



SCHOLAR COACH

Boutique Academic Coaching & Tutoring

AP PHYSICS 2: ALGEBRA-BASED · COURSE REVIEW AND PRACTICE

AP Physics 2: Algebra-Based **Mastery.**

Every unit on the exam: concise concept review, worked examples, exam-style writing, practice problems with full explanations, a mixed practice test, and a reference section.

INSIDE

Words in Context · Inferences · Command of Evidence · Central Ideas and Details · Function of a Sentence · Algebra · Advanced Math · Problem-Solving and Data Analysis · Geometry and Trigonometry · Three Full-Length Practice Exams with Explanations



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Course review, practice, and a mixed test.

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Start Here: The Exam, Your Plan, and Your Tools

How the AP Physics 2: Algebra-Based exam is built, how points are earned, and how to use this book.

F Start Here: The Exam, Your Plan, and Your Tools

How AP Physics 2 is structured and scored, which units carry the most weight, what each free-response type rewards, how to plan your weeks, and the math habits to build.

Why This Matters

Knowing the shape of the exam before you study saves weeks. This chapter explains how AP Physics 2: Algebra-Based is built, which units count most, what each free-response type rewards, how to pace your preparation, and the habits that keep algebra from costing you points.

The 2027 Exam

The exam is on **Thursday, May 6, 2027 (Session 1)**. It is a hybrid digital exam: you view and answer questions in the testing app, and handwrite free-response work in a paper booklet. A calculator is permitted throughout, and reference information (a table of constants and an equation sheet) is provided. Check the current reference sheet with your teacher because conventions are updated from time to time.

Section	Questions	Time	Weight
I: Multiple choice	42, single correct answer, some in sets with a shared stimulus	85 minutes	50%
II: Free response	4 questions	95 minutes	50%

Unit Weighting

The course framework numbers its units 9 to 15 and this book follows that numbering. Weights are the share of the multiple-choice section.

Unit	Title	Weight
9	Thermodynamics	15-18%
10	Electric Force, Field, and Potential	15-18%
11	Electric Circuits	15-18%
12	Magnetism and Electromagnetism	12-15%
13	Geometric Optics	12-15%
14	Waves, Sound, and Physical Optics	12-15%
15	Modern Physics	12-15%

Thermodynamics, electric force and potential, and circuits together make up roughly half of the multiple choice. Spend your first weeks there and keep returning to them.

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Skills the Exam Measures

- ◆ **Creating representations:** free-body-style diagrams, circuit and ray diagrams, labeled graphs with units and scales, and qualitative sketches of curves.
- ◆ **Mathematical routines:** derive a symbolic expression, calculate with units, compare scenarios, and predict factors of change from a functional dependence.
- ◆ **Scientific argumentation:** design an experimental procedure, apply a law to make a claim, and justify the claim with evidence.

The Four Free-Response Types

Type	What it asks	Where points are earned
Mathematical routines	Derive and solve with algebra	Reference equation first, substitutions shown, symbolic answer in the given variables, units
Translation between representations	Move between a diagram, graph, equation and words	Correct vectors or curve shapes, labeled axes, justification that points back to an earlier part
Experimental design and analysis	Plan a measurement and analyze data	Variables to measure, a minimal procedure, a way to reduce uncertainty, a linearized graph whose slope gives the target quantity
Qualitative/quantitative translation	Connect a claim in words to a calculation	Reasoning in sentences plus numbers; state what is held constant

On the 2025 exam these four questions appeared in the order above and were worth 10, 12, 10 and 8 points. Treat those values as a guide and expect similar proportions. Each type has a few predictable ways to lose points: drawing pressure as a force in a free-body diagram, forgetting to say 'n and T constant', plotting a graph whose slope cannot give the target, or adding parallel resistances directly.

A Sign Convention to Know Early

Work and the first law

This book uses the course convention: positive work is work done ON a system, so $\Delta U = Q + W$ and $W = -P\Delta V$ at constant pressure.

Many textbooks write $\Delta U = Q - W_{\text{by}}$. The physics is the same; only the symbol's meaning changes. Always state which convention you are using.

Study Plans

Week	4-week plan	8-week plan
1	Units 9 and 10	Unit 9
2	Unit 11 and Unit 12	Unit 10
3	Units 13 and 14	Unit 11
4	Unit 15, practice test, FR clinic	Unit 12 and a review of 9-11
5		Unit 13
6		Unit 14
7		Unit 15
8		Practice test, FR clinic, weak topics

In either plan, read a unit's Core Ideas first, then cover the solution and work each example before checking it, then do the practice items under time. Study about four to five days a week. Redo missed items two days later.

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How to Use This Book

- 1 Read Core Ideas with a pencil. Copy each boxed equation from memory at the end of the section.
- 2 Work every example before reading its solution, then compare your wording with the model.
- 3 Use 'Write It Like a 5' to learn what full-credit sentences sound like, then answer the Quick Check without notes.
- 4 Do the practice items in two passes: multiple choice timed at about two minutes each, then free response written out fully.

Math Habits That Earn Points

- ◆ Write the reference equation, then substitute with subscripts. Convert to kelvin and SI units first.
- ◆ For factors of change, name what stays constant and use ratios instead of recomputing.
- ◆ Carry units through every line; scientific notation and prefixes (micro = 10 to the minus 6) are common slips.
- ◆ Draw graphs with labeled axes, scales and a straight-edge best-fit line, and say what the slope or area means.
- ◆ Make a rough estimate afterward: is the speed, voltage or force the right order of magnitude?

Quick Check

Q1: What must be held constant in $P_1 V_1 / T_1 = P_2 V_2 / T_2$? A: The amount of gas, and temperatures must be in kelvin. Q2: Are parallel resistances added directly? A: No, add the reciprocals. Q3: Which free-response type asks you to plan a measurement? A: Experimental design and analysis.

Practice

PRACTICE Start Here Check

Show your work. A calculator is allowed.

1. A gas sample's temperature rises from 200 K to 800 K. The rms speed of its molecules changes by a factor of
(A) 4 (B) 2 (C) 1.4 (D) 16
2. A 6.0 V ideal battery is connected across a $5.0 \mu\text{F}$ capacitor. The charge on one plate is
(A) 30 microcoulombs (B) 60 microcoulombs (C) 0.83 microcoulombs
(D) 1.2 microcoulombs
3. A 12 V ideal battery is connected to a 4.0 ohm resistor and a 12 ohm resistor in parallel.
(a) Calculate the equivalent resistance and the battery current.
(b) Calculate the current in each resistor and the total power.
4. A gas in a cylinder is compressed to half its volume while it is held at constant temperature by a water bath.
(a) Sketch the process on a P-V diagram with labeled axes and an arrow.
(b) State the signs of W, Q and the change in internal energy, with the convention you use.

Answers and Explanations

F Start Here: The Exam, Your Plan, and Your Tools

PRACTICE · START HERE CHECK

1. B. $v_{\text{rms}} \propto \sqrt{T}$, and T is multiplied by 4, so the speed doubles. Choice A treats speed as proportional to T .
2. A. $Q = C\Delta V = (5.0)(6.0) = 30 \mu\text{C}$ on each plate. Choice B doubles the charge by counting both plates.
3. (a) $1/R = 1/4.0 + 1/12 = 1/3.0$, so $R_{\text{eq}} = 3.0 \Omega$ and $I = 12/3.0 = 4.0 \text{ A}$.
(b) Each resistor has 12 V: $I = 12/4.0 = 3.0 \text{ A}$ and $12/12 = 1.0 \text{ A}$ (sum 4.0 A). $P = \varepsilon I = 48 \text{ W}$ (36 W plus 12 W).
4. (a) A curve of the form P proportional to $1/V$ (a concave-up hyperbola), starting at a point (V, P) and ending at $(V/2, 2P)$, with an arrow pointing toward smaller volume. A straight line would lose credit.
(b) Using the convention that positive work is done on the gas: W is positive (compression), the change in internal energy is zero (constant T for an ideal gas), so $Q = -W$ is negative: energy leaves as heat to the bath.

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Unit 9: Thermodynamics

Exam weighting: about 15-18% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

9 Thermodynamics

Kinetic theory, the ideal gas law, heat transfer, the first law with P-V diagrams, specific heat, conduction, and entropy, with an explicit sign convention for work.

Why This Matters

Thermodynamics carries about 15-18% of the multiple-choice section, and a gas question is a free-response favorite. Students lose points by using Celsius in the gas law, mixing up the sign of work, and treating entropy as 'disorder' without saying whether the system is isolated.

Core Ideas

Sign convention used in this book

First law and work: our convention

$\Delta U = Q + W$, where Q is energy added to the gas as heat and W is the work done ON the gas.

At constant pressure $W = -P \Delta V$. Compression ($\Delta V < 0$) gives $W > 0$; expansion gives $W < 0$.

In general $|W|$ is the area under the P-V curve: positive for compression, negative for expansion.

Conventions vary: many textbooks write $\Delta U = Q - W_{\text{by}}$, with W_{by} the work done BY the gas. The physics is identical because $W_{\text{by}} = -W$. We follow the course convention (positive work is work done on the system); always check which one a source uses.

9.1 Kinetic Theory of Temperature and Pressure

An ideal gas is a collection of point-like molecules that move freely and collide elastically with each other and the walls. **Pressure** is the average force per area from those collisions: it rises when molecules hit harder (higher speed) or more often (more molecules per volume). **Temperature** measures the average translational kinetic energy per molecule.

Kinetic theory relations

$\bar{K} = \frac{1}{2} m \bar{v}^2 = \frac{3}{2} k_B T$ (T in kelvin; the same for every ideal gas at the same T).

$v_{\text{rms}} = \sqrt{\bar{v}^2} = \sqrt{3k_B T / m}$, so lighter molecules are faster at the same T.

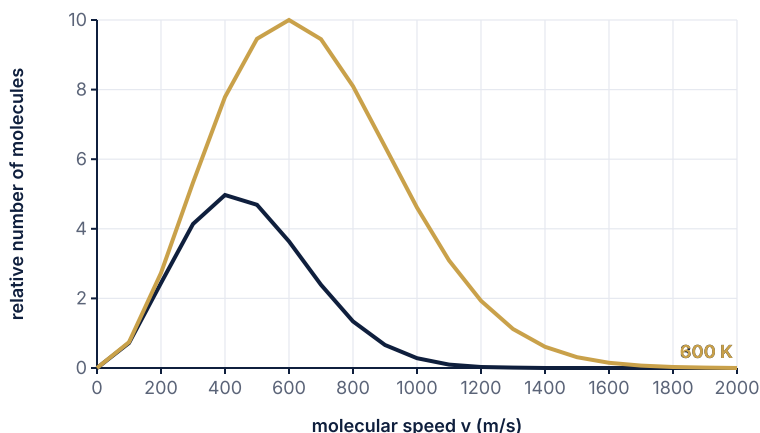
$k_B = 1.38 \times 10^{-23} \text{ J/K}$ and $R = N_A k_B = 8.31 \text{ J/(mol K)}$.

Speeds in a gas are not all equal. The Maxwell-Boltzmann distribution (qualitative only) is a lopsided hump with a tail of fast molecules. Raising the temperature moves the peak to higher speed and lowers and broadens the curve; the area (the number of molecules) is unchanged.

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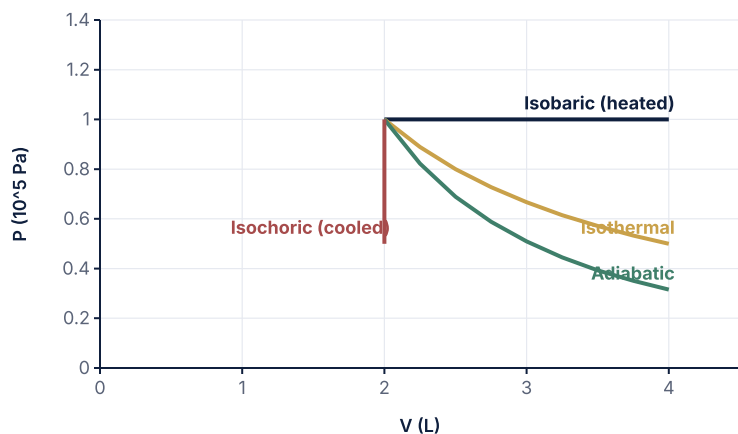
Qualitative speed distributions for the same gas. The hotter curve is lower, broader and peaks at a higher speed; the area (the number of molecules) is the same.

Common error: equal temperature means equal average kinetic energy, not equal speed. Helium and neon at 300 K have the same $\bar{K}$, but helium is faster by $\sqrt{20.2/4.00} \approx 2.2$.

9.2 The Ideal Gas Law

The ideal gas law: $PV = nRT = Nk_B T$. Use absolute pressure, volume in cubic meters and temperature in kelvin ($T_K = T_C + 273$). For fixed n , $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$; hold the other variables fixed and read off the proportionality. Doubling the Kelvin temperature at constant volume doubles the pressure; doubling the Celsius reading does not.

For a monatomic ideal gas, internal energy is the total translational kinetic energy: $U = \frac{3}{2}nRT = \frac{3}{2}PV$. It depends only on temperature, so any isothermal process has $\Delta U = 0$.



Four expansions or changes that all start at the same state ($V = 2$ L, $P = 1.0 \times 10^5$ Pa). On a P-V diagram $1 \text{ L} \times 10^5 \text{ Pa} = 100 \text{ J}$. The isotherm lies above the adiabat because the adiabat cools the gas.

9.3 Thermal Energy Transfer and Equilibrium

Thermal energy moves spontaneously from higher to lower temperature by **conduction** (collisions between particles in contact), **convection** (bulk motion of a fluid) and **radiation** (electromagnetic waves, no medium needed). Objects in thermal contact reach **thermal equilibrium** when their temperatures match, and the net transfer is then zero. In an isolated pair, the energy lost by the hot object equals the energy gained by the cold one. Temperature is not thermal energy: a bathtub of warm water holds far more thermal energy than a cup of boiling water.

9.4 The First Law of Thermodynamics

The first law is energy conservation: internal energy changes by the heat added plus the work done on the gas. Four special cases are tested.

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Process	What is constant	Work on gas W	Effect on first law
Isovolumetric	V	0 (no volume change)	$\Delta U = Q$
Isobaric	P	$-P \Delta V$	$\Delta U = Q - P \Delta V$
Isothermal	T (so U for ideal gas)	area under curve	$\Delta U = 0$, so $Q = -W$
Adiabatic	$Q = 0$ (insulated or very fast)	area under curve	$\Delta U = W$

On a P - V diagram the area under the path is the magnitude of the work; moving right (expansion) gives $W < 0$, moving left (compression) gives $W > 0$. For a closed cycle, the net work is the enclosed area (Example 3). A clockwise cycle has net $W < 0$ on the gas (the gas does net work, like a heat engine), and since $\Delta U = 0$ over a cycle, net $Q = -W > 0$.

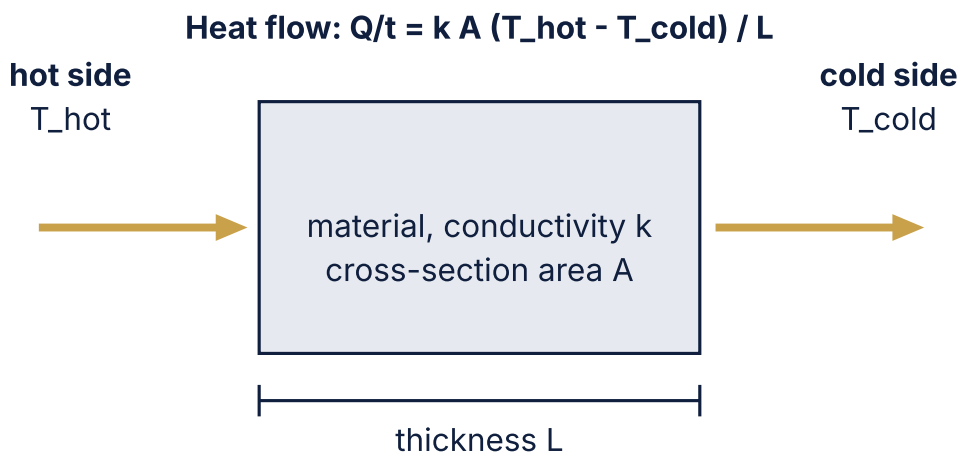
9.5 Specific Heat and Thermal Conductivity

Heat and conduction equations

$Q = mc\Delta T$ (specific heat c treated as constant); the sign of Q follows the sign of ΔT when no phase change occurs.

Calorimetry (isolated system): $\sum Q = 0$, so $m_1c_1(T_f - T_1) + m_2c_2(T_f - T_2) = 0$.

Conduction: $\frac{Q}{\Delta t} = \frac{kA\Delta T}{L}$, where ΔT is the temperature difference across the thickness L .



Conduction through a slab. Only the temperature difference across the thickness matters, and the rate grows with A and k and shrinks with L .

Use the temperature difference across the material, not either surface temperature. The rate doubles if A or k doubles and halves if L doubles. For a gas, the heat needed for a given ΔT depends on the process: at constant pressure some energy leaves as expansion work.

9.6 Entropy and the Second Law of Thermodynamics

Entropy is a state function that grows with the number of microscopic arrangements consistent with a macroscopic state. The second law: the entropy of an isolated system never decreases. Irreversible processes, such as heat flowing from hot to cold, a gas expanding into a vacuum, or friction, increase total entropy. The entropy of a closed system can decrease if energy leaves it (a freezer cooling food), but the surroundings then gain at least as much. Heat flowing spontaneously from cold to hot with no other change is forbidden. The second law is tested qualitatively; no entropy formula is needed.

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Worked Examples

EXAMPLE 1: SPEED AND ENERGY OF MOLECULES

Nitrogen molecules ($m = 4.65 \times 10^{-26}$ kg) are at 300 K. (a) Find the average translational kinetic energy per molecule. (b) Find v_{rms} . (c) By what factor does v_{rms} change if the temperature rises to 1200 K?

SOLUTION

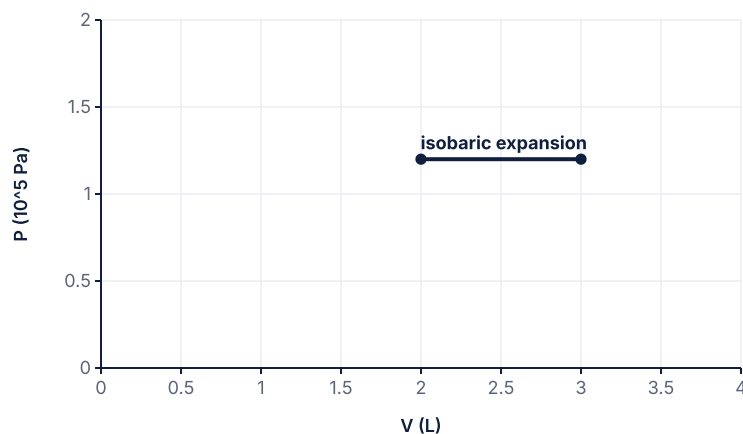
$$(a) \bar{K} = \frac{3}{2}k_B T = 1.5(1.38 \times 10^{-23})(300) = 6.21 \times 10^{-21} \text{ J.}$$

$$(b) v_{\text{rms}} = \sqrt{2\bar{K}/m} = \sqrt{2(6.21 \times 10^{-21})/(4.65 \times 10^{-26})} = 517 \text{ m/s.}$$

$$(c) v_{\text{rms}} \propto \sqrt{T}, \text{ so quadrupling } T \text{ doubles the speed, giving about } 1030 \text{ m/s.}$$

EXAMPLE 2: HEATING A GAS AT CONSTANT PRESSURE

A monatomic ideal gas at 1.2×10^5 Pa and $V = 2.0 \times 10^{-3} \text{ m}^3$ is at 300 K. It is heated at constant pressure until the volume is $3.0 \times 10^{-3} \text{ m}^3$. Find the final temperature, the work done on the gas, the change in internal energy, and the heat added.



The area under the line, $1.2 \times 1.0 = 1.2$ ($\times 100$ J), is 120 J of work.

SOLUTION

At constant P and n, V/T is constant: $T_2 = 300(3.0/2.0) = 450$ K.

$$W = -P\Delta V = -(1.2 \times 10^5)(1.0 \times 10^{-3}) = -120 \text{ J (expansion, so the gas does 120 J of work).}$$

$$\Delta U = \frac{3}{2}\Delta(PV) = 1.5(1.2 \times 10^5)(1.0 \times 10^{-3}) = +180 \text{ J.}$$

$$Q = \Delta U - W = 180 + 120 = +300 \text{ J: } 180 \text{ J raises the internal energy and } 120 \text{ J leaves as expansion work.}$$

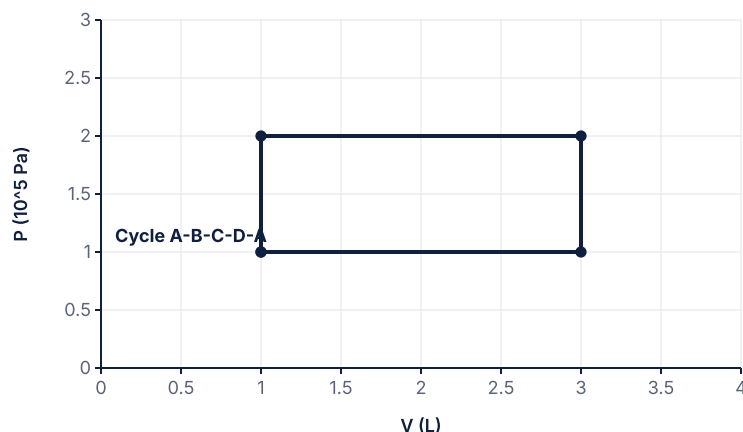
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EXAMPLE 3: A CYCLE ON A P-V DIAGRAM

The cycle in the graph has A = (1 L, 1.0), B = (1 L, 2.0), C = (3 L, 2.0), D = (3 L, 1.0), with pressure in 10^5 Pa. (a) Find the work done on the gas in each stage and the net work. (b) Find the net heat. (c) Find T_C/T_A .



Corners: A (1 L, 1.0), B (1 L, 2.0), C (3 L, 2.0), D (3 L, 1.0), with pressure in units of 10^5 Pa. The cycle runs A to B to C to D to A, clockwise.

SOLUTION

Use $1 \text{ L} \times 10^5 \text{ Pa} = 100 \text{ J}$. A to B and C to D are isovolumetric: $W = 0$.

B to C: $W = -(2.0 \times 10^5)(2.0 \times 10^{-3}) = -400 \text{ J}$ (expansion).

D to A: $W = +(1.0 \times 10^5)(2.0 \times 10^{-3}) = +200 \text{ J}$ (compression). Net $W = -200 \text{ J}$, the enclosed area.

Over a cycle $\Delta U = 0$, so net $Q = +200 \text{ J}$.

$T \propto PV$, so $T_C/T_A = (2.0 \times 3)/(1.0 \times 1) = 6$.

EXAMPLE 4: CALORIMETRY

A 0.150 kg aluminum block ($c = 900 \text{ J/(kg K)}$) at 95 deg C is dropped into 0.400 kg of water ($c = 4186 \text{ J/(kg K)}$) at 20 deg C in an insulated cup. Find the final temperature and the energy transferred. Neglect the cup.

Object	m (kg)	c (J/kg K)	mc (J/K)	T _i (deg C)
Aluminum	0.150	900	135	95
Water	0.400	4186	1674	20

SOLUTION

Isolated system: $m_A c_A (T_f - 95) + m_w c_w (T_f - 20) = 0$.

$m_A c_A = 135 \text{ J/K}$ and $m_w c_w = 1674 \text{ J/K}$, so $T_f = \frac{135(95) + 1674(20)}{135 + 1674} = 25.6 \text{ deg C}$.

Energy transferred = $135(95 - 25.6) = 9.4 \times 10^3 \text{ J}$. The final temperature is near the water's because water has the much larger mc .

Write It Like a 5**Model Response**

Prompt: A gas in a piston-cylinder is compressed slowly while in contact with a large water bath at constant temperature. (a) Explain how the pressure changes. (b) State the signs of W , Q and ΔU and justify each. Response: (a) The temperature is constant, so the molecules' average kinetic energy and speed do not change. In a smaller volume they strike the walls more often, so the average force per area increases; equivalently $PV = nRT$ with n and T constant gives $P \propto 1/V$, so pressure rises. (b) The piston pushes inward, so work is done on the gas: $W > 0$. The internal energy of an ideal gas depends only on T , so $\Delta U = 0$. From $\Delta U = Q + W$, $Q = -W < 0$: energy leaves the gas as heat to the water bath.

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What the grader looks for

Constant T linked to unchanged molecular kinetic energy; collision-frequency argument or $PV = nRT$ with n and T constant; $W > 0$ for compression with a reason; $\Delta U = 0$ because U depends only on T ; $Q < 0$ from the first law with the direction of heat flow; consistent sign convention.

Coach's Playbook

- 1 Convert to kelvin and SI units before touching the gas law.
- 2 Name the process, write the reference equation, then substitute with subscripts.
- 3 On a P - V sketch draw the process arrow; an inverse relationship is a curve, never a straight line.
- 4 For the first law, list W , Q and U with signs and check the sum. State what is held constant.

Common traps

1. Using Celsius in $PV = nRT$ or in a ratio. 2. Writing $\Delta U = Q + W$ with $W = +P\Delta V$ for an expansion; here an expansion has negative work on the gas. 3. Using a surface temperature instead of the difference ΔT across the thickness in conduction. 4. Claiming a refrigerator or living cell violates the second law because its entropy decreases; it is not isolated, and total entropy still rises.

Quick Check

Q1: A gas is compressed adiabatically. Temperature? A: It rises, since $Q = 0$ and $W > 0$ give $\Delta U > 0$. Q2: At constant V , gas at 27 deg C has its absolute pressure doubled. New temperature? A: 600 K = 327 deg C. Q3: What does the area of a clockwise P - V loop represent? A: Net work done by the gas.

Challenge Corner

One mole of monatomic gas at 300 K is heated at constant volume until the pressure triples, then expanded isothermally back to the original pressure. (a) Find the heat added in the first step. (b) Is heat added or removed in the second step? Solution: (a) At constant V , $T \propto P$, so $T_2 = 900$ K and $Q = \Delta U = 1.5(1)(8.31)(600) = 7.5 \times 10^3$ J. (b) Isothermal: $\Delta U = 0$ and the gas does work ($W < 0$), so $Q = -W > 0$: heat is added.

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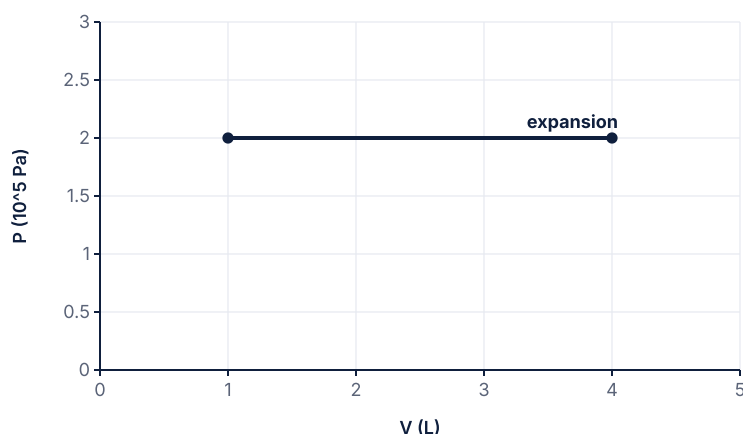
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Practice

PRACTICE Unit 9 Practice

Show your work. A calculator is allowed. Justify conclusions. Work done ON the gas is positive.

- A sealed rigid tank of gas is at 27 deg C and pressure P . It is heated to 327 deg C. What is the new pressure, assuming ideal gas behavior?
 (A) 12P (B) 1.1P (C) 2P (D) 0.5P
- A sample of helium (mass m) and a sample of a heavier gas (mass $4m$) are both at 400 K. Which statement is correct?
 (A) The helium molecules have the greater rms speed, and the two have equal average kinetic energy.
 (B) The heavy molecules have the greater average kinetic energy.
 (C) The two gases have equal rms speed because temperature is equal.
 (D) The helium molecules have the greater average kinetic energy.
- A gas expands at a constant pressure of 2.0×10^5 Pa from 1.0 L to 4.0 L (graph) while absorbing 900 J of heat. What is the change in its internal energy?



Constant-pressure expansion.

- (A) +1500 J (B) -300 J (C) +900 J (D) +300 J
- Slab A conducts heat at rate H for a given temperature difference. Slab B is made of the same material but has three times the cross-sectional area and twice the thickness. For the same temperature difference, the rate through slab B is
 (A) $6H$ (B) $1.5H$ (C) $3H$ (D) $0.67H$
- A refrigerator cools the food inside it, so the entropy of the food decreases. Which statement is correct?
 (A) The surroundings gain at least as much entropy as the food loses, so the total entropy does not decrease.
 (B) The process violates the second law and cannot occur.
 (C) The entropy of the food cannot decrease unless the refrigerator is isolated.
 (D) Entropy is conserved, so the surroundings lose the same amount.
- A gas in an insulated cylinder is compressed rapidly by a piston. For the gas, which row is correct?
 (A) $Q = 0$, $W < 0$, temperature falls (B) $Q > 0$, $W > 0$, temperature constant
 (C) $Q = 0$, $W > 0$, temperature rises (D) $Q < 0$, $W = 0$, temperature falls
- A cylinder holds 0.30 mol of a monatomic ideal gas. State 1 is $P_1 = 3.0 \times 10^5$ Pa and $V_1 = 2.0 \times 10^{-3}$ m³. The gas is cooled at constant volume until $P_2 = 1.0 \times 10^5$ Pa (state 2), then compressed at constant pressure to $V_3 = 1.0 \times 10^{-3}$ m³ (state 3).
 (a) Calculate the temperature in state 1.
 (b) Calculate the heat added to the gas in the first step and state whether energy enters or leaves.
 (c) Calculate the work done on the gas in the second step and explain its sign.
 (d) Calculate the net heat exchanged over both steps.

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8. A student studies how the pressure of a fixed amount of air in a rigid sealed flask depends on temperature. The flask has volume $1.0 \times 10^{-3} \text{ m}^3$ and sits in a water bath whose temperature can be changed. A pressure sensor is attached.
 - (a) Describe a procedure, including how you reduce uncertainty, and list the measurements.
 - (b) The data are: 20 deg C, 97.7 kPa; 40 deg C, 103.7 kPa; 60 deg C, 110.9 kPa; 80 deg C, 117.5 kPa; 100 deg C, 123.7 kPa. State what to plot to get a straight line and what the slope represents.
 - (c) The best-fit slope is 329 Pa/K. Calculate the amount of gas in moles.
 - (d) Describe what the line predicts for the pressure at 0 K and how it relates to the idea of absolute zero.
9. Two identical-material rods connect a hot reservoir at 100 deg C to a cold reservoir at 0 deg C. Rod A has $k = 400 \text{ W/(m K)}$, cross-sectional area $1.0 \times 10^{-4} \text{ m}^2$ and length 0.50 m. Rod B has twice the length and twice the area of rod A.
 - (a) Calculate the rate of heat flow through rod A and the energy transferred in 600 s.
 - (b) Without recalculating from scratch, predict the rate through rod B and justify.
 - (c) In terms of entropy, explain why the heat flows from hot to cold and not the reverse.
10. A student claims: 'When the volume of a gas is halved at constant temperature, the pressure doubles because the molecules speed up.'
 - (a) Identify the error and give the correct kinetic-theory explanation.
 - (b) Sketch qualitative speed distributions for the same gas at temperatures $T_1 < T_2$ on one set of axes, and state two ways they differ.
 - (c) A mixture contains helium (4.00 u) and neon (20.2 u) at equal temperature. Calculate the ratio $v_{\text{rms,He}}/v_{\text{rms,Ne}}$ and compare their average kinetic energies.

Answers Unit 9: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

9 Thermodynamics

PRACTICE · UNIT 9 PRACTICE

1. C. Convert to kelvin: 300 K to 600 K, so at constant volume the pressure doubles. Choice A comes from the ratio 327/27 using Celsius, which is invalid.
2. A. At equal T the average translational kinetic energy is the same for all ideal gases, $\frac{3}{2}k_B T$, so the lighter helium must have the larger speed since $v_{\text{rms}} = \sqrt{3k_B T/m}$. Choice C confuses equal energy with equal speed.
3. D. $W = -P\Delta V = -(2.0 \times 10^5)(3.0 \times 10^{-3}) = -600 \text{ J}$, so $\Delta U = Q + W = 900 - 600 = +300 \text{ J}$. Choice A adds the work with the wrong sign, as if the work were done on the gas.
4. B. Rate $\propto A/L$, so the factor is $3/2 = 1.5$. Choice A multiplies by thickness instead of dividing; choice D inverts the area-to-thickness ratio.
5. A. A refrigerator is not isolated; electrical energy runs the compressor and heat is dumped into the room, which gains more entropy than the food loses. Choice C wrongly says a subsystem cannot lose entropy; the law applies to the isolated total.
6. C. Insulated or rapid means no time for heat exchange, so $Q = 0$. The piston does positive work on the gas, so $\Delta U = W > 0$ and the temperature rises. Choice A would describe an expansion.
7. (a) $T_1 = P_1 V_1 / (nR) = (3.0 \times 10^5)(2.0 \times 10^{-3}) / (0.30 \times 8.31) = 241 \text{ K}$. [1 point equation, 1 point value]
 (b) Constant volume, so $W = 0$ and $Q = \Delta U = \frac{3}{2}V\Delta P = 1.5(2.0 \times 10^{-3})(1.0 \times 10^5 - 3.0 \times 10^5) = -600 \text{ J}$. Negative: 600 J of energy leaves the gas.
 (c) $W = -P\Delta V = -(1.0 \times 10^5)(1.0 \times 10^{-3} - 2.0 \times 10^{-3}) = +100 \text{ J}$. Positive because the surroundings compress the gas.
 (d) Second step: $\Delta U = 1.5(P_3 V_3 - P_2 V_2) = 1.5(100 - 200) = -150 \text{ J}$, so $Q_2 = \Delta U - W = -150 - 100 = -250 \text{ J}$. Net $Q = -600 + (-250) = -850 \text{ J}$ (energy leaves).
8. (a) Fill the flask with air at room temperature and seal it so n is fixed; the volume is rigid so V is constant. Place it in the bath with the sensor, change the bath temperature in steps (about 20 deg C steps), wait several minutes at each setting for thermal equilibrium of gas, water and thermometer, then record T and P. Reduce uncertainty by repeating each setting, stirring the bath, and using many temperatures over a wide range.
 (b) Plot P (vertical) against absolute temperature T in kelvin (horizontal). From $P = (nR/V)T$ the slope is nR/V .
 (c) $n = \text{slope} \times V/R = 329 \times 1.0 \times 10^{-3} / 8.31 = 0.040 \text{ mol}$.
 (d) Extrapolating the line to $P = 0$ gives an intercept near -276 deg C (about 0 K), consistent with absolute zero at -273 deg C where ideal-gas molecules would exert no pressure. In kelvin the intercept is within a few kelvin of zero.
9. (a) $\frac{Q}{\Delta t} = \frac{kA\Delta T}{L} = \frac{400(1.0 \times 10^{-4})(100)}{0.50} = 8.0 \text{ W}$. In 600 s: $Q = 8.0 \times 600 = 4800 \text{ J}$.
 (b) Rate $\propto A/L$: doubling both leaves the ratio unchanged, so rod B also carries 8.0 W.
 (c) The hot reservoir loses entropy when it gives up energy, but the cold reservoir gains more entropy from receiving the same energy at a lower temperature, so the total entropy of the isolated pair increases. Heat flowing from cold to hot would decrease the total entropy, which the second law forbids.

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10. (a) The speed does not change because T is constant. Halving V puts the same molecules in half the space, so they hit the walls twice as often, giving twice the average force per area. Equivalently $PV = nRT$ with n and T constant gives $P \propto 1/V$.
- (b) Both curves are humps with long tails to high speed; the T_2 curve has its peak at a higher speed, is lower and broader, and both have the same area (same number of molecules). Axes labeled speed and number of molecules.
- (c) $v \propto 1/\sqrt{m}$, so $v_{\text{He}}/v_{\text{Ne}} = \sqrt{20.2/4.00} = 2.25$. The average kinetic energies are equal, $\frac{3}{2}k_B T$, at the same temperature.

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Unit 10: Electric Force, Field, and Potential

Exam weighting: about 15-18% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

10 Electric Force, Field, and Potential

Coulomb's law, charging, fields, potential energy and potential, capacitors, and energy conservation for charged particles.

Why This Matters

Unit 10 is about 15-18% of the multiple-choice section and feeds every circuit and magnetism problem that follows. Points are lost by treating potential like a vector, by dropping the sign of a charge in $\Delta U = q\Delta V$, by mixing up $E \propto 1/r^2$ with $V \propto 1/r$, and by forgetting what stays fixed in a capacitor (the voltage or the charge).

Core Ideas

10.1 Electric Charge and Electric Force

Charge is quantized in units of $e = 1.60 \times 10^{-19}$ C: the electron has charge $-e$ and the proton $+e$. Like charges repel and unlike charges attract. Two point charges exert equal and opposite forces on each other (Newton's third law), whatever their sizes.

Coulomb's law

$F = k \frac{|q_1 q_2|}{r^2}$, with $k = 9.0 \times 10^9$ N m²/C², along the line joining the charges.

Direction: repulsive for like signs, attractive for unlike signs. With several charges, add the force vectors (superposition).

Inverse-square: doubling r divides F by 4; doubling either charge doubles F .

Use the magnitude formula for the size and decide the direction from the signs.

10.2 Conservation of Electric Charge and the Process of Charging

Net charge is conserved. In **conductors** charges move freely; in **insulators** they stay put. Objects can be charged by **friction** (electrons transferred when two materials rub), by **conduction** (contact with a charged object shares charge, giving the same sign; identical spheres share equally), and by **induction** (charged object brought near, a path to ground provided and then removed, giving the opposite sign without contact). A neutral insulator near a charged rod becomes **polarized**: its molecules shift slightly, so it is attracted but stays neutral overall.

Induction with a negative rod and an isolated metal sphere: bring the rod near (electrons shift to the far side), ground the sphere (electrons flow off), remove the ground, then remove the rod. The sphere keeps a net positive charge. Only electrons move in solids.

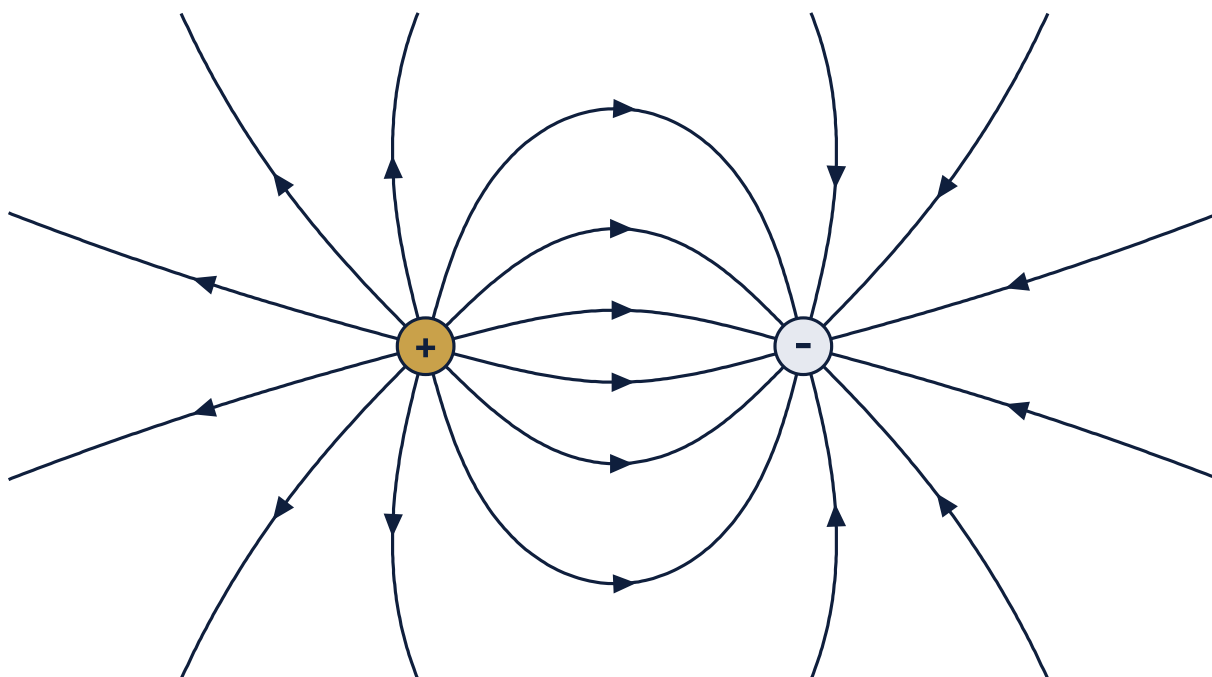
10.3 Electric Fields

The **electric field** at a point is the force per unit positive test charge, $\vec{E} = \vec{F}/q$, in N/C. The force on a charge is $\vec{F} = q\vec{E}$, so it is along $\vec{E}$ for a positive charge and opposite for a negative one. A point charge Q produces a field of magnitude $E = k|Q|/r^2$ pointing away from $+Q$ and toward $-Q$. Fields from several sources add as vectors.

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Electric field lines of equal and opposite point charges. Lines leave the positive charge, enter the negative charge, never cross, and are densest where the field is strongest.

Field lines start on positive charge, end on negative charge, are tangent to $\vec{E}$, never cross, and are closer together where the field is stronger. Inside a conductor in equilibrium the field is zero and excess charge sits on the surface. Between large parallel plates the field is uniform, $E = \Delta V/d$, from the positive plate to the negative plate.

10.4 Electric Potential Energy

Electric potential energy is stored in a system of charges. For two point charges, $U = k q_1 q_2 / r$ with zero at infinite separation, and signs matter: $U > 0$ for like charges (repulsive), $U < 0$ for unlike charges (attractive). It equals the work an external agent does assembling the charges from infinity. For several charges, add U over every pair. If the field pushes a charge along its motion, U decreases.

10.5 Electric Potential

Electric potential

$V = U/q$ (volts = J/C). For a point charge, $V = kQ/r$ with the sign of Q , taking $V = 0$ at infinity. Potentials from several sources add as scalars.

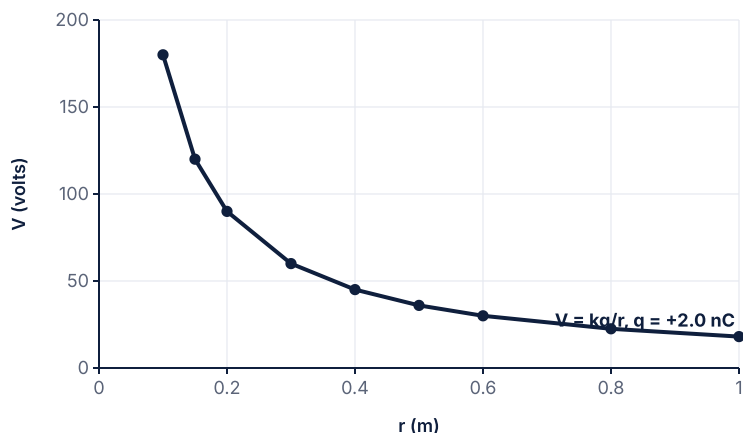
$\Delta U = q \Delta V$ with q carrying its sign. A positive charge gains U moving to higher V ; a negative charge gains U moving to lower V .

$\vec{E}$ points toward decreasing V , and in a uniform field $E = |\Delta V|/d$. Equipotential surfaces are perpendicular to field lines, and no net work is done moving a charge along one.

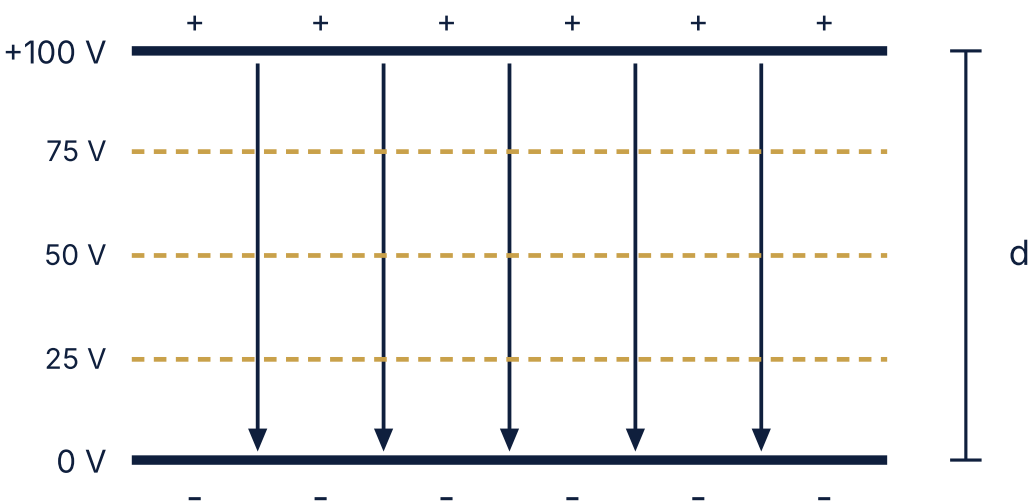
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Potential of an isolated $+2.0 \text{ nC}$ point charge (zero at infinity). V falls as $1/r$, while the field strength falls as $1/r^2$.



Uniform field between parallel plates (edge effects ignored). Field lines point from high to low potential and cross the dashed equipotentials at right angles. With $d = 4.0 \text{ cm}$, $E = 100 \text{ V} / 0.040 \text{ m} = 2500 \text{ V/m}$.

Only differences in V are physical. One electron-volt, $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$, is the energy of an elementary charge moved through 1 V .

10.6 Capacitors

A **capacitor** stores energy in the field between two conductors carrying $+Q$ and $-Q$. Capacitance $C = Q/\Delta V$ (farads) depends on geometry. For parallel plates $C = \kappa\epsilon_0 A/d$, with $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N m}^2)$ and dielectric constant $\kappa \geq 1$. Q is the magnitude on one plate.

Capacitor relations

$$U = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2 = Q^2/(2C).$$

Parallel: same ΔV , $C_{eq} = C_1 + C_2 + \dots$ and charges add. Series: same Q , $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$ and voltages add.

If a capacitor stays connected to a battery, ΔV is fixed; if it is isolated, Q is fixed. Decide which before applying a change to d , A or κ .

10.7 Conservation of Electric Energy

For a charged particle moving where only the electric force does work, energy is conserved: $\Delta K + \Delta U = 0$, so $\frac{1}{2}mv^2 = -q\Delta V$ when released from rest. In a uniform field the particle also has constant acceleration $a = qE/m$, so kinematics works too. A proton moving through a 500 V drop gains 500 eV .

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Worked Examples

EXAMPLE 1: COULOMB'S LAW

Charges $+3.0 \mu\text{C}$ and $-5.0 \mu\text{C}$ are 0.30 m apart. Find the force on each. What is the force if the separation is tripled?

SOLUTION

$$F = k|q_1q_2|/r^2 = (9.0 \times 10^9)(3.0 \times 10^{-6})(5.0 \times 10^{-6})/(0.30)^2 = 1.5 \text{ N}.$$

The charges are unlike, so the force is attractive: each feels 1.5 N toward the other (equal and opposite, by Newton's third law).

At 0.90 m , $F \propto 1/r^2$ falls by a factor of 9 , so $F = 0.17 \text{ N}$.

EXAMPLE 2: SUPERPOSITION OF FIELDS

Charges $+4.0 \text{ nC}$ at $x = 0$ and -4.0 nC at $x = 0.20 \text{ m}$ lie on the x -axis. Find the field at the midpoint and the force on a $+1.0 \text{ nC}$ charge placed there.

SOLUTION

Each charge is 0.10 m from the midpoint, so each makes $E = kQ/r^2 = (9.0 \times 10^9)(4.0 \times 10^{-9})/(0.10)^2 = 3600 \text{ N/C}$.

Away from the positive charge and toward the negative charge are both the $+x$ direction, so the fields add: $E = 7200 \text{ N/C}$ toward $+x$.

$$F = qE = (1.0 \times 10^{-9})(7200) = 7.2 \times 10^{-6} \text{ N toward } +x.$$

EXAMPLE 3: A PROTON IN A UNIFORM FIELD

A proton is released from rest at the positive plate of the parallel plates drawn in section 10.5, where the plates differ by 100 V and are 4.0 cm apart. Find the field, the force, and the speed on reaching the negative plate.

SOLUTION

$$E = \Delta V/d = 100/0.040 = 2500 \text{ V/m, directed from } + \text{ to } - \text{ plate.}$$

$$F = eE = (1.6 \times 10^{-19})(2500) = 4.0 \times 10^{-16} \text{ N, along the field.}$$

Energy conservation: $\frac{1}{2}mv^2 = e\Delta V$, so $v = \sqrt{2(1.6 \times 10^{-19})(100)/(1.67 \times 10^{-27})} = 1.4 \times 10^5 \text{ m/s}$. (Check with $a = F/m = 2.4 \times 10^{11} \text{ m/s}^2$ and $v^2 = 2ad$: same result.)

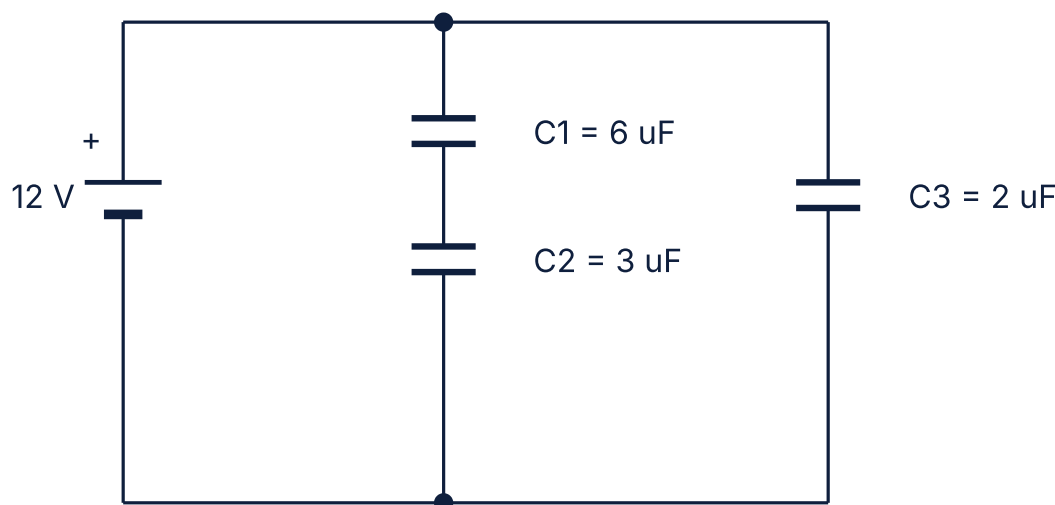
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EXAMPLE 4: CAPACITOR NETWORK

For the network shown, find the equivalent capacitance, the total charge from the battery, the charge on and voltage across each capacitor, and the total stored energy.



Capacitor network for Example 4: C1 and C2 in series, that branch in parallel with C3, all connected to a 12 V battery.

SOLUTION

C1 and C2 in series: $C_s = (6 \times 3)/(6 + 3) = 2.0 \mu\text{F}$. This branch is in parallel with C3: $C_{eq} = 2.0 + 2.0 = 4.0 \mu\text{F}$.

Total charge $Q = C_{eq}\Delta V = (4.0)(12) = 48 \mu\text{C}$. C3 has 12 V across it, so $Q_3 = 24 \mu\text{C}$. The series branch also holds $24 \mu\text{C}$ on each of C1 and C2.

$\Delta V_1 = 24/6 = 4.0 \text{ V}$ and $\Delta V_2 = 24/3 = 8.0 \text{ V}$; they add to 12 V.

$U = \frac{1}{2}C_{eq}(\Delta V)^2 = 0.5(4.0 \times 10^{-6})(144) = 2.88 \times 10^{-4} \text{ J}$.

Write It Like a 5**Model Response**

Prompt: A positive point charge Q is fixed. Point A is a distance r from Q and point B is a distance $2r$. (a) Compare the field and potential at A and B. (b) A small negative charge $-q$ is released from rest at B. Describe its motion and energy. Response: (a) For a point charge $E = kQ/r^2$, so at B the distance is doubled and the field is one-fourth as large: $E_B = E_A/4$. Potential is $V = kQ/r$, so $V_B = V_A/2$; both are positive and the potential is higher at A. (b) The field points away from Q , so the force on $-q$ is toward Q and the charge accelerates toward A. Moving to higher potential, $\Delta U = (-q)\Delta V < 0$, so its potential energy decreases and its kinetic energy increases by the same amount: $\frac{1}{2}mv^2 = q(V_A - V_B)$.

What the grader looks for

Correct factors for E (1/4) and V (1/2) from formulas written first; force direction on the negative charge from the field direction; $\Delta U = q\Delta V$ with the sign of q ; decreasing U linked to increasing K .

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Coach's Playbook

- 1 Separate vectors (force, field) from scalars (potential energy, potential): add vectors by components, scalars with signs.
- 2 Put the sign of every charge into U and V formulas, then check the answer against the physical picture.
- 3 For a capacitor, write down what is held fixed (battery: V; disconnected: Q) before calculating.

Common traps

1. Scaling V as $1/r^2$ or E as $1/r$: field strength goes as $1/r^2$ but potential goes as $1/r$. 2. Dropping the negative sign of an electron in $\Delta U = q\Delta V$ and predicting the wrong direction of motion. 3. Saying that a negative charge ends up with the same sign as the rod in induction. 4. Writing a capacitor's charge as the sum of both plates, or using Q on one plate with $2Q$ elsewhere in $Q = C\Delta V$.

Quick Check

Q1: Two like charges are released from rest. What happens to their total kinetic energy and potential energy? A: K increases and U decreases by the same amount. Q2: Which way does the field point relative to potential? A: Toward lower potential. Q3: Plate separation is halved with the battery connected. Charge? A: It doubles (C doubles, V fixed).

Challenge Corner

Three equal charges $+2.0$ nC sit at the corners of an equilateral triangle of side 0.10 m. How much work must an external agent do to assemble them from infinity? Solution: Bringing in the first charge takes no work. The second gains $U = kq^2/r = 9.0 \times 10^9 (2.0 \times 10^{-9})^2 / 0.10 = 3.6 \times 10^{-7}$ J. The third interacts with two charges, adding $2(3.6 \times 10^{-7})$ J. Total $W = 3(3.6 \times 10^{-7}) = 1.1 \times 10^{-6}$ J.

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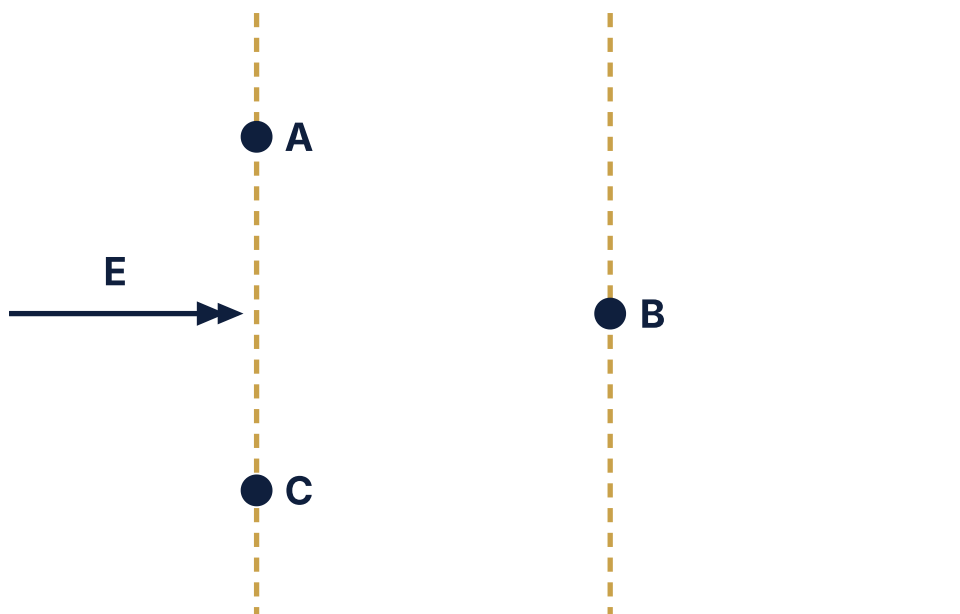
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Practice

PRACTICE Unit 10 Practice

Show your work. A calculator is allowed. Justify conclusions.

- Two small spheres carry charges q_1 and q_2 and exert a force of magnitude F on each other at separation r . If q_1 is doubled and the separation is halved, the new force is
 (A) $2F$ (B) $4F$ (C) $16F$ (D) $8F$
- A neutral, isolated metal sphere is brought near, but does not touch, a negatively charged rod. The sphere is briefly connected to ground, the ground wire is removed, and then the rod is taken away. The sphere is left
 (A) positively charged, because electrons left through the ground
 (B) negatively charged, because electrons moved from the rod
 (C) neutral, because the rod never touched it
 (D) positively charged, because protons moved in from the ground
- The figure shows a uniform electric field with dashed equipotential lines. How do the electric potentials at A, B and C compare?



Uniform electric field pointing right. Dashed lines are equipotentials. Points A and C lie on the same equipotential line.

- (A) $V_B > V_A = V_C$ (B) $V_A > V_B > V_C$ (C) $V_A = V_C > V_B$
 (D) $V_A = V_B = V_C$
- A parallel-plate capacitor is connected to a battery. With the battery still connected, the plate separation is doubled. The charge on the plates
 (A) doubles (B) is halved (C) stays the same
 (D) is reduced to one-fourth
 - An electron is released from rest in a region where the potential at its starting point is -100 V and the potential at another point is $+50\text{ V}$. When it reaches that point, its kinetic energy has
 (A) lost $2.4 \times 10^{-17}\text{ J}$ (B) gained $8.0 \times 10^{-18}\text{ J}$ (C) gained $1.6 \times 10^{-17}\text{ J}$
 (D) gained $2.4 \times 10^{-17}\text{ J}$
 - At distance r from a positive point charge the potential is 400 V and the field strength is 800 V/m . At distance $2r$, the potential and field strength are
 (A) 200 V and 400 V/m (B) 200 V and 200 V/m (C) 100 V and 200 V/m
 (D) 400 V and 200 V/m

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7. Charge $Q_1 = +6.0$ nC is at $x = 0$ and $Q_2 = -3.0$ nC is at $x = 0.30$ m on the x-axis.
 - (a) Calculate the electric field (magnitude and direction) at the midpoint $x = 0.15$ m.
 - (b) Find the position on the x-axis where the field is zero, and explain why no such point exists between the charges or to the left of Q_1 .
 - (c) Calculate the electric force on an electron placed at the midpoint.
8. A student wants to determine the capacitance of an unlabeled capacitor using a variable DC supply, a voltmeter, and a device that measures the charge delivered to the capacitor.
 - (a) Describe an experimental procedure, including how to reduce uncertainty.
 - (b) State what to plot so that the slope gives C.
 - (c) For voltages 1.5, 3.0, 4.5, 6.0, 7.5 V the measured charges are 71, 141, 213, 281, 353 microcoulombs. Estimate C from the data.
 - (d) Explain why plotting charge versus $1/V$ would not give a straight line.
9. A parallel-plate capacitor has $C_0 = 2.0$ μF and is charged by a 12 V battery.
 - (a) Calculate the charge and stored energy.
 - (b) With the battery still connected, a dielectric of constant 3.0 fills the space between the plates. Calculate the new charge and energy.
 - (c) Instead, the battery is disconnected first and then the dielectric is inserted. Calculate the new voltage and energy.
 - (d) Explain, using energy, why the stored energy decreases in part (c) but increases in part (b).
10. The electric potential along a line in a region of uniform field is measured: $V = 20, 16, 12, 8, 4$ V at $x = 0, 2, 4, 6, 8$ cm.
 - (a) Determine the magnitude and direction of the electric field.
 - (b) Calculate the force on an electron placed in the field and state its direction.
 - (c) Calculate the change in the electron's electric potential energy and the work needed by an external agent when it is moved slowly from $x = 0$ to $x = 8$ cm.
 - (d) Describe in words or a sketch the equipotentials and field lines.

Answers Unit 10: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

10 Electric Force, Field, and Potential

PRACTICE · UNIT 10 PRACTICE

1. D. Doubling a charge multiplies F by 2 and halving r multiplies F by 4, so the force is 8F. Choice C (16F) comes from squaring the charge factor as well; here only one charge changes.
2. A. The rod repels electrons toward the ground connection. Removing the ground while the rod is near traps a deficit of electrons, so the sphere ends up positive. Choice D fails because protons in solids do not move.
3. C. The field points toward decreasing potential, so moving right lowers V. A and C lie on the same equipotential, and B is farther along the field. Choice A has the trend reversed.
4. B. The battery keeps ΔV fixed. Doubling d halves C, so $Q = C\Delta V$ is halved. Choice C would be correct only if the capacitor had been disconnected.
5. D. $\Delta U = q\Delta V = (-1.6 \times 10^{-19})(150) = -2.4 \times 10^{-17}$ J, so $\Delta K = +2.4 \times 10^{-17}$ J. Choice A ignores the sign of q; choices B and C use 50 V and 100 V instead of the 150 V difference.
6. B. V falls as $1/r$ (400 to 200 V) and E falls as $1/r^2$ (800 to 200 V/m). Choice A halves the field instead of quartering it.
7. (a) Each charge is 0.15 m from the midpoint. $E_1 = (9.0 \times 10^9)(6.0 \times 10^{-9})/0.15^2 = 2400$ N/C, pointing away from Q_1 (+x). $E_2 = (9.0 \times 10^9)(3.0 \times 10^{-9})/0.15^2 = 1200$ N/C, pointing toward Q_2 (+x). Net $E = 3600$ N/C in the +x direction.
 (b) Between the charges both fields point +x, so they add and cannot cancel. Left of Q_1 , the field from Q_1 (larger charge, closer) always dominates. To the right of Q_2 , the field from Q_1 points +x and the field from Q_2 points -x, and they can cancel. At distance d from Q_2 : $6.0/(0.30 + d)^2 = 3.0/d^2$, giving $0.30 + d = \sqrt{2}d$, so $d = 0.724$ m and $x = 1.02$ m.
 (c) $F = eE = (1.6 \times 10^{-19})(3600) = 5.8 \times 10^{-16}$ N, toward -x because the electron is negative.
8. (a) Connect the capacitor to the supply set at a chosen voltage; read the voltage on the voltmeter and the charge on the charge device. Discharge the capacitor fully between trials. Take five or more voltages across the available range and repeat each setting to average. Allow the capacitor to reach the full voltage before reading. (b) Plot Q on the vertical axis and ΔV on the horizontal axis; $Q = C\Delta V$ so the slope is C. (c) The slope of the best-fit line is about 47 microcoulombs per volt, so $C \approx 47$ μF . (d) Since $Q = C\Delta V$, a graph of Q against $1/V$ is a hyperbola, $Q = C/(1/V)$, which is curved; only Q against V is a straight line through the origin.

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9. (a) $Q_0 = C_0 \Delta V = 24 \mu\text{C}$; $U_0 = \frac{1}{2} C_0 \Delta V^2 = 1.44 \times 10^{-4} \text{ J}$.
(b) $C = 3.0(2.0) = 6.0 \mu\text{F}$, V fixed at 12 V, so $Q = 72 \mu\text{C}$ and $U = 4.3 \times 10^{-4} \text{ J}$.
(c) Charge stays 24 microcoulombs and $C = 6.0$ microfarads, so $\Delta V = Q/C = 4.0 \text{ V}$ and $U = Q^2/2C = 4.8 \times 10^{-5} \text{ J}$.
(d) In (b) the battery supplies extra charge and energy, so U rises. In (c) the system is isolated and the dielectric is pulled into the gap by the field, which does positive work, so the stored energy falls (to 48 microjoules, from 144); the lost energy appears as kinetic energy of the dielectric or as work done on whoever holds it back.
10. (a) V decreases 4 V for every 2 cm, so $E = 4/0.02 = 200 \text{ V/m}$, pointing toward $+x$ (decreasing potential).
(b) $F = eE = (1.6 \times 10^{-19})(200) = 3.2 \times 10^{-17} \text{ N}$, in the $-x$ direction since the electron is negative.
(c) $\Delta V = 4 - 20 = -16 \text{ V}$, so $\Delta U = q\Delta V = (-1.6 \times 10^{-19})(-16) = +2.6 \times 10^{-18} \text{ J}$. An external agent must do $+2.6 \times 10^{-18} \text{ J}$ because the electron moves against the force.
(d) Equipotentials are equally spaced planes perpendicular to x . Field lines are parallel straight lines pointing toward $+x$, as between parallel plates with the left one positive.

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Unit 11: Electric Circuits

Exam weighting: about 15-18% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

11 Electric Circuits

Current, resistance, power, series and parallel circuits, Kirchhoff's rules, and the charging and discharging of RC circuits.

Why This Matters

Circuits are 15-18% of the multiple-choice section and a recurring free-response theme. Points are lost by adding parallel resistances directly, using one resistor's R in $P = V^2/R$ for the whole circuit, skipping the loop rule, and mis-sketching what a capacitor does at the start and the end of charging.

Core Ideas

11.1 Electric Current

Current is the rate at which charge crosses a cross-section: $I = \Delta Q / \Delta t$, in amperes (C/s). By convention it points the way positive charge would move, from the positive terminal around the circuit to the negative one, although in metals electrons carry the charge the other way. Charge is conserved and cannot pile up, so the current is the same at every point in a single-path wire. A lamp does not use up current; it converts energy.

11.2 Simple Circuits

A battery is a source of **emf** $\mathcal{E}$ that raises the potential energy of charge, so each coulomb gains $\mathcal{E}$ joules. Current needs a complete loop. Ammeters are placed in series and read the current through them; voltmeters connect in parallel across a component and read the potential difference. Unless told otherwise, batteries are ideal (no internal resistance), ammeters have zero resistance and voltmeters have infinite resistance. A real battery has an internal resistance r , so its terminal voltage is $\Delta V = \mathcal{E} - Ir$, which sags as the current grows.

11.3 Resistance, Resistivity, and Ohm's Law

Resistance relations

Ohm's law: $\Delta V = IR$ (R in ohms). A resistor is **ohmic** if R is constant, giving a straight V - I line; a bulb filament is not, because R rises as it heats.

Resistance of a wire: $R = \rho L / A$, where ρ is the resistivity of the material, L the length and A the cross-sectional area. Longer is more resistive; thicker is less.

Doubling the diameter multiplies A by 4 and divides R by 4.

11.4 Electric Power

Power

$P = I\Delta V = I^2 R = (\Delta V)^2 / R$, in watts. Choose the form whose two quantities you know for that resistor.

Energy: $E = Pt$. A kilowatt-hour is 3.6×10^6 J.

Series resistors carry the same I , so the larger R dissipates more power. Parallel resistors share the same V , so the smaller R dissipates more.

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11.5 Compound Direct Current (DC) Circuits

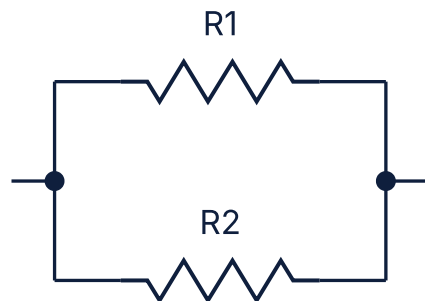
Resistors in **series** add: $R_s = R_1 + R_2 + \dots$, and the voltages divide in proportion to R . Resistors in **parallel** follow $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$; the equivalent is smaller than the smallest branch, and the currents divide inversely to R . For a compound circuit: reduce to one equivalent resistance, find the battery current, then work back outward, finding the voltage across each combination and the current in each branch. Adding a resistor in parallel lowers the total resistance and raises the battery current but leaves the voltage across the existing branches unchanged.

Series: same current



$$R_{\text{series}} = R_1 + R_2$$

Parallel: same voltage



$$1/R_{\text{par}} = 1/R_1 + 1/R_2$$

Series resistors share one path; parallel resistors share the same two nodes.

11.6 Kirchhoff's Loop Rule

Energy conservation for a charge going around any closed loop: the sum of potential changes is zero, $\sum \Delta V = 0$. Pick a direction around the loop. Crossing a battery from its negative to positive terminal is $+\varepsilon$ and from positive to negative is $-\varepsilon$. Crossing a resistor in the direction of the assumed current is $-IR$ and against it is $+IR$. A negative current in the answer simply means the real current is opposite to the assumed arrow.

11.7 Kirchhoff's Junction Rule

Charge does not accumulate at a junction, so $\sum I_{\text{in}} = \sum I_{\text{out}}$. With unknown currents in several branches, write one junction equation and enough independent loop equations to match the number of unknowns, then solve the simultaneous algebraic equations.

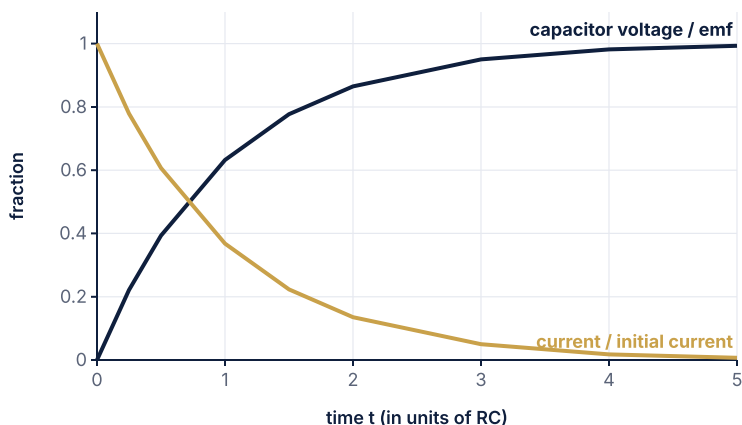
11.8 Resistor-Capacitor (RC) Circuits

A capacitor in series with a resistor charges or discharges gradually. For an initially uncharged capacitor connected to a battery: at the instant the circuit closes the capacitor has no voltage, so it behaves like a wire and $I = \varepsilon/R$; after a long time the capacitor voltage equals ε , the current is zero and the capacitor behaves like an open gap. In between, the current falls and the capacitor voltage rises with the shapes shown, and the loop rule $\varepsilon = IR + Q/C$ holds at every instant. The time scale is set by RC : a larger R or C slows the process, but the final charge $C\varepsilon$ depends only on C and the emf. When a charged capacitor discharges through a resistor, current starts at its largest value and falls to zero while the voltage decays to zero. In the steady state of any DC circuit, no current flows through a capacitor branch.

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Charging an initially uncharged capacitor. The current starts at its maximum, emf/R , and falls toward zero while the capacitor voltage rises toward the emf. Only the shapes matter for the exam.

Worked Examples

EXAMPLE 1: CURRENT AND RESISTANCE OF A WIRE

A copper wire carries 3.0 A for 2.0 minutes. (a) How many electrons pass a cross-section? The wire is 20 m long with diameter 1.0 mm and $\rho = 1.7 \times 10^{-8} \Omega \cdot \text{m}$. (b) Find its resistance. (c) What is the resistance if the diameter is doubled?

SOLUTION

(a) $\Delta Q = I\Delta t = 3.0(120) = 360 \text{ C}$, so $N = 360/(1.6 \times 10^{-19}) = 2.3 \times 10^{21}$ electrons.

(b) $A = \pi(0.50 \times 10^{-3})^2 = 7.85 \times 10^{-7} \text{ m}^2$, so $R = \rho L/A = (1.7 \times 10^{-8})(20)/(7.85 \times 10^{-7}) = 0.43 \Omega$.

(c) Doubling the diameter quadruples A , so $R = 0.43/4 = 0.11 \Omega$.

EXAMPLE 2: SERIES CIRCUIT AND POWER

A 12 V ideal battery drives a 4.0Ω and an 8.0Ω resistor in series. Find the current, each voltage, and each power.

SOLUTION

$R_s = 12 \Omega$, so $I = 12/12 = 1.0 \text{ A}$ in both resistors.

$\Delta V_1 = IR_1 = 4.0 \text{ V}$ and $\Delta V_2 = 8.0 \text{ V}$; they add to 12 V (loop rule).

$P_1 = I^2 R_1 = 4.0 \text{ W}$ and $P_2 = 8.0 \text{ W}$; total 12 W = ϵI . The larger resistor dissipates more power in series.

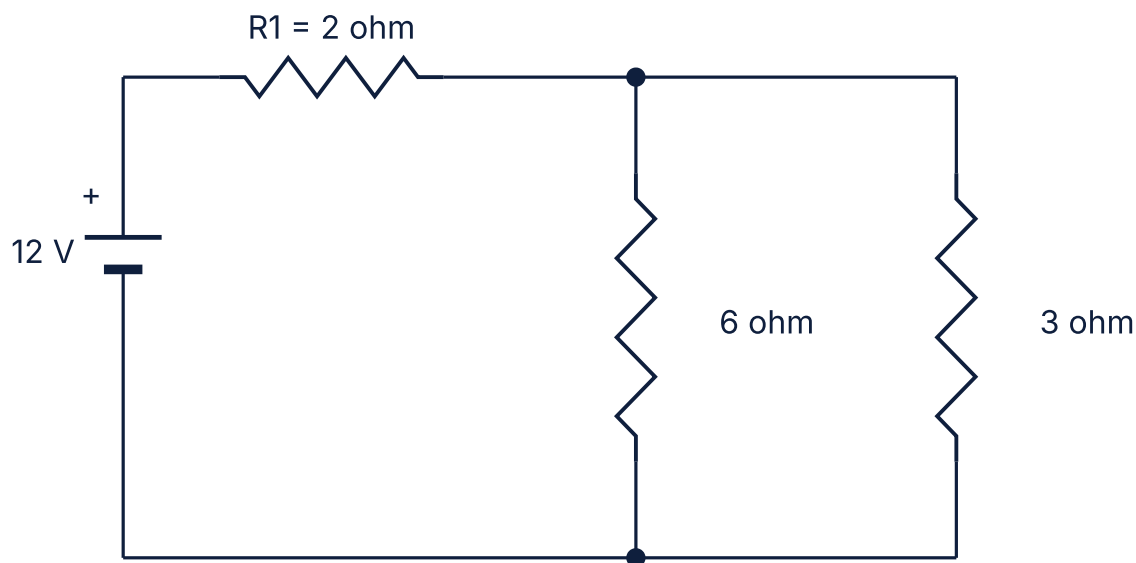
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EXAMPLE 3: COMPOUND CIRCUIT

For the circuit shown (12 V ideal battery), find the battery current, the voltage across the 6 ohm resistor, and the current in each parallel branch.



Compound circuit for Example 3: R_1 in series with a parallel pair. Conventional current leaves the positive terminal.

SOLUTION

Parallel pair: $1/R_p = 1/6 + 1/3 = 1/2$, so $R_p = 2.0 \Omega$. Total $R = 2.0 + 2.0 = 4.0 \Omega$.

Battery current $I = 12/4.0 = 3.0 \text{ A}$, which also flows through R_1 , dropping $3.0(2.0) = 6.0 \text{ V}$.

Voltage across the parallel pair = $12 - 6.0 = 6.0 \text{ V}$. Branch currents: $6.0/6 = 1.0 \text{ A}$ and $6.0/3 = 2.0 \text{ A}$, which add to 3.0 A (junction rule).

Total power = $12(3.0) = 36 \text{ W}$.

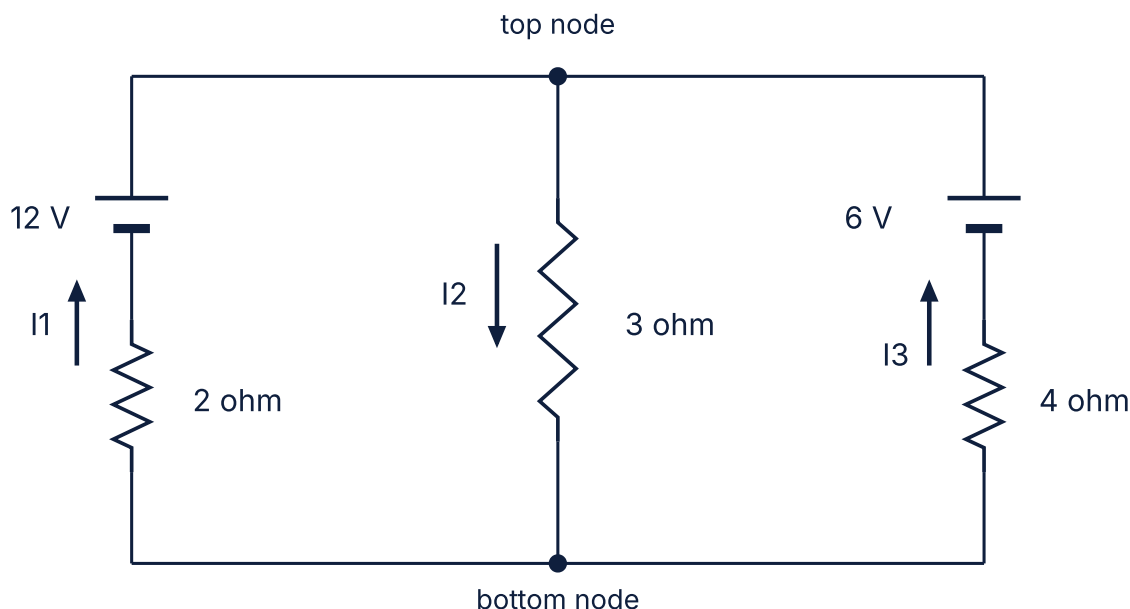
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EXAMPLE 4: KIRCHHOFF'S RULES

Find the three currents in the circuit shown, using the assumed directions.



Two-battery circuit for Example 4. Each battery has its positive terminal up. Assumed current directions: I_1 and I_3 upward, I_2 downward.

SOLUTION

Junction (top node): $I_1 + I_3 = I_2$.

Left loop (up branch 1, down the middle): $12 - 2I_1 - 3I_2 = 0$. Right loop (up branch 3, down the middle): $6 - 4I_3 - 3I_2 = 0$.

Substitute $I_1 = (12 - 3I_2)/2$ and $I_3 = (6 - 3I_2)/4$ into the junction equation: $7.5 - 2.25I_2 = I_2$, so $I_2 = 2.31$ A. Then $I_1 = 2.54$ A and $I_3 = -0.23$ A.

The negative sign means the real current in the 6 V branch is downward: that battery is being charged. Check: $2.54 - 0.23 = 2.31$ A.

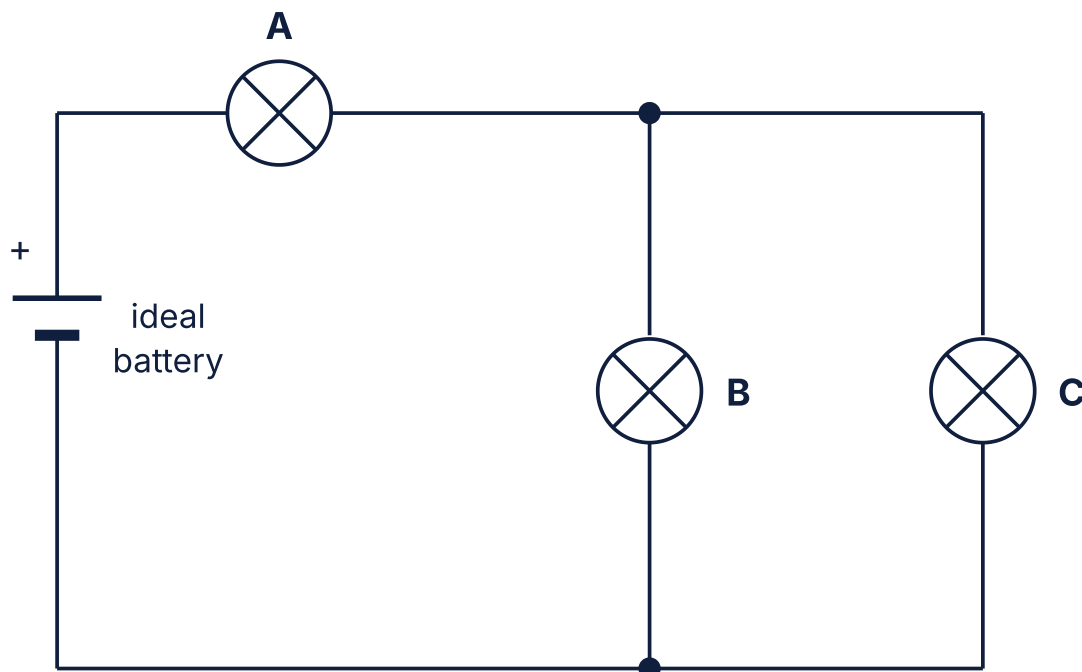
Write It Like a 5**Model Response**

Prompt: The circuit shows three identical bulbs, each of resistance R , on an ideal battery (A in series with B and C in parallel). Bulb C is unscrewed. Compare the brightness of A and B before and after, and justify. Response: Brightness increases with power, $P = V^2/R$. Before: B and C in parallel give $R/2$, so the total is $3R/2$ and $I = 2\varepsilon/(3R)$. Bulb A carries all of it, so $V_A = 2\varepsilon/3$, and the parallel pair shares $\varepsilon/3$, so $V_B = \varepsilon/3$. After removing C the total resistance is $2R$, $I = \varepsilon/(2R)$, so $V_A = V_B = \varepsilon/2$. A's voltage falls from $2\varepsilon/3$ to $\varepsilon/2$, so its power falls to $(3/4)^2 = 56\%$ of the earlier value and A dims. B's voltage rises from $\varepsilon/3$ to $\varepsilon/2$, so its power rises by a factor $(3/2)^2 = 2.25$ and B brightens.

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Three identical bulbs: A in series with the parallel pair B and C.

What the grader looks for

Uses $P = V^2/R$ or I^2R with the same bulb resistance; finds equivalent resistance before and after; gets the voltage across each bulb from the loop rule or voltage division; compares numerically and states dims or brightens for each bulb with a reason.

Coach's Playbook

- 1 Redraw the circuit and label each node; mark which elements share a current (series) and which share a voltage (parallel).
- 2 Reduce to one equivalent resistance, solve for the battery current, then expand back out. Check that branch currents add at each junction and voltages add around each loop.
- 3 For power, choose I^2R in series and V^2/R in parallel. For a real battery use $\Delta V = \mathcal{E} - Ir$.
- 4 For RC circuits, state the capacitor's role at $t = 0$ (a wire if uncharged) and at long times (an open gap).

Common traps

1. Adding parallel resistances directly instead of using reciprocals. 2. Using one resistor's R with the battery voltage in $P = V^2/R$ for a different part of the circuit. 3. Skipping the loop rule and forgetting that a real battery's terminal voltage drops when more current flows. 4. Estimating capacitor charge as current times time when the current is changing, or claiming that the capacitor's current stays constant while charging.

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Quick Check

Q1: Two resistors, 10 ohm and 40 ohm, are in parallel. Equivalent resistance? A: $400/50 = 8 \Omega$. Q2: What current flows through a capacitor branch long after the switch closes in a DC circuit? A: Zero. Q3: A 1500 W heater runs on 120 V. What current does it draw? A: $P/V = 12.5 \text{ A}$.

Challenge Corner

A 12.0 V battery with internal resistance 1.0 ohm powers a 5.0 ohm bulb. A second identical 5.0 ohm bulb is then added in parallel. Compare the power in the first bulb before and after. Solution: Before: $I = 12/(5 + 1) = 2.0 \text{ A}$, terminal voltage $= 12 - 2.0(1.0) = 10.0 \text{ V}$, $P = 10.0^2/5.0 = 20 \text{ W}$. After: external resistance 2.5 ohm, $I = 12/3.5 = 3.43 \text{ A}$, terminal voltage $= 12 - 3.43 = 8.57 \text{ V}$, so each bulb gets $P = 8.57^2/5.0 = 14.7 \text{ W}$. The first bulb dims because the extra current drops more voltage across the internal resistance.

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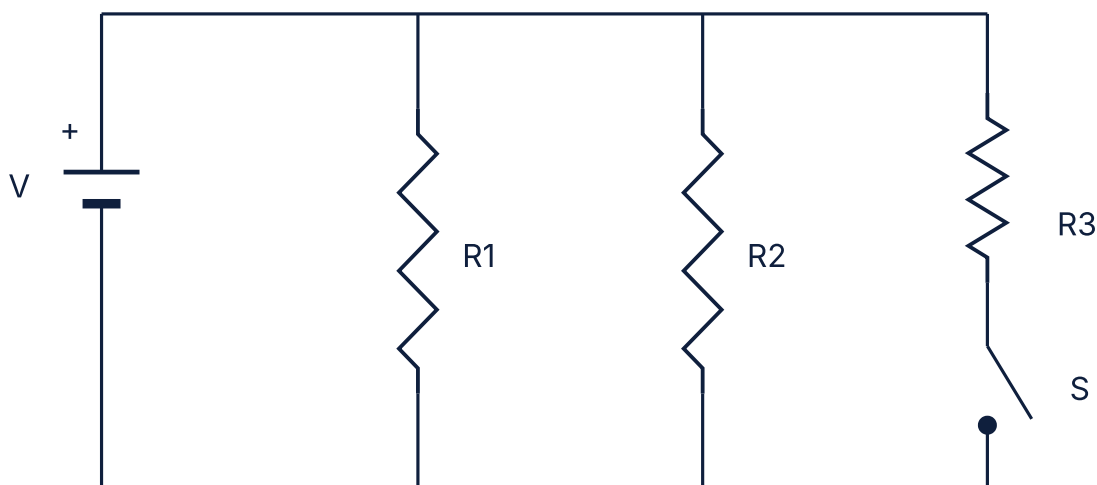
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Practice

PRACTICE Unit 11 Practice

Show your work. A calculator is allowed. Justify conclusions. Assume ideal batteries and meters, and conventional current, unless stated.

1. A wire has resistance R . A second wire of the same material is twice as long and has twice the diameter. Its resistance is
(A) $R/2$ (B) R (C) $2R$ (D) $R/4$
2. A 60 W bulb and a 100 W bulb, both rated for 120 V, are connected in series to a 120 V source. Which statement is correct?
(A) The 100 W bulb is brighter, because it has the higher power rating.
(B) The 60 W bulb is brighter, because it has the larger resistance and the same current.
(C) They are equally bright, because the current is the same.
(D) The 100 W bulb is brighter, because it has the larger resistance.
3. In the figure, the switch S is closed. Which statement about the currents is correct?



An ideal battery drives R_1 and R_2 in parallel. A third resistor R_3 can be added in parallel by closing switch S (shown open).

- (A) The currents in R_1 and R_2 decrease, and the battery current is unchanged.
(B) The currents in R_1 and R_2 increase, and the battery current decreases.
(C) The currents in R_1 and R_2 are unchanged, and the battery current is unchanged.
(D) The currents in R_1 and R_2 are unchanged, and the battery current increases.
4. At a junction, wires carrying 3.0 A and 4.0 A enter and two other wires leave, one carrying 2.0 A. The current in the fourth wire is
(A) 9.0 A (B) 5.0 A (C) 1.0 A (D) 2.0 A
5. An uncharged capacitor C is connected in series with a resistor R and a battery. R is doubled while C and the emf are unchanged. Compared with the original circuit, the capacitor now
(A) reaches a larger final charge, more quickly
(B) reaches half the final charge, at the same rate
(C) reaches the same final charge, but more slowly
(D) reaches the same final charge, at the same rate
6. A heater is rated 1500 W at 120 V. Its resistance in operation is closest to
(A) 9.6 ohm (B) 12.5 ohm (C) 0.080 ohm (D) 180 ohm

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7. A 9.0 V ideal battery is connected to a 10 ohm resistor R_1 in series with a parallel combination of a 20 ohm resistor R_2 and a 30 ohm resistor R_3 .
 - (a) Calculate the equivalent resistance and the battery current.
 - (b) Calculate the voltage across and current in each resistor.
 - (c) Calculate the total power supplied by the battery.
 - (d) R_3 burns out and becomes an open circuit. Determine whether the power dissipated in R_2 increases or decreases, with numbers.
8. A student has several lengths of nichrome wire of diameter 0.40 mm and wants to find the resistivity of nichrome using a battery, an ammeter, a voltmeter and a ruler.
 - (a) Describe a procedure, including how to reduce uncertainty, and the measurements to record.
 - (b) The measured resistances for lengths 0.20, 0.40, 0.60, 0.80 and 1.00 m are 1.78, 3.46, 5.30, 6.95 and 8.80 ohm. State what to plot and how to get the resistivity from the graph.
 - (c) Calculate the resistivity from the best-fit slope of 8.77 ohm per meter.
 - (d) Explain why a very large current should be avoided.
9. A 12 V ideal battery charges an initially uncharged 500 microfarad capacitor through a 2.0 kilo-ohm resistor.
 - (a) Calculate the current immediately after the circuit is closed.
 - (b) Calculate the final charge and the final energy stored in the capacitor.
 - (c) Calculate the work done by the battery and the energy dissipated in the resistor.
 - (d) Sketch qualitatively the current as a function of time, and state how the curve changes if R is doubled.
10. A bulb is tested: the measured potential differences and currents are $V = 1, 2, 4, 6, 8$ volts and $I = 0.10, 0.17, 0.28, 0.37, 0.45$ amperes.
 - (a) Calculate R at each voltage.
 - (b) State whether the bulb is ohmic, citing the evidence, and describe the shape of the V-I graph.
 - (c) Calculate the power at 6 V.
 - (d) Explain the trend physically.

Answers Unit 11: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

11 Electric Circuits

PRACTICE · UNIT 11 PRACTICE

1. A. $R = \rho L/A$. Doubling L doubles R; doubling the diameter multiplies A by 4 and divides R by 4. The net factor is $2/4 = 1/2$. Choice D ($R/4$) forgets the doubled length.
2. B. Rated resistance is V^2/P : 240 ohm for the 60 W bulb and 144 ohm for the 100 W bulb. In series the same current gives $P = I^2 R$, larger for the 60 W bulb. Choice A uses the rated power, which applies only at 120 V across each bulb.
3. D. R_1 and R_2 stay connected across the same ideal battery voltage, so their currents do not change. R_3 adds a new branch, so the battery supplies more current. Choice A treats the battery as a fixed-current source.
4. B. The junction rule gives $3.0 + 4.0 = 2.0 + I$, so $I = 5.0$ A leaving. Choice A (9.0 A) adds every current; choice C (1.0 A) subtracts the wrong way.
5. C. The final charge Q_f does not depend on R. A larger R reduces the current and lengthens the time scale RC. Choice D neglects the effect of R on the charging time.
6. A. $R = V^2/P = 14400/1500 = 9.6 \Omega$. Choice B is the current in amperes (12.5 A), not a resistance.
7. (a) $R_p = 1/(1/20 + 1/30) = 12 \Omega$, so $R_{eq} = 10 + 12 = 22 \Omega$ and $I = 9.0/22 = 0.41$ A.
 (b) R_1 : $I = 0.41$ A, $\Delta V = 4.1$ V. Parallel voltage = $9.0 - 4.1 = 4.9$ V. R_2 : $4.9/20 = 0.25$ A; R_3 : $4.9/30 = 0.16$ A (sum 0.41 A).
 (c) $P = \varepsilon I = 9.0(0.409) = 3.7$ W (equal to the sum of the three resistor powers: $1.67 + 1.20 + 0.80$ W).
 (d) With R_3 open the total is $10 + 20 = 30 \Omega$ and $I = 0.30$ A through R_2 , so $P_2 = I^2 R_2 = (0.30)^2(20) = 1.8$ W, up from $4.91^2/20 = 1.2$ W. It increases: with R_3 gone the battery current falls from 0.41 A to 0.30 A, so R_1 drops only 3.0 V instead of 4.1 V, leaving 6.0 V across R_2 instead of 4.9 V.
8. (a) Connect a battery, ammeter (series) and voltmeter (across the wire section) to a length of wire. Measure the diameter with a micrometer at several places and average. For each length, measure the length with the ruler, then V and I; compute $R = V/I$. Use a small current and short connection times so the wire does not heat, and repeat each length. (b) Plot R against L. Since $R = (\rho/A)L$, the slope is ρ/A , so $\rho = \text{slope} \times A$. (c) $A = \pi(0.20 \times 10^{-3})^2 = 1.26 \times 10^{-7} \text{ m}^2$, so $\rho = 8.77 \times 1.26 \times 10^{-7} = 1.1 \times 10^{-6} \Omega \text{ m}$. (d) Large currents heat the wire, raising its resistance (and its temperature changes its resistivity), so the data would no longer follow a constant slope.

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9. (a) $I_0 = \varepsilon/R = 12/2000 = 6.0 \times 10^{-3}$ A, since the uncharged capacitor acts like a wire.
(b) $Q = C\varepsilon = (500 \times 10^{-6})(12) = 6.0 \times 10^{-3}$ C. $U = \frac{1}{2}C\varepsilon^2 = 0.5(500 \times 10^{-6})(144) = 0.036$ J.
(c) Work by the battery = $Q\varepsilon = 0.072$ J. Dissipated in R = $0.072 - 0.036 = 0.036$ J, half the battery's work.
(d) The current starts at 6.0 mA and decreases smoothly and concavely upward toward zero. If R is doubled, the curve starts lower (3.0 mA) and falls more slowly; the area under the curve (the charge) stays 6.0 mC.
10. (a) $R = V/I$: 10.0, 11.8, 14.3, 16.2 and 17.8 ohm.
(b) Not ohmic: R is not constant (it rises with voltage), and the V-I graph curves, with current rising less than proportionally. An ohmic resistor would give a straight line through the origin.
(c) $P = IV = 0.37(6.0) = 2.2$ W.
(d) More voltage means more current and more power dissipated, so the filament heats up. A hotter metal has more frequent collisions between electrons and ions, so its resistance increases.

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Unit 12: Magnetism and Electromagnetism

Exam weighting: about 12-15% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

12 Magnetism and Electromagnetism

Fields from magnets and currents, forces on moving charges and wires, and how a changing flux induces an emf: four topics, each with a right-hand rule to master.

Why This Matters

Unit 12 is worth 12-15% of the multiple-choice score. Students lose the most points on right-hand-rule slips, on putting the wrong quantity in a formula (wire length belongs in the force, not the field), and on Lenz's law justifications.

Core Ideas

12.1 Magnetic Fields

A magnetic field $\vec{B}$ (unit: tesla, T) exists around magnets and moving charges. Outside a magnet the field lines run from north to south; they never start or stop, because isolated magnetic poles (monopoles) have not been found. Magnetic fields come from moving charge, so an electric current makes a field. Earth acts like a bar magnet with a field near 5×10^{-5} T.

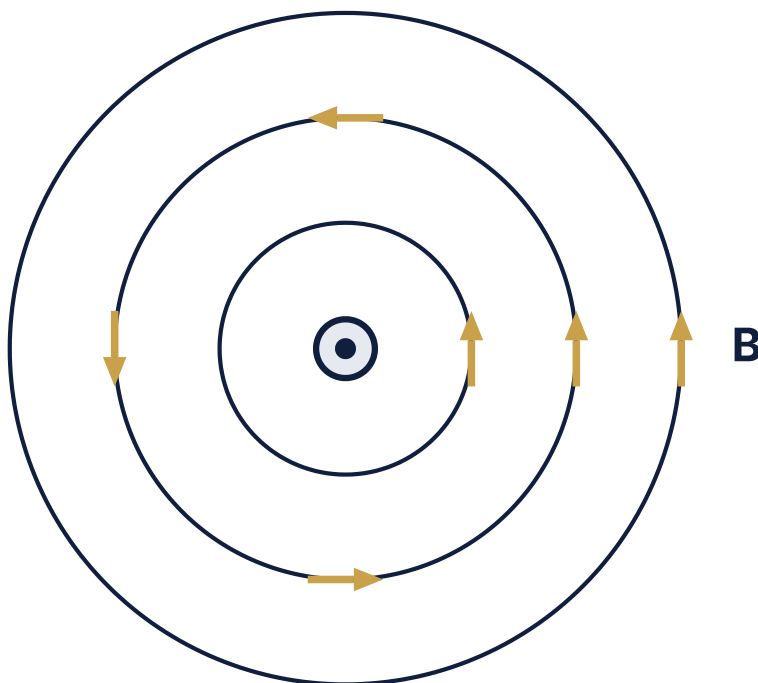
A long straight wire carrying current I makes a field that circles the wire. Its magnitude at distance r is $B = \mu_0 I / (2\pi r)$ with $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$. The wire's length does not appear. For a loop or solenoid, curl the fingers of the right hand along the current: the thumb points along the field inside, and more current or more turns per meter makes it stronger.

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Current I out of the page (dot)



Rings are closer to the wire where B is larger

Right-hand rule for a long straight wire: point the thumb along the current (out of the page) and the fingers curl counterclockwise, the direction of B .

Field of a long straight wire

$$B = \frac{\mu_0 I}{2\pi r}, \text{ so } B \propto I \text{ and } B \propto 1/r.$$

Direction: thumb along conventional current, fingers curl the way $\vec{B}$ circulates.

Fields from several wires add as vectors: same direction add, opposite directions subtract.

12.2 Magnetism and Moving Charges

A charge q moving with velocity $\vec{v}$ in a field feels the force $F = |q|vB \sin \theta$, where θ is the angle between $\vec{v}$ and $\vec{B}$. The force is perpendicular to both. For a positive charge use the right hand: fingers along $\vec{v}$, curl toward $\vec{B}$, thumb gives $\vec{F}$. For a negative charge, reverse the result. A charge at rest, or moving parallel to the field ($\theta = 0$), feels no magnetic force.

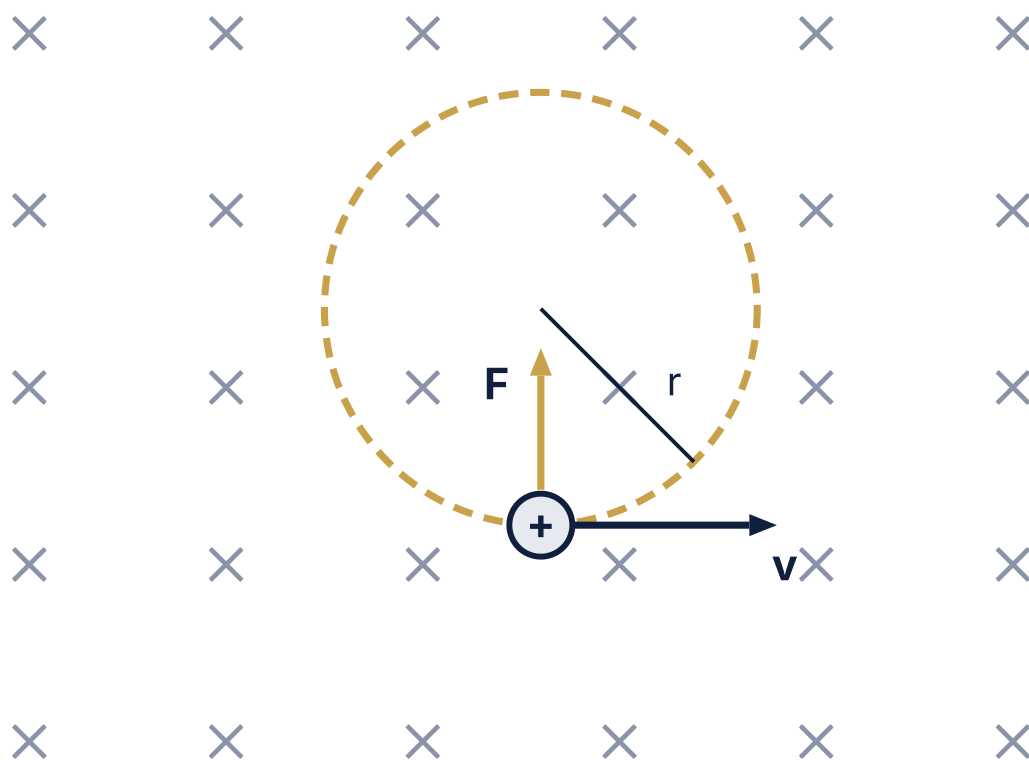
Because $\vec{F}$ is always perpendicular to $\vec{v}$, the magnetic force does no work. It changes the direction of the velocity but never the speed or kinetic energy. When $\vec{v} \perp \vec{B}$ the force supplies the centripetal force: $qvB = mv^2/r$, so $r = mv/(qB)$ and the period $T = 2\pi m/(qB)$ does not depend on speed. If an electric force balances the magnetic force, $qE = qvB$, then only speed $v = E/B$ passes straight through (a velocity selector).

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B into page (x)



A positive charge moving right in a field directed into the page feels an upward force ($\mathbf{v} \times \mathbf{B}$, thumb-fingers-palm rule), so it curves up and circles counterclockwise at constant speed.

Charged particle in a uniform field ($\mathbf{v}$ perpendicular to $\mathbf{B}$)

$F = |q|vB$, always toward the center of the circle.

$r = \frac{mv}{|q|B}$: faster or heavier particles make bigger circles; stronger B makes smaller circles.

Speed and kinetic energy stay constant; only direction changes.

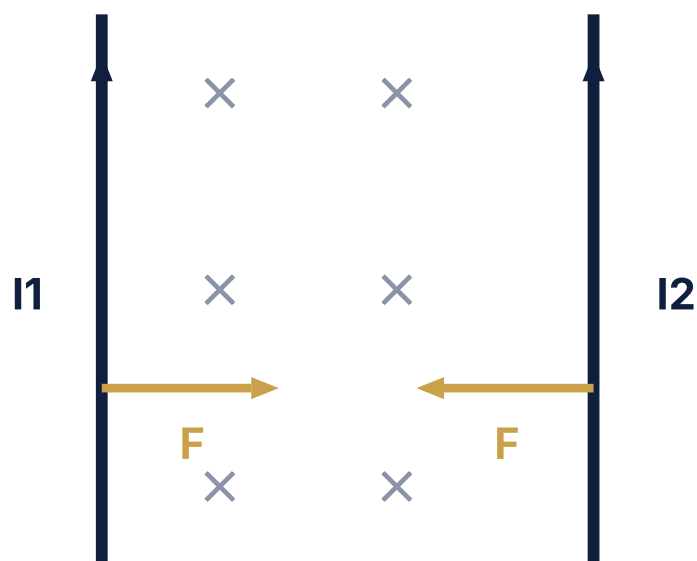
12.3 Magnetism and Current-Carrying Wires

A current is moving charge, so a wire of length L carrying I at angle θ to a field feels $F = ILB \sin \theta$. The direction follows the same right-hand rule with the fingers along the conventional current. Two parallel wires exert forces on each other: wire 1 makes a field at wire 2, and that field pushes on wire 2. The force on a length L is $F = \mu_0 I_1 I_2 L / (2\pi d)$. Parallel currents attract; antiparallel currents repel. The two forces are equal in size and opposite in direction even when the currents differ, because they are an action-reaction pair. In this formula L belongs to the force and the separation d replaces r .

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Crosses: B of wire 1 (into page) at wire 2

Parallel currents attract. The field of wire 1 at wire 2 points into the page; $F = I L B$ for wire 2 then points toward wire 1, and by Newton's third law the forces are equal and opposite.

12.4 Electromagnetic Induction and Faraday's Law

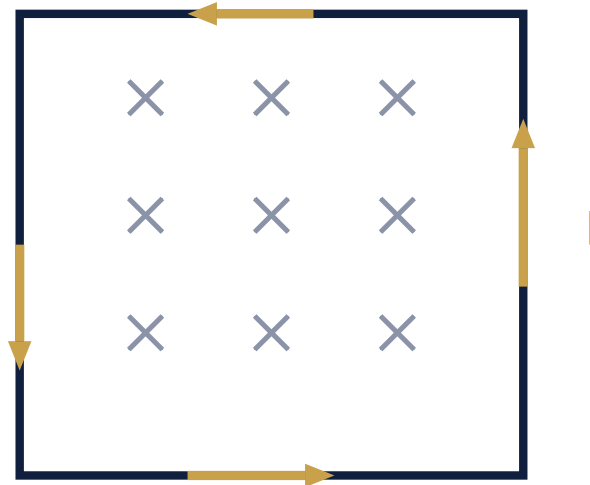
Magnetic flux through a flat loop is $\Phi = BA \cos \theta$, where θ is the angle between $\vec{B}$ and the normal to the loop (unit: weber, $\text{Wb} = \text{T m}^2$). Faraday's law: an emf is induced whenever the flux changes, $\varepsilon = -\Delta\Phi/\Delta t$. Flux can change by changing B , the area, or the angle. A steady flux, however large, induces nothing; the emf depends on the rate of change, which is the slope of a flux-versus-time graph.

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B into page, increasing



Induced I counterclockwise: induced B out of page

Lenz's law: the flux into the page is growing, so the induced current makes a field out of the page inside the loop, which requires a counterclockwise current.

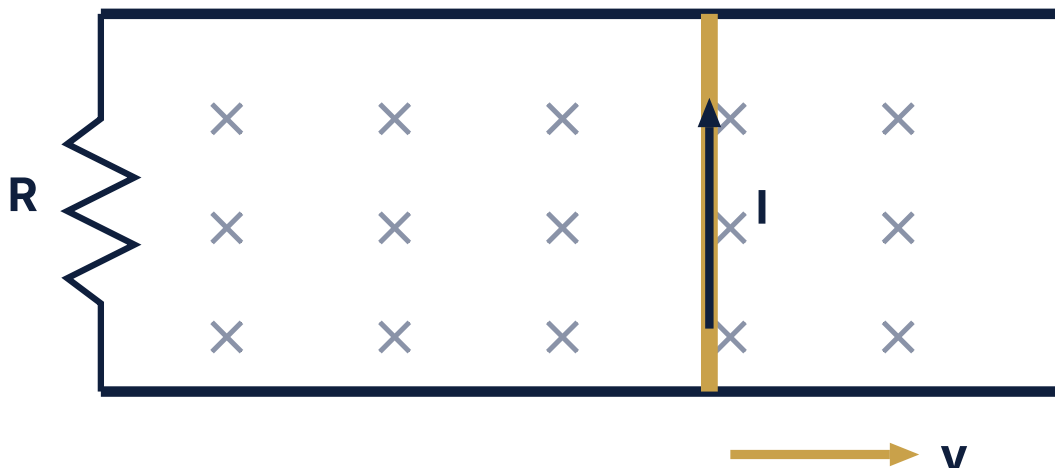
The minus sign is **Lenz's law**: the induced current produces a field that opposes the change in flux. It does not oppose the external field itself. If the flux is increasing, the induced field points opposite to the external field; if it is decreasing, it points the same way. Then use the right-hand rule to find the current. A conducting rod of length L moving at speed v perpendicular to B is a special case: $\mathcal{E} = BLv$, since the area swept grows at rate Lv .

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Rod of length L slides on rails in B (into page)



Motional emf: charges in the rod feel a magnetic force along the rod, so the rod acts as a battery of emf $B L v$. The loop area grows, flux into the page rises, and the induced current is counterclockwise.

Induction routine

1. Flux direction and trend: increasing, decreasing, or constant?
2. Induced field: opposite to the external field if increasing, the same direction if decreasing.
3. Right-hand rule gives the current direction; $|\varepsilon| = |\Delta\Phi|/\Delta t$ gives the size; $I = \varepsilon/R$.

Worked Examples

EXAMPLE 1: FIELD OF A WIRE

A long straight wire carries 12 A. Find the magnitude of the field 4.0 cm from it, and compare it with Earth's field of about 5×10^{-5} T.

SOLUTION

$$B = \mu_0 I / (2\pi r) = (4\pi \times 10^{-7})(12) / (2\pi \times 0.040) = 6.0 \times 10^{-5} \text{ T.}$$

This is slightly larger than Earth's field. Doubling the distance would halve it to 3.0×10^{-5} T.

EXAMPLE 2: PROTON IN A FIELD

A proton ($q = 1.6 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg) moves at 3.0×10^5 m/s perpendicular to a 0.25 T field. Find the force and the radius of its path.

SOLUTION

$$F = qvB = (1.6 \times 10^{-19})(3.0 \times 10^5)(0.25) = 1.2 \times 10^{-14} \text{ N.}$$

$$\text{This force is centripetal: } r = mv/(qB) = (1.67 \times 10^{-27})(3.0 \times 10^5) / [(1.6 \times 10^{-19})(0.25)] = 1.3 \times 10^{-2} \text{ m.}$$

As in the figure, the proton circles counterclockwise at constant speed.

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EXAMPLE 3: ANTIPARALLEL WIRES

Two parallel wires 5.0 cm apart carry 25 A and 40 A in opposite directions. Find the force on a 2.0 m length of either wire and say whether it attracts or repels.

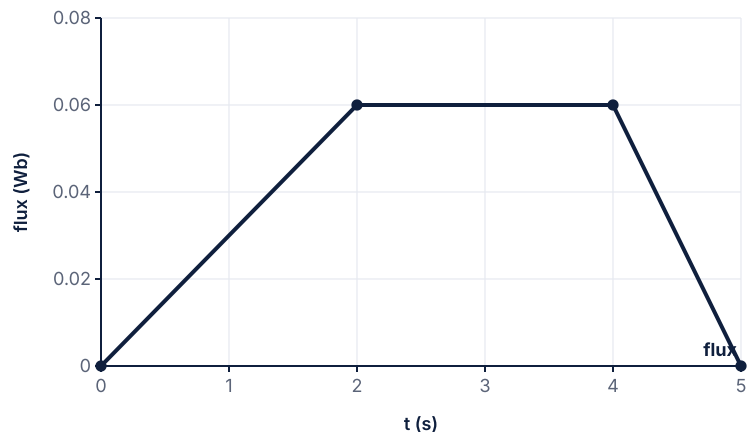
SOLUTION

$$F = \mu_0 I_1 I_2 L / (2\pi d) = (2 \times 10^{-7})(25)(40)(2.0)/0.050 = 8.0 \times 10^{-3} \text{ N.}$$

Antiparallel currents repel. Each wire feels this same force, directed away from the other, even though the currents differ.

EXAMPLE 4: INDUCED EMF AND CURRENT

A rectangular loop 0.20 m by 0.25 m and resistance 0.80 ohm lies perpendicular to a field into the page. The field rises uniformly from 0.20 T to 0.60 T in 0.50 s. Find the emf, the current, and its direction.



Magnetic flux through a loop.

SOLUTION

$$A = 0.050 \text{ m}^2, \Delta\Phi = A\Delta B = (0.050)(0.40) = 0.020 \text{ Wb.}$$

$$|\mathcal{E}| = \Delta\Phi/\Delta t = 0.020/0.50 = 0.040 \text{ V, so } I = \mathcal{E}/R = 0.040/0.80 = 0.050 \text{ A.}$$

The flux into the page is increasing, so the induced field points out of the page; the current is counterclockwise.

Write It Like a 5**Model Response**

Prompt: A circular loop lies in the plane of the page in a uniform field directed out of the page. The field magnitude decreases steadily to zero. Determine the direction of the induced current and justify your answer. The flux through the loop is out of the page, and it decreases because B decreases while A and the angle stay fixed. By Lenz's law the induced current must oppose this decrease in flux. To oppose a decrease in out-of-page flux, the induced magnetic field inside the loop must point out of the page, the same direction as the external field. By the right-hand rule, fingers curled counterclockwise make a thumb pointing out of the page, so the induced current is counterclockwise.

What the grader looks for

(1) Identifies the flux direction and says whether its magnitude is increasing or decreasing. (2) Links the induced field direction to that change, not just to the external field direction. (3) Applies the right-hand rule and states a clear final direction. (4) Refers to a specific cause (B changing), not 'the magnet'.

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Coach's Playbook

- ◆ Write the equation first, then substitute with units, keeping subscripts for each wire.
- ◆ For every direction question, draw $\vec{v}$ or I , draw $\vec{B}$, then do the hand rule slowly. Reverse for a negative charge.
- ◆ For circular motion, start from $qvB = mv^2/r$ and derive r .
- ◆ Lenz's law: decide increasing or decreasing first. The sign of the answer follows.

Common traps

1. Putting wire length in the field formula or the separation in the force formula for a single wire. 2. Using the wrong wire's current or claiming the two parallel-wire forces differ because the currents differ. 3. Saying the induced current opposes the external field instead of opposing the change in flux. 4. Saying a magnetic force speeds a particle up; it does no work.

Quick Check

Q1. By what factor does the field of a straight wire change if the current triples and the distance triples? A: unchanged ($3/3 = 1$). Q2. Does a magnetic field exert a force on a positive charge at rest? A: No, force needs a velocity component perpendicular to B . Q3. A loop sits in a constant, large field. What emf is induced? A: Zero, because the flux is not changing.

Challenge Corner

Singly charged ions of mass 20 u and 22 u ($1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$) are accelerated from rest through 2000 V and enter a 0.50 T field perpendicular to their velocity. How far apart do they land after half a circle? Solution: $qV = \frac{1}{2}mv^2$ gives $v = \sqrt{2qV/m}$, and $r = mv/(qB) = \sqrt{2mV/q}/B$. For 20 u, $r = 0.0576 \text{ m}$; for 22 u, $r = 0.0604 \text{ m}$. They land at distances $2r$ from the entry point: 0.1152 m and 0.1209 m, so they are separated by about $5.6 \times 10^{-3} \text{ m}$, or 5.6 mm.

Practice

PRACTICE Unit 12 Practice

Show your work. A calculator is allowed. Justify conclusions.

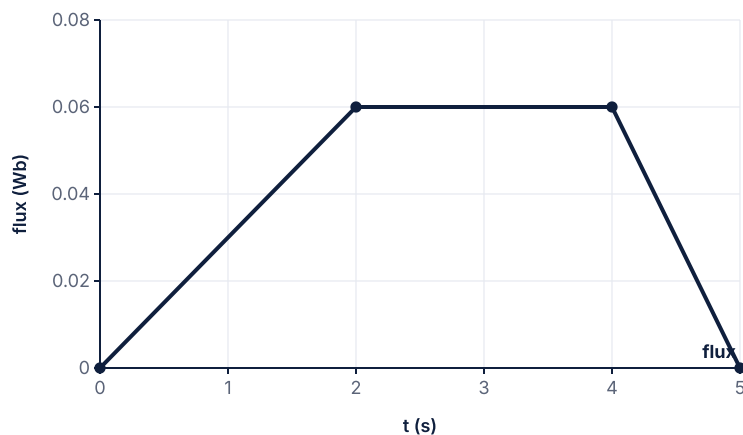
1. Point 1 is a distance r from a long straight wire carrying current I . Point 2 is a distance $2r$ from a different long straight wire carrying current $3I$. The field at point 2 compared with the field at point 1 is
 (A) 0.67 times as large (B) 1.5 times as large (C) 3 times as large
 (D) 6 times as large
2. An electron moves to the right across the page through a uniform magnetic field directed out of the page. The magnetic force on the electron points
 (A) into the page (B) out of the page
 (C) toward the bottom of the page (D) toward the top of the page
3. A proton moves in a circle of radius r in a uniform magnetic field with period T . If the proton's speed is doubled (field unchanged, velocity still perpendicular to the field), the radius and period become
 (A) $2r$ and T (B) $2r$ and $2T$ (C) r and $2T$ (D) $4r$ and T
4. Two long parallel wires carry currents I and $3I$ in the same direction. Which statement about the magnetic force between them is correct?
 (A) The wire with $3I$ feels a force three times larger, and the wires attract.
 (B) The forces have equal magnitude, and the wires repel.
 (C) The forces have equal magnitude, and the wires attract.
 (D) The wire with $3I$ feels a force three times larger, and the wires repel.

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5. A circular loop lies in the plane of the page in a uniform magnetic field directed into the page, and the field magnitude is decreasing. Which statement about the induced current is correct?
- (A) Counterclockwise, since the induced field must point opposite the external field.
 (B) Counterclockwise, since the flux is decreasing.
 (C) There is no current, since the direction of the field does not change.
 (D) Clockwise, since the induced field must point into the page to oppose the loss of flux.
6. The graph shows magnetic flux through a loop. During which interval is the magnitude of the induced emf largest?



Magnetic flux through a loop.

- (A) 0 to 2 s
 (B) 2 to 4 s
 (C) 4 to 5 s
 (D) The emf is the same in 0 to 2 s and 4 to 5 s
7. A proton is accelerated from rest through a potential difference of 500 V, then enters a region of uniform magnetic field 0.40 T perpendicular to its velocity ($q = 1.6 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg).
- (a) Calculate the proton's speed on entering the field.
 (b) Calculate the radius of its circular path.
 (c) A student says the field increases the proton's kinetic energy because the proton accelerates. Explain what is wrong with this claim.
8. A student measures the field B of a long straight wire carrying a steady 10 A at several perpendicular distances r with a magnetic field sensor.
- (a) Describe a procedure, including one way to reduce uncertainty in the data.
 (b) State what quantities to graph so the data form a straight line, and what the slope represents.
 (c) The data are: r (m) = 0.020, 0.040, 0.060, 0.080 and B (μ T) = 101, 49, 34, 26. Use the data to find a value for μ_0 .
9. A conducting rod of length 0.40 m slides to the right at a constant 2.5 m/s along two horizontal rails in a uniform field of 0.60 T directed into the page. The rails are joined at the left end by a 3.0 ohm resistor; the rod and rails have negligible resistance.
- (a) Calculate the emf. (b) Calculate the current and state its direction through the rod (up or down the page).
 (c) Calculate the external force needed to keep the speed constant.
 (d) Show that the power delivered by that force equals the power dissipated in the resistor.
10. Two long parallel wires 0.12 m apart carry 6.0 A and 2.0 A in the same direction.
- (a) Calculate the force per unit length between them and state whether it attracts or repels.
 (b) Find the point on the line between the wires where the net field is zero, measured from the 6.0 A wire.
 (c) Is there any point between the wires where the net field would be zero if the 2.0 A current were reversed? Justify.

Answers

Unit 12: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

12 Magnetism and Electromagnetism

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1. B. Field goes as $1/r$, so the factor is $3/2 = 1.5$. Choice D multiplies the distance factor instead of dividing; choice A inverts the ratio.
2. D. For a positive charge, v along $+x$ and B along $+z$ give a force along $-y$ (toward the bottom). An electron is negative, so the force reverses to the top of the page. Choice C is the answer for a proton.
3. A. From $qvB = mv^2/r$, $r = mv/(qB)$ is proportional to v , so r doubles. The period $T = 2\pi r/v = 2\pi m/(qB)$ does not depend on speed. Choice B wrongly assumes the longer path takes longer.
4. C. Each wire's field acts on the other with $F = \mu_0 I_1 I_2 L/(2\pi d)$, the same expression for both wires, so the forces are equal (Newton's third law). Parallel currents attract. Choice A assumes the larger current feels the larger force, but both forces contain the same product $I \times I$.
5. D. The flux into the page is decreasing, so the induced field must add to it, pointing into the page, which needs a clockwise current. Choice A ties the induced field to the external field direction rather than to the change; choice B states the right trend but the wrong direction.
6. C. The emf magnitude is the slope of the flux graph. Slopes are 0.03 Wb/s , 0 and -0.06 Wb/s , so the largest magnitude is from 4 to 5 s . Choice B confuses large flux with large emf.
7. (a) 2 points: $qV = \frac{1}{2}mv^2$ gives $v = \sqrt{2(1.6 \times 10^{-19})(500)/(1.67 \times 10^{-27})} = 3.1 \times 10^5 \text{ m/s}$.
 (b) 2 points: $qvB = mv^2/r$ so $r = mv/(qB) = (1.67 \times 10^{-27})(3.1 \times 10^5)/[(1.6 \times 10^{-19})(0.40)] = 8.1 \times 10^{-3} \text{ m}$.
 (c) 2 points: The magnetic force is perpendicular to the velocity at every instant, so it does no work ($W = Fd \cos 90^\circ = 0$). By the work-energy theorem the kinetic energy and speed stay constant; the acceleration is centripetal and changes only direction.
8. (a) 2 points: Place the sensor at a measured distance from the wire, record B , repeat at several distances with the current held fixed at 10 A . Reduce uncertainty by taking several readings at each distance and averaging, or by orienting the sensor along the field direction and subtracting Earth's background reading.
 (b) 2 points: $B = (\mu_0 I/2\pi)(1/r)$, so graph B (vertical) against $1/r$ (horizontal). The slope is $\mu_0 I/(2\pi)$.
 (c) 2 points: $1/r = 50, 25, 16.7, 12.5$ per m. A best-fit line through the points has slope about $2.0 \times 10^{-6} \text{ T m}$. Then $\mu_0 = 2\pi(\text{slope})/I = 2\pi(2.0 \times 10^{-6})/10 = 1.3 \times 10^{-6} \text{ T m/A}$, consistent with $4\pi \times 10^{-7}$ within the scatter.
9. (a) 1 point: $\varepsilon = BLv = (0.60)(0.40)(2.5) = 0.60 \text{ V}$.
 (b) 2 points: $I = \varepsilon/R = 0.60/3.0 = 0.20 \text{ A}$. Positive charges in the rod feel $q\vec{v} \times \vec{B}$ upward (v right, B into page), so the current is up the page through the rod (counterclockwise around the loop, as Lenz's law requires since the flux into the page grows).
 (c) 2 points: The field exerts a force $F = ILB = (0.20)(0.40)(0.60) = 0.048 \text{ N}$ to the left, so a 0.048 N force to the right is needed.
 (d) 1 point: $P = Fv = (0.048)(2.5) = 0.12 \text{ W}$ and $P = I^2R = (0.20)^2(3.0) = 0.12 \text{ W}$.
10. (a) 2 points: $F/L = \mu_0 I_1 I_2/(2\pi d) = (2 \times 10^{-7})(6.0)(2.0)/0.12 = 2.0 \times 10^{-5} \text{ N/m}$; parallel currents attract.
 (b) 2 points: Between the wires the fields point in opposite directions. Setting $6.0/x = 2.0/(0.12 - x)$ gives $x = 0.09 \text{ m}$ from the 6.0 A wire (and 0.03 m from the 2.0 A wire).
 (c) 2 points: No. With the currents opposite, the two fields point in the same direction between the wires and add, so they cannot cancel there; the null point would lie outside, beyond the smaller current.

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Unit 13: Geometric Optics

Exam weighting: about 12-15% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

13 Geometric Optics

The ray model of light: reflection, mirror and lens images, and refraction. One pair of equations and three principal rays handle every case.

Why This Matters

Geometric optics is 12-15% of the multiple-choice score. The math is short, but points are lost on sign conventions, on drawing rays that do not follow the three rules, and on saying that the frequency of light changes in glass. Check every problem twice: a ray diagram for the picture, the equation for the number.

Core Ideas

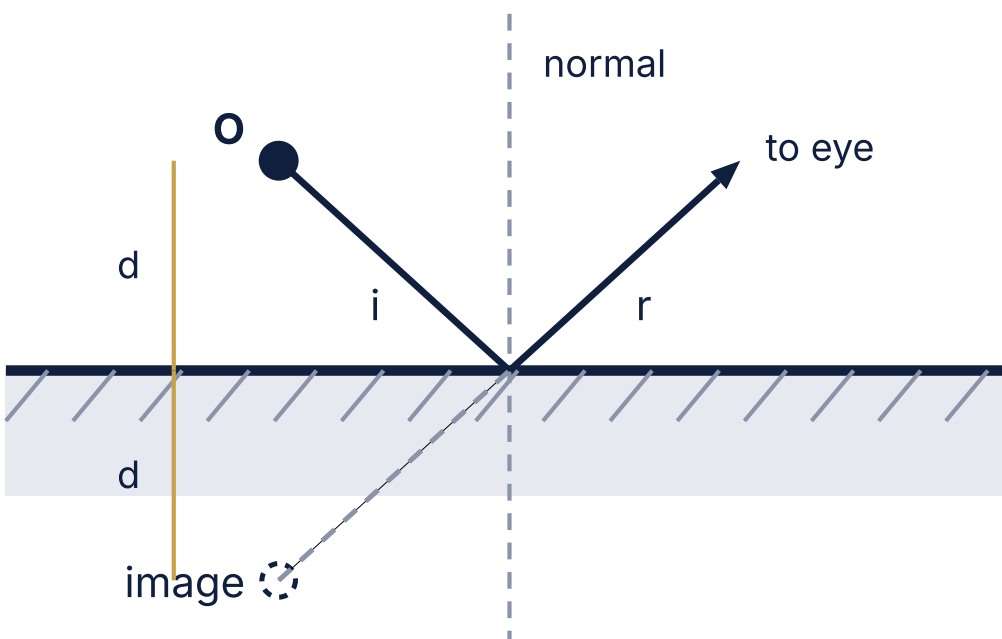
13.1 Reflection

Light travels in straight lines (rays) in a uniform medium. At a smooth surface the **law of reflection** says the angle of incidence equals the angle of reflection, $\theta_i = \theta_r$, with both angles measured from the **normal**, the line perpendicular to the surface, not from the surface itself. Rough surfaces reflect each ray by the same law but send rays in many directions (diffuse reflection). A plane mirror forms a **virtual** image: light only appears to come from it. The image is upright, the same size as the object, and as far behind the mirror as the object is in front.

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Plane mirror: angle of incidence equals angle of reflection (both measured from the normal). The image is virtual, as far behind the mirror as the object is in front.

13.2 Images Formed by Mirrors

A spherical mirror has a focal length $f = R/2$, half its radius of curvature. A concave mirror brings parallel rays to a real focus; a convex mirror spreads them out as if from a point behind it. Use two or three principal rays from the top of the object: parallel to the axis, then reflected through F (or as if from F); through F, then reflected parallel; and through the center C, reflected straight back. Where reflected rays cross is a real image; where their backward extensions cross is a virtual image.

Mirror and thin-lens equations

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \text{ and } m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}.$$

Use the table of signs below; the same equations serve mirrors and lenses.

$|m| > 1$ means enlarged; $m < 0$ means inverted.

Quantity	Positive when	Negative when
f (mirror)	concave	convex
f (lens)	converging (thicker in middle)	diverging (thinner in middle)
d_i	real image (in front of mirror, past the lens)	virtual image (behind mirror, same side as object for a lens)
m	image upright	image inverted

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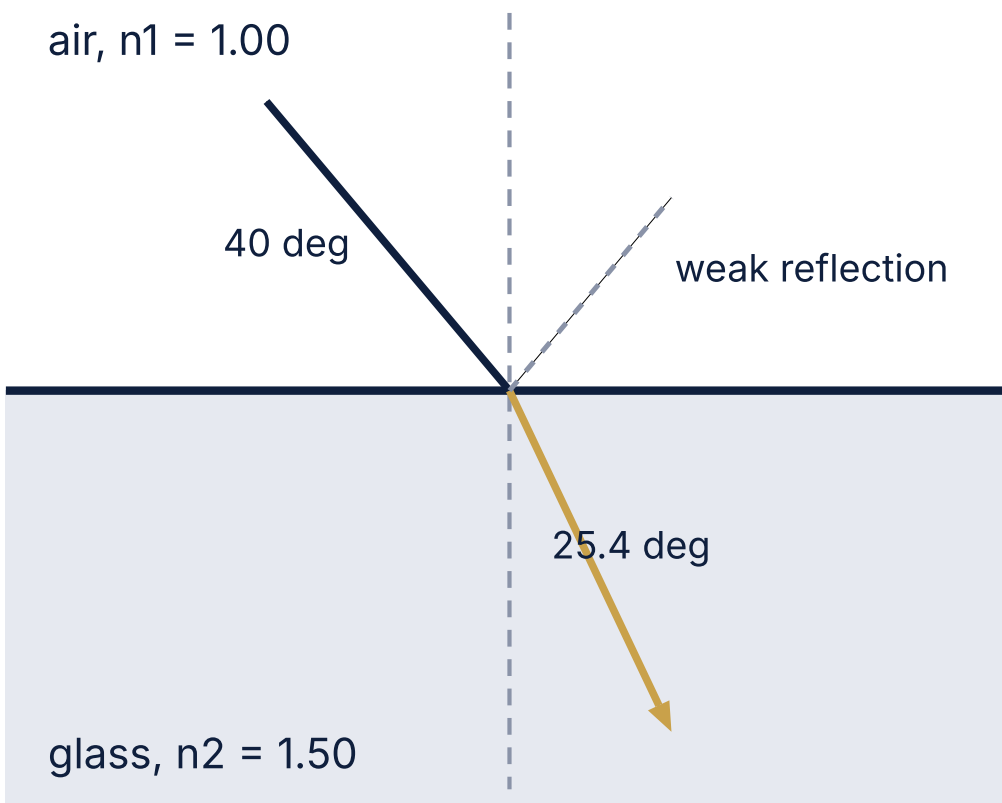
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Object position (concave mirror)	Image	Size
$do > 2f$ (beyond C)	real, inverted	smaller
$f < do < 2f$	real, inverted	larger
$do < f$	virtual, upright	larger

13.3 Refraction

Light slows down in matter. The **index of refraction** is $n = c/v$, with $c = 3.00 \times 10^8$ m/s and $n \geq 1$. When light crosses a boundary its **frequency stays the same** (set by the source), while speed and wavelength change: $\lambda = \lambda_0/n$ where λ_0 is the vacuum wavelength. **Snell's law** gives the direction: $n_1 \sin \theta_1 = n_2 \sin \theta_2$. Going into higher n the ray bends toward the normal; going into lower n it bends away. A ray along the normal is not deflected.



Refraction from air into glass. The ray bends toward the normal because it slows down. Angles are measured from the normal.

Refraction relations

$n = c/v$, $\lambda = \lambda_0/n$, f unchanged.

$n_1 \sin \theta_1 = n_2 \sin \theta_2$; critical angle $\sin \theta_c = n_2/n_1$ ($n_1 > n_2$).

If light travels toward a lower index and $\sin \theta_2$ would exceed 1, no refracted ray exists: **total internal reflection** occurs for angles of incidence above the critical angle, $\sin \theta_c = n_2/n_1$ with $n_1 > n_2$. Fiber optics relies on this. Because n depends slightly on wavelength, violet light bends more than red in glass (dispersion), which is why a prism spreads white light.

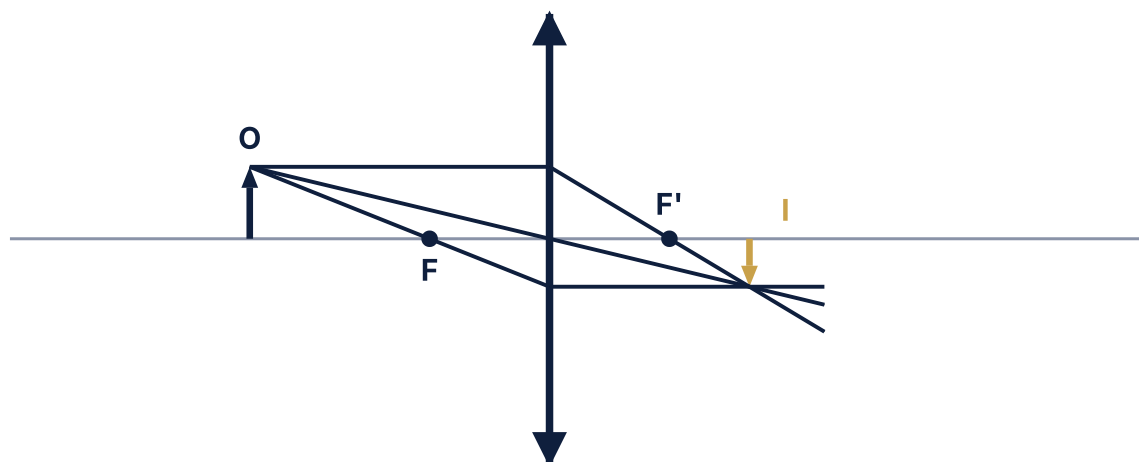
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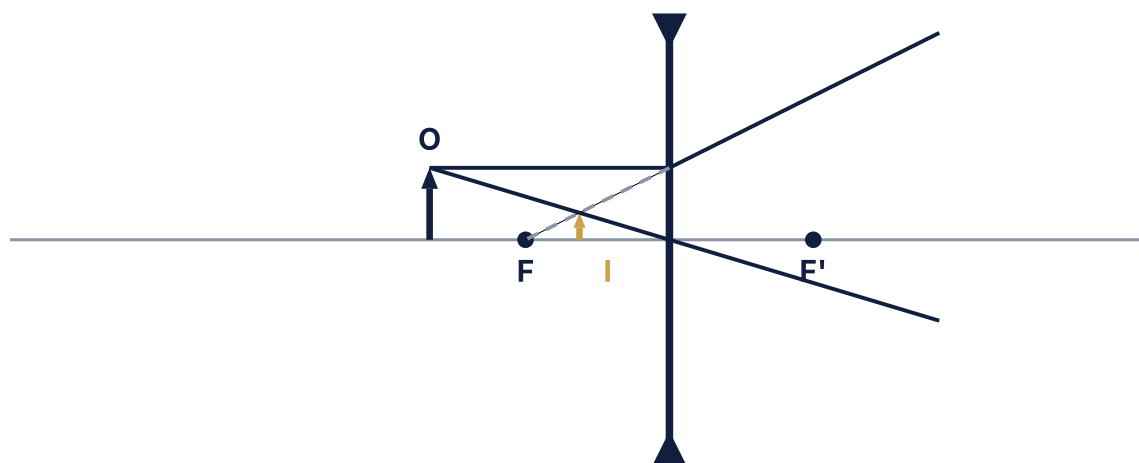
13.4 Images Formed by Lenses

Thin lenses use the same equation and magnification formula. A converging lens (positive f) sends rays parallel to the axis through the far focal point; a diverging lens (negative f) spreads them as if from the near focal point. The three rays are: parallel then through F' (or away from F); through the center, undeviated; through F then parallel. A real image forms on the opposite side from the object and can be caught on a screen; a virtual image forms on the same side as the object and cannot. A converging lens gives a real inverted image when $d_o > f$ and a virtual upright enlarged image when $d_o < f$ (the magnifying glass). A diverging lens gives only a virtual, upright, smaller image for a real object.



$$d_o = 25 \text{ cm} \quad d_i = 16.7 \text{ cm} \quad f = 10 \text{ cm}$$

Converging lens, $f = 10 \text{ cm}$, object 6 cm tall at 25 cm. Three rays (parallel then through F' , through the center, through F then parallel) meet at 16.7 cm: real, inverted, 4.0 cm tall.



$$d_o = 20 \text{ cm} \quad d_i = -7.5 \text{ cm} \quad f = -12 \text{ cm}$$

Diverging lens, $f = -12 \text{ cm}$, object 6 cm tall at 20 cm. The parallel ray leaves as if from F ; the center ray goes straight. The rays diverge, so the image (dashed extensions) is virtual, upright, 7.5 cm from the lens on the object side, 2.25 cm tall.

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Worked Examples

EXAMPLE 1: PLANE MIRROR

A student 2.4 m from a plane mirror walks toward it at 1.5 m/s. (a) Where is the image? (b) How fast does the image approach the student? (c) A ray strikes the mirror making 35 degrees with the surface. Find the angle of reflection and the angle between the incident and reflected rays.

SOLUTION

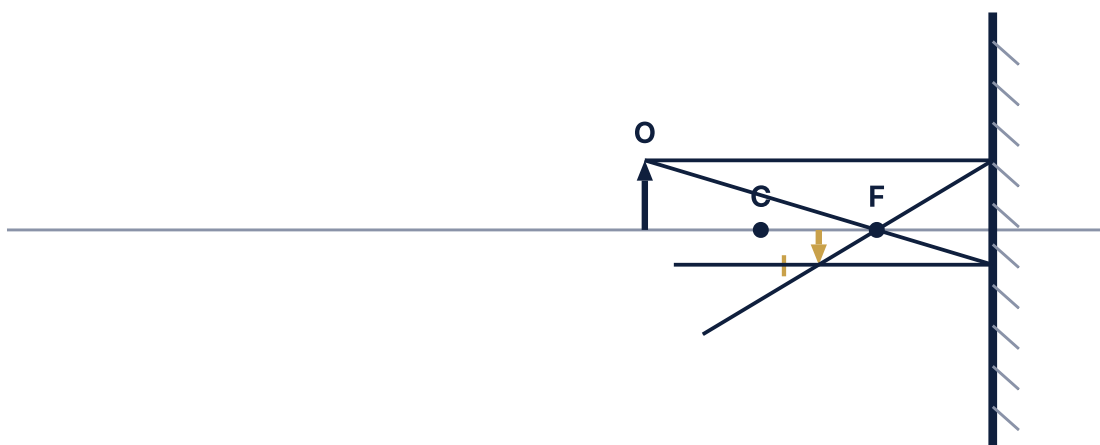
(a) The image is 2.4 m behind the mirror, 4.8 m from the student.

(b) The image moves toward the mirror at 1.5 m/s while the student moves toward it at 1.5 m/s, so the gap closes at 3.0 m/s.

(c) The angle of incidence from the normal is $90 - 35 = 55$ degrees, so the angle of reflection is 55 degrees and the rays differ in direction by $2 \times 55 = 110$ degrees.

EXAMPLE 2: CONCAVE MIRROR

A concave mirror has $f = 10$ cm. A 6.0 cm object is 30 cm away. Find the image, then repeat with the object 6.0 cm from the mirror.



$$d_o = 30 \text{ cm} \quad d_i = 15 \text{ cm} \quad f = 10 \text{ cm}$$

Concave mirror, $f = 10$ cm, object 6 cm tall at 30 cm. Ray 1 leaves parallel to the axis and reflects through F; ray 2 passes through F and reflects parallel. They cross 15 cm in front of the mirror, giving a real, inverted image 3 cm tall.

SOLUTION

$1/d_i = 1/f - 1/d_o = 1/10 - 1/30 = 2/30$, so $d_i = 15$ cm. $m = -15/30 = -0.50$, so $h_i = -3.0$ cm: real, inverted, smaller (see figure).

For $d_o = 6.0$: $1/d_i = 1/10 - 1/6 = -1/15$, so $d_i = -15$ cm, $m = +2.5$, $h_i = +15$ cm. The negative d_i means virtual: upright and enlarged, behind the mirror. The object is inside F, so the rays never meet.

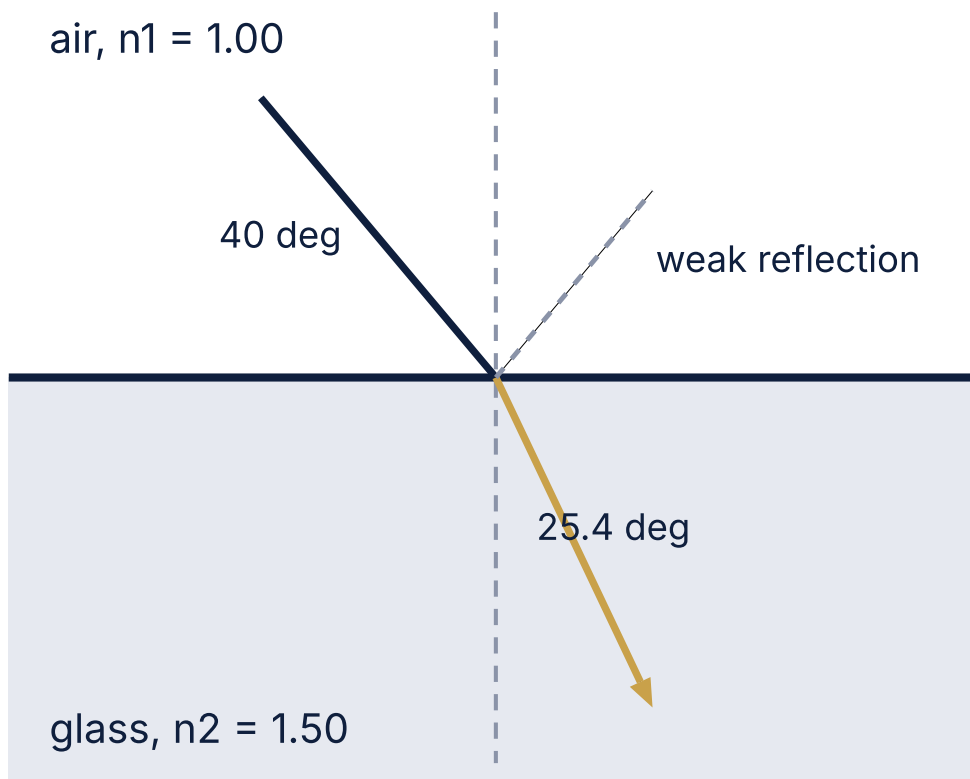
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EXAMPLE 3: REFRACTION AND SPEED

A ray in air strikes glass ($n = 1.50$) at 40 degrees from the normal. Find the refraction angle, the speed in glass, and the wavelength in glass of light with vacuum wavelength 600 nm. Could light inside the glass undergo total internal reflection at the glass-air surface?



Refraction from air into glass. The ray bends toward the normal because it slows down. Angles are measured from the normal.

SOLUTION

$\sin \theta_2 = (1.00) \sin 40^\circ / 1.50 = 0.428$, so $\theta_2 = 25.4^\circ$.

$v = c/n = 2.00 \times 10^8$ m/s and $\lambda = 600/1.50 = 400$ nm. The frequency, 5.0×10^{14} Hz, is unchanged.

Inside the glass, $\sin \theta_c = 1.00/1.50$, so $\theta_c = 41.8^\circ$. Light striking the surface at a larger angle is totally reflected.

EXAMPLE 4: A PROJECTOR LENS

A converging lens with $f = 15$ cm must form a real image magnified 3.0 times on a screen. Find the object and screen distances from the lens.

SOLUTION

Real and magnified 3 times means $m = -3$, so $d_i = 3d_o$.

$\frac{1}{15} = \frac{1}{d_o} + \frac{1}{3d_o} = \frac{4}{3d_o}$, so $d_o = 20$ cm and $d_i = 60$ cm.

Check with the equation: $1/20 + 1/60 = 4/60 = 1/15$. The object sits just beyond F, and the screen is far away.

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Model Response

Prompt: A beam of light travels from air into a glass block at an angle to the normal. Describe how the speed, wavelength and frequency of the light change and explain the change in direction. The frequency is unchanged because it is set by the source and the number of waves arriving at the boundary per second must equal the number leaving. The speed decreases, $v = c/n$, because glass has a higher index of refraction than air. Since $v = f\lambda$ and f is constant, the wavelength also decreases, $\lambda = \lambda_0/n$. Snell's law, $n_{\text{air}} \sin \theta_1 = n_{\text{glass}} \sin \theta_2$, with $n_{\text{glass}} > n_{\text{air}}$ gives $\sin \theta_2 < \sin \theta_1$, so the ray bends toward the normal.

What the grader looks for

(1) States that frequency is unchanged and gives a reason. (2) States speed and wavelength both decrease, with $n = c/v$ or $v = f\lambda$. (3) Uses Snell's law with the correct ordering of the indices to reach 'toward the normal'. (4) Measures angles from the normal.

Coach's Playbook

- 1 Draw the axis, the mirror or lens, F and the object to scale before computing.
- 2 Compute d_i and m with the equation, keeping the sign of every quantity.
- 3 Draw two of the three principal rays and check that the picture matches the signs.
- 4 State the image in four words: real or virtual, upright or inverted, larger or smaller, where.
- 5 For refraction, label n_1 and n_2 on the diagram first, then predict the bend before using Snell's law.

Common traps

1. Measuring angles from the surface instead of the normal. 2. Using $1/f = 1/d_o + 1/d_i$ as $f = d_o + d_i$ or forgetting to invert at the end. 3. Saying the frequency or the color changes in glass; only speed and wavelength change. 4. Drawing rays that bend at the wrong place: for a thin lens, bend at the lens plane, not at the object.

Quick Check

Q1. Where does a concave mirror with $f = 15$ cm focus a distant star? A: 15 cm in front of the mirror, on the axis. Q2. A mirror or lens with a negative focal length: which types? A: Convex mirror, diverging lens. Q3. Does the frequency of light change as it enters water? A: No, only the speed and wavelength.

Challenge Corner

A converging lens of focal length 20 cm is placed between a lit object and a screen 100 cm apart. Find every lens position that gives a sharp image on the screen and the magnification in each case. Solution: $d_o + d_i = 100$ and $1/d_o + 1/d_i = 1/20$ give $d_o d_i = 2000$. So $d_o(100 - d_o) = 2000$, i.e. $d_o^2 - 100d_o + 2000 = 0$, $d_o = 72.4$ cm or 27.6 cm. Then $d_i = 27.6$ cm ($m = -0.38$, reduced) or 72.4 cm ($m = -2.6$, enlarged). Two positions work, and the object and image distances swap.

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Practice

PRACTICE Unit 13 Practice

Show your work. A calculator is allowed. Justify conclusions.

1. An object is 3.0 m in front of a plane mirror. What is the distance between the object and its image?
(A) 3.0 m (B) 1.5 m (C) 6.0 m (D) 9.0 m
2. A concave mirror has focal length 20 cm. An object is placed 30 cm in front of it. The image is
(A) real, inverted, 12 cm from the mirror (B) real, inverted, 60 cm from the mirror
(C) virtual, upright, 60 cm behind the mirror (D) virtual, upright, 12 cm behind the mirror
3. A ray in water ($n = 1.33$) strikes the water-air boundary at 30 degrees from the normal. The angle of the refracted ray in air, measured from the normal, is closest to
(A) 22 degrees (B) 30 degrees (C) 48 degrees
(D) 42 degrees
4. Light passes from air into glass. Which row correctly describes its speed, wavelength and frequency in the glass?
(A) speed decreases, wavelength decreases, frequency unchanged
(B) speed decreases, wavelength unchanged, frequency decreases
(C) speed decreases, wavelength increases, frequency unchanged
(D) speed, wavelength and frequency all decrease
5. A converging lens has $f = 12$ cm. An object is placed 8.0 cm from the lens. The image is
(A) real, inverted, 24 cm on the far side of the lens
(B) virtual, upright, 4.8 cm on the object side
(C) virtual, upright, 24 cm on the object side, three times the object height
(D) real, inverted, 4.8 cm on the far side of the lens
6. An optical fiber has a core of index 1.50 surrounded by cladding of index 1.40. What is the smallest angle of incidence, measured from the normal at the core-cladding boundary, for total internal reflection?
(A) 21 degrees (B) 69 degrees (C) 42 degrees
(D) 90 degrees
7. A converging lens has focal length 20 cm. A 4.0 cm tall object is 30 cm from the lens.
(a) Calculate the image distance and the image height, including sign.
(b) Describe the image (real or virtual, upright or inverted).
(c) The object is moved to 15 cm from the lens. Calculate the new image distance and magnification and describe the image.
(d) Explain why a screen can display the image in part (a) but not in part (c).
8. A student wants to determine the focal length of a converging lens using a light source with an object slit, the lens, a screen, and a meter stick.
(a) Describe a procedure that gives several pairs of object and image distances, and one way to reduce uncertainty in locating the image.
(b) Explain what to graph so the data fall on a straight line and how f is found from the line.
(c) Measured data: d_o (cm) = 20, 25, 30, 40, 50 and d_i (cm) = 61, 37, 30, 24, 21.5. Use the data to find f .
9. Light in a glass slab ($n = 1.60$) strikes the boundary with water ($n = 1.33$) at 60 degrees from the normal.
(a) Calculate the critical angle for light going from glass to water.
(b) Determine whether the light refracts into the water or is totally reflected, and justify.
(c) If the angle of incidence is reduced to 50 degrees, calculate the angle of refraction in the water.
(d) Compare the speed of the light in the glass and in the water.
10. A concave mirror forms a real image twice as tall as an object that is 12 cm from the mirror.
(a) Find the image distance, the focal length and the radius of curvature.
(b) The object is moved to 20 cm from the mirror. Find the new image distance and say whether the image is larger or smaller than the object.
(c) A student claims that replacing the concave mirror with a convex mirror of the same radius could also produce a real image. Evaluate the claim.

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Answers Unit 13: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

13 Geometric Optics

PRACTICE · UNIT 13 PRACTICE

1. C. The image is as far behind the mirror as the object is in front, so the separation is $3.0 + 3.0 = 6.0$ m. Choice A forgets that the image is behind the mirror.
2. B. $1/d_i = 1/20 - 1/30 = 1/60$, so $d_i = +60$ cm, real and inverted ($m = -2$). Choice A uses $d_i = d_o f/(d_o + f)$, which is not the equation.
3. D. $n_1 \sin(30) = n_2 \sin(\theta)$: $\sin(\theta) = 1.33 \times 0.50 = 0.665$, $\theta = 41.7$ degrees, bending away from the normal. Choice A divides by 1.33 instead (going from air into water), and choice C is the complement of the correct angle.
4. A. Frequency is set by the source and is unchanged. The speed falls to c/n and, since $v = f \lambda$, the wavelength falls too. Choice C reverses the wavelength change.
5. C. $1/d_i = 1/12 - 1/8 = -1/24$, so $d_i = -24$ cm: virtual, same side as the object, and $m = -d_i/d_o = +3$. The object is inside the focal length. Choice A would be right only for an object beyond F .
6. B. $\sin(\theta_c) = n_2/n_1 = 1.40/1.50 = 0.933$, so $\theta_c = 69$ degrees. Choice A is the angle measured from the boundary surface rather than the normal, and choice C wrongly uses air outside.
7. (a) 3 points: $1/d_i = 1/20 - 1/30 = 1/60$, $d_i = 60$ cm; $m = -60/30 = -2$; $h_i = -8.0$ cm.
 (b) 1 point: real, inverted (and enlarged).
 (c) 3 points: $1/d_i = 1/20 - 1/15 = -1/60$, $d_i = -60$ cm; $m = +4$, $h_i = +16$ cm; virtual, upright, enlarged, on the object side.
 (d) 1 point: A screen can only display a real image, where rays actually converge; in (c) the rays diverge after the lens and only appear to come from a point.
8. (a) 2 points: Set the source and screen far apart. Place the lens at several positions, adjust the screen until the image is sharp, and record d_o and d_i with the meter stick. To reduce uncertainty, find the screen position of sharpest focus several times (from both directions) and average, or use a larger object and measure the sharpness range.
 (b) 2 points: Graph $1/d_i$ against $1/d_o$ since $1/d_i = 1/f - 1/d_o$. The slope should be -1 and the vertical intercept is $1/f$.
 (c) 2 points: $1/d_o = 0.050, 0.040, 0.0333, 0.025, 0.020 \text{ cm}^{-1}$ and $1/d_i = 0.0164, 0.0270, 0.0333, 0.0417, 0.0465 \text{ cm}^{-1}$. The best-fit line has slope about -1.00 and intercept about 0.0667 cm^{-1} , so $f = 1/0.0667 = 15$ cm.
9. (a) 1 point: $\sin \theta_c = 1.33/1.60 = 0.831$, so $\theta_c = 56.2$ degrees.
 (b) 2 points: 60 degrees exceeds 56.2 degrees, and light is going into a lower index, so total internal reflection occurs and no light enters the water.
 (c) 2 points: $1.60 \sin 50^\circ = 1.33 \sin \theta_2$ gives $\sin \theta_2 = 0.9215$, $\theta_2 = 67.2$ degrees. Since 50 is below the critical angle, refraction occurs.
 (d) 1 point: $v = c/n$: glass 1.88×10^8 m/s, water 2.26×10^8 m/s. The light is faster in water, which has the lower index.
10. (a) 3 points: Real and twice as tall means $m = -2$, so $d_i = 2d_o = 24$ cm. $1/f = 1/12 + 1/24 = 3/24$, so $f = 8.0$ cm and $R = 2f = 16$ cm.
 (b) 2 points: $1/d_i = 1/8 - 1/20 = 3/40$, $d_i = 13.3$ cm; $m = -13.3/20 = -0.67$, so the image is smaller (the object is now beyond C at 16 cm).
 (c) 1 point: The claim is false. A convex mirror has $f < 0$, so $1/d_i = 1/f - 1/d_o$ is negative for every positive d_o ; the image is always virtual, upright and smaller for a real object.

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Unit 14: Waves, Sound, and Physical Optics

Exam weighting: about 12-15% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

14 Waves, Sound, and Physical Optics

How waves travel, reflect, shift in frequency, and interfere: from pulses on a string to standing waves, double slits, gratings and thin films.

Why This Matters

Waves, sound and physical optics are 12-15% of the multiple-choice score. Students reverse the wavelength and frequency relationships for red and violet light, misread the order m , and forget that frequency is fixed by the source. Think path difference and phase every time.

Core Ideas

14.1 Properties of Wave Pulses and Waves

A wave transfers energy through a medium without transferring matter. In a **transverse** wave the medium moves perpendicular to the direction of travel (string, light); in a **longitudinal** wave it moves along it (sound). The speed of a wave is set by the medium, never by how hard you shake the source: on a string $v = \sqrt{F_T/\mu}$, so more tension is faster and a heavier string (μ larger) is slower. Overlapping pulses add their displacements (**superposition**) and then continue unchanged.

14.2 Periodic Waves

A source oscillating with frequency f and period $T = 1/f$ makes a periodic wave with $v = f\lambda$. A displacement-position graph shows λ and amplitude; a displacement-time graph at one point shows T . Intensity (power per area) is proportional to amplitude squared, and for a point source it falls as $1/r^2$. Sound in air travels near 343 m/s; each 10 dB is a factor of 10 in intensity.

14.3 Boundary Behavior of Waves and Polarization

At a boundary part of a wave reflects and part transmits; the frequency never changes, but speed and wavelength do. A pulse reflects inverted from a fixed end and upright from a free end. Going from a light string to a heavy one (slower), the reflected pulse is inverted and the transmitted pulse upright; heavy to light, the reflected pulse is upright. **Polarization** is possible only for transverse waves: a polarizing filter passes the electric-field component along its axis. Unpolarized light loses half its intensity in one filter; a second filter at angle θ gives $I = I_0 \cos^2 \theta$ (I_0 = intensity reaching it).

14.4 Electromagnetic Waves

Light, radio and X-rays are transverse electromagnetic waves with electric and magnetic fields perpendicular to each other and to the travel direction. They need no medium and move in vacuum at $c = 3.00 \times 10^8$ m/s, with $c = f\lambda$. In order of increasing frequency: radio, microwave, infrared, visible (about 700 nm red to 400 nm violet), ultraviolet, X-ray, gamma.

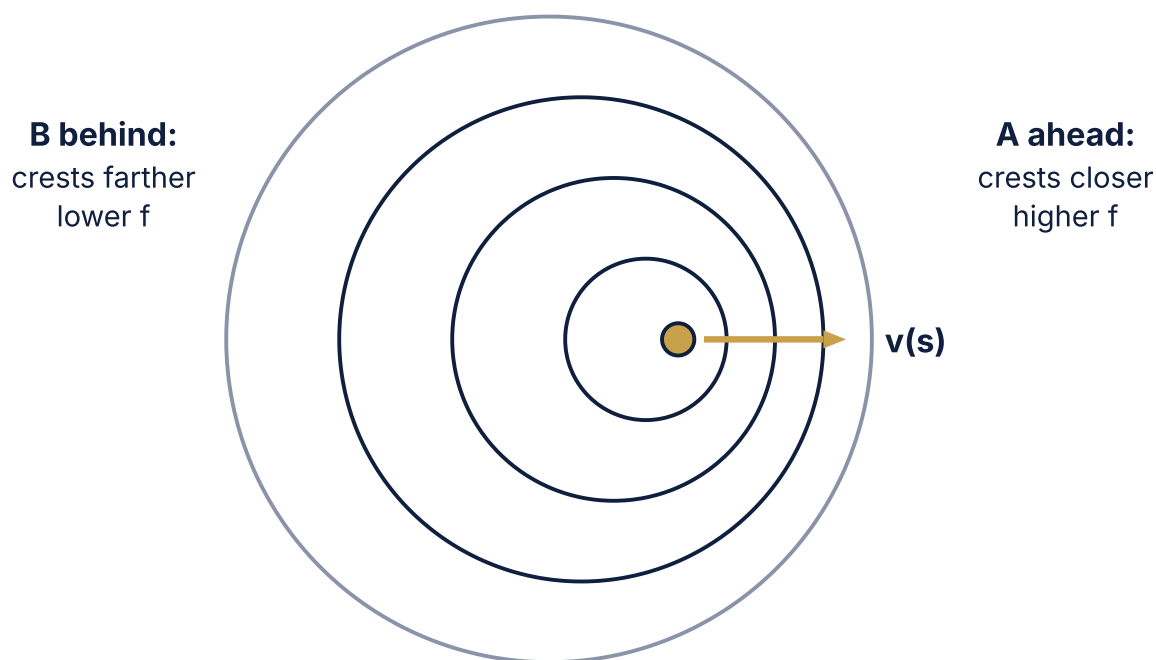
14.5 The Doppler Effect

When a source and observer move relative to each other, the observed frequency shifts. Approach raises it; recession lowers it. The wave speed in the medium is unchanged.

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Source speed = 0.4 v . Circles are crests emitted at equal time steps

Wavefronts from a moving source: each circle is centered where the source was when that crest was emitted.

Doppler effect for sound

$$f_o = f_s \frac{v \pm v_o}{v \mp v_s} \text{ with } v \text{ the speed of sound.}$$

Use the upper signs (+ in the numerator, - in the denominator) when observer and source approach each other; the lower signs when they recede.

Light shifts the same way: approach gives a blueshift, recession a redshift.

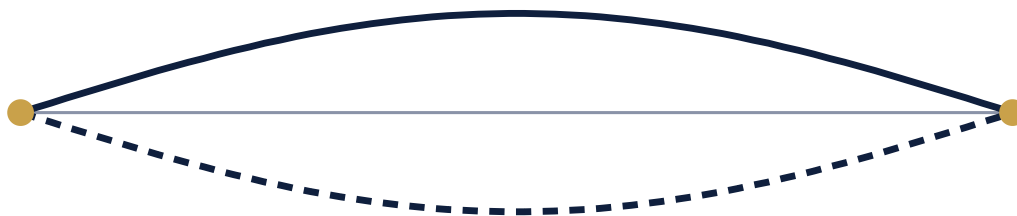
14.6 Wave Interference and Standing Waves

Two coherent waves arriving at a point add. If the path difference is $\Delta r = m\lambda$ ($m = 0, 1, 2, \dots$) the waves are in phase and interfere constructively; if $\Delta r = (m + \frac{1}{2})\lambda$ they cancel. A wave and its reflection in the same medium form a **standing wave** with fixed nodes and antinodes. Only wavelengths that fit the boundaries survive (resonance). A string fixed at both ends, or a pipe open at both ends, has $\lambda_n = 2L/n$ and $f_n = nv/(2L)$ for $n = 1, 2, 3, \dots$. A pipe closed at one end has $\lambda_n = 4L/n$ and $f_n = nv/(4L)$ for odd n only.

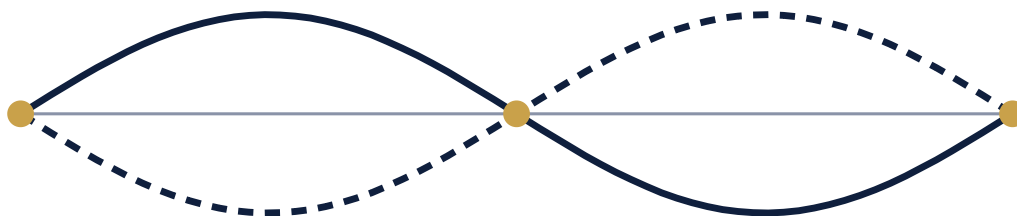
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$$n = 1: L = \lambda/2$$



$$n = 2: L = \lambda$$



$$n = 3: L = 3\lambda/2$$

Gold dots: nodes. $f(n) = n f(1)$

Standing waves on a string fixed at both ends; solid and dashed curves are the extremes of the motion.

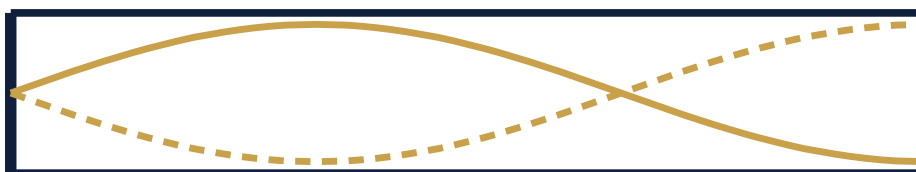
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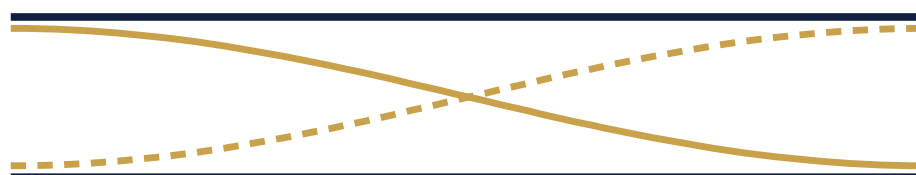
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Closed-open, $n = 1$: $L = \lambda/4$



Closed-open, $n = 3$: $L = 3\lambda/4$



Open-open, $n = 1$: $L = \lambda/2$

Curves show displacement amplitude of the air

Standing sound waves: a closed end is a displacement node, an open end an antinode.

14.7 Diffraction

Waves spread when they pass through an opening or around an obstacle, and the effect is strongest when the opening width a is comparable to or smaller than λ . For a single slit the central bright band is widest, with dark fringes where $a \sin \theta = m\lambda$ ($m = 1, 2, \dots$). A narrower slit or a longer wavelength gives a broader pattern.

14.8 Double-Slit Interference and Diffraction Gratings

Two narrow slits a distance d apart act as coherent sources. Bright fringes appear where $d \sin \theta = m\lambda$ ($m = 0, \pm 1, \pm 2, \dots$ is the order); dark fringes where $d \sin \theta = (m + \frac{1}{2})\lambda$. For small angles, with the screen a distance L away, the fringe at order m lies at $y_m = m\lambda L/d$ and adjacent fringes are separated by $\Delta y = \lambda L/d$. The spacing is **directly proportional** to λ and L and inversely proportional to d ; red gives wider spacing than violet. A diffraction grating has many slits per millimeter, so d is the reciprocal of the line density. It obeys the same equation but gives sharp, bright maxima, and white light spreads into spectra with red farthest from the center.

Double slit and grating

Bright: $d \sin \theta = m\lambda$. Dark: $d \sin \theta = (m + \frac{1}{2})\lambda$. Highest order: $m < d/\lambda$.

Small angles: $\Delta y = \frac{\lambda L}{d}$. For the distance across N spacings, divide by N .

Frequency is set by the source; in a medium $\lambda = \lambda_0/n$.

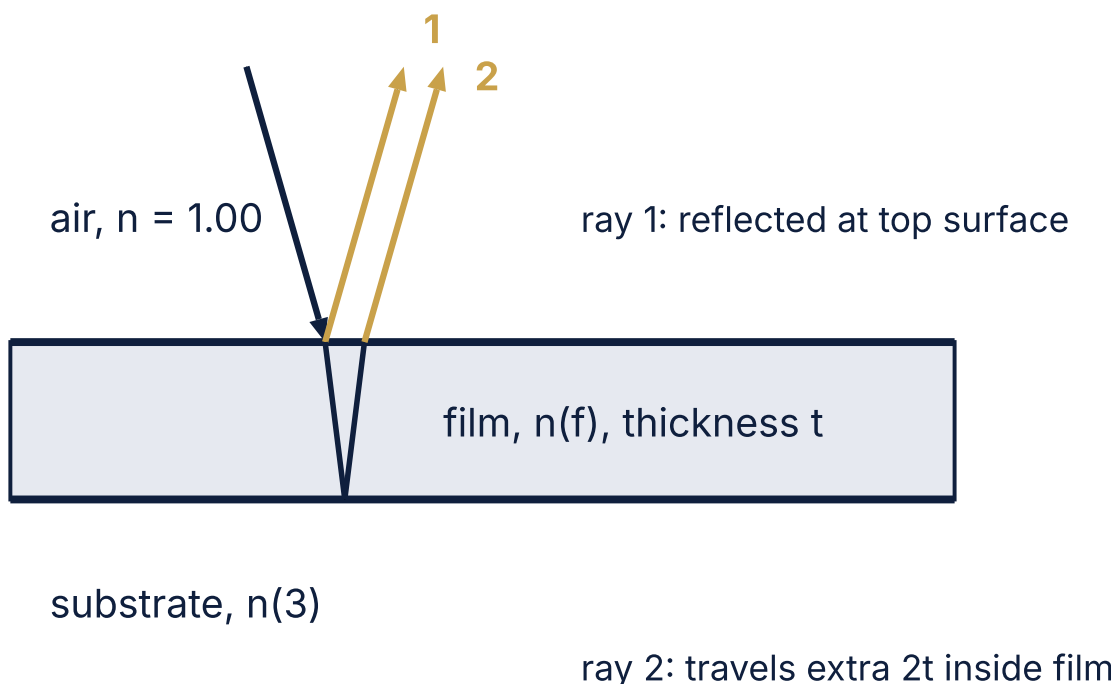
14.9 Thin-Film Interference

Light reflecting from the top and bottom of a film of thickness t and index n_f interferes. Two things set the phase: the extra path $2t$ (near-normal incidence), measured in the film wavelength $\lambda_f = \lambda/n_f$, and a half-cycle flip on reflection where the index increases (no flip where it decreases).

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A reflection off a higher- n medium flips the wave (half cycle)

Thin-film interference (angles exaggerated): reflected waves 1 and 2 differ by the extra path $2t$ and any phase flips.

Thin-film conditions (normal incidence)

One reflection flips (e.g. soap film in air): constructive $2t = (m + \frac{1}{2})\lambda_f$, destructive $2t = m\lambda_f$.

Zero or two reflections flip (e.g. coating with n between air and glass): constructive $2t = m\lambda_f$, destructive $2t = (m + \frac{1}{2})\lambda_f$.

A film much thinner than λ_f in air appears dark; a minimum-thickness anti-reflective coating has $t = \lambda_f/4 = \lambda/(4n_f)$.

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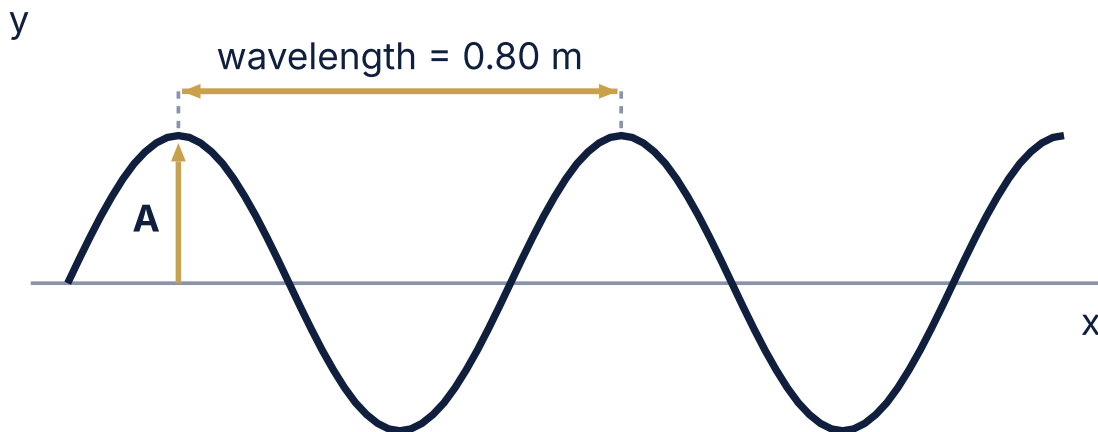
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Worked Examples

EXAMPLE 1: WAVE ON A STRING

The graph above shows a wave of frequency 15 Hz on a string with linear density 0.020 kg/m. Find the wave speed and the tension. If the tension is quadrupled, what are the new wave speed and wavelength?



Snapshot of a transverse wave on a string: distance between crests is one wavelength, 0.80 m.

SOLUTION

$$v = f\lambda = (15)(0.80) = 12 \text{ m/s.}$$

$$v = \sqrt{F_T/\mu} \text{ gives } F_T = \mu v^2 = (0.020)(12)^2 = 2.88 \text{ N.}$$

Quadrupling F_T doubles v to 24 m/s. The frequency is set by the source and stays 15 Hz, so $\lambda = v/f = 1.6 \text{ m}$.

EXAMPLE 2: PASSING AMBULANCE

A siren emits 700 Hz. The ambulance moves at 30 m/s past a stationary listener; $v_{\text{sound}} = 343 \text{ m/s}$. Find the frequency heard as it approaches and as it recedes, and the wavelength ahead of it.

SOLUTION

$$\text{Approaching: } f_o = 700 \frac{343}{343 - 30} = 767 \text{ Hz.}$$

$$\text{Receding: } f_o = 700 \frac{343}{343 + 30} = 644 \text{ Hz.}$$

Ahead of the source the wavelength is $(v - v_s)/f_s = 313/700 = 0.447 \text{ m}$, shorter than 0.490 m at rest.

EXAMPLE 3: RESONANCE IN STRINGS AND PIPES

A guitar string of length 0.65 m has fundamental frequency 220 Hz. (a) Find the wave speed and the third harmonic. (b) A pipe 0.50 m long is closed at one end. Find its two lowest resonant frequencies and the fundamental if it were open at both ends ($v = 343 \text{ m/s}$).

SOLUTION

$$\text{(a) } \lambda_1 = 2L = 1.30 \text{ m, } v = f\lambda = (220)(1.30) = 286 \text{ m/s; } f_3 = 3(220) = 660 \text{ Hz.}$$

$$\text{(b) Closed-open: } f_1 = v/(4L) = 343/2.0 = 171.5 \text{ Hz and } f_3 = 3f_1 = 514.5 \text{ Hz (no even harmonics).}$$

$$\text{Open at both ends: } f_1 = v/(2L) = 343 \text{ Hz.}$$

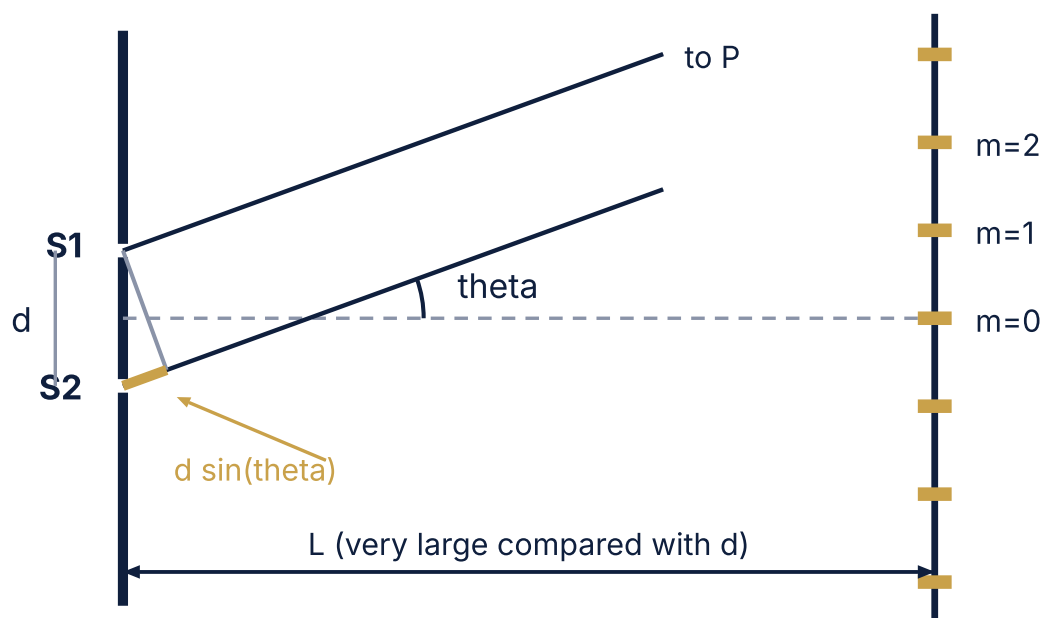
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EXAMPLE 4: DOUBLE SLIT AND GRATING

Light of wavelength 600 nm illuminates two slits 0.20 mm apart; the screen is 2.0 m away. (a) Find the fringe spacing and the position of the third bright fringe. (b) Repeat the spacing for 420 nm light. (c) A grating has 500 lines per mm. Find the first-order angle for 600 nm and the number of bright maxima.



Double-slit geometry, not to scale: the lower ray travels an extra $d \sin(\theta)$ to reach a distant screen.

SOLUTION

(a) $\Delta y = \lambda L / d = (6.0 \times 10^{-7})(2.0) / (2.0 \times 10^{-4}) = 6.0 \times 10^{-3} \text{ m} = 6.0 \text{ mm}$. $y_3 = 3(6.0) = 18 \text{ mm}$.

(b) $\Delta y = (4.2 \times 10^{-7})(2.0) / (2.0 \times 10^{-4}) = 4.2 \text{ mm}$: shorter wavelength, smaller spacing.

(c) $d = 1/500 \text{ mm} = 2.0 \times 10^{-6} \text{ m}$. $\sin \theta_1 = \lambda / d = 0.30$, $\theta_1 = 17.5^\circ$. Since $d / \lambda = 3.33$, $m \leq 3$: 7 maxima.

Write It Like a 5**Model Response**

Prompt: In a double-slit experiment with red laser light, bright fringes appear on a screen. The red laser is replaced by a violet laser and nothing else changes. Predict the change in the fringe spacing and justify your answer. The fringe spacing decreases. Bright fringes occur where the path difference from the slits is $d \sin \theta = m\lambda$. For small angles the fringe spacing is $\Delta y = \lambda L / d$, which is directly proportional to wavelength with L and d unchanged. Violet light has a shorter wavelength than red light, so the path difference needed for each fringe is smaller and the fringes lie closer together.

What the grader looks for

(1) States the correct direction (spacing decreases). (2) Starts from the equation, $d \sin \theta = m\lambda$ or $\Delta y = \lambda L / d$. (3) States which quantities are constant. (4) Correct ordering of the wavelengths of red and violet light, with no mix-up with frequency.

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Coach's Playbook

- ◆ Frequency belongs to the source; when a wave changes medium, ask only how v and λ change.
- ◆ For interference, write the path-difference condition first and say what m is.
- ◆ Standing waves: count antinodes to get n , then $L = n\lambda/2$ (string, open pipe) or $L = n\lambda/4$, odd n (closed pipe).
- ◆ Thin films: list every boundary, mark flips, then pick the condition.
- ◆ Between fringe 1 and fringe 7 there are 6 spacings.

Common traps

1. Saying the shorter wavelength needs a longer path difference or gives wider fringes. 2. Misreading order m or the number of spacings between two bright bands. 3. Using the wrong sign in the Doppler formula; check that approaching gives a higher frequency. 4. Forgetting phase flips in thin films, or using the wavelength in air instead of λ/n inside the film.

Quick Check

Q1. A source sends 40 Hz waves with wavelength 5.0 m. What is the speed? A: 200 m/s. Q2. How many nodes (including the fixed ends) does the $n = 3$ standing wave on a string have? A: 4. Q3. Why do soap films look dark where they are thinnest? A: Only one reflection flips, and the extra path is almost zero, so the two reflections cancel.

Challenge Corner

A soap film ($n = 1.33$) in air is 350 nm thick. Which visible wavelengths are strongly reflected at normal incidence, and which are weakest? Solution: One reflection flips, so constructive reflection needs $2nt = (m + \frac{1}{2})\lambda$ with $2nt = 2(1.33)(350) = 931$ nm. Then $\lambda = 931/(m + \frac{1}{2})$ gives 1862 nm (infrared), 621 nm (orange-red) and 372 nm (ultraviolet): only 621 nm is strongly reflected. Destructive: $\lambda = 931/m$ gives 466 nm (blue) for $m = 2$, so blue is weakest.

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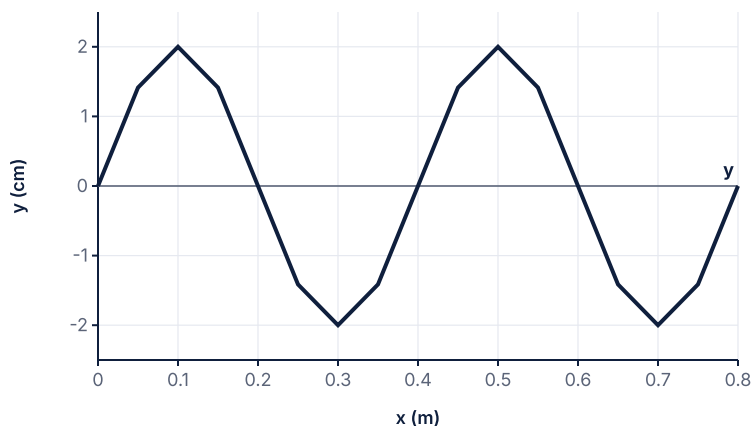
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Practice

PRACTICE Unit 14 Practice

Show your work. A calculator is allowed. Justify conclusions.

1. The graph shows a snapshot of a wave of frequency 50 Hz traveling along a string. What is the wave speed?



Wave snapshot; the wave has frequency 50 Hz.

- (A) 10 m/s (B) 20 m/s (C) 40 m/s (D) 125 m/s
2. A source of sound moves toward a stationary observer at constant speed. Compared with the source at rest, the observer measures
- (A) a larger wave speed and the same frequency
 (B) a higher frequency emitted by the source
 (C) a shorter wavelength, a higher frequency, and the same wave speed
 (D) a longer wavelength, a lower frequency, and the same wave speed
3. A pipe closed at one end has a fundamental resonance of 125 Hz. Which of the following is also a resonant frequency of this pipe?
- (A) 250 Hz (B) 500 Hz (C) 1000 Hz (D) 375 Hz
4. In a double-slit experiment, the slit separation is doubled and the distance to the screen is halved, with the same light. The spacing between adjacent bright fringes becomes
- (A) one-fourth as large (B) one-half as large (C) unchanged
 (D) four times as large
5. A soap film ($n = 1.33$) in air is viewed in reflected light. Where the film is much thinner than the wavelength of light it looks dark because
- (A) both reflected waves flip phase and cancel
 (B) neither reflected wave flips phase, so they add
 (C) only the reflection from the front surface flips phase, so the two reflections are out of phase
 (D) the film absorbs all of the light
6. Unpolarized light of intensity I_0 passes through a polarizer, then through a second polarizer whose axis is 60 degrees from the first. The final intensity is
- (A) $I_0/4$ (B) $I_0/8$ (C) $I_0/2$ (D) zero
7. A string of length 1.20 m is fixed at both ends and driven by a vibrator. The tension is 18.4 N. A student records the resonant frequencies: $n = 1$ to 5 gives f (Hz) = 41, 80, 121, 160, 199.
- (a) Describe how the student could find the resonant frequencies precisely and reduce uncertainty.
 (b) State what to plot so the data are linear and what the slope represents.
 (c) Use the data to determine the wave speed on the string.
 (d) Use the speed and the tension to find the linear density of the string.

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8. A laser shines on two slits 0.25 mm apart. On a screen 3.0 m away, the distance from the center of the 1st bright fringe on one side to the center of the 7th bright fringe on the same side is 4.5 cm.
 - (a) Calculate the wavelength of the light.
 - (b) The whole apparatus is submerged in water ($n = 1.33$). Calculate the new fringe spacing.
 - (c) Explain, using path difference, why the spacing would be smaller if the laser were replaced by a violet one in air.
9. A siren of frequency 500 Hz moves toward a stationary observer at 20 m/s. The speed of sound is 340 m/s.
 - (a) Calculate the frequency the observer hears.
 - (b) Calculate the wavelength of the sound reaching the observer, and the wavelength behind the siren.
 - (c) Now the siren is at rest and the observer moves toward it at 20 m/s. Calculate the frequency heard and explain, in terms of wavelength and wave speed, why it differs from part (a).
10. A lens is coated with a thin transparent layer of index 1.25 on glass of index 1.52. The coating is designed to minimize reflection of light of wavelength 520 nm in air at normal incidence.
 - (a) Calculate the wavelength of this light in the coating.
 - (b) Explain which reflections flip phase.
 - (c) Calculate the minimum coating thickness.
 - (d) Explain why the coating reduces but does not eliminate reflection for light of other wavelengths.

Answers Unit 14: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

14 Waves, Sound, and Physical Optics

PRACTICE · UNIT 14 PRACTICE

1. B. Reading the graph, one full cycle spans 0.40 m, so $v = f\lambda = (50)(0.40) = 20$ m/s. Choice A uses half a wavelength, and choice C uses 0.80 m, which is two wavelengths.
2. C. A moving source does not change the speed of the wave in the medium. It compresses the crests ahead of it, giving shorter wavelength and higher observed frequency. Choice B is wrong because the frequency the source emits is unchanged.
3. D. For a pipe closed at one end only odd multiples of the fundamental occur: 125, 375, 625 Hz. Choice A (250 Hz) is an even harmonic that fits an open-open pipe but not this one.
4. A. Spacing is $\lambda L/d$. Doubling d halves it and halving L halves it again, so it becomes 1/4 as large. Choice B changes only one of the two quantities.
5. C. The reflection at the air-film surface goes to a higher index, so it flips by a half cycle; the reflection at the film-air back surface goes to a lower index and does not flip. With a negligible extra path, the two are out of phase and cancel. Choice A requires both boundaries to be low-to-high.
6. B. The first polarizer passes $I_0/2$. Malus's law for the second gives $(I_0/2) \cos^2(60^\circ) = (I_0/2)(1/4) = I_0/8$. Choice A forgets the halving by the first polarizer.
7. (a) 2 points: Drive the string at slowly changing frequency and find each frequency that gives the largest amplitude with clear nodes and antinodes; repeat several times for each mode and average, and count the number of antinodes to identify n . A frequency counter or smaller frequency steps near resonance reduces uncertainty.
 (b) 2 points: $f_n = nv/(2L)$, so plot f (vertical) against n (horizontal). The slope is $v/(2L)$.
 (c) 3 points: The best-fit line has slope about 39.6 Hz and passes close to the origin. $v = 2L \times \text{slope} = 2(1.20)(39.6) = 95$ m/s.
 (d) 1 point: $\mu = F_T/v^2 = 18.4/(95)^2 = 2.0 \times 10^{-3}$ kg/m.
8. (a) 3 points: The 1st to 7th bright fringe spans 6 spacings, so $\Delta y = 4.5/6 = 0.75$ cm $= 7.5 \times 10^{-3}$ m. $\lambda = \Delta y d/L = (7.5 \times 10^{-3})(2.5 \times 10^{-4})/3.0 = 6.25 \times 10^{-7}$ m $= 625$ nm.
 (b) 2 points: In water $\lambda' = 625/1.33 = 470$ nm, so $\Delta y' = \lambda' L/d = 5.6$ mm (a factor $1/1.33$ smaller).
 (c) 2 points: A bright fringe needs path difference $d \sin \theta = m\lambda$. Violet light has a shorter wavelength, so each order needs a smaller path difference and therefore a smaller angle; with L and d fixed, $\Delta y = \lambda L/d$ shrinks in proportion to λ .
9. (a) 2 points: $f_o = 500(340)/(340 - 20) = 531$ Hz.
 (b) 2 points: Ahead: $\lambda = (v - v_s)/f_s = 320/500 = 0.64$ m. Behind: $(340 + 20)/500 = 0.72$ m.
 (c) 3 points: $f_o = 500(340 + 20)/340 = 529$ Hz. This is slightly lower than 531 Hz. When the source moves, the wavelength in the air shortens to 0.64 m while the wave moves at 340 m/s toward the observer, so $f = 340/0.64$. When the observer moves, the wavelength stays 0.68 m but the observer meets the crests at the relative speed 360 m/s, so $f = 360/0.68$. The two are different functions of v_s and v_o , so they agree only for small speeds.

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10. (a) 1 point: $\lambda_f = 520/1.25 = 416$ nm.

(b) 2 points: Reflection off the air-coating boundary goes to a higher index (1.00 to 1.25), so it flips; reflection off the coating-glass boundary goes to higher index again (1.25 to 1.52) so it also flips. Both flip, so the flips cancel out.

(c) 2 points: For destructive interference with both flipped, $2t = \lambda_f/2$, so $t = \lambda_f/4 = 104$ nm.

(d) 1 point: At other wavelengths the extra path $2t$ is not an odd multiple of half the wavelength in the coating, so the reflected waves only partially cancel.

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Unit 15: Modern Physics

Exam weighting: about 12-15% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

15 Modern Physics

Photons, matter waves, atomic energy levels, blackbody radiation, the photoelectric and Compton effects, and nuclear mass-energy and decay, all with clear energy bookkeeping.

Why This Matters

Modern physics is 12-15% of the multiple-choice section and a favorite for paragraph-style reasoning. Points are lost by using c as an electron speed, by setting electron energy equal to photon energy (forgetting the work function), and by mixing up what intensity changes and what frequency changes.

Core Ideas

15.1 Quantum Theory and Wave-Particle Duality

Light shows interference (wave behavior) yet delivers energy in lumps called **photons** (particle behavior). Matter shows wave behavior too: a particle of momentum p has a **de Broglie wavelength**. Diffraction and interference reveal waves; one-at-a-time, localized events such as photoelectron emission reveal particles.

Photon and matter-wave relations

Photon energy: $E = hf = \frac{hc}{\lambda}$, with $h = 6.63 \times 10^{-34} \text{ J s} = 4.14 \times 10^{-15} \text{ eV s}$

Convenient form: $hc \approx 1240 \text{ eV nm}$, so $E(\text{eV}) = 1240/\lambda(\text{nm})$

Photon momentum: $p = \frac{h}{\lambda} = \frac{E}{c}$ Matter wave: $\lambda = \frac{h}{p} = \frac{h}{mv}$ (slow particles)

$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$. Electron mass $9.11 \times 10^{-31} \text{ kg}$.

Never use c as an electron speed. An electron has kinetic energy $\frac{1}{2}mv^2$ and wavelength $h/(mv)$; larger momentum means shorter wavelength.

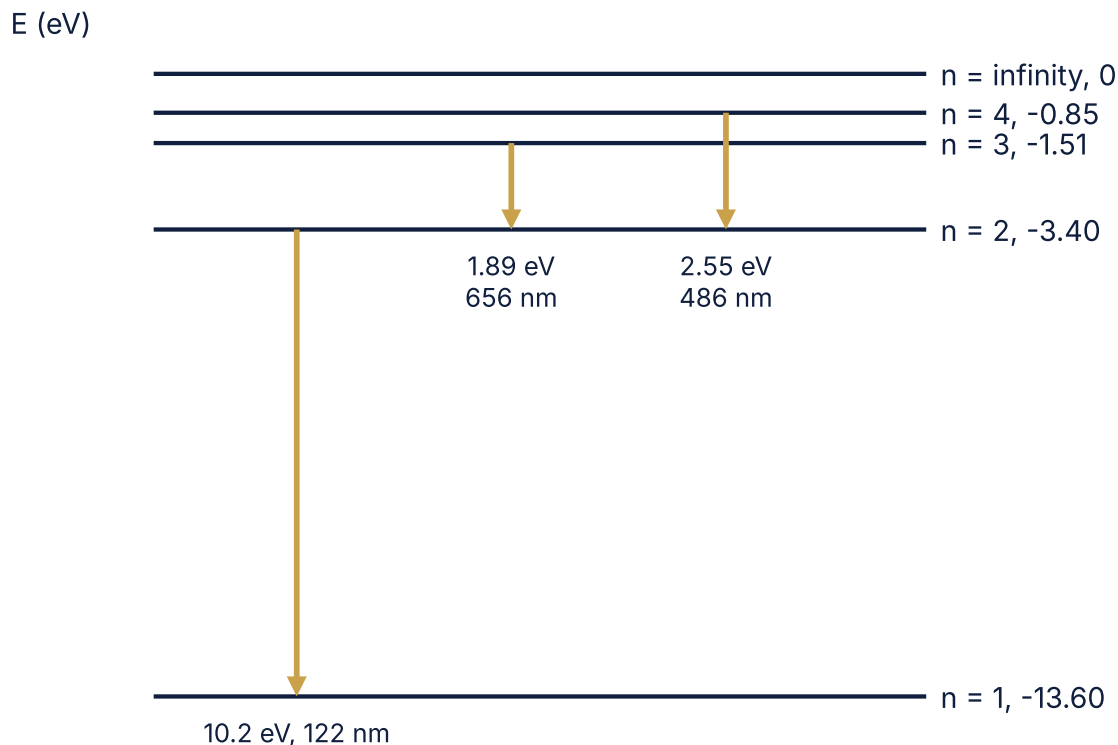
15.2 The Bohr Model of Atomic Structure

In the Bohr model an electron occupies only certain **energy levels**. Energies are negative because the electron is bound; zero is the ionization limit (free electron at rest). For hydrogen, $E_n = -13.6 \text{ eV}/n^2$. The lowest level is the **ground state**; higher ones are **excited states**.

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Hydrogen energy levels drawn to scale (16 px per eV). Each downward arrow is one emitted photon; its energy equals the drop between levels.

15.3 Emission and Absorption Spectra

A hot, low-density gas emits only certain wavelengths: an **emission spectrum** of bright lines. White light passing through a cool gas of the same element loses photons whose energies equal the gaps, leaving dark lines at the same wavelengths: an **absorption spectrum**. A photon must match a gap exactly (or exceed the ionization energy, with the extra becoming kinetic energy), but a colliding electron can excite an atom with any kinetic energy at least as big as the gap. Downward arrows in the diagram: bigger gap, higher frequency, shorter wavelength.

15.4 Blackbody Radiation

Every object emits a continuous thermal spectrum. As temperature rises, total radiated power increases and the peak wavelength moves shorter (dull red to white). Classical wave theory predicted unlimited short-wavelength emission, which is not observed; Planck fixed this by proposing energy quanta hf . Treat this qualitatively.

15.5 The Photoelectric Effect

Light can eject electrons from a metal. Each electron absorbs one photon, and a minimum energy, the **work function** ϕ , frees it; the rest becomes kinetic energy. The wave model cannot explain why nothing is emitted below a **threshold frequency** however intense the light, why emission has no delay, or why maximum kinetic energy depends on frequency, not intensity.

Photoelectric equations

$$K_{\max} = hf - \phi \quad \text{Threshold: } f_0 = \phi/h$$

$$\text{Stopping potential: } eV_s = K_{\max}$$

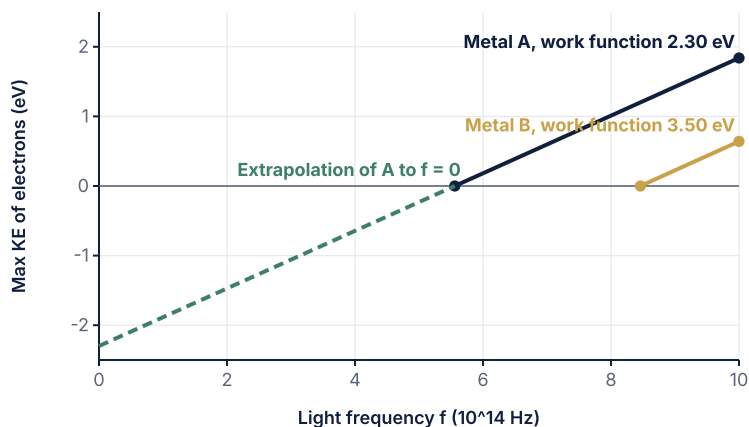
Higher intensity at fixed f above threshold: more photons per second, so more electrons per second (larger current), same $K_{\max}$.

Higher frequency: larger $K_{\max}$; current is unchanged if the photon rate is held fixed.

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Max kinetic energy versus frequency. Slope = h (4.14×10^{-15} eV s) for every metal; x-intercept = threshold frequency; extended y-intercept = minus the work function.

15.6 Compton Scattering

An X-ray photon scattered by a nearly free electron emerges with a longer wavelength while the electron recoils. This shows photons carry momentum h/λ and that the collision conserves both energy and (vector) momentum. The scattered photon has lower frequency but still moves at c .

15.7 Fission, Fusion, and Nuclear Decay

The strong force binds nuclei. The **binding energy** is the mass defect (separated nucleons minus nucleus) times c^2 . In **fission** a heavy nucleus splits; in **fusion** light nuclei join. Both release energy because the products have greater binding energy per nucleon, so total mass falls and the lost mass appears as kinetic energy and radiation.

Mass-energy and decay

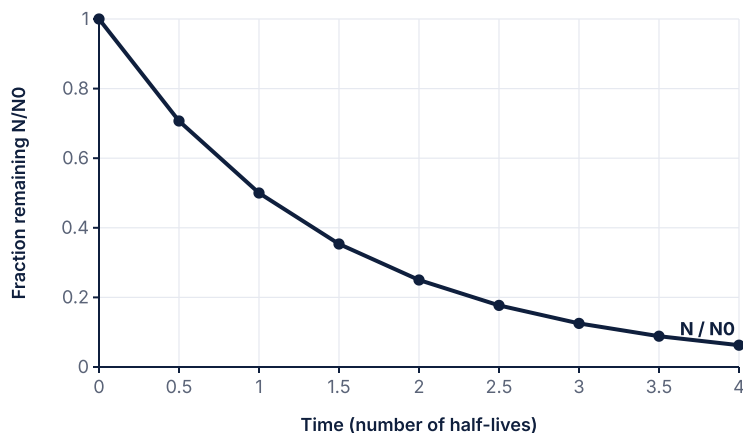
$$E = (\Delta m)c^2, \text{ and } 1 \text{ u} = 931.5 \text{ MeV}/c^2$$

Radioactive decay: a nucleus spontaneously changes into one or more different nuclei, or drops to a lower energy level of the same nucleus.

Every decay conserves charge (total Z), nucleon number (total A), energy and momentum.

Half-life $T_{1/2}$: time for half of a large sample to decay. $N = N_0(1/2)^{t/T_{1/2}}$

When one nucleus decays is random; for a large sample the fraction remaining is predictable, and activity (decays per second) falls in step with N .



Exponential decay: each half-life removes half of what is left. Values: 1, 0.5, 0.25, 0.125, 0.0625 at 0, 1, 2, 3, 4 half-lives.

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15.8 Types of Radioactive Decay

Decay	Emitted	Change in nucleus	Example
Alpha	helium-4 nucleus	Z down 2, A down 4	${}_{88}^{226}\text{Ra} \rightarrow {}_{86}^{222}\text{Rn} + {}_2^4\text{He}$
Beta-minus	electron + antineutrino	Z up 1, A same (neutron becomes proton)	${}_{6}^{14}\text{C} \rightarrow {}_{7}^{14}\text{N} + e^{-} + \bar{\nu}$
Beta-plus	positron + neutrino	Z down 1, A same (proton becomes neutron)	${}_{11}^{22}\text{Na} \rightarrow {}_{10}^{22}\text{Ne} + e^{+} + \nu$
Gamma	high-energy photon	same nucleus, lower energy level; Z , A unchanged	${}^{60}\text{Ni}^{*} \rightarrow {}^{60}\text{Ni} + \gamma$

Balance A and Z (charge) separately. The beta electron is created in the decay.

Worked Examples

PHOTOELECTRIC EFFECT NUMBERS

Light of wavelength 250 nm strikes a metal with work function 3.60 eV. (a) Find the maximum kinetic energy of the ejected electrons and the stopping potential. (b) Find the threshold wavelength. (c) What happens to the current and to the maximum kinetic energy if the light intensity is doubled?

SOLUTION

(a) Photon energy $E = 1240/250 = 4.96$ eV. $K_{\max} = hf - \phi = 4.96 - 3.60 = 1.36$ eV, so $V_s = 1.36$ V.

(b) At threshold $hf_0 = \phi$, so $\lambda_0 = 1240/3.60 = 344$ nm. Any wavelength longer than 344 nm ejects nothing.

(c) Twice as many photons arrive each second, so twice as many electrons are ejected and the current doubles. Each photon still has 4.96 eV, so $K_{\max}$ stays 1.36 eV.

TRANSITIONS IN A FICTIONAL ATOM

A fictional atom has levels at -8.0 (ground), -3.0 and -1.0 eV. (a) Which photon energies can a ground-state atom absorb? (b) Find the energies and wavelengths of all photons that an $n = 3$ atom can emit. (c) Can a 6.0 eV electron excite a ground-state atom?

SOLUTION

(a) Gaps from $n = 1$: 5.0 eV (to $n = 2$) and 7.0 eV (to $n = 3$). Only these, or any photon of at least 8.0 eV (ionization), are absorbed; 6.0 eV is not.

(b) 3 to 2: 2.0 eV, 620 nm. 2 to 1: 5.0 eV, 248 nm. 3 to 1: 7.0 eV, 177 nm.

(c) Yes: 5.0 eV goes to the atom ($n = 2$) and the electron leaves with 1.0 eV. It cannot reach $n = 3$ (needs 7.0 eV).

HALF-LIFE OF A SAMPLE

A radioactive isotope has a half-life of 12 days. A sample starts at 80.0 g. (a) How much remains after 36 days? (b) After 30 days? (c) A detector reads 640 counts per minute at $t = 0$; when does it read 40?

SOLUTION

(a) 36 days is 3 half-lives: $80.0 \times (1/2)^3 = 10.0$ g.

(b) 2.5 half-lives: $80.0 \times 2^{-2.5} = 14.1$ g (between 20.0 and 10.0 g, as expected).

(c) 640 to 40 is a factor of 16 = 2^4 : 4 half-lives = 48 days.

ENERGY RELEASED IN FUSION

Deuterium (2.014102 u) and tritium (3.016049 u) fuse to give helium-4 (4.002603 u) and a neutron (1.008665 u). Find the energy released per reaction in MeV and joules ($1 \text{ u} = 931.5 \text{ MeV}/c^2$) and say where it comes from.

SOLUTION

Mass before = 5.030151 u; after = 5.011268 u; $\Delta m = 0.018883$ u.

$E = 0.018883 \times 931.5 = 17.6$ MeV. In joules: $17.6 \times 10^6 \times 1.60 \times 10^{-19} = 2.82 \times 10^{-12}$ J.

The products are more tightly bound per nucleon, so mass is lower afterwards; the missing mass becomes kinetic energy of the products.

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Write It Like a 5

Model Response

Prompt: A metal has work function 2.30 eV. Beam A is dim 400 nm light; beam B is very intense 700 nm light. Does either eject electrons? Justify. Response: A 400 nm photon has energy $1240/400 = 3.10$ eV, more than 2.30 eV, so one photon can free an electron; beam A ejects electrons with $K_{max} = 3.10 - 2.30 = 0.80$ eV. A 700 nm photon has only $1240/700 = 1.77$ eV. Each electron absorbs one photon, so none gets enough energy, and more intensity only means more photons that each fall short. Beam B ejects nothing. Only A gives a current; brightening A raises the current but leaves K_{max} at 0.80 eV.

What the grader looks for

(1) Photon energy computed with units and compared with the work function. (2) One photon, one electron. (3) $K_{max} = hf - \phi$. (4) Intensity changes photon number, not photon energy. (5) A conclusion about the current for each beam.

Coach's Playbook

- 1 Decide: photon (hf), particle (kinetic energy, h/p), or nucleus (A , Z , mass).
- 2 Use 1240 eV nm for photons; use joules only when mass or speed is involved.
- 3 Write an energy balance in words: photon in = energy to free or raise + energy left over.
- 4 For decay balance A and Z , name the emitted particle, and count halvings.

Common traps

(1) Setting electron energy equal to the whole photon energy. (2) Using c as the electron speed. (3) Saying bright light below threshold will eventually eject electrons. (4) Forgetting that gamma decay leaves the same nucleus at a lower energy level, or applying half-life to one nucleus.

Quick Check

Q1: Photon energy of 500 nm light? A1: $1240/500 = 2.48$ eV. Q2: Heating an object makes its peak emission wavelength do what? A2: Move to a shorter wavelength, with more total power. Q3: Ra-226 emits an alpha particle; what are Z and A of the daughter? A3: $Z = 86$ and $A = 222$ (radon-222).

Challenge Corner

Hydrogen atoms in $n = 3$ emit light onto a metal with work function 2.20 eV. Which lines eject electrons, and what is the largest stopping potential? Solution: The lines are 3 to 2 (1.89 eV), 2 to 1 (10.20 eV), 3 to 1 (12.09 eV). The first is below 2.20 eV and ejects nothing. The others give $K_{max} = 8.00$ eV and $12.09 - 2.20 = 9.89$ eV. The largest stopping potential is 9.89 V.

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Practice

PRACTICE Unit 15 Practice

Show your work. A calculator is allowed. Justify conclusions.

- A metal has a threshold frequency of 5.0×10^{14} Hz. It is illuminated by very intense light of frequency 4.0×10^{14} Hz. Which statement is correct?
 - Electrons are ejected with a small kinetic energy because the light is intense.
 - Electrons are ejected after a delay while the metal absorbs enough energy.
 - No electrons are ejected, because each photon has less energy than the work function.
 - Electrons are ejected only if the light is also made more intense.
- An electron moves at 2.0×10^6 m/s. Its de Broglie wavelength is closest to
 - 3.6×10^{-10} m
 - 2.4×10^{-12} m
 - 7.3×10^{-10} m
 - 3.6×10^{-7} m
- An atom has energy levels at -6.0 eV (ground state), -2.5 eV and -1.0 eV. Which photon energy could be absorbed by an atom in its ground state?
 - 2.0 eV
 - 3.5 eV
 - 4.5 eV
 - 5.5 eV
- A carbon-14 nucleus ($Z = 6$, $A = 14$) undergoes beta-minus decay. The daughter nucleus has
 - $Z = 5$, $A = 14$
 - $Z = 6$, $A = 13$
 - $Z = 7$, $A = 13$
 - $Z = 7$, $A = 14$
- A radioactive sample registers 640 counts per minute. After 36 minutes it registers 80 counts per minute. The half-life is
 - 9 min
 - 12 min
 - 18 min
 - 36 min
- An X-ray photon scatters from a free electron, which recoils. Compared with the incident photon, the scattered photon has
 - a longer wavelength and a lower frequency
 - a shorter wavelength and a higher frequency
 - the same wavelength but a lower intensity
 - a longer wavelength and a lower speed
- A student measures the stopping potential V_s for a metal photocathode at five light frequencies.

f (10^{14} Hz): 6.0, 7.0, 8.0, 9.0, 10.0

V_s (V): 0.18, 0.60, 1.01, 1.42, 1.84

 - Write an equation that relates V_s to f and explain which quantities the slope and the vertical intercept represent.
 - Plot the data (V_s on the vertical axis), draw a best-fit line with a straightedge, and use it to determine the work function in eV and the threshold frequency.
 - Describe one change to the procedure that would reduce the uncertainty in the slope.
- An electron is accelerated from rest through a potential difference of 150 V.
 - Calculate its kinetic energy in joules and its momentum.
 - Calculate its de Broglie wavelength.
 - A crystal has atomic spacing of about 0.2 nm. Explain why a beam of these electrons could show diffraction from the crystal while a beam of visible light could not be used the same way to resolve the atoms.
- Sodium-24 is a beta-minus emitter with a half-life of about 15 hours. A sample has an activity of 800 MBq at $t = 0$.
 - Write the decay equation for Na-24 ($Z = 11$), identifying the daughter.
 - Complete a table of activity at $t = 0, 15, 30, 45$ h and sketch the activity versus time.
 - A student says that after 30 hours every nucleus in the sample has decayed, since two half-lives is the whole time. Evaluate this claim.
- A positron and an electron, both essentially at rest, annihilate and produce two photons.
 - Calculate the total energy released in MeV, using the electron rest energy 0.511 MeV for each particle.
 - Calculate the wavelength of each photon.
 - Explain why two photons, and not one, must be produced.

Answers Unit 15: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

15 Modern Physics

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PRACTICE · UNIT 15 PRACTICE

1. C. Each electron absorbs one photon, and a 4.0×10^{14} Hz photon has less energy than the work function. Choice A and D treat intensity as able to make up for low frequency, which is the wave-model error.
2. A. $\lambda = h/(mv) = 6.63 \times 10^{-34} / (9.11 \times 10^{-31} \times 2.0 \times 10^6) = 3.6 \times 10^{-10}$ m. Choice B comes from using the speed of light for the electron.
3. B. The gaps from the ground state are 3.5 eV and 5.0 eV, and any photon of 6.0 eV or more ionizes it. Only 3.5 eV matches. A 5.5 eV photon is tempting because it lies between the gaps and below ionization, but it matches neither.
4. D. In beta-minus decay a neutron becomes a proton, so Z rises by 1 and A stays 14, giving nitrogen-14. Choice A reverses the sign (that is beta-plus).
5. B. A drop from 640 to 80 is a factor of 8, which is 3 half-lives, so each is $36/3 = 12$ min. Choice C divides by 2 (a single halving) instead of counting 3.
6. A. The electron gains kinetic energy and momentum, so the photon loses energy and its wavelength increases. Choice D confuses frequency with speed; photons always travel at c in a vacuum.
7. (a) 1 pt: $eV_s = hf - \phi$, so $V_s = (h/e)f - \phi/e$. 1 pt: slope = h/e ; vertical intercept = $-\phi/e$ (the horizontal intercept is f_0). (b) 1 pt: labeled axes with units and scales, points plotted. 1 pt: straight best-fit line. 1 pt: slope about 4.1×10^{-15} V s (so $h = \text{slope} \times e = 6.6 \times 10^{-34}$ J s, consistent with the accepted value). 1 pt: intercept about -2.3 V gives $\phi \approx 2.3$ eV; $f_0 = 2.3/4.14 \times 10^{-15} \approx 5.6 \times 10^{14}$ Hz. A least-squares fit of the data gives slope 4.14×10^{-15} V s and intercept -2.30 V. (c) 1 pt: for example, use more frequencies spread over a wider range (a longer baseline reduces the fractional slope uncertainty) or repeat each stopping-potential measurement and average.
8. (a) 1 pt: $K = eV = 1.60 \times 10^{-19} \times 150 = 2.40 \times 10^{-17}$ J. 1 pt: $p = \sqrt{2mK} = \sqrt{2(9.11 \times 10^{-31})(2.40 \times 10^{-17})} = 6.61 \times 10^{-24}$ kg m/s (non-relativistic is fine at this energy). (b) 1 pt: $\lambda = h/p = 6.63 \times 10^{-34} / 6.61 \times 10^{-24} = 1.00 \times 10^{-10}$ m = 0.100 nm. (c) 1 pt: diffraction is strong when the wavelength is comparable to the spacing; 0.100 nm is the same order as 0.2 nm. 1 pt: visible light has wavelengths of 400-700 nm, thousands of times larger than the spacing, so it cannot resolve the atoms.
9. (a) 2 pts: ${}^{24}_{11}\text{Na} \rightarrow {}^{24}_{12}\text{Mg} + e^- + \bar{\nu}$ (A = 24 and charge balance: $11 = 12 + (-1)$). (b) 1 pt: 800, 400, 200, 100 MBq. 1 pt: a curve that decreases and levels toward zero without touching it, with labeled axes and the points plotted. (c) 2 pts: the claim is wrong; each half-life removes half of the remaining nuclei, so after 30 h one quarter (25%) is still undecayed; it takes many half-lives before almost none are left, and individual nuclei decay at random times.
10. (a) 1 pt: total rest energy is $2 \times 0.511 = 1.022$ MeV (the mass is converted to energy, $E = mc^2$). (b) 1 pt: each photon carries 0.511 MeV. 1 pt: $\lambda = hc/E = 1240 \text{ eV nm} / 5.11 \times 10^5 \text{ eV} = 2.43 \times 10^{-3} \text{ nm} = 2.43 \times 10^{-12} \text{ m}$. (c) 2 pts: the initial total momentum is zero; a single photon would carry momentum E/c and violate momentum conservation, so two photons must move in opposite directions with equal energy so the net momentum is zero.

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Mixed Practice Test

A mixed set of multiple-choice and free-response questions across all units, with answers, explanations, and scoring guidance.

T Mixed Practice Test

Twelve multiple-choice questions and three free-response questions drawn from all seven units, weighted like the exam.

Time: about 50 minutes. Part A: items 1-12, multiple choice (Units 9-15; two each from Units 9, 10, 11, 14 and 15, one each from Units 12 and 13). Part B: items 13-15, free response, 26 practice points. Use a calculator and write your reasoning. Score with the guide that follows; point values here are practice values.

PRACTICE TEST Mixed Practice Test

Part A: choose the best answer. Part B: free response. Show equations, substitutions and units, and justify every claim.

- The absolute temperature of an ideal gas rises from 200 K to 800 K. By what factor does the rms speed of its molecules change?
(A) 1/2 (B) 2 (C) 4 (D) 16
- During a process, 300 J of work is done on a gas and 120 J of thermal energy leaves the gas. Using the convention that work on the system is positive, the change in internal energy of the gas is
(A) -420 J (B) -180 J (C) +180 J (D) +420 J
- A parallel-plate capacitor remains connected to a battery while the plate separation is doubled. The energy stored in the capacitor
(A) is halved (B) is doubled (C) is unchanged (D) is quartered
- A proton is accelerated from rest through a potential difference of 200 V. Its final speed is closest to
(A) 1.4×10^5 m/s (B) 8.4×10^6 m/s (C) 3.8×10^{10} m/s (D) 2.0×10^5 m/s
- A 12 V ideal battery is connected in series with a 4 ohm resistor and a parallel pair of 6 ohm and 12 ohm resistors. What power is dissipated in the 12 ohm resistor?
(A) 12 W (B) 3 W (C) 6 W (D) 9 W
- An uncharged capacitor is connected in series with a resistor and a battery by closing a switch. Which describes the circuit at the instant after closing and a very long time later?
(A) Instantly: current zero. Later: current maximum.
(B) Instantly: capacitor voltage equal to the battery voltage. Later: current zero.
(C) Instantly: current is battery voltage divided by R. Later: current zero and capacitor voltage equal to the battery voltage.
(D) Instantly and later: current is battery voltage divided by R.
- A proton moves in the +x direction (to the right) through a uniform magnetic field directed into the page (-z). The magnetic force on the proton points
(A) toward +y (up the page) (B) toward -y (down the page)
(C) toward +x (D) out of the page
- An object is 30 cm from a thin converging lens of focal length 10 cm. The image is
(A) real, upright, 7.5 cm from the lens (B) virtual, upright, 15 cm from the lens
(C) virtual, inverted, 30 cm from the lens (D) real, inverted, 15 cm from the lens
- A string of length 1.2 m is fixed at both ends and vibrates in its third harmonic. The wave speed on the string is 48 m/s. The frequency is
(A) 20 Hz (B) 60 Hz (C) 40 Hz (D) 120 Hz

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10. In a double-slit experiment, red light of wavelength 650 nm is replaced by blue light of wavelength 450 nm, with the slits and the screen unchanged. The spacing between adjacent bright fringes is multiplied by about
 (A) 1.44 (B) 1.00 (C) 0.69 (D) 0.48
11. Light of frequency f ejects photoelectrons from a metal with maximum kinetic energy K . When the frequency is doubled to $2f$, the maximum kinetic energy is
 (A) unchanged (B) exactly $2K$
 (C) less than $2K$, but more than K (D) greater than $2K$
12. Which transition in a hydrogen atom emits the photon with the shortest wavelength?
 (A) $n = 3$ to $n = 1$ (B) $n = 4$ to $n = 2$ (C) $n = 2$ to $n = 1$
 (D) $n = 4$ to $n = 3$
13. [10 points] A 12 V ideal battery is connected to a 20 ohm resistor R_1 in series with a parallel combination of $R_2 = 30$ ohm and $R_3 = 60$ ohm. A 5.0 microfarad capacitor is connected across R_3 and has been in the circuit a very long time.
 (a) [2 points] Calculate the equivalent resistance of the circuit.
 (b) [2 points] Calculate the current supplied by the battery.
 (c) [2 points] Calculate the power dissipated in R_3 .
 (d) [2 points] Calculate the charge on and the energy stored in the capacitor.
 (e) [2 points] R_2 is removed. Does the power dissipated in R_1 increase, decrease, or stay the same? Justify with a calculation or a reasoned argument.
14. [8 points] An ideal gas at point A has pressure 2.0×10^5 Pa, volume 1.0×10^{-3} m³ and temperature 300 K. It expands at constant pressure to point B, where the volume is 3.0×10^{-3} m³, while 900 J of thermal energy is added to it. The amount of gas is constant.
 (a) [2 points] Calculate the work done on the gas from A to B, with its sign.
 (b) [2 points] Calculate the change in internal energy from A to B.
 (c) [2 points] Calculate the temperature at B.
 (d) [2 points] On a P-V sketch, draw the isothermal curve through A out to volume 3.0×10^{-3} m³. Explain, using areas, how the magnitude of work done by the gas for the isothermal expansion compares with the A to B process.
15. [8 points] Coherent light of wavelength 650 nm passes through two narrow slits separated by 0.20 mm. A screen is 2.0 m from the slits.
 (a) [2 points] Calculate the spacing between adjacent bright fringes on the screen.
 (b) [2 points] The light is replaced by violet light of wavelength 420 nm. Calculate the new spacing, and explain in terms of path difference why the fringes move closer together.
 (c) [2 points] Calculate the path difference at the third bright fringe ($m = 3$) for the 650 nm light.
 (d) [2 points] With 650 nm light, the screen is moved to 1.0 m. State how the fringe spacing changes and justify.

Key**Mixed Practice Test: Answers, Explanations, and Scoring****T Mixed Practice Test****PRACTICE TEST · MIXED PRACTICE TEST**

1. B. Unit 9. $v_{rms} \propto \sqrt{T}$, so quadrupling T doubles the speed. Choice C is the tempting error of treating speed as proportional to T (that is the behavior of average kinetic energy).
2. C. Unit 9. $\Delta U = Q + W = -120 + 300 = +180$ J. Choice B flips the sign of the work, which would be the convention for work done by the gas.
3. A. Unit 10. $C = \epsilon_0 A/d$ halves while V stays fixed by the battery, so $U = \frac{1}{2} CV^2$ is halved. Choice C would apply if the charge, not the voltage, were fixed.
4. D. Unit 10. $eV = \frac{1}{2} mv^2$ gives $v = \sqrt{2eV/m} = 1.96 \times 10^5$ m/s. Choice A drops the factor 2, and choice B uses the electron mass.
5. B. Unit 11. The parallel pair is 4 ohm, so the total is 8 ohm and $I = 1.5$ A. The pair has $V = 1.5 \times 4 = 6$ V, so $P = 6^2/12 = 3$ W. Choice A uses the full 12 V across the resistor; D is the power in the series resistor.
6. C. Unit 11. The uncharged capacitor acts like a wire at first, so $I = V/R$; when fully charged it acts like an open circuit with the capacitor voltage equal to the battery voltage. Choice B mixes up the two stages.
7. A. Unit 12. $\vec{F} = q\vec{v} \times \vec{B}$ with $\hat{x} \times (-\hat{z}) = +\hat{y}$, and q is positive, so the force is up the page. Choice B is the result for an electron, or for using the wrong hand rule.
8. D. Unit 13. $1/s' = 1/f - 1/s = 1/10 - 1/30$, so $s' = 15$ cm on the far side; $m = -s'/s = -0.5$, so the image is real and inverted. Choice B would require the object to be inside the focal length.
9. B. Unit 14. $\lambda = 2L/3 = 0.80$ m and $f = v/\lambda = 48/0.80 = 60$ Hz. Choice A is the fundamental (20 Hz) and ignores the harmonic number; C uses $\lambda = L$.
10. C. Unit 14. Fringe spacing $\Delta y = \lambda L/d$ is directly proportional to wavelength, so the factor is $450/650 = 0.69$. Choice A inverts the ratio; D squares it.

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11. D. Unit 15. $K = hf - \phi$, so the new value is $2hf - \phi = 2K + \phi$, which is more than $2K$. Choice B forgets the work function, which does not double.
12. A. Unit 15. The energy gaps are 12.09, 2.55, 10.2 and 0.66 eV, and shortest wavelength means largest photon energy. Choice C is tempting because it ends at $n = 1$ but is smaller than the gap from $n = 3$.
13. (a) 1 pt: $R_{23} = (1/30 + 1/60)^{-1} = 20 \text{ ohm}$. 1 pt: $R_{eq} = 20 + 20 = 40 \text{ ohm}$. (b) 1 pt: $I = V/R_{eq}$. 1 pt: $12/40 = 0.30 \text{ A}$. (c) 1 pt: voltage across the parallel pair $= 0.30 \times 20 = 6.0 \text{ V}$. 1 pt: $P_3 = V^2/R_3 = 36/60 = 0.60 \text{ W}$. (d) 1 pt: the capacitor is in parallel with R_3 , so $V = 6.0 \text{ V}$ and $Q = CV = 5.0 \times 10^{-6} \times 6.0 = 3.0 \times 10^{-5} \text{ C}$. 1 pt: $U = \frac{1}{2}CV^2 = 9.0 \times 10^{-5} \text{ J}$. (e) 1 pt: with R_2 removed, the resistance is $20 + 60 = 80 \text{ ohm}$ and $I = 0.15 \text{ A}$ (the capacitor carries no steady current). 1 pt: $P_1 = I^2 R_1 = 0.45 \text{ W}$, down from $0.30^2 \times 20 = 1.8 \text{ W}$, so it decreases (the total resistance rises so the current falls). The justification must say that the same battery drives less current through the larger total resistance.
14. (a) 1 pt: work on the gas $W = -P\Delta V$. 1 pt: $-2.0 \times 10^5 \times 2.0 \times 10^{-3} = -400 \text{ J}$ (negative because the gas does work on its surroundings). (b) 1 pt: $\Delta U = Q + W$. 1 pt: $900 + (-400) = +500 \text{ J}$. (c) 1 pt: $PV = nRT$ with n constant and P constant gives $T_B/T_A = V_B/V_A = 3$. 1 pt: $T_B = 900 \text{ K}$. (d) 1 pt: a concave-up decreasing curve from A ($1.0\text{e-}3, 2.0\text{e}5$) down to about $0.67\text{e}5 \text{ Pa}$ at $3.0\text{e-}3 \text{ m}^3$ (curve must not be straight and must not touch the axes), lying below the horizontal A to B line. 1 pt: the work magnitude equals the area under each curve; the isothermal area ($PV \ln 3 \approx 220 \text{ J}$) is less than the rectangle (400 J), because the pressure falls along the isotherm while it stays at $2.0\text{e}5 \text{ Pa}$ on the isobar.
15. (a) 1 pt: $\Delta y = \lambda L/d$ (small-angle double-slit relation). 1 pt: $(650 \times 10^{-9})(2.0)/(0.20 \times 10^{-3}) = 6.5 \times 10^{-3} \text{ m} = 6.5 \text{ mm}$. (b) 1 pt: $(420 \times 10^{-9})(2.0)/(0.20 \times 10^{-3}) = 4.2 \text{ mm}$. 1 pt: bright fringes occur where the path difference is $m\lambda$; for a shorter wavelength, a given path difference $m\lambda$ is reached at a smaller angle, so adjacent bright fringes are closer. (c) 1 pt: bright fringe condition $d \sin \theta = m\lambda$. 1 pt: $3 \times 650 \text{ nm} = 1950 \text{ nm} = 1.95 \times 10^{-6} \text{ m}$. (d) 1 pt: spacing is proportional to L , so it halves. 1 pt: new spacing 3.25 mm , since $\Delta y = \lambda L/d$ with L halved and the other quantities unchanged.

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Exam-style free-response questions with point-by-point scoring guides and full-credit model responses.

X Free-Response Clinic

Four full free-response questions in the exam's four styles plus one short extra, with point-by-point scoring guides and model responses.

How Section II Works

Section II has four questions in 95 minutes, 50% of the score, with a 3-hour exam. That is about 24 minutes per question, and you are scored on the reasoning shown, not just the final number. The questions below follow the four styles in the order of the exam and add a short extra. All point values are practice values: use the scoring guide, not the actual exam, to estimate your performance.

Question	Style	Practice points	What it rewards
1	Mathematical Routines	10	Starting from an equation, symbolic work, units, a conclusion
2	Translation Between Representations	12	Diagrams, derivations, graph sketches, justification linking parts
3	Experimental Design and Analysis	10	Procedure, uncertainty reduction, linearization, graph, slope meaning
4	Qualitative/Quantitative Translation	8	Reasoning in words that agrees with the calculation
Short	Extra practice	5	Concise claim and evidence

Scoring habits that earn points: write the governing equation first, show substitutions with units, label every axis, draw vectors from a dot, state conditions such as 'n and T are constant', and refer to earlier parts by letter. Use $g = 9.8 \text{ m/s}^2$ and the reference values given in each question.

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PRACTICE Free-Response Clinic

Write full answers, then score each part with the guide. A calculator is allowed. Show equations, substitutions and units, and justify each claim.

1. Question 1: Mathematical Routines [10 points]

A proton (charge 1.60×10^{-19} C, mass 1.67×10^{-27} kg) enters a region of uniform magnetic field of magnitude 0.040 T with speed 2.4×10^5 m/s, perpendicular to the field. Magnetic force is the only force.

(a) [2 points] Starting from the magnetic force and Newton's second law, derive an expression for the radius r of the circular path in terms of m , v , q and B .

(b) [2 points] Calculate r .

(c) [2 points] Show that the period of the motion does not depend on the proton's speed, and calculate it.

A separate square loop of side 0.10 m and resistance 0.50 ohm lies flat on the page in a field directed into the page. The field magnitude falls uniformly from 0.40 T to zero in 0.20 s.

(d) [2 points] Calculate the magnitude of the average induced emf and current.

(e) [2 points] State the direction of the induced current (clockwise or counterclockwise as seen by the reader) and justify your answer.

2. Question 2: Translation Between Representations [12 points]

A vertical cylinder of cross-sectional area 2.0×10^{-3} m² is closed on top by a frictionless piston of mass 4.0 kg, open to the atmosphere (1.0×10^5 Pa) above. The gas below the piston has height 0.30 m and temperature 300 K, and the piston is in equilibrium. Use $g = 9.8$ m/s².

(a) [2 points] Draw a free-body diagram of the piston as a dot. Label each force with a name and direction (do not use pressure as a force).

(b) [2 points] Derive an expression for the gas pressure and calculate it.

(c) [2 points] The gas is heated slowly until the piston is 0.45 m high. Calculate the final temperature, and state what must be assumed.

(d) [2 points] The piston is then locked and the gas cooled to 300 K. Calculate the final pressure.

(e) [2 points] Sketch the whole process on a P-V diagram with labeled points and arrows, from the initial state to the final state.

(f) [2 points] For the whole process, find the net work done on the gas and the net thermal energy transferred to the gas.

3. Question 3: Experimental Design and Analysis [10 points]

A student wants to determine the focal length of a thin converging lens using a small bright object, a lens, a screen, a meter stick, and a lens holder on an optical bench.

(a) [2 points] Describe a procedure that gives the image distance for several object distances. State the independent and dependent variables, what is measured, and how the sharp image is identified.

(b) [2 points] Describe one way to reduce the uncertainty of the measured image distance.

(c) [2 points] State which two quantities to plot so that a straight line results, and say how the focal length is found from the slope or intercept.

The student obtained this data.

s (cm): 20, 25, 30, 40, 60

s' (cm): 59.5, 37.0, 30.5, 23.5, 20.0

(d) [2 points] Calculate $1/s$ and $1/s'$ for each point (in 1/cm), plot $1/s'$ versus $1/s$, and draw the best-fit line.

(e) [2 points] Determine the focal length from your line.

4. Question 4: Qualitative/Quantitative Translation [8 points]

Light passes through a diffraction grating with 500 lines per millimeter. A screen shows bright lines at angles θ measured from the central maximum.

(a) [2 points] Violet light (about 420 nm) and red light (about 650 nm) are used in turn. Without calculating, state which color has its first-order line at the larger angle, and justify using the condition for constructive interference.

(b) [2 points] Calculate the angle of the first-order line for 600 nm light.

(c) [2 points] What is the highest order that can be observed with 600 nm light? Show your reasoning.

(d) [2 points] The grating is replaced by one with more lines per millimeter. Explain how the angle of the first-order line for 600 nm light changes.

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5. Short extra: Electric Field and Potential [5 points]

A $+3.0$ nC point charge and a -3.0 nC point charge are 0.40 m apart. Point M is at the midpoint of the line joining them. Use $k = 8.99 \times 10^9$ N m²/C² and take $V = 0$ at infinity.

(a) [2 points] Find the magnitude and direction of the electric field at M.

(b) [1 point] Find the electric potential at M.

(c) [2 points] A student says that since $V = 0$ at M the electric field there must also be zero. Evaluate this claim.

Key

Free-Response Clinic: Scoring Guides and Model Responses



Free-Response Clinic

PRACTICE · FREE-RESPONSE CLINIC

1. (a) 1 pt: the magnetic force is perpendicular to v and supplies the centripetal force: $qvB = mv^2/r$. 1 pt: $r = mv/(qB)$. (b) 1 pt: substitution $r = (1.67 \times 10^{-27})(2.4 \times 10^5)/[(1.60 \times 10^{-19})(0.040)]$. 1 pt: $r = 6.3 \times 10^{-2}$ m (0.0626 m). (c) 1 pt: $T = 2\pi r/v = 2\pi m/(qB)$, in which v has cancelled. 1 pt: $T = 1.64 \times 10^{-6}$ s. (d) 1 pt: $\mathcal{E} = \Delta\Phi/\Delta t = (0.40)(0.10)^2/0.20 = 0.020$ V. 1 pt: $I = \mathcal{E}/R = 0.040$ A. (e) 1 pt: the flux into the page is decreasing, so the induced current makes a field that opposes the decrease, which means into the page. 1 pt: by the right-hand rule, an into-the-page field is produced by a clockwise current, so the current is clockwise. (Reason from the change in flux, not from the direction of the original field alone.)

Model: Magnetic force supplies the centripetal force, so $qvB = mv^2/r$ and $r = mv/(qB) = 0.063$ m. $T = 2\pi m/(qB) = 1.6$ microseconds, independent of v . $\mathcal{EMF} = 0.020$ V, $I = 0.040$ A, clockwise to restore the decreasing inward flux.

2. (a) 1 pt: three forces drawn from the dot: weight mg down, force from atmosphere down, force from gas up (labeled as forces, with the gas force longest). 1 pt: no pressure arrows and no unlabeled 'applied' force. (b) 1 pt: equilibrium gives $P_{\text{gas}}A = P_{\text{atm}}A + mg$, so $P = P_{\text{atm}} + mg/A$. 1 pt: $1.0 \times 10^5 + 39.2/(2.0 \times 10^{-3}) = 1.196 \times 10^5$ Pa. (c) 1 pt: pressure and amount of gas are constant, so V/T is constant: $T_2 = 300 \times 0.45/0.30 = 450$ K. 1 pt: state that n and P are constant (the piston is free, so the gas pressure stays 1.196×10^5 Pa). (d) 1 pt: at constant volume P/T is constant. 1 pt: $P_3 = 1.196 \times 10^5 \times 300/450 = 7.97 \times 10^4$ Pa. (e) 1 pt: a horizontal line to the right from ($V_1 = 6.0\text{e-}4$, $1.196\text{e}5$) to ($V_2 = 9.0\text{e-}4$, $1.196\text{e}5$), with arrow. 1 pt: a vertical line down to ($9.0\text{e-}4$, $7.97\text{e}4$), with arrow, and the end state labeled; final point lies on the same isotherm as the start ($T = 300$ K), below and to the right of the start. (f) 1 pt: only the isobaric step involves work: $W_{\text{on}} = -P\Delta V = -(1.196 \times 10^5)(3.0 \times 10^{-4}) = -35.9$ J. 1 pt: the gas returns to 300 K so $\Delta U = 0$ and $Q = -W = +35.9$ J into the gas (net).

Model: The piston is in equilibrium, so $P^*A = P_{\text{atm}}A + mg$ and $P = 1.0\text{e}5 + 39.2/0.0020 = 1.20\text{e}5$ Pa. At constant P and n , V/T is constant, so $T_2 = 450$ K. Cooling at constant volume gives $P_3 = 7.97\text{e}4$ Pa. Net work on the gas is -35.9 J; because T is back to 300 K, $\Delta U = 0$ and 35.9 J of thermal energy entered the gas.

3. (a) 1 pt: the object distance s is the independent variable; the image distance s' is dependent; place the object at several measured positions in front of the lens and move the screen until the image is sharp. 1 pt: measure s and s' from the lens (center) with the meter stick; the image is sharpest where moving the screen either way blurs it. (Excess steps such as wiring a lamp earn nothing.) (b) 1 pt: for each s , find the sharp image position several times (approaching from both directions) and average. 1 pt: it narrows the range over which the image looks sharp; alternatively use a bigger range of s or a darkened room to sharpen the image. (c) 1 pt: use $1/s' = 1/f - 1/s$, so plot $1/s'$ on the vertical axis against $1/s$ on the horizontal axis. 1 pt: the slope is -1 and the vertical intercept is $1/f$. (d) 1 pt: $1/s = 0.0500, 0.0400, 0.0333, 0.0250, 0.0167$; $1/s' = 0.0168, 0.0270, 0.0328, 0.0426, 0.0500$. 1 pt: labeled axes with units (1/cm), scale, points plotted, a straightedge line through the trend. (e) 1 pt: the vertical intercept is about 0.067 per cm (the fit gives slope -1.00 and intercept 0.0670). 1 pt: $f = 1/0.067 = 14.9$ cm, about 15 cm.

Model: I vary s and find the sharp image position s' with a meter stick, repeating each position and averaging. I plot $1/s'$ against $1/s$: the slope is -1 and the intercept $1/f$. My line crosses at 0.067 per cm, so $f = 15$ cm.

4. (a) 1 pt: red has the larger angle. 1 pt: constructive interference requires $d \sin \theta = m\lambda$; for the same d and m , a longer wavelength needs a larger $\sin \theta$ (violet, being shorter, has a smaller angle). (b) 1 pt: $d = 1/(500 \text{ mm}^{-1}) = 2.0 \times 10^{-6}$ m and $\sin \theta = \lambda/d$. 1 pt: $\sin \theta = 600 \times 10^{-9}/2.0 \times 10^{-6} = 0.30$, so $\theta = 17.5$ degrees. (c) 1 pt: $\sin \theta \leq 1$ so $m \leq d/\lambda = 3.33$. 1 pt: the highest observed order is $m = 3$ (m must be an integer). (d) 1 pt: more lines per mm means a smaller d . 1 pt: $\sin \theta = \lambda/d$ increases, so the angle becomes larger and the lines spread farther apart.

Model: Red has the larger first-order angle because $d \sin(\theta) = m\lambda$ gives a larger $\sin(\theta)$ for longer wavelength. For 600 nm, $\sin(\theta) = 0.30$, $\theta = 17.5$ degrees, and the maximum order is 3. A smaller d gives a larger angle.

5. (a) 1 pt: each charge produces a field of $kq/r^2 = (8.99 \times 10^9)(3.0 \times 10^{-9})/(0.20)^2 = 674$ N/C, and both point toward the negative charge, so they add. 1 pt: $E = 1.35 \times 10^3$ N/C directed toward the -3.0 nC charge. (b) 1 pt: potential is a scalar: $V = kq/r - kq/r = 0$. (c) 1 pt: the claim is wrong; the field is related to how the potential changes with position, not to its value. 1 pt: here the potential is zero at M but drops from positive near the $+3.0$ nC charge to negative near the other, so there is a field directed toward decreasing potential (toward the negative charge).

Model: At M both fields point toward the negative charge, so $E = 2(674) = 1.35\text{e}3$ N/C. The potentials cancel, $V = 0$, but a zero potential does not imply zero field because the field depends on the potential gradient.

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Reference: Formulas, Skills Checklist, and Glossary

Quick-reference pages to review the night before the exam.

R1 Equations, Constants, and Conventions

A unit-by-unit reference to the constants, relationships and sign conventions used in this book, with a note on what each means.

How to Use This Page

An equation sheet and table of constants are provided on the exam, but you must know what each relationship means, when it applies and which sign convention it uses. Symbols here follow common usage; check your reference sheet for its exact notation. Unless noted, a calculator is allowed, $g = 9.8 \text{ m/s}^2$ is used in this book, conventional current is used, and batteries and meters are ideal.

Constants Used in This Book

Constant	Value
Boltzmann constant k_B	$1.38 \times 10^{-23} \text{ J/K}$
Gas constant R	8.31 J/(mol K)
Avogadro's number	$6.02 \times 10^{23} \text{ per mol}$
Coulomb constant k	$8.99 \times 10^9 \text{ N m}^2/\text{C}^2$
Elementary charge e	$1.60 \times 10^{-19} \text{ C}$
Vacuum permittivity ϵ_0	$8.85 \times 10^{-12} \text{ C}^2/(\text{N m}^2)$
Permeability μ_0	$4\pi \times 10^{-7} \text{ T m/A}$
Speed of light c	$3.00 \times 10^8 \text{ m/s}$
Planck's constant h	$6.63 \times 10^{-34} \text{ J s} = 4.14 \times 10^{-15} \text{ eV s}$
Electron / proton mass	$9.11 \times 10^{-31} \text{ kg} / 1.67 \times 10^{-27} \text{ kg}$
Atomic mass unit	$1 \text{ u} = 931.5 \text{ MeV}/c^2; 1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$

Equations by Unit

Unit 9: Thermodynamics. Positive work means work done on the system.

Relationship	Meaning and sign notes
$PV = nRT = Nk_B T$	Ideal gas; T in kelvin
$K_{avg} = \frac{3}{2}k_B T; v_{rms} = \sqrt{3k_B T/m}$	Average translational energy depends only on T
$\Delta U = Q + W$	$Q > 0$ heat in; $W > 0$ work done on gas
$W = -P\Delta V$ (constant P)	Compression gives $W > 0$; $ W $ is the area under the P-V curve
$Q = mc\Delta T; \frac{Q}{\Delta t} = \frac{kA\Delta T}{L}$	ΔT is the difference across the material

Unit 10: Electric force, field and potential. Electron charge is $-e$; the field points away from positive charge.

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Relationship	Meaning and sign notes
$F = k \frac{q_1 q_2}{r^2}$	Like charges repel; apply to magnitudes, then assign direction
$\vec{E} = \vec{F}/q; E = kq/r^2$	Force on a negative charge is opposite to $\vec{E}$
$V = kq/r; U = kq_1 q_2 / r$	Scalars; $V = 0$ at infinity; signs are kept
$\Delta V = -Ed; \Delta U = q\Delta V$	Uniform field; the field points toward decreasing potential
$C = Q/\Delta V; C = \kappa\epsilon_0 A/d; U = \frac{1}{2}C\Delta V^2$	Q is the magnitude on one plate

Unit 11: Circuits.

Relationship	Meaning and sign notes
$I = \Delta q/\Delta t; V = IR; R = \rho L/A$	Conventional current from + to -
$P = IV = I^2 R = V^2/R$	Use the voltage across that element
$R_s = \sum R; 1/R_p = \sum 1/R$	Capacitors are the reverse: parallel adds
$\sum \Delta V = 0; \sum I_{in} = \sum I_{out}$	Loop rule: gain at battery, drop across resistor along the current
$\tau = RC$	Charging or discharging is exponential (qualitative)

Unit 12: Magnetism. Right-hand rule for positive charge or conventional current.

Relationship	Meaning and sign notes
$F = qvB \sin \theta; F = ILB \sin \theta$	Force is perpendicular to both $\vec{v}$ (or I) and $\vec{B}$
$B = \mu_0 I/(2\pi r)$	Long straight wire
$r = mv/(qB)$	Circular path when $\vec{v} \perp \vec{B}$
$\Phi = BA \cos \theta; \mathcal{E} = \Delta \Phi/\Delta t ; \mathcal{E} = BLv$	Lenz: the induced current opposes the change in flux

Unit 13: Geometric optics. Real images have positive s' ; virtual images negative.

Relationship	Meaning and sign notes
$\theta_i = \theta_r; n = c/v$	Angles from the normal
$n_1 \sin \theta_1 = n_2 \sin \theta_2; \sin \theta_c = n_2/n_1$	Critical angle needs $n_1 > n_2$
$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}; m = -\frac{s'}{s}$	Converging: $f > 0$; $m < 0$ means inverted; for a mirror $f = R/2$

Unit 14: Waves.

Relationship	Meaning and sign notes
$v = f\lambda; T = 1/f$	Speed is set by the medium
$f_{obs} = f \frac{v \pm v_o}{v \mp v_s}$	Closing speeds raise frequency
$\lambda_n = 2L/n$ (fixed-fixed, open-open)	Closed-open pipe: odd harmonics only, $\lambda = 4L/n$
$d \sin \theta = m\lambda; \Delta y \approx \lambda L/d$	Slits and gratings, bright fringes
$a \sin \theta = m\lambda$ ($m = 1, 2, \dots$)	Single-slit dark fringes
$2t = (m + \frac{1}{2})\lambda_{film}$ or $m\lambda_{film}$	Choose by counting phase flips; $\lambda_{film} = \lambda/n$

Unit 15: Modern physics.

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Relationship	Meaning and sign notes
$E = hf = hc/\lambda$; $p = h/\lambda$	Photons; $hc \approx 1240 \text{ eV nm}$
$\lambda = h/p$	Matter waves
$K_{max} = hf - \phi$	$K_{max} = eV_s$
$E_n = -13.6 \text{ eV}/n^2$; $\Delta E = hf$	Hydrogen; photon energy equals level difference
$E = \Delta m c^2$; $N = N_0(1/2)^{t/T_{1/2}}$	Mass defect releases energy

R2 Skills Checklist and Glossary

A self-check of the ten exam skills and a glossary of key AP Physics 2 terms.

Skills Checklist

Tick each skill only when you can do it without notes, then test yourself on a past question that uses it.

Skill	I can...
1.A Representations	draw free-body diagrams, circuits, ray diagrams and energy-level diagrams with labels
1.B Quantitative graphs	plot data with scales and units, draw a straightedge best-fit line, find slope and intercept
1.C Qualitative graphs	sketch inverse, exponential-like and sinusoidal curves with correct trends and no axis-touching errors
2.A Derivations	start from a reference equation and solve symbolically in the given variables
2.B Calculations	compute with correct units, scientific notation and reasonable estimates
2.C Comparisons	compare two scenarios using ratios
2.D Predictions	predict how a quantity changes when another is scaled
3.A Procedures	design a minimal experiment with variables, measurements and uncertainty reduction
3.B Apply a model	use a law to support a claim with stated conditions
3.C Justify	connect claim, evidence and reasoning in complete sentences

Habits Before You Submit

- ◆ Is the sign convention for work and charge stated?
- ◆ Are units written on every answer?
- ◆ Does the explanation refer to the specific earlier part?
- ◆ Are conditions such as constant n and T stated?

Glossary

Term	Meaning
Absolute zero	0 K, the lowest possible temperature
Activity	Number of nuclear decays per second
Alpha decay	Emission of a helium-4 nucleus
Amplitude	Maximum displacement from equilibrium
Beta decay	Nuclear change of a neutron into a proton (minus) or reverse (plus), with a lepton pair
Binding energy	Energy needed to separate a nucleus into nucleons
Blackbody	Idealized object whose thermal spectrum depends only on temperature
Capacitance	Charge stored per volt, $C = Q/V$
Centripetal force	Net force toward the center of circular motion
Constructive interference	Waves meeting in phase, giving a larger amplitude

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Term	Meaning
Converging lens	Lens that bends parallel rays toward a focal point
Critical angle	Smallest incidence angle giving total internal reflection
Diffraction	Spreading of waves passing an opening or edge
Doppler effect	Change in observed frequency from relative motion
Electric field	Force per unit positive test charge
Electric potential	Potential energy per unit charge
Emf	Energy per charge supplied by a source
Entropy	Measure of the number of microstates; never decreases for an isolated system
Equipotential	A surface of constant potential
Faraday's law	Induced emf equals the rate of change of magnetic flux
Fission	A heavy nucleus splitting into lighter ones
Fusion	Light nuclei combining into a heavier one
Gamma ray	High-energy photon from a nucleus
Half-life	Time for half of a sample to decay
Harmonic	Standing-wave pattern at a natural frequency
Ideal gas	Gas of point particles with no intermolecular forces
Internal energy	Total microscopic energy of a system
Isobaric	At constant pressure
Isothermal	At constant temperature
Isovolumetric	At constant volume
Kirchhoff's junction rule	Current in equals current out at a junction
Kirchhoff's loop rule	Potential changes around a closed loop sum to zero
Lenz's law	Induced current opposes the change in flux
Magnetic flux	B times area times cosine of angle
Mass defect	Mass of separated nucleons minus the nucleus
Photoelectric effect	Ejection of electrons from a metal by light
Photon	Quantum of light with energy hf
Polarization	Restriction of transverse oscillations to one direction
Real image	Image where light rays actually converge
Refraction	Bending of a wave at a boundary
Resistivity	Material property in $R = \rho L/A$
rms speed	Square root of the mean of the squared speeds
Specific heat	Energy per mass per kelvin to change temperature
Standing wave	Fixed pattern of nodes and antinodes
Stopping potential	Voltage that just halts the fastest photoelectrons
Threshold frequency	Lowest frequency that can eject electrons
Time constant	RC , the characteristic time of an RC circuit
Total internal reflection	Complete reflection at a boundary with a smaller index
Virtual image	Image from which rays only appear to diverge

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Term	Meaning
Wave-particle duality	Light and matter show both wave and particle behavior
Wavelength	Distance between repeating points of a wave
Work function	Minimum energy to free an electron from a metal
Compton scattering	Photon scattering with a longer wavelength after losing energy to an electron
Thin-film interference	Interference between reflections at two surfaces of a thin layer
Node	Point of zero amplitude in a standing wave

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