoyant forces are equal, and the larger density of water is balanced by a smaller submerged volume.
5. Scoring guide (practice values): (a) 2 points: continuity $A_1 v_1 = A_2 v_2$ (1); $v_2 = 1.5(4.0) = 6.0$ m/s (1).
 (b) 2 points: horizontal, so $P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$ (1); $= 500(36 - 2.25) = 1.7 \times 10^4$ Pa lower (1).
 Model response: the water speeds up in the narrow section, so the pressure there is 1.7×10^4 Pa lower than in the wide section.

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Reference: Formulas, Skills Checklist, and Glossary

Quick-reference pages to review the night before the exam.

R1 Equations, Constants, and Sign Conventions

A quick reference for the relationships, constants and conventions used throughout AP Physics 1, by unit, with when to use each.

How to Use This Page

An equation sheet and a table of constants are provided on the exam, but the layout can change, so practice with your teacher's current copy. This page restates the relationships in our own words, grouped by unit, with the sign conventions that decide your answers. Always substitute with units and copy each equation exactly; dropping a square is a common lost point.

Constants and Conventions

Item	Value or rule
Gravitational field near Earth	$g = 10 \text{ m/s}^2$ (exam default); 9.8 is also accepted
Universal gravitation constant	$G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$
Atmospheric pressure	$P_0 \approx 1.0 \times 10^5 \text{ Pa}$ (1 atm is about $1.01 \times 10^5 \text{ Pa}$)
Density of water	1000 kg/m^3
Differences	Δ means final minus initial
Direction	In one dimension, the sign of a component carries direction; choose and state a positive direction
Defaults	inertial frame, negligible air resistance, positive work is work done on the system
Rotation	counterclockwise positive unless you say otherwise

Equations by Unit

Unit	Relationship	Use
1 Kinematics	$v = v_0 + at$; $\Delta x = v_0 t + \frac{1}{2}at^2$; $v^2 = v_0^2 + 2a\Delta x$	Constant acceleration in one dimension; apply separately to x and y components
2 Forces	$\sum \vec{F} = m\vec{a}$; $F_g = mg$; $f_k = \mu_k N$; $f_s \leq \mu_s N$; $F_s = -kx$; $a_c = v^2/r$; $F_g = Gm_1m_2/r^2$	Newton's second law, friction, springs, circular motion, gravitation
3 Energy	$K = \frac{1}{2}mv^2$; $W = F \Delta x \cos \theta$; $\Delta K = W_{\text{net}}$; $U_g = mg\Delta y$; $U_s = \frac{1}{2}kx^2$; $P = \Delta E / \Delta t = Fv \cos \theta$	Work-energy theorem, conservation, power
4 Momentum	$\vec{p} = m\vec{v}$; $\vec{J} = \vec{F}\Delta t = \Delta\vec{p}$; $\sum \vec{p}$ constant with no net external force	Collisions and impulse; elastic keeps K , perfectly inelastic sticks
5 Rotation	$\omega = \omega_0 + \alpha t$; $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$; $v = r\omega$; $\tau = rF \sin \theta$; $I = \sum mr^2$; $\sum \tau = I\alpha$	Angular kinematics, torque, rotational inertia, equilibrium ($\sum \tau = 0$)
6 Rotating systems	$K_{\text{rot}} = \frac{1}{2}I\omega^2$; $W = \tau\Delta\theta$; $L = I\omega$; $\Delta L = \tau\Delta t$; rolling without slipping $v_{\text{cm}} = r\omega$; circular orbit $v = \sqrt{GM/r}$	Angular momentum conservation, rolling energy, satellites

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Unit	Relationship	Use
7 Oscillations	$T = 1/f$; $T_s = 2\pi\sqrt{m/k}$; $T_p = 2\pi\sqrt{L/g}$; $E = \frac{1}{2}kA^2$; $v_{\max} = A\omega$; $\omega = 2\pi f$	Spring-mass and small-angle pendulum
8 Fluids	$\rho = m/V$; $P = F/A$; $P = P_0 + \rho gh$; $F_b = \rho Vg$; $A_1v_1 = A_2v_2$; $P + \frac{1}{2}\rho v^2 + \rho gy = \text{const}$	Density, static pressure, buoyancy, flow

Linearizing Experiments

Relationship	Plot (vertical vs horizontal)	Slope equals
$T_s = 2\pi\sqrt{m/k}$	T^2 vs m	$4\pi^2/k$
$T_p = 2\pi\sqrt{L/g}$	T^2 vs L	$4\pi^2/g$
$v^2 = v_0^2 + 2a\Delta x$	v^2 vs Δx (from rest)	$2a$
$F_s = kx$	F vs x	k
$W = \rho_m Vg$, $F_b = \rho_w Vg$	W vs F_b	ρ_m/ρ_w

Put the quantity you control on the horizontal axis, draw one straight best-fit line with a straightedge, and take the slope from two points on the line.

Sign and Setup Habits

- ◆ Pick one positive direction per axis and keep it for every quantity in the problem.
- ◆ Springs: the force is opposite the displacement from equilibrium; the sign of x is not the force direction.
- ◆ Energy: choose the system, then say what does work on it; friction is external to a block-only system.
- ◆ Torque about a chosen axis: a force through the axis gives zero torque.
- ◆ Fluids: use the fluid's density in $F_b = \rho Vg$; weight and buoyant force point in opposite directions.

R2 Skills Checklist and Glossary

Check your readiness on the ten exam skills and review the key terms of AP Physics 1.

Skills Checklist

Rate yourself 1 to 3 on each skill, then complete the prompt in writing. Revisit anything below 3 before the exam. The exam asks for the bolded task verbs (justify, derive, determine), so use them when you practice.

Skill	I can...	Self-check prompt
1.A Diagrams	I can draw free-body diagrams, schematics and tables.	Draw all forces on a hanging submerged block.
1.B Graphs	I can plot data with scales, units and a best-fit line.	Plot T^2 against L from data.
1.C Sketches	I can sketch qualitative graphs.	Sketch x , v and a against t for a spring.
2.A Derive	I can derive symbolic expressions.	Derive the pendulum period from forces.
2.B Calculate	I can calculate with units.	Find P at 30 m depth.
2.C Compare	I can compare across scenarios.	Compare tension in two fluids.
2.D Predict	I can predict factors of change.	Predict T if m is quadrupled.
3.A Design	I can design a procedure.	Find g with a pendulum and a stopwatch.
3.B Apply	I can apply a law to make a claim.	Use momentum for a collision.
3.C Justify	I can justify with evidence.	Justify with Newton's second law.

- ◆ Content check: I can state every unit's key relationships and when each applies.
- ◆ Habit check: I label graph axes with quantity and unit, use $g = 10 \text{ m/s}^2$ and write substitutions.
- ◆ Reflect: Which skill cost me the most points on practice tests? What one habit will I change this week?

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Glossary

Term	Meaning
Acceleration	Rate of change of velocity
Amplitude	Maximum displacement from equilibrium
Angular acceleration	Rate of change of angular velocity
Angular impulse	Torque multiplied by the time it acts
Angular momentum	$I\omega$; the rotational counterpart of momentum
Angular velocity	Rate of change of angular position
Buoyant force	Upward force on a submerged object equal to the weight of displaced fluid
Center of mass	Point where the system's mass acts as if concentrated
Centripetal acceleration	v^2/r toward the circle's center
Conservation of energy	Total energy of an isolated system stays constant
Continuity equation	$A_1v_1 = A_2v_2$ for incompressible flow
Density	Mass per unit volume
Displacement	Change in position, a vector
Elastic collision	Collision in which kinetic energy is conserved
Equilibrium	State of zero net force (and zero net torque)
Fluid	A substance that flows: liquid or gas
Free-body diagram	Diagram of all external forces on one object
Frequency	Cycles per second, in hertz
Friction, kinetic	Force opposing sliding, $\mu_k N$
Friction, static	Force opposing the start of sliding, up to $\mu_s N$
Gauge pressure	Pressure above atmospheric, ρgh
Gravitational field	Force per unit mass, $g = GM/r^2$
Hooke's law	Spring force proportional to displacement, $-kx$
Impulse	Force times time, equal to change in momentum
Inelastic collision	Collision in which kinetic energy is not conserved
Inertia	Resistance to a change in motion
Kinetic energy	Energy of motion, $\frac{1}{2}mv^2$
Lever arm	Perpendicular distance from the axis to the line of force
Momentum	Mass times velocity, a vector
Net force	Vector sum of all external forces
Normal force	Contact force perpendicular to a surface
Period	Time for one complete cycle
Potential energy	Energy of position or configuration
Power	Rate at which work is done or energy is transferred
Pressure	Force per unit area, F/A
Projectile	Object moving under gravity alone
Restoring force	Force directed back toward equilibrium

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Term	Meaning
Rolling without slipping	Motion with $v_{cm} = r\omega$ and no sliding at contact
Rotational inertia	Resistance to angular acceleration, $\sum mr^2$
Scalar	Quantity with magnitude only
Simple harmonic motion	Motion under a force $-kx$
Speed	Magnitude of velocity
System	The chosen object or objects being analyzed
Tension	Pulling force transmitted by a string
Spring potential energy	Energy stored in a stretched or compressed spring, $\frac{1}{2}kx^2$
Torque	$rF \sin \theta$; twisting effect of a force
Translational motion	Motion of the center of mass
Vector	Quantity with magnitude and direction
Velocity	Rate of change of position, a vector
Weight	Gravitational force on an object, mg
Work	Energy transferred by a force along a displacement
Work-energy theorem	Net work equals change in kinetic energy
Pascal's principle	Pressure change in an enclosed fluid is transmitted undiminished
Bernoulli's equation	Energy conservation per volume for an ideal fluid
Linearization	Plotting so that the data form a straight line
Best-fit line	One straight line that follows the trend of the data

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