



SCHOLAR COACH

Boutique Academic Coaching & Tutoring

AP PHYSICS C: ELECTRICITY AND MAGNETISM · COURSE REVIEW AND PRACTICE

AP Physics C: Electricity and Magnetism **Mastery.**

Every unit on the exam: concise concept review, worked examples, exam-style writing, practice problems with full explanations, a mixed practice test, and a reference section.

INSIDE

Words in Context · Inferences · Command of Evidence · Central Ideas and Details · Function of a Sentence · Algebra · Advanced Math · Problem-Solving and Data Analysis · Geometry and Trigonometry · Three Full-Length Practice Exams with Explanations



Table of Contents.

Course review, practice, and a mixed test.

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Start Here: The Exam, Your Plan, and Your Tools

How the AP Physics C: Electricity and Magnetism exam is built, how points are earned, and how to use this book.

F Start Here: Exam Overview and Study Plan

What the AP Physics C: E&M exam looks like, how the free-response questions are scored, how the units are weighted, and how to use this book with a 4-week or 8-week plan.

Why This Matters

AP Physics C: Electricity and Magnetism is the calculus-based second course, equivalent to a semester of college physics, and it assumes the reasoning habits from Mechanics: define a system, choose a model, derive before you substitute. The exam rewards a clean chain of laws far more than a lone number. This book is organized around that: each unit teaches the official topics, models full-credit writing, and drills the traps that cost points.

Exam Format

Section	Questions	Time	Share
I. Multiple choice	42 (discrete items and short sets)	85 minutes	50%
II. Free response	4	95 minutes	50%

The exam is Wednesday, May 5, 2027 (Session 2) and is hybrid digital: you answer the multiple-choice section on a device and write free-response answers by hand in a booklet. A calculator (four-function, scientific, or graphing) is allowed in both sections, and reference information is provided; confirm the current version of the reference sheet with your teacher and learn where each quantity lives before test day. The 2027 changes are to the question count and timings; the content and skills are unchanged.

Unit	Title	MC weight
8	Electric Charges, Fields, and Gauss's Law	15-25%
9	Electric Potential	10-20%
10	Conductors and Capacitors	10-15%
11	Electric Circuits	15-25%
12	Magnetic Fields and Electromagnetism	10-20%
13	Electromagnetic Induction	10-20%

The Four Free-Response Questions

The four free-response questions come in a fixed order and each tests a different mix of skills. Practice each type separately, because the way you earn points differs.

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Q	Type	Points	What earns credit
1	Mathematical Routines	10	A multistep derivation, often with an integral, that may end in a number. Show setup, integral, limits, result.
2	Translation Between Representations	12	Graphs or bar charts with labeled axes and scales, plus comparing quantities with equations.
3	Experimental Design and Analysis	10	A procedure, measured versus calculated quantities, linearized graph, and slope tied to a physical quantity.
4	Qualitative/Quantitative Translation	8	A claim reasoned in words, then a derivation, with the two connected.

Pacing: the multiple-choice section allows about 2 minutes per question, and the free-response section about 24 minutes per question on average, but the 12-point question is longer than the 8-point one. Spend your time in proportion to points, and when you are stuck, write the governing law and the symmetry argument; partial credit is awarded for each correct step, and a blank earns nothing.

Three skill groups are scored: creating representations (diagrams, quantitative and qualitative graphs), mathematical routines (derive, calculate, compare, predict), and argumentation (design a procedure, apply a law to a claim, justify with data or principles). On a justification, name the principle and the quantities it connects; a correct number with no reasoning earns partial credit at best.

How Points Are Earned

Habits that earn points

Start from a law or definition, not from the last line of a formula you remember.

Define every symbol and each coordinate direction; draw the picture.

Show an integral or derivative before evaluating it, with limits.

Carry units through and check that dimensions match.

Justify claims with equal-quantity comparisons ("same charge, same speed, different distance").

Typical errors: the wrong Gaussian surface, mixed charge densities, field lines that do not match symmetry, a graph that is the wrong shape (E falling as $1/r$, not linearly), emf confused with current, and slopes computed from one point and the origin without justification. Each unit's trap list names the ones that belong to it.

Study Plans

Week	4-week plan	Do this
1	Units 8 and 9	Read, work examples, finish both practice sets; redo missed items after two days.
2	Units 10 and 11	Practice circuits daily; do one timed free-response set.
3	Units 12 and 13	Focus on right-hand rules and Faraday/Lenz reasoning.
4	Mixed test and clinic	Take the mixed test timed; fix weak units; do the free-response clinic.

Week	8-week plan	Do this
1	Unit 8	Coulomb, fields, integrals; symmetry arguments for Gauss.
2	Unit 9	Potential, energy conservation, graphs both ways.
3	Unit 10	Conductors and capacitors; dielectrics.
4	Unit 11	Circuits, Kirchhoff, RC.
5	Unit 12	Magnetic force, Biot-Savart, Ampere.
6	Unit 13	Induction, inductors, LR and LC.
7	Mixed test	Timed test; sort errors by unit.
8	Clinic and review	Free-response clinic; reference pages; light review.

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How to Use This Book

Each unit follows the same path: Core Ideas, Worked Examples, a model free-response answer with the grader's checklist, a playbook of traps, quick checks, and ten practice problems with full solutions. Attempt an example before reading the solution. After practice, keep a log of every missed item with the cause (concept, setup, algebra, unit, wording) so you can see patterns. The back of the book has a mixed test, a free-response clinic, and two reference pages for equations, skills, and vocabulary.

Key Math Habits

- ◆ Calculus: integrate over elements ($dq = \lambda dx$, σdA , ρdV); differentiate to relate quantities ($E = -dV/dx$, $I = dQ/dt$, $\mathcal{E} = -d\Phi/dt$).
- ◆ Symmetry first: decide what cancels before integrating.
- ◆ Check limits: far away, small, or large parameters should recover a known result.
- ◆ Estimate orders of magnitude; use unit analysis as a quick filter.
- ◆ Keep vectors as vectors: components, directions, and the right-hand rule.

Warm-Up Practice

The four items below check three habits: units, derivatives, and a short derivation. Full solutions follow.

Practice

PRACTICE Warm-Up Practice

Show your work. A calculator is allowed. Justify conclusions.

- Which of the following has the units of an electric field? Here k is the Coulomb constant, Q a charge, and x and R lengths.

(A) $\frac{kQx}{x^2 + R^2}$

(B) $\frac{kQ}{(x^2 + R^2)^{3/2}}$

(C) $\frac{kQx}{(x^2 + R^2)^{3/2}}$

(D) $\frac{kQx^2}{(x^2 + R^2)^{3/2}}$
- In a region the potential is $V(x) = 5.0 - 3.0x^2$, with V in volts and x in meters. The x component of the electric field at $x = 0.50$ m is

(A) $+3.0$ V/m

(B) -3.0 V/m

(C) $+4.25$ V/m

(D) -1.5 V/m
- A point charge $q = +2.0$ nC is fixed at the origin.
 - Starting from Coulomb's law, derive the field $E(r)$ and calculate its magnitude at $r = 0.50$ m.
 - By what factor does the field magnitude change if the distance is doubled? Justify with the formula.
 - Sketch E versus r qualitatively and state its shape.
- A student wants to test whether the field of a long charged rod falls off as $1/r$.
 - Describe a measurement procedure (instruments, what is varied, what is held fixed).
 - State what to plot to get a straight line and what the slope represents.
 - Give one source of error and how it would show up in the graph.

Answers and Explanations

F Start Here: Exam Overview and Study Plan

PRACTICE · WARM-UP PRACTICE

- C. Electric field has units of kQ/L^2 . The choice $kQx/(x^2 + R^2)^{3/2}$ has one length over three, giving kQ/L^2 . Choices A and D give kQ/L and choice B gives kQ/L^3 , so they cannot be fields. It is also the ring formula you derive in Unit 8.
- A. $E_x = -dV/dx = 6.0x = 3.0$ V/m, in the $+x$ direction. The minus sign must be applied once, and the value of V itself (4.25) is not the field.
- (a) Coulomb's law gives the force on a test charge q_0 as $F = kqq_0/r^2$ directed away from q . The field is $E = F/q_0 = kq/r^2$. At 0.50 m: $E = (8.99 \times 10^9)(2.0 \times 10^{-9})/(0.50)^2 = 72$ N/C, radially outward (positive charge).
 (b) $E \propto 1/r^2$, so doubling r multiplies E by $1/4$ (18 N/C).
 (c) The curve starts very large near the charge and decreases steeply, always positive, approaching zero for large r : a $1/r^2$ shape, not a straight line.

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4. (a) Charge the rod; place a small field probe (an electrometer with a test sphere or a force sensor on a small known charge q_0) at several measured distances r from the rod's axis, in the middle of the rod to avoid end effects. Keep the rod's charge fixed (use the same charging method and take readings quickly), and vary only r . Measure $E = F/q_0$ at each distance.
- (b) If $E = \lambda/(2\pi\epsilon_0 r)$, plot E versus $1/r$: a straight line through the origin with slope $\lambda/(2\pi\epsilon_0)$, so λ follows from the slope.
- (c) Charge leakage in humid air makes later readings lower, so points taken at larger r (if taken last) fall below the line; a rod that is not long compared with r makes the field fall faster than $1/r$, curving the graph downward at large r .

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Unit 8: Electric Charges, Fields, and Gauss's Law

Exam weighting: about 15-25% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

8 Electric Charges, Fields, and Gauss's Law

Charge, Coulomb's law, and fields of point charges and continuous distributions, then flux and Gauss's law, with integrals and symmetry arguments.

Why This Matters

Unit 8 carries 15-25% of the multiple-choice section and feeds nearly every free-response question, because fields, flux, and symmetry return in capacitors, circuits, and magnetism. Points are lost in three places: vectors added as if they were scalars, Gaussian surfaces chosen without a symmetry argument, and integrals started without defining the small element (dq , its position, and the component that survives). Calculus is the tool throughout; this unit uses $k = 1/(4\pi\epsilon_0) = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$ and $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N m}^2)$.

Core Ideas

8.1 Electric Charge and Electric Force

Charge comes in two kinds and is **quantized**: every free charge is an integer multiple of $e = 1.60 \times 10^{-19} \text{ C}$. Two point charges exert equal and opposite forces on each other; the model treats an object as a point whenever its size does not affect the analysis. The force on charge 2 due to charge 1, with $\hat{r}$ pointing from 1 to 2, is

Coulomb force and superposition

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \text{ (like charges repel, unlike attract)}$$

$$\vec{F}_{\text{net}} = \sum_i \vec{F}_i: \text{ forces from several charges add as vectors, one pair at a time.}$$

Doubling the distance cuts the force by 4; doubling one charge doubles the force.

Add by components: resolve each force, sum x and y separately, then rebuild magnitude and angle.

8.2 Conservation of Electric Charge and the Process of Charging

Net charge is conserved: charge is moved, never created or destroyed. In **conductors** some charges move freely; in **insulators** they do not. *Friction* and *conduction* (touching) transfer charge; two identical conducting spheres that touch share the total equally.

Induction charges an object with no net transfer from the charging body: a charged rod near a neutral conductor polarizes it, grounding lets charge flow on or off, and removing the ground before the rod leaves the conductor with charge opposite to the rod. A neutral insulator near a rod polarizes slightly and is attracted.

8.3 Electric Fields

The electric field at a point is the force per unit charge on a small positive test charge placed there: $\vec{E} = \vec{F}/q$. The test charge is small enough not to disturb the sources. For a point source charge Q , $E = kQ/r^2$, directed away from positive and toward negative charge. Field lines start on positive charge, end on negative, never cross, and crowd where the field is stronger. A charge q in a field feels $\vec{F} = q\vec{E}$: along $\vec{E}$ for positive q , opposite for negative q . Fields superpose as vectors, which is the source of most mistakes.

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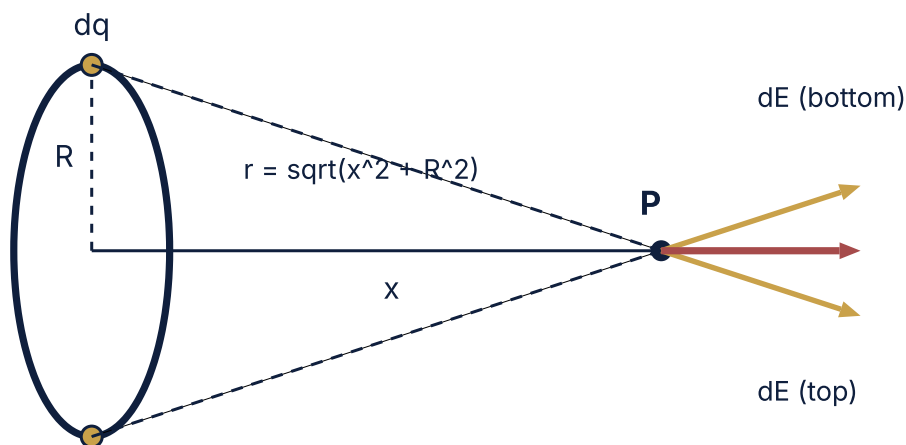
8.4 Electric Fields of Charge Distributions

A continuous distribution is cut into elements dq , each treated as a point charge, and the contributions are integrated. The charge element depends on the geometry: $dq = \lambda dx$ on a line, σdA on a surface, ρdV in a volume (linear, surface, and volume densities; do not mix them).

Procedure for a field by integration

1. Draw the object, the field point, and one element dq ; label its position.
2. Write $dE = k dq/r^2$ and resolve it; use symmetry to cancel components.
3. Express dq and r with one integration variable.
4. Integrate with limits that cover the whole object; then check units and limiting cases (far away it should look like a point charge).

A ring of charge Q and radius R , at a point on its axis a distance x from the center, has $dE_x = k dq \cos \theta / (x^2 + R^2)$ with $\cos \theta = x / \sqrt{x^2 + R^2}$. Every element has the same r and $\cos \theta$, so the integral of dq is just Q :



Ring of charge seen edge-on at an angle: the perpendicular parts of dE from opposite elements cancel, leaving only the axial part (dark arrow).

Standard results (verify limits as you use them)

Ring, on axis: $E = \frac{kQx}{(x^2 + R^2)^{3/2}}$ (zero at the center, kQ/x^2 far away)

Disk, uniform σ , on axis: $E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{x}{\sqrt{x^2 + R^2}} \right)$

Rod of length L , perpendicular bisector at distance y : $E = \frac{kQ}{y\sqrt{y^2 + L^2/4}}$

Infinite line: $E = \frac{\lambda}{2\pi\epsilon_0 r}$; infinite sheet: $E = \frac{\sigma}{2\epsilon_0}$

8.5 Electric Flux

Electric flux measures how much field passes through a surface: $\Phi_E = \int \vec{E} \cdot d\vec{A}$. For a flat area in a uniform field, $\Phi_E = EA \cos \theta$, where θ is the angle between $\vec{E}$ and the area vector (perpendicular to the surface). For a closed surface the area vector points outward, so outgoing field gives positive flux and incoming field gives negative flux. Flux is a scalar with units $\text{N m}^2/\text{C}$. It is zero when the field is parallel to the surface.

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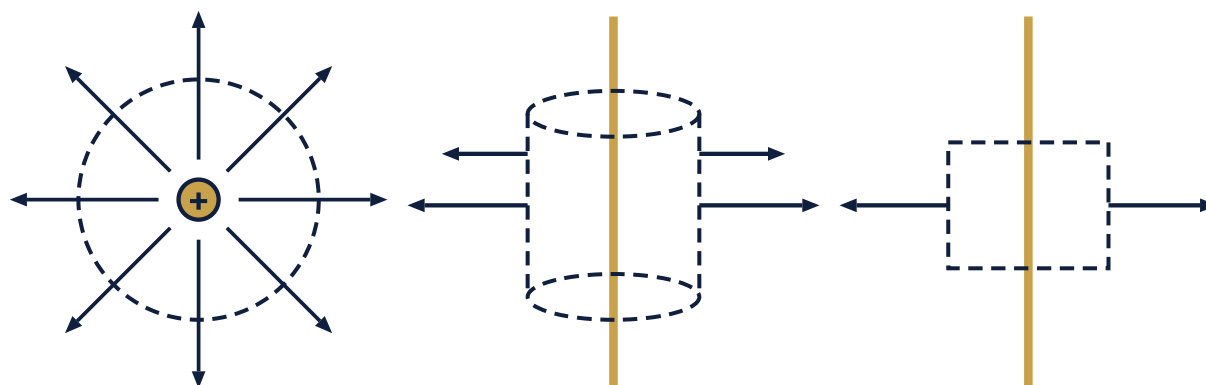
8.6 Gauss's Law

Gauss's law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \text{ for any closed surface, with } Q_{enc} \text{ the net charge inside.}$$

Always true; it is only useful for finding E when symmetry makes E constant (or zero) over each piece of the surface.

Charges outside the surface contribute to $\vec{E}$ on it but not to the net flux. Use Gauss's law to find E only for spherical, cylindrical, or planar symmetry, and in the free response you must state the symmetry argument: why the field has the assumed direction and depends on only one coordinate. Then choose a surface on which $\vec{E}$ is perpendicular and constant, or parallel (zero flux).



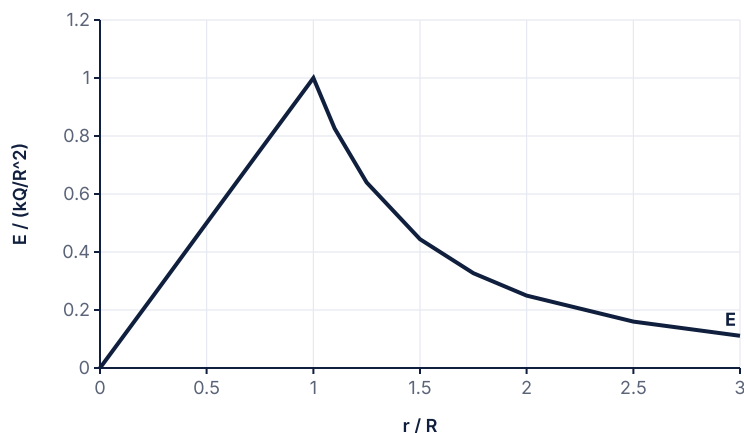
Sphere: $A = 4\pi r^2$

Cylinder: side $A = 2\pi r L$

Plane: caps $A + A = 2A$

Gauss surfaces matched to symmetry: field lines cross the sphere, the side of the cylinder, or the two caps of the pillbox, always perpendicular to the surface.

With volume density ρ that varies, find the enclosed charge by integrating ρdV over shells ($dV = 4\pi r^2 dr$ for a sphere), not by multiplying ρ by a volume. For a uniform solid insulating sphere, $E = kQr/R^3$ inside and kQ/r^2 outside.



Uniformly charged solid insulating sphere: linear inside, $1/r^2$ outside.

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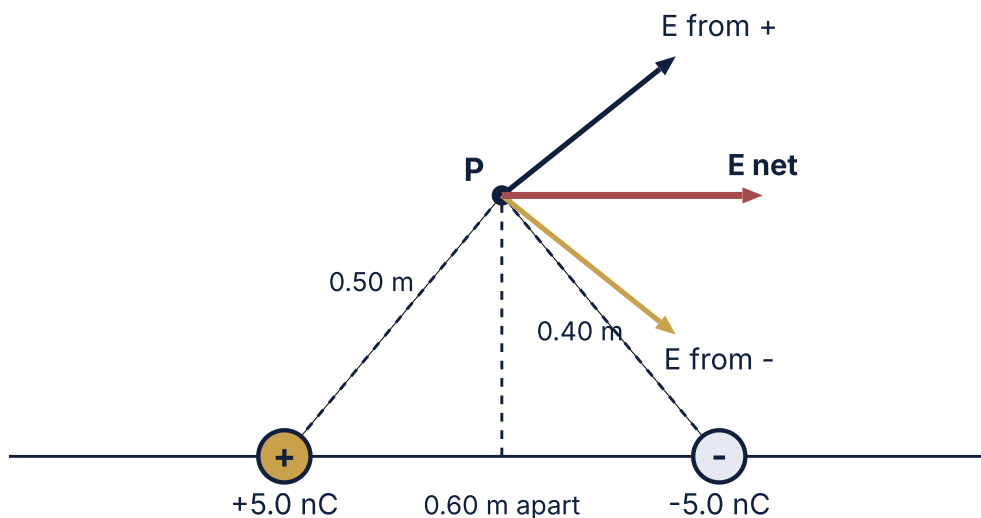
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Worked Examples

EXAMPLE 1: SUPERPOSITION OF TWO POINT-CHARGE FIELDS

A charge of $+5.0 \text{ nC}$ sits at $x = -0.30 \text{ m}$ and a charge of -5.0 nC at $x = +0.30 \text{ m}$, both on the x axis. Find the electric field at the point $P = (0, 0.40 \text{ m})$.



Example 1 geometry: each charge is 0.50 m from P ; the y components cancel and the x components add.

SOLUTION

Each charge is $\sqrt{0.30^2 + 0.40^2} = 0.50 \text{ m}$ from P , so each makes a field of magnitude $E = kq/r^2 = (8.99 \times 10^9)(5.0 \times 10^{-9})/0.25 = 180 \text{ N/C}$.

The $+5.0 \text{ nC}$ charge pushes away: components $(+0.6, +0.8) \times 180$. The -5.0 nC charge pulls toward itself: components $(+0.6, -0.8) \times 180$.

E_y cancels. $E_x = 2(180)(0.6) = 216 \text{ N/C}$, so $\vec{E} = 216 \text{ N/C}$ in the $+x$ direction. Adding magnitudes (360 N/C) is the classic error.

EXAMPLE 2: RING OF CHARGE BY INTEGRATION

A ring of radius $R = 0.10 \text{ m}$ carries $Q = +12 \text{ nC}$ uniformly. (a) Derive E on the axis at distance x . (b) Evaluate it at $x = 0.20 \text{ m}$. (c) At what x is the field largest?

SOLUTION

(a) $dE_x = \frac{k dq}{x^2 + R^2} \cdot \frac{x}{\sqrt{x^2 + R^2}}$. The factors multiplying dq are the same for every element, so $E = \frac{kx}{(x^2 + R^2)^{3/2}} \oint dq = \frac{kQx}{(x^2 + R^2)^{3/2}}$, directed along the axis away from the ring.

(b) $E = (8.99 \times 10^9)(12 \times 10^{-9})(0.20)/(0.05)^{3/2} = 1.93 \times 10^3 \text{ N/C}$. A point charge of 12 nC at 0.20 m would give 2.70 kN/C .

(c) Set $dE/dx = 0$: $(x^2 + R^2) - 3x^2 = 0$, so $x = R/\sqrt{2} = 0.071 \text{ m}$, where $E_{\max} = 4.15 \times 10^3 \text{ N/C}$.

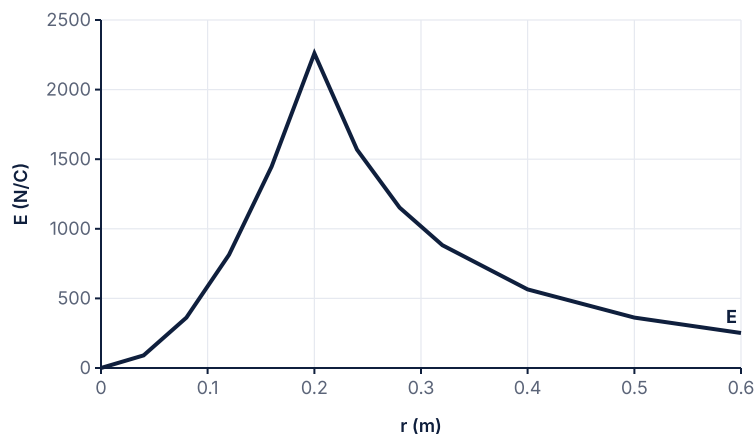
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EXAMPLE 3: GAUSS'S LAW WITH A NONUNIFORM SPHERE

A solid insulating sphere of radius $R = 0.20$ m has volume charge density $\rho = Ar$ with $A = 2.0 \times 10^{-6}$ C/m⁴. Find the total charge and the field at $r = 0.10$ m and $r = 0.40$ m.



Example 3: E rises as r^2 inside and falls as $1/r^2$ outside; the peak is at the surface.

SOLUTION

Symmetry is spherical, so use a sphere of radius r . Enclosed charge: $Q_{enc} = \int_0^r (As)(4\pi s^2) ds = \pi Ar^4$.

Total: $Q = \pi AR^4 = \pi(2.0 \times 10^{-6})(0.20)^4 = 1.0 \times 10^{-8}$ C.

Inside, $E(4\pi r^2) = \pi Ar^4/\epsilon_0$, so $E = Ar^2/(4\epsilon_0)$; at 0.10 m, $E = 565$ N/C.

Outside, $E = kQ/r^2 = 565$ N/C at 0.40 m. The two values match because 0.10 m is $R/2$ (inside field is 1/4 of the surface value) and 0.40 m is $2R$ (outside field is also 1/4 of it). The peak, at the surface, is 2.26×10^3 N/C.

EXAMPLE 4: DISK OF CHARGE AND THE LIMITS

A thin disk of radius $R = 0.12$ m carries uniform $\sigma = 5.0$ nC/m². Derive the axial field, then find it at $x = 0.05$ m and $x = 0.30$ m, and compare the far value to a point charge.

SOLUTION

Cut the disk into rings of radius s , width ds , charge $dq = \sigma 2\pi s ds$. Each ring contributes $dE = \frac{k dq x}{(x^2 + s^2)^{3/2}}$.

$$E = \frac{\sigma x}{2\epsilon_0} \int_0^R \frac{s ds}{(x^2 + s^2)^{3/2}} = \frac{\sigma x}{2\epsilon_0} \left[\frac{1}{x} - \frac{1}{\sqrt{x^2 + R^2}} \right] = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{x}{\sqrt{x^2 + R^2}} \right).$$

$\sigma/(2\epsilon_0) = 282$ N/C. At $x = 0.05$ m: $E = 282(1 - 0.05/0.13) = 174$ N/C. At $x = 0.30$ m: $E = 282(1 - 0.30/0.3231) = 20.2$ N/C.

Point-charge estimate at 0.30 m with $Q = \sigma\pi R^2 = 2.26 \times 10^{-10}$ C: $kQ/x^2 = 22.6$ N/C, about 12% too high because the disk is not yet small compared with 0.30 m. As $x \rightarrow 0$ the field approaches $\sigma/2\epsilon_0$, the infinite-sheet value.

Write It Like a 5**Model Response**

Prompt: A solid insulating sphere of radius R carries total charge $+Q$ spread uniformly through its volume. Use Gauss's law to derive the magnitude of the electric field at a distance $r < R$ from the center, and justify your choice of surface. The charge distribution is spherically symmetric, so the field at any point can only point radially (any sideways component would single out a direction that the charges do not have) and its magnitude can depend only on r . I choose a Gaussian sphere of radius $r < R$, concentric with the charge. On it, E is perpendicular to the surface and constant, so the flux is $E(4\pi r^2)$. The enclosed charge is the volume fraction: $Q_{enc} = Q(r^3/R^3)$, because the density $Q/((4/3)\pi R^3)$ is uniform. Gauss's law gives $E(4\pi r^2) = Qr^3/(\epsilon_0 R^3)$, so $E = Qr/(4\pi\epsilon_0 R^3) = kQr/R^3$, directed radially outward. It grows linearly with r and matches kQ/R^2 at the surface, as it must.

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What the grader looks for

1. A symmetry argument naming direction and dependence, not just 'spherical'. 2. A concentric Gaussian sphere through the field point. 3. Flux $E(4\pi r^2)$ with the reason E is constant and perpendicular. 4. Enclosed charge for the surface radius, not the whole sphere. 5. Final expression with direction.

Coach's Playbook

- 1 Name the geometry and decide: point sum, integral, or Gauss's law.
- 2 For an integral, sketch one element, write its distance and angle, and cross out canceled components before integrating.
- 3 For Gauss, write the symmetry argument in a sentence, then flux, then Q_{enc} for each region.
- 4 Check limits (far away a bounded object gives kQ/r^2) and give direction and units.

Common traps

1. Using the volume or cross-section instead of only the curved side of a cylindrical surface, or leaving out the length L . 2. Mixing linear, surface and volume density, or using the full sphere's charge for a surface that is smaller. 3. Treating a vector sum of fields as a sum of magnitudes. 4. Sketching E versus r as a straight line or parabola outside a charged cylinder when it falls as $1/r$; or applying Gauss's law where no symmetry exists (a finite rod, a short cylinder).

Quick Check

Q1. What is the flux through a closed surface that encloses no net charge? A: Zero, even if the field on the surface is nonzero. Q2. A -3.0 nC charge sits in a 400 N/C field. What is the force magnitude and direction? A: $F = qE = 1.2 \times 10^{-6}$ N, opposite to the field. Q3. A line of charge: distance triples. E changes by what factor? A: One third, since $E \propto 1/r$.

Challenge Corner

A thin rod of length $L = 0.50$ m carries $Q = 6.0$ nC uniformly. Find the field on the line of the rod at a distance $d = 0.10$ m beyond one end. Place the origin at the field point; the rod covers x from d to $d + L$ with $dq = (Q/L) dx$, and all fields point along the axis. $E = \int_d^{d+L} \frac{k(Q/L) dx}{x^2} = \frac{kQ}{L} \left(\frac{1}{d} - \frac{1}{d+L} \right) = \frac{kQ}{d(d+L)}$. Numerically $E = (8.99 \times 10^9)(6.0 \times 10^{-9})/(0.10 \times 0.60) = 899$ N/C, pointing away from the rod (for positive Q). For $d \gg L$ this becomes kQ/d^2 , as required.

Practice**PRACTICE Unit 8 Practice**

Show your work. A calculator is allowed. Justify conclusions.

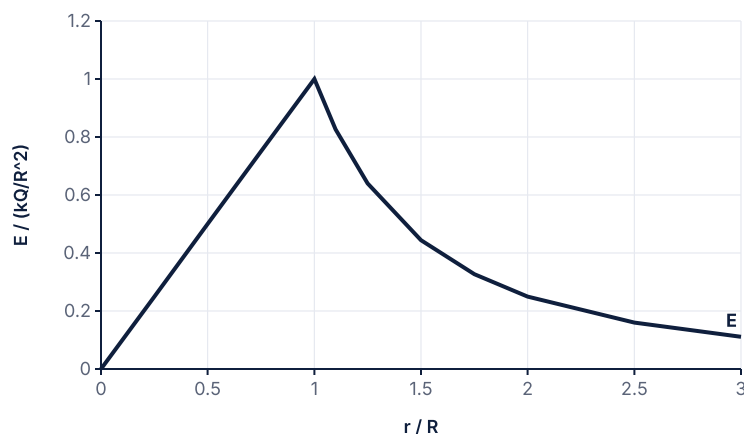
1. Two small spheres carry charges q and q and exert a force of magnitude F on each other. The charge on one sphere is doubled and the separation is tripled. The new force magnitude is
 (A) $2F/3$ (B) $2F/9$ (C) $4F/9$ (D) $6F$
2. Point charges $+Q$ and $+4Q$ are a distance d apart. At what point on the line between them is the net electric field zero?
 (A) $d/2$ from $+Q$ (B) $d/5$ from $+Q$ (C) $d/3$ from $+Q$
 (D) $2d/3$ from $+Q$
3. A point charge $+q$ is at the center of a cube with sides of length s . The electric flux through one face of the cube is
 (A) $q/(6\epsilon_0)$ (B) q/ϵ_0
 (C) $q/(6s^2\epsilon_0)$ (D) 0 because the field lines are bent

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4. The graph shows the magnitude of the electric field versus distance from the center for a charged object with spherical symmetry. The object is most likely



Electric field magnitude versus distance from the center.

- (A) a thin spherical shell of charge
(C) a long uniformly charged cylinder
- (B) a solid conducting sphere carrying charge
(D) a uniformly charged solid insulating sphere
5. A neutral metal sphere is on an insulating stand. A negatively charged rod is brought near, not touching. The sphere is momentarily touched by a grounded wire, the wire is removed, and then the rod is removed. The sphere ends up
(A) negatively charged, because electrons moved from the rod
(B) neutral, because the wire removed the induced charge
(C) positively charged, because electrons left through the ground
(D) positively charged, because protons came from the wire
6. A long straight uniformly charged wire produces a field of magnitude E_0 at a distance d . At a distance $2d$ the field magnitude is
(A) $E_0 / 4$
(B) $E_0 / 2$
(C) $E_0 / \sqrt{2}$
(D) $E_0 / 8$
7. A thin rod of length L lies along the x axis, centered at the origin, with total charge Q uniformly distributed. Point P is on the y axis at distance y from the rod.
(a) Draw the rod, an element dq at position x , and the field it makes at P. Explain by symmetry why only the y component of the net field survives.
(b) Derive an expression for the electric field at P. Show the integral before evaluating it. You may use $\int dx/(x^2 + y^2)^{3/2} = x/(y^2 \sqrt{x^2 + y^2})$.
(c) Show that the result reduces to the point-charge field when $y \gg L$, and to $\lambda/(2\pi\epsilon_0 y)$ for a very long rod with density $\lambda = Q/L$.
(d) Evaluate the field for $Q = 8.0 \text{ nC}$, $L = 0.60 \text{ m}$, and $y = 0.40 \text{ m}$.
8. A solid insulating sphere of radius R carries charge $+Q$ uniformly distributed throughout its volume. A thin concentric insulating spherical shell of radius $2R$ carries charge $-Q$ spread uniformly over its surface.
(a) Explain why the field is radial and depends only on r , and state the Gaussian surface you will use.
(b) Derive $E(r)$ for $r < R$, $R < r < 2R$, and $r > 2R$.
(c) Sketch E versus r from 0 to $3R$, labeling the values at R and $2R$.
(d) The field changes abruptly at $r = 2R$. Show that the size of the change equals $|\sigma|/\epsilon_0$, where σ is the surface charge density on the shell.

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9. Two identical small conducting spheres are given equal charge q by touching them together after one was charged. One sphere hangs from a force sensor and the other is held on an insulating rod at a measured center-to-center distance d . The table shows the force magnitude measured at several separations.
- d (cm): 2.0, 3.0, 4.0, 5.0, 6.0, 8.0
 F (mN): 9.10, 3.95, 2.21, 1.46, 0.98, 0.57
- (a) Describe a procedure that makes the charge on each sphere the same for every trial, and name one source of error in the charge during the experiment.
- (b) The student wants a straight-line graph. State what to plot on each axis and what the slope represents in terms of k and q .
- (c) Calculate the values you would plot, draw the best-fit line through the origin, and use its slope to find q .
- (d) At the smallest separations the measured force is expected to differ from the point-charge prediction. Explain why, and say which direction the discrepancy goes for these like charges.
10. Two identical conducting spheres A and B, small compared with their separation, carry charges $+8.0$ nC and -2.0 nC. Their centers are 0.30 m apart.
- (a) Find the force magnitude and say whether it is attractive or repulsive.
- (b) The spheres are touched together and returned to their original separation. Find the new charge on each and the new force. Justify the charge each sphere ends with.
- (c) A third identical neutral sphere C is touched first to A in its original $+8.0$ nC state, then to B in its original -2.0 nC state, and removed. Find the final charges and the force between A and B at 0.30 m.
- (d) Explain, without equations, why the force in (b) is smaller than in (a) even though the total charge is the same in both cases.

Answers Unit 8: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

8 Electric Charges, Fields, and Gauss's Law

PRACTICE · UNIT 8 PRACTICE

1. B. Force scales as q_1q_2/r^2 : doubling one charge gives a factor 2, tripling r gives $1/9$, so $2F/9$. Choosing $2F/3$ forgets that distance is squared; $4F/9$ squares the charge factor.
2. C. The fields are equal and opposite where $kQ/x^2 = k(4Q)/(d-x)^2$, so $(d-x) = 2x$ and $x = d/3$. The point $d/2$ would work only for equal charges; $d/5$ takes the charge ratio 4 instead of its square root 2.
3. A. Gauss's law gives q/ϵ_0 through the whole cube, and by symmetry each of the six faces carries one sixth. Choice B is the whole-cube flux; a flux has no $1/s^2$ because field times area already cancels the distance.
4. D. Inside the field rises linearly, which requires charge spread through the volume; outside it falls as $1/r^2$, so the source is spherical. A thin shell or conducting sphere would give zero field inside; a cylinder falls as $1/r$ outside.
5. C. The rod repels electrons in the sphere to the far side; grounding lets them leave. With the ground removed first, the leftover charge is net positive and spreads out when the rod leaves. No protons are transferred, and the sphere is not neutral.
6. B. By Gauss's law with a cylindrical surface, $E = \lambda/(2\pi\epsilon_0 r)$, which is inversely proportional to r , so doubling r halves E . The $1/4$ answer applies to a point charge, not a line.
7. (a) Each element $dq = \lambda dx$ at position x produces a field at P directed away from it. The element at $-x$ is the mirror image across the y axis, so the x components cancel in pairs and only dE_y survives; the net field points along $+y$ for positive charge. [diagram expected]
- (b)
$$dE_y = \frac{k\lambda dx}{x^2 + y^2} \cdot \frac{y}{\sqrt{x^2 + y^2}} \cdot E = 2k\lambda y \int_0^{L/2} \frac{dx}{(x^2 + y^2)^{3/2}} = 2k\lambda y \left[\frac{x}{y^2 \sqrt{x^2 + y^2}} \right]_0^{L/2} = \frac{2k\lambda(L/2)}{y \sqrt{y^2 + L^2/4}} = \frac{kQ}{y \sqrt{y^2 + L^2/4}}$$
- (c) For $y \gg L$, $\sqrt{y^2 + L^2/4} \rightarrow y$ so $E \rightarrow kQ/y^2$. For $L \rightarrow \infty$ at fixed λ , $E = 2k\lambda(L/2)/(y \cdot L/2) = 2k\lambda/y = \lambda/(2\pi\epsilon_0 y)$.
- (d) $\sqrt{0.40^2 + 0.30^2} = 0.50$ m, so $E = (8.99 \times 10^9)(8.0 \times 10^{-9})/(0.40 \times 0.50) = 360$ N/C in the $+y$ direction (away from the rod).
8. (a) The charge is spherically symmetric, so there is no preferred direction other than radial and the magnitude can depend only on r ; the Gaussian surface is a sphere of radius r concentric with the charge, on which E is perpendicular and constant, so $\Phi = E(4\pi r^2)$.
- (b) $r < R$: $Q_{enc} = Qr^3/R^3$, so $E = kQr/R^3$ radially outward. $R < r < 2R$: $Q_{enc} = Q$, so $E = kQ/r^2$ outward. $r > 2R$: $Q_{enc} = Q - Q = 0$, so $E = 0$.
- (c) Straight line from 0 up to kQ/R^2 at $r = R$; then a $1/r^2$ curve falling to $kQ/(4R^2)$ at $r = 2R$; then an abrupt drop to zero and zero beyond.
- (d) The jump is $kQ/(4R^2)$. Shell density $\sigma = -Q/(4\pi(2R)^2)$, so $|\sigma|/\epsilon_0 = Q/(16\pi\epsilon_0 R^2) = kQ/(4R^2)$. They agree, as expected for a field that drops by $|\sigma|/\epsilon_0$ across a surface charge.

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9. (a) The model is $F = kq^2/d^2$ for equal charges, so keep q constant: charge both spheres, touch them together, and separate them (identical conductors share equally); take all readings quickly with a fixed order of distances and measure d from center to center. Source of error: charge slowly leaks into humid air, so q falls during the run (also induced charges on nearby objects).
- (b) Plot F (vertical) versus $1/d^2$ (horizontal). The graph is a straight line through the origin with slope kq^2 .
- (c) $1/d^2$ (m^{-2}): 2500, 1111, 625, 400, 278, 156. Force in N: $9.10\text{e-}3$, $3.95\text{e-}3$, $2.21\text{e-}3$, $1.46\text{e-}3$, $0.98\text{e-}3$, $0.57\text{e-}3$. The best fit through the origin has slope about $3.6 \times 10^{-6} \text{ N m}^2$ (the points lie within about 3% of the line). Then $q = \sqrt{\text{slope}/k} = \sqrt{3.6 \times 10^{-6}/8.99 \times 10^9} = 2.0 \times 10^{-8} \text{ C} = 20 \text{ nC}$.
- (d) At small separations the spheres are not point charges: each sphere's charge redistributes on the surface (induction), moving the charges to the far sides of the two spheres. For like charges the effective separation is larger than the center-to-center distance, so the measured force is smaller than the point-charge prediction.
10. (a) $F = k|q_A q_B|/r^2 = (8.99 \times 10^9)(8.0 \times 10^{-9})(2.0 \times 10^{-9})/(0.30)^2 = 1.6 \times 10^{-6} \text{ N}$, attractive (unlike charges).
- (b) Total charge $+6.0 \text{ nC}$ is conserved, and identical conductors share it equally, so each has $+3.0 \text{ nC}$. $F = (8.99 \times 10^9)(3.0 \times 10^{-9})^2/(0.30)^2 = 9.0 \times 10^{-7} \text{ N}$, repulsive.
- (c) C touches A: each has $+4.0 \text{ nC}$. C then touches B: total $+4.0 + (-2.0) = +2.0 \text{ nC}$, shared equally, so C and B each have $+1.0 \text{ nC}$. Final: A = $+4.0 \text{ nC}$, B = $+1.0 \text{ nC}$, C = $+1.0 \text{ nC}$. $F_{AB} = (8.99 \times 10^9)(4.0 \times 10^{-9})(1.0 \times 10^{-9})/(0.30)^2 = 4.0 \times 10^{-7} \text{ N}$, repulsive.
- (d) The force depends on the product of the charge magnitudes, not their sum. Before the touch the product is $8.0 \times 2.0 = 16 \text{ nC}^2$; after, $3.0 \times 3.0 = 9 \text{ nC}^2$. The total charge (sum) is unchanged, but force depends on the product of magnitudes. Opposite signs let a small sum hide a large product (16 nC^2), while equal sharing gives 9 nC^2 .

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Unit 9: Electric Potential

Exam weighting: about 10-20% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

9 Electric Potential

Electric potential energy and potential, the integral and derivative links between E and V , and conservation of energy for charges moving in electric fields.

Why This Matters

Unit 9 is worth 10-20% of multiple choice, but its ideas run through capacitors, circuits, and induction. Exam questions ask you to move between energy, potential, and field in both directions: integrate $\vec{E}$ to get V , differentiate V to get E , and use conservation of energy for speeds. Students lose points by treating V like a vector, by thinking zero field means zero potential, and by dropping signs in $\Delta U = q \Delta V$. Take $V = 0$ at infinity for an isolated point charge.

Core Ideas

9.1 Electric Potential Energy

Electric potential energy U belongs to a *system* of charges, not to one particle. When the electric force does work W as the charges rearrange, $\Delta U = -W_{\text{field}}$, and $W_{\text{field}} = \int \vec{F} \cdot d\vec{r}$. An external agent that moves a charge slowly (no change in kinetic energy) does $W_{\text{ext}} = +\Delta U$. Because the Coulomb force is conservative, the work between two configurations is independent of path.

Potential energy of point charges ($U = 0$ at infinite separation)

Two charges: $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$, found from $U(r) = -\int_{\infty}^r F dr$.

Several charges: add the term for every pair once, $U = \sum_{i < j} k q_i q_j / r_{ij}$. Keep each charge's sign.

$U > 0$ for like charges (work was needed to assemble); $U < 0$ for a bound arrangement of unlike charges.

For three charges, count three pairs, not three terms with one charge each. Equivalent thinking: bring the charges in from infinity one at a time. The first costs nothing, the second works against the first, and the third works against both.

9.2 Electric Potential

Electric potential is potential energy per unit charge: $V = U/q$ in volts (J/C). It describes the *space* around the sources and is a scalar, so potentials from several sources add as ordinary numbers (with signs). The potential difference between points is defined through the field:

Linking field and potential

$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$ and $\Delta U = q \Delta V$.

$E_x = -\frac{dV}{dx}$ (generally $\vec{E} = -\nabla V$): the field points toward decreasing V , and its magnitude is the steepness of V .

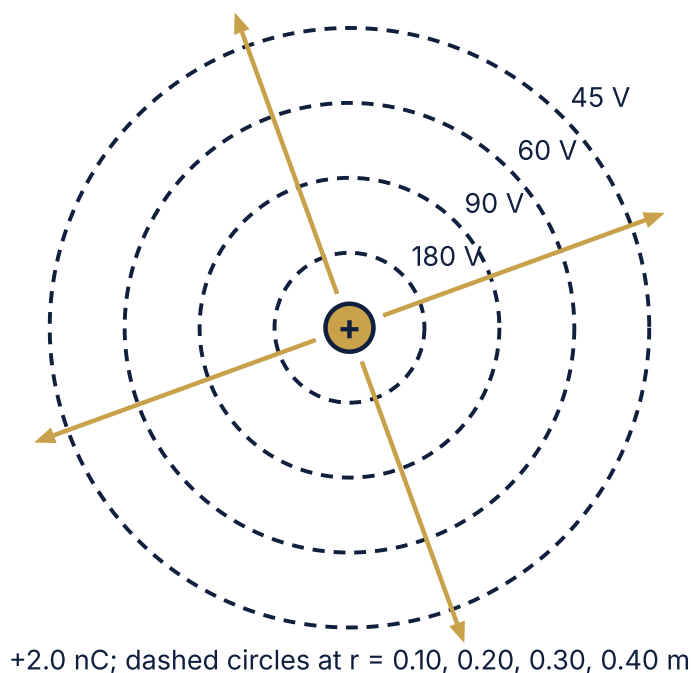
A point charge: $V = kq/r$. Continuous distribution: $V = k \int dq/r$ (a scalar integral, no components).

The field has no component along an **equipotential** (a surface of constant V), so field lines cross equipotentials at right angles and no work is done moving a charge along one. Equipotentials of a point charge are spheres:

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Equipotentials of a point charge (dashed) and its field lines (gold): the lines cross every equipotential at a right angle and point toward lower V .

Because V is a scalar, integrals for potential are often easier than for field. For a ring of radius R with charge Q , every element is the same distance $\sqrt{x^2 + R^2}$ from an axial point, so $V = kQ/\sqrt{x^2 + R^2}$. Differentiating, $E_x = -dV/dx = kQx/(x^2 + R^2)^{3/2}$, which matches the field found by direct integration in Unit 8. Only axial values of V give the axial component of E ; you cannot find a sideways component from V along the axis alone.

Source	Potential ($V = 0$ at infinity unless noted)
Point charge or sphere, outside	kQ/r
Ring, on axis	$kQ/\sqrt{x^2 + R^2}$
Disk, uniform σ , on axis	$\frac{\sigma}{2\epsilon_0} (\sqrt{x^2 + R^2} - x)$
Uniform solid insulating sphere, inside	$\frac{kQ(3R^2 - r^2)}{2R^3}$
Long line charge, between r_a and r_b	$V_b - V_a = -\frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_b}{r_a}$ (no zero at infinity)

Two common confusions. Zero field does not imply zero potential: at the center of a ring $E = 0$ but $V = kQ/R$. And zero potential does not imply zero field: at the midpoint of a dipole $V = 0$ but $E \neq 0$. The field depends on the *slope* of V , not its value.

In a **uniform** field, E_x is constant, so V changes linearly: $V(x) = V_0 - E_x x$. Moving a distance d along the field lowers the potential by Ed , which gives the handy forms $E = |\Delta V|/d$ and units of V/m equal to N/C. A graph of V against position is then a straight line whose slope is $-E_x$; a curved graph means a nonuniform field, with steeper parts where the field is stronger.

9.3 Conservation of Electric Energy

With only electric forces doing work, mechanical energy is conserved: $K_i + U_i = K_f + U_f$, or $\Delta K = -q\Delta V$. A positive charge speeds up moving toward lower V ; a negative charge speeds up moving toward higher V . A charge e accelerated through 1 V gains one electron-volt, $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$, so the speed follows from $\frac{1}{2}mv^2 = |q||\Delta V|$.

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Energy-conservation procedure

1. Name the system and confirm no non-electric work enters (fixed source charges, no friction).
2. Write $K_i + qV_i = K_f + qV_f$ with each V from the sources at that position.
3. Solve for v ; use $|q||\Delta V|$ for the kinetic-energy change and decide direction from sign arguments.
4. Check signs: kinetic energy cannot be negative, which fixes the turning points and closest approach.

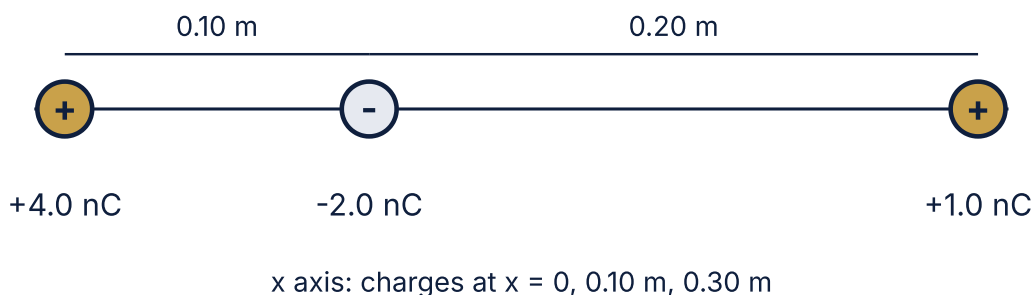
Because kinetic energy gained depends only on $q\Delta V$, different particles through the same potential difference gain the same energy if their charges match, but lighter ones end up faster: an electron outruns a proton by $\sqrt{m_p/m_e} \approx 43$.

Graphs make the idea visible: the gap between the total-energy line and the potential-energy curve is the kinetic energy, and the turning points are where the curve meets the line.

Worked Examples

EXAMPLE 1: ENERGY OF A THREE-CHARGE ARRANGEMENT

Point charges of $+4.0$ nC, -2.0 nC, and $+1.0$ nC lie on a line at $x = 0$, 0.10 m, and 0.30 m. Find the total electric potential energy of the system, and say what its sign means.



Example 1: three point charges on a line.

SOLUTION

Three pairs: $U_{12} = k(4.0)(-2.0) \times 10^{-18}/0.10 = -7.19 \times 10^{-7}$ J; $U_{13} = k(4.0)(1.0) \times 10^{-18}/0.30 = +1.20 \times 10^{-7}$ J; $U_{23} = k(-2.0)(1.0) \times 10^{-18}/0.20 = -0.90 \times 10^{-7}$ J.

$$U = -7.19 \times 10^{-7} + 1.20 \times 10^{-7} - 0.90 \times 10^{-7} = -6.89 \times 10^{-7} \text{ J.}$$

The negative sign means the system is bound: the charges will not fly apart by themselves, and about 6.9×10^{-7} J must be supplied from outside to separate them to infinity.

EXAMPLE 2: RING OF CHARGE, V AND E

A ring of radius 0.10 m carries $+12$ nC. (a) Find V at the center and at $x = 0.20$ m on the axis. (b) Find E_x at $x = 0.20$ m from V . (c) How much work must an external agent do to bring a $+2.0$ nC charge slowly from far away to $x = 0.20$ m?

SOLUTION

(a) $V(x) = kQ/\sqrt{x^2 + R^2}$. Center: $kQ/R = (8.99 \times 10^9)(12 \times 10^{-9})/0.10 = 1.08 \times 10^3$ V. At 0.20 m: $\sqrt{0.04 + 0.01} = 0.2236$ m, so $V = 482$ V.

(b) $E_x = -\frac{d}{dx} \frac{kQ}{\sqrt{x^2 + R^2}} = \frac{kQx}{(x^2 + R^2)^{3/2}} = 1.93 \times 10^3$ N/C along $+x$, in agreement with Unit 8 and pointing toward decreasing V .

(c) $W_{ext} = q\Delta V = (2.0 \times 10^{-9})(482 - 0) = 9.65 \times 10^{-7}$ J, positive because the charges repel.

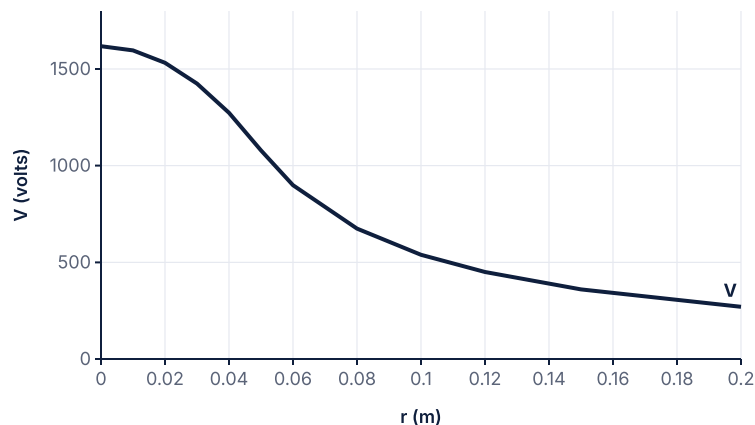
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EXAMPLE 3: V FROM E FOR A CHARGED SPHERE

A solid insulating sphere of radius $R = 5.0$ cm carries $Q = +6.0$ nC uniformly. Using $E = kQr/R^3$ inside and kQ/r^2 outside, find V at the surface and at the center.



Example 3: $V(r)$ for a uniformly charged solid sphere is rounded (parabolic) inside and a $1/r$ curve outside; the slope is zero at the center and the two pieces join smoothly at R .

SOLUTION

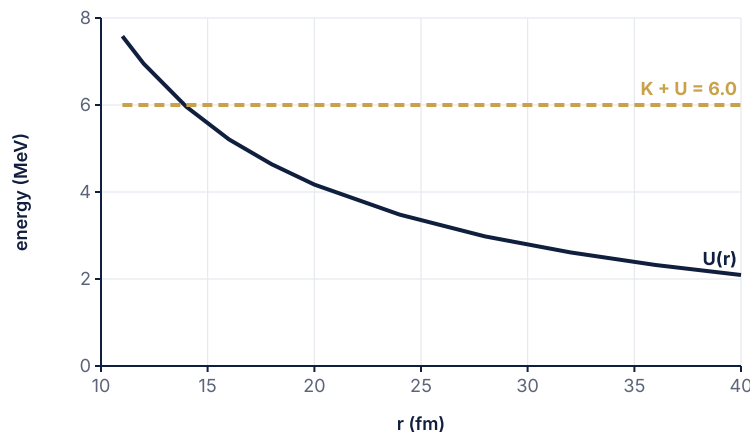
Outside, $V(r) = \int_r^\infty kQ/s^2 ds$ gives kQ/r . At the surface: $V(R) = (8.99 \times 10^9)(6.0 \times 10^{-9})/0.050 = 1.08 \times 10^3$ V.

Inside, $V(0) - V(R) = -\int_R^0 E dr = \int_0^R \frac{kQr}{R^3} dr = \frac{kQ}{2R} = 539$ V.

So $V(0) = 3kQ/(2R) = 1.62 \times 10^3$ V, and for $r < R$, $V(r) = kQ(3R^2 - r^2)/(2R^3)$. The center is higher than the surface because a positive test charge there would be pushed outward through the whole sphere. The slope is zero at $r = 0$, where $E = 0$, but $V \neq 0$.

EXAMPLE 4: CLOSEST APPROACH OF AN ALPHA PARTICLE

An alpha particle (charge $+2e$, mass 6.64×10^{-27} kg) with 6.0 MeV of kinetic energy comes from far away straight at a copper nucleus (charge $+29e$), which is fixed. (a) Find the closest approach. (b) Find its speed at $r = 30$ fm $= 3.0 \times 10^{-14}$ m.



Example 4: potential energy of an alpha particle near a copper nucleus. The dashed line is the total energy; kinetic energy is the gap between the lines.

SOLUTION

(a) At closest approach $K = 0$: $K_i = k(2e)(29e)/r_{min}$, so $r_{min} = (8.99 \times 10^9)(58)(1.60 \times 10^{-19})^2/(6.0 \times 10^6 \times 1.60 \times 10^{-19}) = 1.39 \times 10^{-14}$ m $= 13.9$ fm.

(b) $U(30 \text{ fm}) = 83.4 \text{ MeV fm}/30 \text{ fm} = 2.78 \text{ MeV}$, so $K = 6.0 - 2.78 = 3.22 \text{ MeV} = 5.15 \times 10^{-13}$ J.

$v = \sqrt{2K/m} = \sqrt{2(5.15 \times 10^{-13})/6.64 \times 10^{-27}} = 1.25 \times 10^7$ m/s. (Far away it moves at 1.70×10^7 m/s.) The nucleus is treated as fixed because it is much heavier.

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Write It Like a 5

Model Response

Prompt: In a region the electric potential along the x axis is $V(x) = 8.0x^2 - 6.0x$, with V in volts and x in meters. Find the electric field, locate where it is zero, and describe the motion of a proton released from rest at $x = 0.60$ m. The field is $E_x = -dV/dx = -(16x - 6.0) = 6.0 - 16x$, in V/m. It is zero at $x = 0.375$ m. The second derivative of V is positive there, so V has a minimum, $V = -1.125$ V. At $x = 0.60$ m, $E_x = -3.6$ V/m, so the force on the proton is in the $-x$ direction and it moves toward lower V . Energy is conserved: it gains kinetic energy until $x = 0.375$ m, where $K = e(0.405 \text{ V}) = 0.405 \text{ eV}$, a speed of $8.8e3 \text{ m/s}$, then slows as V rises again. It stops where $V(x) = V(0.60) = -0.72 \text{ V}$, which gives $x = 0.15$ m, and returns. The proton oscillates between $x = 0.15$ m and $x = 0.60$ m.

What the grader looks for

1. E found as the negative derivative, with the sign kept. 2. The zero of E tied to an extremum of V , with a reason for minimum. 3. Direction of force from the sign of E_x and the charge. 4. Energy conservation written with potential differences. 5. Turning points found from $V(x) = V(0.60)$, giving a clear claim of oscillation.

Coach's Playbook

- 1 Decide the direction of the question: field to potential means integrate; potential to field means differentiate.
- 2 Pick the zero of potential and say so.
- 3 Keep signs in $U = kq_1q_2/r$ and in $\Delta U = q\Delta V$; check against the story (does the charge speed up or slow down?).
- 4 On a graph of $V(x)$, read E from the slope (opposite sign); on a graph of $E(x)$, read ΔV from the area (opposite sign).
- 5 Estimate before you calculate: speed from $\frac{1}{2}mv^2 = |q||\Delta V|$.

Common traps

1. Treating potential as a vector, or adding potentials from different sources with their directions. 2. Assuming $E = 0$ means $V = 0$ (or the reverse). 3. Dropping the minus sign in $E = -dV/dx$, or forgetting that negative charges move toward higher V . 4. Counting only three terms for three charges' energy, or using q alone instead of the pair product kq_1q_2/r .

Quick Check

Q1. What is the potential energy of two $+2.0 \text{ nC}$ charges 0.10 m apart? A: $kq^2/r = 3.6 \times 10^{-7} \text{ J}$. Q2. An electron is accelerated through 100 V from rest. Its kinetic energy? A: 100 eV , $1.6 \times 10^{-17} \text{ J}$. Q3. V is constant along a line. What is E along that line? A: Zero ($E_x = -dV/dx = 0$).

Challenge Corner

A very long charged wire has $\lambda = 3.0 \text{ nC/m}$. A proton is released from rest at $r_a = 2.0 \text{ cm}$. Find its speed at $r_b = 10 \text{ cm}$. From Gauss's law $E = \lambda/(2\pi\epsilon_0 r)$, so $V_a - V_b = \int_{r_a}^{r_b} E dr = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_b}{r_a} = 2(8.99 \times 10^9)(3.0 \times 10^{-9}) \ln 5 = 86.8 \text{ V}$. The proton moves toward lower potential and gains $K = e(86.8 \text{ V}) = 86.8 \text{ eV}$: $v = \sqrt{2(86.8)(1.60 \times 10^{-19})/1.67 \times 10^{-27}} = 1.29 \times 10^5 \text{ m/s}$. Note a long wire has no finite potential at infinity, so only differences make sense.

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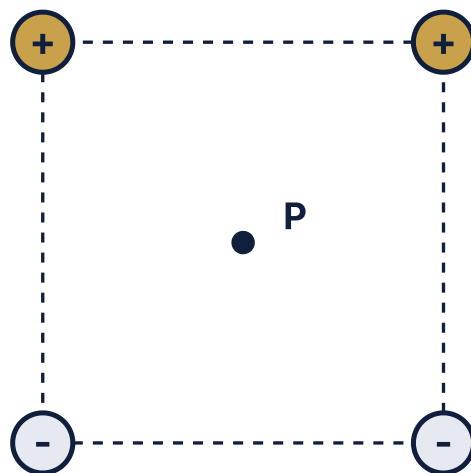
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Practice

PRACTICE Unit 9 Practice

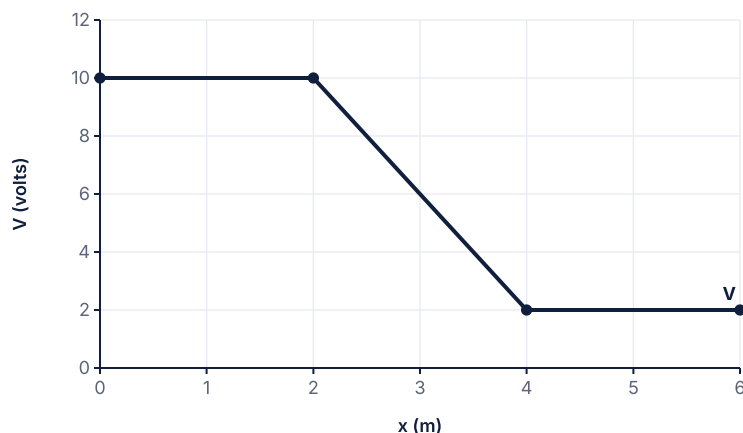
Show your work. A calculator is allowed. Justify conclusions.

- Three identical point charges $+q$ sit at the corners of an equilateral triangle of side a . The electric potential energy of the system is
 (A) kq^2 / a (B) $2kq^2 / a$ (C) $3kq^2 / a$ (D) $6kq^2 / a$
- Four point charges of equal magnitude are at the corners of a square: $+q$ at both top corners and $-q$ at both bottom corners. At the center of the square, the electric potential and electric field are



Four equal-magnitude charges at the corners of a square; P is the center.

- (A) $V = 0$ and $E = 0$ (B) $V = 0$ and $E \neq 0$ (C) $V \neq 0$ and $E = 0$
 (D) $V \neq 0$ and $E \neq 0$
- The potential along the x axis is shown. The x component of the electric field at $x = 3$ m is



Potential along the x axis.

- (A) $+4$ V/m (B) -4 V/m (C) $+6$ V/m (D) 0
- A proton starts from rest and is accelerated through a potential difference of 500 V. Its final speed is closest to
 (A) 2.2×10^5 m/s (B) 1.3×10^7 m/s (C) 9.6×10^{10} m/s
 (D) 3.1×10^5 m/s

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5. An electron is released from rest in a region where the electric field is uniform. Which statement is correct?
- (A) It moves toward higher potential and its electric potential energy decreases.
 (B) It moves toward lower potential and its electric potential energy decreases.
 (C) It moves toward higher potential and its electric potential energy increases.
 (D) It stays at the same potential because the force is constant.
6. A thin ring of radius R carries total charge Q distributed uniformly. With $V = 0$ at infinity, the field and potential at the center of the ring are
- (A) $E = 0$ and $V = 0$ (B) $E = kQ/R^2$ and $V = kQ/R$ (C) $E = 0$ and $V = kQ/R$
 (D) $E = kQ/R^2$ and $V = 0$
7. A thin disk of radius R carries a uniform surface charge density σ . Point P is on the axis through the disk center, a distance x from the disk, with $V = 0$ at infinity.
- (a) Consider a ring of radius s and width ds within the disk. Write its charge dq and its contribution dV at P.
 (b) Integrate to find $V(x)$. Show the integral and the antiderivative.
 (c) Use $E_x = -dV/dx$ to obtain E_x and check that it approaches $\sigma/2\epsilon_0$ as $x \rightarrow 0^+$.
 (d) Evaluate V and E_x for $\sigma = 5.0 \text{ nC/m}^2$, $R = 0.12 \text{ m}$, and $x = 0.05 \text{ m}$.
8. The graph shows the x component of the electric field along the x axis from 0 to 0.12 m. The field is zero for $x < 0$, and the potential at $x = 0$ is $V = 0$. The graph points are: $E_x = 0$ at $x = 0$, 800 V/m at $x = 0.04 \text{ m}$, 800 V/m at $x = 0.08 \text{ m}$, and 0 at $x = 0.10 \text{ m}$ and beyond, joined by straight segments.
- (a) Find V at $x = 0.04 \text{ m}$, 0.08 m , and 0.12 m . Explain how you used the graph.
 (b) Sketch V versus x for $0 \leq x \leq 0.12 \text{ m}$, noting the shape of each segment (straight or curved) and whether each is concave up or down.
 (c) An electron is released from rest at $x = 0.06 \text{ m}$. Describe its motion and calculate its speed at $x = 0$.
 (d) Explain why the electron's speed does not change for $x < 0$.
9. A student studies the potential between two long parallel metal bars (electrodes) painted on conductive paper, 20.0 cm apart, connected to a 9.0 V supply. The bar at $x = 0$ is at 0 V. With a voltmeter, the student measures the potential at several positions along a line perpendicular to the bars.
- x (cm): 2, 4, 6, 8, 10, 12, 14, 16, 18
 V (V): 0.95, 1.80, 2.75, 3.60, 4.55, 5.40, 6.25, 7.25, 8.10
- (a) Describe a procedure for taking these data, including how to connect the voltmeter and one way to reduce random error.
 (b) Plot V versus x (describe the axes, scales, and the best-fit line) and use the slope to find the magnitude of the electric field. Explain how the slope is related to E .
 (c) Does the data support a uniform field? Cite two features of the data.
 (d) Use your result to find the force on a proton placed in this field, and explain why the measured field near the ends of the bars could differ from the middle.
10. Two fixed point charges, each $+Q = +5.0 \text{ nC}$, lie on the x axis at $x = +a$ and $x = -a$ with $a = 0.10 \text{ m}$. A small bead with charge $-q = -1.0 \text{ nC}$ and mass $m = 1.0 \times 10^{-4} \text{ kg}$ is threaded on a frictionless insulating rod along the y axis and released from rest at $y_0 = 0.20 \text{ m}$.
- (a) Without calculation, claim whether the bead moves toward the origin or away from it, and justify using the direction of the electric force.
 (b) Derive an expression for the speed of the bead as it passes the origin and evaluate it.
 (c) Describe the motion of the bead after it passes the origin, using a statement about potential energy.
 (d) At the origin the net electric field is zero, but the bead's speed is at its maximum. Explain why this is consistent.

Answers**Unit 9: Answers and Explanations**

Read every explanation, including the ones for problems you answered correctly.

9 Electric Potential

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- C. Count three pairs, each kq^2/a , for $3kq^2/a$. Choosing kq^2/a counts one pair only, and $6kq^2/a$ double counts each pair.
- B. By symmetry the $+q$ and $-q$ terms in $\sum kq/r$ cancel, so $V = 0$. The fields of the two positive charges and two negative charges all have a downward component at P (pushed away from the top, pulled toward the bottom), so they add and $E \neq 0$. Zero potential does not mean zero field.
- A. $E_x = -dV/dx$; the slope between 2 m and 4 m is $(2 - 10)/2 = -4$ V/m, so $E_x = +4$ V/m. Answer B has the wrong sign; $+6$ V/m reads the value of V at $x = 3$ m rather than its slope.
- D. $\frac{1}{2}mv^2 = eV$ gives $v = \sqrt{2eV/m} = 3.1 \times 10^5$ m/s. The 2.2×10^5 value forgets the factor 2; 1.3×10^7 uses the electron mass; 9.6×10^{10} is v^2 .
- A. A negative charge is pushed against the field, which points toward lower potential, so the electron moves toward higher V . Its $U = qV$ is lower there since $q < 0$. The tempting statement that it moves to lower potential confuses positive and negative charges.
- C. By symmetry the field components along the axis cancel at the center, so $E = 0$, but every element is a distance R away, so $V = kQ/R \neq 0$. The tempting $V = 0$ confuses zero field with zero potential.
- (a) $dq = \sigma 2\pi s ds$ and each point of the ring is a distance $\sqrt{x^2 + s^2}$ from P, so $dV = \frac{k dq}{\sqrt{x^2 + s^2}} = \frac{\sigma s ds}{2\epsilon_0 \sqrt{x^2 + s^2}}$.
 (b) $V = \frac{\sigma}{2\epsilon_0} \int_0^R \frac{s ds}{\sqrt{x^2 + s^2}} = \frac{\sigma}{2\epsilon_0} \left[\sqrt{x^2 + s^2} \right]_0^R = \frac{\sigma}{2\epsilon_0} (\sqrt{x^2 + R^2} - x)$ (for $x > 0$).
 (c) $E_x = -\frac{dV}{dx} = -\frac{\sigma}{2\epsilon_0} \left(\frac{x}{\sqrt{x^2 + R^2}} - 1 \right) = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{x}{\sqrt{x^2 + R^2}} \right)$. As $x \rightarrow 0^+$ this approaches $\sigma/2\epsilon_0$, the field of an infinite sheet.
 (d) $\sigma/2\epsilon_0 = 282.5$ V/m. $V = 282.5(0.13 - 0.05) = 22.6$ V. $E_x = 282.5(1 - 0.05/0.13) = 174$ N/C, directed away from the disk (for positive σ).
- (a) $\Delta V = -\int E_x dx$, so $-(\text{area under the graph})$. 0 to 0.04: triangle $\frac{1}{2}(0.04)(800) = 16$, so $V(0.04) = -16$ V. Next, rectangle $800 \times 0.04 = 32$, so $V(0.08) = -48$ V. Next, triangle $\frac{1}{2}(0.02)(800) = 8$, giving -56 V at $x = 0.10$ m; no field after that, so $V(0.12) = -56$ V.
 (b) Starting at 0, V falls with slope that grows in magnitude (curved, concave down) to -16 V; then a straight line of slope -800 V/m down to -48 V; then a curve flattening (concave up) to -56 V at 0.10 m; then flat at -56 V. V never increases because $E_x \geq 0$.
 (c) The force on the electron is $-eE_x$, in the $-x$ direction wherever $E_x > 0$, so it accelerates toward smaller x and keeps accelerating until it leaves the field at $x = 0$. $V(0.06) = -48 + 800(0.02) = -32$ V. $\Delta K = -q\Delta V = e(32) = 32$ eV, so $v = \sqrt{2(32)(1.60 \times 10^{-19})/9.11 \times 10^{-31}} = 3.4 \times 10^6$ m/s.
 (d) With $E_x = 0$ for $x < 0$ the net force is zero, so by Newton's first law the electron moves at constant velocity, consistent with V being constant there.
- (a) Place the bars with a fixed supply; connect the voltmeter's common lead to the 0 V bar and move the probe to marked points along a line perpendicular to the bars, reading the voltage at each. Use a ruler to place the probe; take each reading two or three times and average, and keep the probe pressed lightly with the same pressure.
 (b) Vertical axis V (V), horizontal axis x (m, converted from cm), scales starting at zero with uniform spacing; draw one best-fit line with a straightedge. The least-squares slope is about 44.8 V/m (0.448 V/cm), intercept about 0.04 V (consistent with zero). Since $E_x = -dV/dx$, the magnitude of the field is the slope: $E \approx 45$ V/m, pointing from the high-voltage bar toward the low-voltage bar (the $-x$ direction). The expected value is $9.0 \text{ V}/0.200 \text{ m} = 45 \text{ V/m}$.
 (c) Yes: the data lie on a straight line, with residuals no larger than about 0.06 V (random in sign), and a constant increment of about 0.9 V per 2 cm, which means a constant slope and thus constant E .
 (d) $F = eE = (1.60 \times 10^{-19})(44.8) = 7.2 \times 10^{-18}$ N, toward the low-voltage bar. Near the ends of the bars the field lines bulge outward (fringing), so V is not linear there; use positions in the middle region of long bars.
- (a) The two positive charges attract the negative bead. At $(0, y_0)$ the x components cancel (equal and opposite) and the y components both point toward the origin, so the net force on the bead is toward the origin; it accelerates that way.
 (b) Energy conservation: $U_i = -2kQq/\sqrt{a^2 + y_0^2}$, $U_f = -2kQq/a$, and $\frac{1}{2}mv^2 = U_i - U_f$, so $v = \sqrt{\frac{4kQq}{m} \left(\frac{1}{a} - \frac{1}{\sqrt{a^2 + y_0^2}} \right)}$. $U_i = -4.02 \times 10^{-7}$ J and $U_f = -8.99 \times 10^{-7}$ J, so $K = 4.97 \times 10^{-7}$ J and $v = \sqrt{2(4.97 \times 10^{-7})/10^{-4}} = 0.10$ m/s.
 (c) U is lowest at the origin, so the bead passes through, slows as U rises again, and stops at $y = -0.20$ m by symmetry (same U as at the start). It then oscillates between $y = \pm 0.20$ m.
 (d) Speed depends on the kinetic energy gained, which is the potential-energy drop accumulated from the start. The net force is zero at the origin, but only there does the force change from accelerating to decelerating, so the speed is maximum there. $E = 0$ means $dV/dy = 0$ (an extremum of V), not that the speed is zero.

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Unit 10: Conductors and Capacitors

Exam weighting: about 10-15% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

10 Conductors and Capacitors

Conductors in electrostatic equilibrium, charge sharing, capacitance derived from Gauss's law for plates, cylinders and spheres, stored energy, and what dielectrics change when charge or voltage is held fixed.

Why This Matters

Conductors and capacitors carry 10-15% of the multiple-choice section, but their ideas reappear in almost every free-response question about fields, potential, and circuits. A cylindrical capacitor with a dielectric has been used as a full multi-part derivation: choose a Gaussian surface, find E , integrate to get V , and form $C = Q/V$. Students lose points by using the parallel-plate formula for a curved geometry, by forgetting which quantity a dielectric leaves fixed, and by writing that the field is zero inside a conductor without saying it is in electrostatic equilibrium.

Core Ideas

10.1 Electrostatics with Conductors

A conductor contains charges that move freely. In **electrostatic equilibrium** nothing is moving, which forces the conclusions in the box below. If $\vec{E}$ were nonzero inside the material, free charges would still be accelerating. Applying Gauss's law to a closed surface drawn just inside the metal gives zero flux, so the enclosed charge is zero: any net charge sits on the surface.

Conductor in electrostatic equilibrium

$\vec{E} = 0$ at every point inside the conducting material, and the interior carries no net charge.

The entire conductor, surface included, is at one potential V .

Just outside the surface, $\vec{E}$ is perpendicular to the surface with magnitude $E = \sigma/\epsilon_0$ (use a small Gaussian pillbox: flux leaves through the outside face only).

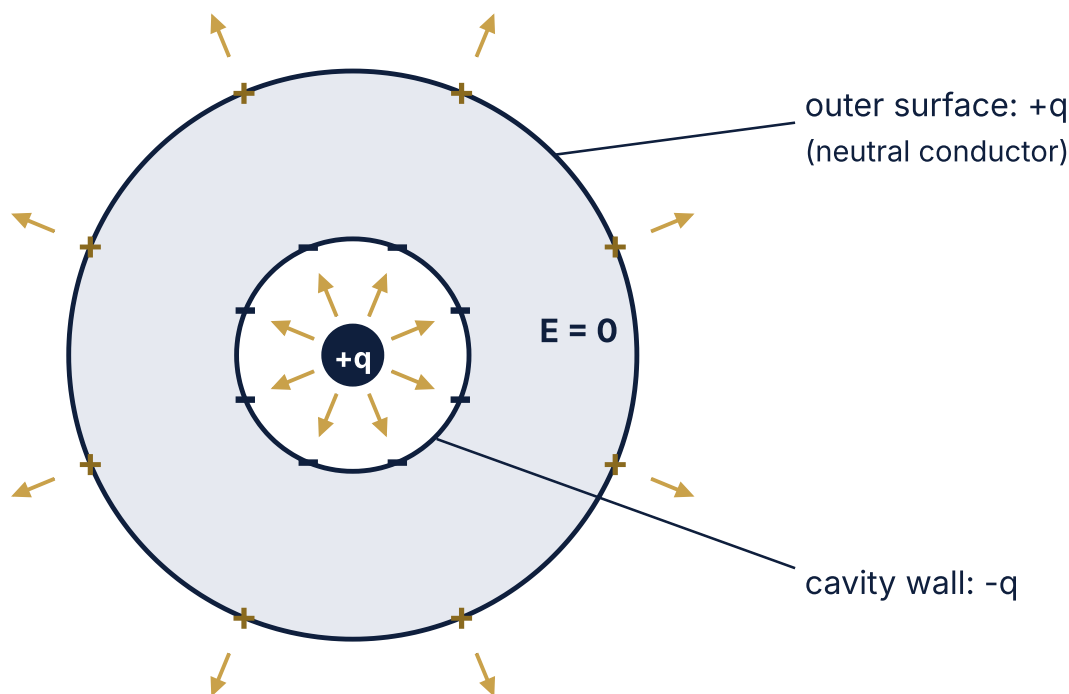
A cavity inside a conductor that holds no charge has no field and no charge on its wall. A charge $+q$ placed in a cavity induces $-q$ on the cavity wall, and the outer surface then carries the conductor's net charge plus q .

The statement 'the field inside a conductor is zero' refers to the metal, not to an empty cavity that holds a charge. Notice also that the outside surface charge is arranged so the field of the whole system vanishes in the metal; the outside world cannot tell where in the cavity the charge sits, only the total.

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Neutral spherical shell with point charge $+q$ at the center of its cavity. The material has $E = 0$; the cavity wall holds $-q$ and the outer surface $+q$.

For a charge $+q$ at the center of a neutral shell with inner radius a and outer radius b , Gauss's law gives $E = kq/r^2$ for $r < a$, $E = 0$ for $a < r < b$, and $E = kq/r^2$ again for $r > b$ (the outer surface returns the same net enclosed charge). The potential, measured from zero at infinity, is kq/r outside, constant at kq/b throughout the metal, and $kq(1/r - 1/a + 1/b)$ inside the cavity.

10.2 Redistribution of Charge between Conductors

When conductors are joined by a wire, charge flows until they share one potential, while the total charge is conserved. Two widely separated spheres of radii R_1 and R_2 each have $V = kQ/R$ to a good approximation, so equal potentials give a clean ratio.

Joined spheres (far apart)

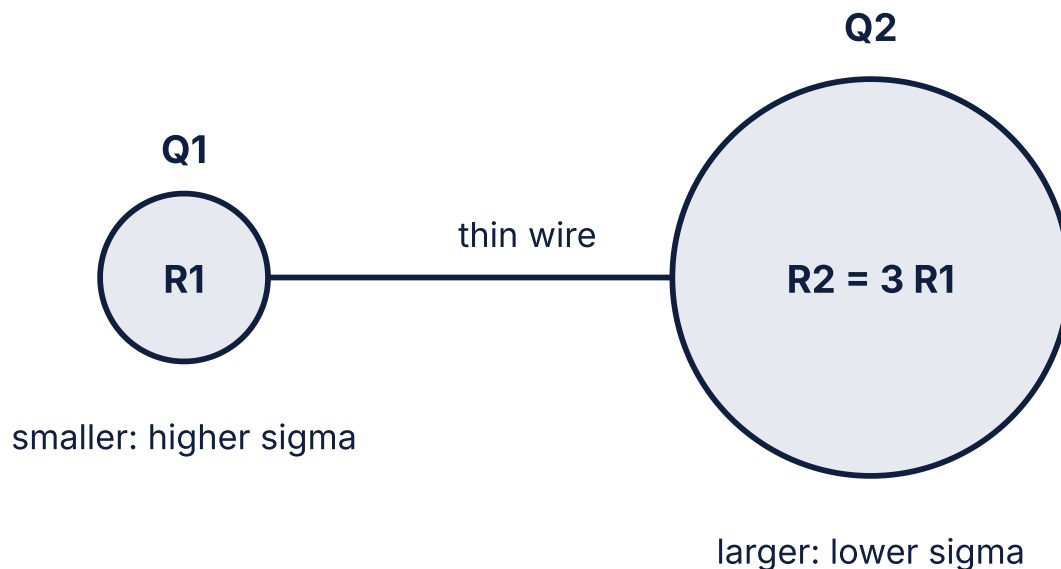
$$\frac{Q_1}{R_1} = \frac{Q_2}{R_2} \text{ and } Q_1 + Q_2 = Q_{\text{total}}.$$

Surface charge density $\sigma = Q/(4\pi R^2)$, so $\sigma_1/\sigma_2 = R_2/R_1$: the smaller sphere has the larger σ and the larger surface field $E = \sigma/\epsilon_0$. Equal potential does not mean equal charge and does not mean equal surface field.

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Two far-apart conducting spheres joined by a thin wire reach the same potential, so $Q1/Q2 = R1/R2$ and $\sigma1/\sigma2 = R2/R1$.

This is why sharp points concentrate field and discharge easily. Identical conductors that touch share their total charge equally.

Grounding a conductor connects it to Earth, which fixes its potential at zero and supplies or removes whatever charge is needed; a grounded shell enclosing a charge has no charge on its outer surface and no field outside it.

10.3 Capacitors

A capacitor is two conductors carrying $+Q$ and $-Q$. Its **capacitance** $C = Q/V$ depends only on geometry (and any dielectric), and it is measured in farads. The method is always the same: put charge Q on the plates, find $\vec{E}$ with Gauss's law, integrate $V = \int \vec{E} \cdot d\vec{\ell}$ between the conductors, then divide.

Capacitances by derivation

Parallel plates (area A , gap d): $E = Q/(\epsilon_0 A)$, $V = Ed$, so $C = \epsilon_0 A/d$.

Cylinder (radii $a < b$, length L): Gauss on a coaxial cylinder gives $E = \lambda/(2\pi\epsilon_0 r)$ with $\lambda = Q/L$ and flux only through the curved side;

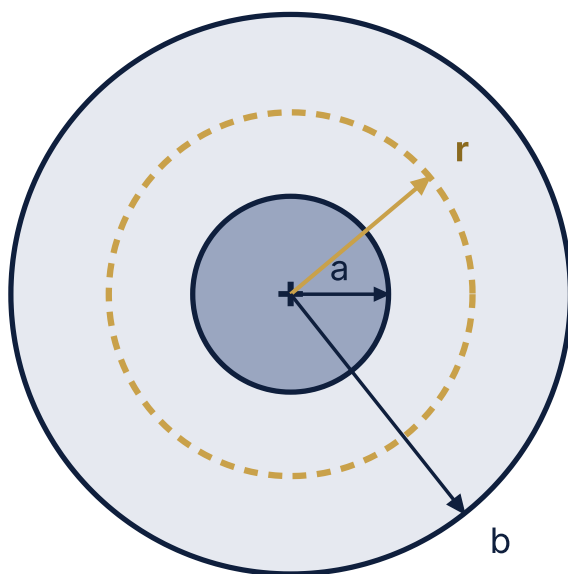
$$V = \int_a^b E dr = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{b}{a}, \text{ so } C = \frac{2\pi\epsilon_0 L}{\ln(b/a)}.$$

Sphere (radii $a < b$): $E = kQ/r^2$ between, so $C = \frac{4\pi\epsilon_0 ab}{b-a}$.

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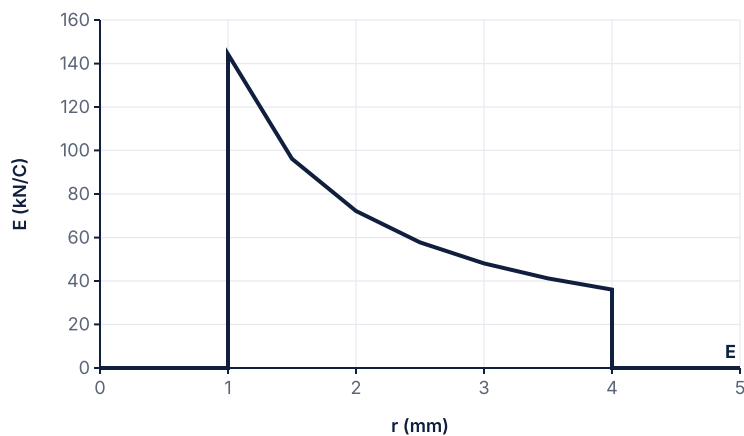
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Gaussian surface:
coaxial cylinder,
radius r , length l

inner: $+\lambda$ per length
outer: $-\lambda$ per length

End view of a cylindrical capacitor: inner conductor radius a , outer conductor radius b , dashed Gaussian cylinder of radius r ($a < r < b$).



Field of the cylindrical capacitor: zero in the inner conductor, proportional to $1/r$ in the gap, zero beyond the outer conductor ($a = 1$ mm, $b = 4$ mm, 8.0 nC per meter).

Capacitors in **parallel** share a voltage and add: $C = C_1 + C_2$. In **series** they carry the same charge and the reciprocals add: $1/C = 1/C_1 + 1/C_2$. To charge a capacitor, work $dU = V dq = (q/C) dq$ is done on each increment of charge, so integrating from 0 to Q gives the stored energy.

Energy in a capacitor

$$U = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV.$$

Energy density in a field: $u = \frac{1}{2} \epsilon_0 E^2$. Integrating u over the region between the plates reproduces $\frac{1}{2} CV^2$; for the nonuniform field of a cylinder, use shells of volume $dV = 2\pi r L dr$.

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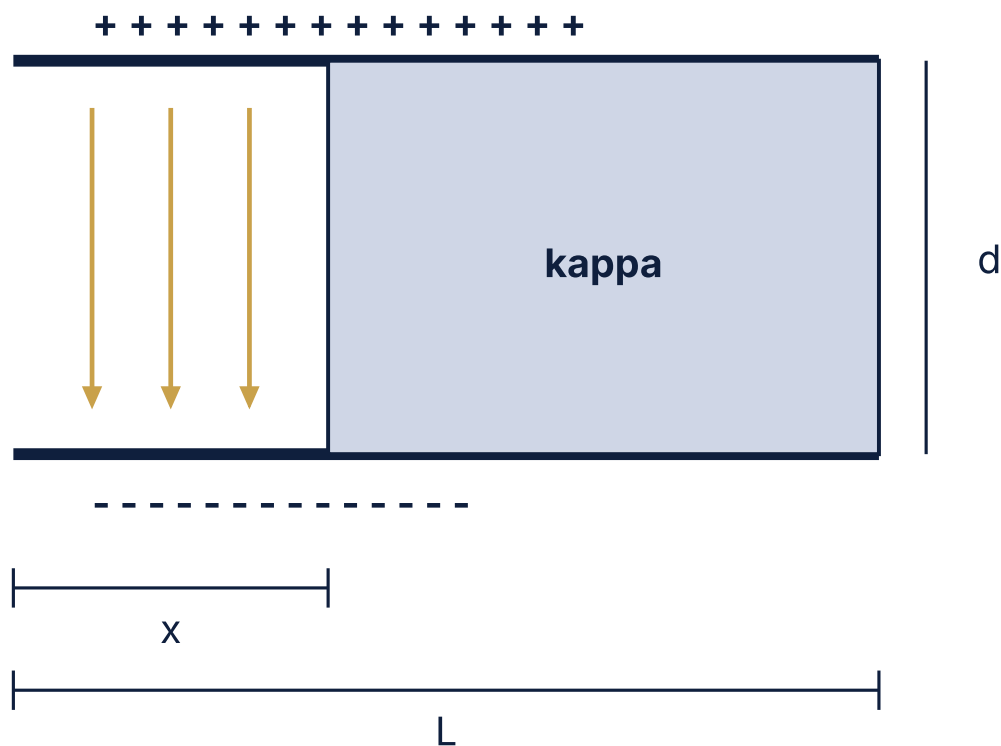
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10.4 Dielectrics

An insulator placed between the plates polarizes: bound charge appears on its faces and its field opposes the free-charge field, so the net field drops to $E = E_0/\kappa$, where $\kappa \geq 1$ is the dielectric constant. A dielectric that fills the whole gap multiplies the capacitance by κ (replace ϵ_0 by $\kappa\epsilon_0$). Treat a capacitor as air-filled unless a dielectric is specified, and state that assumption if a problem is unclear. The bound surface charge density is $\sigma_b = \sigma_f(1 - 1/\kappa)$. What changes on insertion depends entirely on what the circuit holds fixed.

Fully inserted slab (factor)	Isolated, Q fixed	Battery attached, V fixed
Capacitance C	times kappa	times kappa
Charge Q	unchanged	times kappa
Voltage V	divided by kappa	unchanged
Field E	divided by kappa	unchanged
Energy U	divided by kappa	times kappa



Slab of dielectric constant kappa inserted a distance x between plates of length L and width w (into the page). The strip with the slab and the strip without it act as parallel capacitors.

A slab that spans the full gap but only part of the plate length acts as two parallel capacitors: $C(x) = \frac{\epsilon_0 w}{d} [L + (\kappa - 1)x]$. A slab that fills part of the gap thickness and all of the area acts as two capacitors in series.

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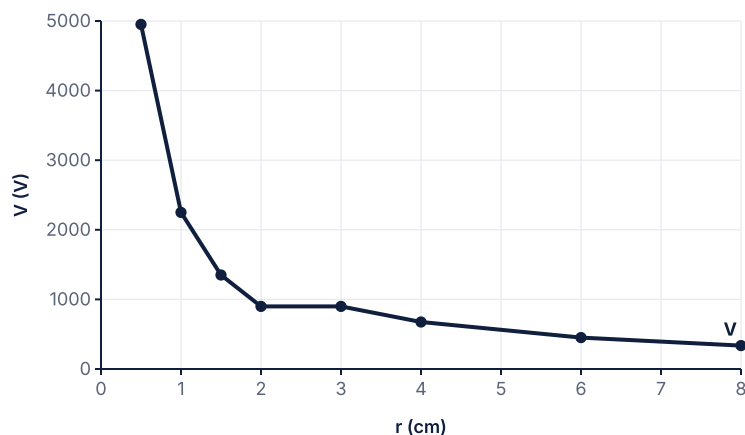
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Worked Examples

EXAMPLE 1: POINT CHARGE INSIDE A CONDUCTING SHELL

A point charge $+3.0 \text{ nC}$ sits at the center of a neutral conducting spherical shell of inner radius $a = 2.0 \text{ cm}$ and outer radius $b = 3.0 \text{ cm}$. Take $V = 0$ at infinity and $k = 9.0 \times 10^9 \text{ N m}^2/\text{C}^2$. (a) Find the charge on each surface. (b) Find E at $r = 1.0, 2.5$ and 4.0 cm . (c) Find V at $r = 1.0 \text{ cm}$ and 4.0 cm .



Potential of the shell system: flat (900 V) between 2 and 3 cm.

SOLUTION

(a) The metal must have $E = 0$, so the enclosed charge for a Gaussian sphere inside it is zero: the inner wall carries -3.0 nC . The shell is neutral, so the outer surface carries $+3.0 \text{ nC}$.

(b) At 1.0 cm : $E = kq/r^2 = (9.0 \times 10^9)(3.0 \times 10^{-9})/(0.010)^2 = 2.7 \times 10^5 \text{ N/C}$, radially outward. At 2.5 cm (inside the metal) $E = 0$. At 4.0 cm the enclosed charge is $+3.0 \text{ nC}$: $E = 1.7 \times 10^4 \text{ N/C}$, radially outward.

(c) $V(4.0 \text{ cm}) = kq/r = 27/0.040 = 675 \text{ V}$. The shell is an equipotential at $kq/b = 900 \text{ V}$. At 1.0 cm , $V = kq(1/r - 1/a + 1/b) = 27(100 - 50 + 33.3) = 2250 \text{ V}$. The graph confirms a flat segment through the metal.

The potential is continuous even though the field jumps from $2.7 \times 10^5 \text{ N/C}$ to zero at the inner wall; the field is the slope, not the value, of V .

EXAMPLE 2: SPHERES JOINED BY A WIRE

Two conducting spheres of radii 2.0 cm and 6.0 cm are far apart. Together they hold $+80 \text{ nC}$. A thin wire joins them. Find the final charge on each, their common potential, and the ratio of surface charge densities.

SOLUTION

Charge conservation: $Q_1 + Q_2 = 80 \text{ nC}$. Equal potentials: $Q_1/R_1 = Q_2/R_2$, so $Q_2 = 3Q_1$.

Then $4Q_1 = 80 \text{ nC}$ gives $Q_1 = 20 \text{ nC}$ and $Q_2 = 60 \text{ nC}$.

$V = kQ_1/R_1 = (9.0 \times 10^9)(20 \times 10^{-9})/0.020 = 9.0 \times 10^3 \text{ V}$; the other sphere gives $(9.0 \times 10^9)(60 \times 10^{-9})/0.060 = 9.0 \times 10^3 \text{ V}$, a consistency check.

$\sigma_1/\sigma_2 = R_2/R_1 = 3$. Numerically $\sigma_1 = 4.0 \times 10^{-6} \text{ C/m}^2$ and $\sigma_2 = 1.3 \times 10^{-6} \text{ C/m}^2$, so the small sphere has the stronger surface field (4.5×10^5 versus $1.5 \times 10^5 \text{ N/C}$).

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EXAMPLE 3: CYLINDRICAL CAPACITOR FROM GAUSS'S LAW

A cylindrical capacitor has inner radius $a = 1.0$ mm, outer radius $b = 4.0$ mm and length $L = 0.50$ m, with air between. It is charged to 200 V. Derive C , then find the charge, the field at the inner conductor, and the stored energy by integrating the energy density.

SOLUTION

Gauss's law on a coaxial cylinder of radius r and length ℓ : $E(2\pi r\ell) = \lambda\ell/\epsilon_0$, so $E = \lambda/(2\pi\epsilon_0 r)$. The end caps contribute no flux.

$$V = \int_a^b E dr = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{b}{a}, \text{ so } C = Q/V = \frac{2\pi\epsilon_0 L}{\ln(b/a)} = \frac{2\pi(8.85 \times 10^{-12})(0.50)}{\ln 4} = 2.0 \times 10^{-11} \text{ F (20 pF).}$$

$$Q = CV = 4.0 \times 10^{-9} \text{ C, so } \lambda = 8.0 \times 10^{-9} \text{ C/m and } E(a) = \lambda/(2\pi\epsilon_0 a) = 1.4 \times 10^5 \text{ N/C.}$$

$$\text{Energy: } U = \int u dV = \int_a^b \frac{1}{2}\epsilon_0 \left(\frac{\lambda}{2\pi\epsilon_0 r} \right)^2 2\pi r L dr = \frac{\lambda^2 L}{4\pi\epsilon_0} \ln \frac{b}{a} = 4.0 \times 10^{-7} \text{ J, which equals } \frac{1}{2}CV^2 = \frac{1}{2}(2.0 \times 10^{-11})(200)^2.$$

EXAMPLE 4: SHARING CHARGE, THEN INSERTING A DIELECTRIC

A $6.0 \mu\text{F}$ capacitor is charged to 12 V and disconnected. It is then connected, plate to plate (+ to +), across an uncharged $3.0 \mu\text{F}$ capacitor. (a) Find the final voltage and the energy lost. (b) The slab $\kappa = 2.0$ is then slid fully into the $3.0 \mu\text{F}$ capacitor. Find the new voltage across the pair.

SOLUTION

Initial charge $Q = CV = 72 \mu\text{C}$, conserved. (a) The pair is in parallel with $C = 9.0 \mu\text{F}$, so $V = 72/9.0 = 8.0$ V. Energy before:

$\frac{1}{2}(6.0)(12)^2 = 432 \mu\text{J}$. After: $\frac{1}{2}(9.0)(8.0)^2 = 288 \mu\text{J}$. The $144 \mu\text{J}$ difference is dissipated as heat and radiation in the connecting wires as charge redistributes; it is not stored anywhere.

(b) The pair is isolated, so total $Q = 72 \mu\text{C}$ still. The second capacitor becomes $6.0 \mu\text{F}$, so $C_{\text{tot}} = 12 \mu\text{F}$ and $V = 72/12 = 6.0$ V. Charge shifts to the larger capacitance: both capacitors now hold $36 \mu\text{C}$, and the energy is $\frac{1}{2}(12)(6.0)^2 = 216 \mu\text{J}$, less than before because the dielectric was pulled in.

Write It Like a 5**Model Response**

Prompt: An air-filled parallel-plate capacitor is charged by a battery and then disconnected. A slab of dielectric constant κ is slid fully between the plates. (a) By what factor does the stored energy change? (b) Does the slab need to be pushed in or does it get pulled in? Justify. (a) After the battery is disconnected the plates are isolated, so the charge Q is unchanged. The slab multiplies the capacitance, $C' = \kappa C$. The energy is $U = Q^2/(2C)$, so $U' = Q^2/(2\kappa C) = U/\kappa$: the energy decreases by a factor of κ . (b) The energy of the system decreases as the slab enters, so the electric field does positive work on the slab. The slab is therefore pulled in, and an outside agent must do negative work (hold it back) to insert it slowly.

What the grader looks for

(1) States that Q is fixed because the capacitor is isolated. (2) States $C' = \kappa C$. (3) Uses $U = Q^2/2C$, not $\frac{1}{2}CV^2$ with the old V , and reaches U/κ . (4) Connects a decrease in energy to positive work by the field and a pull on the slab. A numeric answer with no reason for the fixed quantity loses the justification point.

Coach's Playbook

- 1 Name what is fixed.** Write 'Q fixed' (isolated) or 'V fixed' (battery) before touching any formula.
- 2 Choose the surface or path.** For capacitance by derivation, draw the Gaussian surface, state which part of it carries flux, then integrate E between the conductors.
- 3 Check limits.** A cylinder or sphere formula should reduce to the plate result when the gap is thin; a ratio like b/a must be unitless.
- 4 Use the equipotential.** Joined conductors share V, not charge; solve with $V = kQ/R$ and charge conservation.

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- 5 Keep units.** pF, nF and uF cause most arithmetic slips; convert to farads before substituting.

Common traps

(1) Applying $C = \epsilon_0 A/d$ to a cylinder or sphere. (2) Believing joined spheres end with equal charges rather than equal potentials. (3) Mixing up the isolated and battery-connected cases for a dielectric. (4) Using the flux through the end caps of a cylinder, or the wrong length, so a factor of L or 2π goes missing.

Quick Check

Q1: What is the field inside the material of a conductor in electrostatic equilibrium? A: zero. Q2: A cylindrical capacitor has a and b both tripled with L unchanged; what happens to C ? A: nothing, since $\ln(b/a)$ is unchanged. Q3: A capacitor stays attached to a battery while a dielectric is inserted; what happens to Q ? A: it increases by a factor κ because V is held and C rises.

Challenge Corner

Derive the capacitance of a spherical capacitor with inner radius $a = 5.0$ cm and outer radius $b = 5.5$ cm, and compare it with a plate estimate. Solution: $E = Q/(4\pi\epsilon_0 r^2)$, so $V = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)$ and $C = \frac{4\pi\epsilon_0 ab}{b-a} = \frac{4\pi(8.85 \times 10^{-12})(0.050)(0.055)}{0.0050} = 6.1 \times 10^{-11}$ F. A plate estimate with $d = 0.50$ cm and A taken at the mean radius 5.25 cm is $\epsilon_0 \cdot 4\pi(0.0525)^2/d = 6.1 \times 10^{-11}$ F, agreeing because $d \ll a$. Using the inner-sphere area instead gives 5.6×10^{-11} F, about 9% low.

Practice

PRACTICE Unit 10 Practice

Show your work. A calculator is allowed. Justify conclusions.

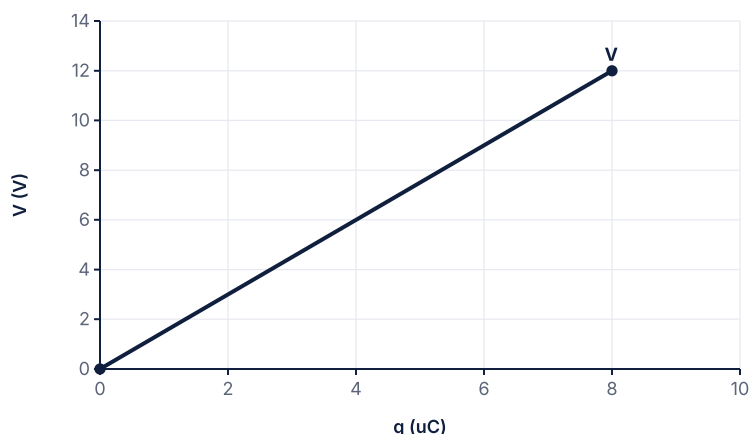
- A neutral conducting spherical shell has a small cavity that contains a point charge $+Q$ placed off center. Which statement correctly describes the charge on the shell?
 - The cavity wall has $-Q$ spread uniformly and the outer surface has $+Q$ spread uniformly.
 - The cavity wall has $-Q$ with a nonuniform distribution, and the outer surface has $+Q$ spread uniformly.
 - The cavity wall has no charge and the outer surface has $+Q$ spread uniformly.
 - The cavity wall has $-Q$ and the outer surface has no charge, because the shell is neutral.
- Two conducting spheres of radii R and $3R$ are far apart and joined by a thin wire. They carry a total charge Q . What is the charge on the larger sphere?
 - $Q/2$
 - $Q/4$
 - $3Q/4$
 - $9Q/10$
- A cylindrical capacitor has inner radius a , outer radius b and length L . If a and b are both doubled while L and the dielectric stay the same, the capacitance
 - is unchanged
 - is doubled
 - is halved
 - increases by a factor of $\ln 2$
- A parallel-plate capacitor is charged and then disconnected from the battery. The plates are slowly pulled apart until the separation doubles (air between). The stored energy
 - is halved
 - is unchanged
 - is quadrupled
 - doubles

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5. A capacitor is charged from zero. The voltage across it as a function of stored charge is shown. How much energy is stored when $q = 8.0 \mu\text{C}$ and $V = 12 \text{ V}$?



Voltage versus stored charge.

- (A) $96 \mu \text{ J}$ (B) $48 \mu \text{ J}$ (C) $24 \mu \text{ J}$ (D) $1.5 \mu \text{ J}$
6. A $6.0 \mu\text{F}$ capacitor and a $3.0 \mu\text{F}$ capacitor are connected in series across an 18 V battery. What is the voltage across the $3.0 \mu\text{F}$ capacitor?
- (A) 6.0 V (B) 9.0 V (C) 12 V (D) 18 V
7. A spherical capacitor has an inner conducting sphere of radius a carrying $+Q$ and a concentric conducting shell of inner radius b carrying $-Q$.
- Use Gauss's law to derive $E(r)$ for $a < r < b$.
 - Integrate to find the potential difference between the conductors.
 - Derive the capacitance C and evaluate it for $a = 2.0 \text{ cm}$ and $b = 6.0 \text{ cm}$.
 - Integrate the energy density over the gap to show that the stored energy is $Q^2/(2C)$.
8. A student measures the capacitance C of a parallel-plate capacitor with square plates of side 10.0 cm for several plate separations d , using a meter that reads in pF.
- d (mm): 1.0, 2.0, 3.0, 4.0, 5.0
 C (pF): 90.5, 46.2, 31.5, 24.1, 19.7
- Describe a procedure that keeps the plates parallel and reduces the uncertainty in d .
 - State what to plot so the data form a straight line, and say what the slope and intercept mean.
 - Use the data to find the slope and then a value of ϵ_0 .
 - The line does not pass through the origin. Give a physical reason and say what the intercept tells you.
9. A $4.0 \mu\text{F}$ capacitor is charged to 9.0 V and a $2.0 \mu\text{F}$ capacitor is charged to 3.0 V . The two are connected with positive plate to negative plate (opposite polarity).
- Calculate the final voltage.
 - Calculate the total energy before and after.
 - Explain where the lost energy goes, and why the amount lost does not depend on the resistance of the connecting wires.
10. A solid conducting sphere of radius 1.0 cm carries $+2.0 \text{ nC}$. It is at the center of a conducting shell with inner radius 3.0 cm and outer radius 4.0 cm ; the shell has net charge -5.0 nC . Use $V = 0$ at infinity and $k = 9.0 \times 10^9 \text{ N m}^2/\text{C}^2$.
- Find the charge on the inner and outer surfaces of the shell.
 - Find E at $r = 2.0 \text{ cm}$ and $r = 5.0 \text{ cm}$, giving directions.
 - Find the potential of the shell and of the inner sphere.
 - The sphere and the shell are then joined by a wire. Describe what charge moves and find the final potential.

Answers Unit 10: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

10 Conductors and Capacitors

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1. B. The wall charge must cancel the charge's field in the metal, so it crowds toward the nearest wall (nonuniform), while the outer surface of a sphere sees no cavity detail and is uniform. Choice A fails because an off-center charge does not induce a uniform pattern; D fails because the neutral conductor must still have a net charge of zero overall, so $+Q$ sits outside.
2. C. Equal potentials give $Q_1/R = Q_2/(3R)$, so $Q_2 = 3Q_1$ and $Q_2 = 3Q/4$. Choice A assumes equal sharing, which holds only for identical spheres; the surface charge density, not the charge, is the quantity inversely related to radius.
3. A. $C = 2\pi\epsilon_0 L / \ln(b/a)$ depends on the ratio b/a , which does not change. The tempting choice B treats the capacitor as a plate capacitor whose size has grown; a cylinder's C depends on the ratio of radii and the length.
4. D. The charge is fixed and $C = \epsilon_0 A/d$ halves, so $U = Q^2/(2C)$ doubles. The extra energy comes from the work done pulling the plates apart against their attraction. Choice A uses $\frac{1}{2}CV^2$ as if V stayed fixed.
5. B. Energy is the area under the V-q graph: $\frac{1}{2}(8.0 \mu\text{C})(12 \text{ V}) = 48 \mu\text{J}$. Choice A (QV) treats the voltage as constant during charging, which double counts because the average voltage is only 6 V.
6. C. Series capacitors carry the same charge: $C_{eq} = 2.0 \mu\text{F}$, $Q = 36 \mu\text{C}$, so $V_3 = Q/C = 12 \text{ V}$ (and 6.0 V across the $6.0 \mu\text{F}$). Choice B assumes the voltage splits equally, which is true only for equal capacitors; the smaller capacitor takes the larger share.
7. (a) Spherical Gaussian surface of radius r : $E(4\pi r^2) = Q/\epsilon_0$, so $E = Q/(4\pi\epsilon_0 r^2)$, radially outward.
 (b) $V = \int_a^b E dr = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)$.
 (c) $C = Q/V = \frac{4\pi\epsilon_0 ab}{b-a}$. For the numbers: $4\pi(8.85 \times 10^{-12})(0.020)(0.060)/0.040 = 3.3 \times 10^{-12} \text{ F}$.
 (d) $U = \int_a^b \frac{1}{2}\epsilon_0 E^2(4\pi r^2) dr = \frac{Q^2}{8\pi\epsilon_0} \int_a^b \frac{dr}{r^2} = \frac{Q^2}{8\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{Q^2}{2C}$.
8. (a) Place identical spacers (for example stacked plastic sheets or a micrometer-measured set of blocks) between the plates so the gap is uniform, measure the plate corners to confirm equal spacing, and take several measurements of each thickness with calipers; use a larger set of d values rather than one measurement. Keep leads short and in the same position so stray capacitance is constant.
 (b) Plot C versus $1/d$. From $C = \epsilon_0 A/d + C_s$, the slope is $\epsilon_0 A$ and the intercept is the stray capacitance C_s .
 (c) Using the end points of the best-fit line: slope = $(90.5 - 19.7) \text{ pF}/(1.00 - 0.20) \text{ mm}^{-1} = 88.5 \text{ pF mm} = 8.85 \times 10^{-14} \text{ F m}$. With $A = (0.100)^2 = 1.00 \times 10^{-2} \text{ m}^2$, $\epsilon_0 = 8.85 \times 10^{-14}/1.00 \times 10^{-2} = 8.85 \times 10^{-12} \text{ F/m}$. (A line fitted through all points gives the same value.)
 (d) The intercept, about 2.0 pF, is stray capacitance from the leads and fringing fields, which adds to the ideal plate capacitance. A line forced through the origin would give a wrong slope.
9. (a) Net charge: $36 \mu\text{C} - 6.0 \mu\text{C} = 30 \mu\text{C}$, since the opposite plates cancel. $C_{eq} = 6.0 \mu\text{F}$, so $V = 30/6.0 = 5.0 \text{ V}$.
 (b) Before: $\frac{1}{2}(4.0)(9.0)^2 + \frac{1}{2}(2.0)(3.0)^2 = 162 + 9 = 171 \mu\text{J}$. After: $\frac{1}{2}(6.0)(5.0)^2 = 75 \mu\text{J}$.
 (c) $96 \mu\text{J}$ is dissipated as thermal energy in the wires and radiated. The final energy is fixed by the conserved charge and the final capacitances, so the loss is the same for any wire resistance; resistance changes only how quickly and how the energy is dissipated.
10. (a) The shell's inner surface must cancel the $+2.0 \text{ nC}$ inside: -2.0 nC . Net -5.0 nC means the outer surface carries -3.0 nC .
 (b) At 2.0 cm the enclosed charge is $+2.0 \text{ nC}$: $E = (9.0 \times 10^9)(2.0 \times 10^{-9})/(0.020)^2 = 4.5 \times 10^4 \text{ N/C}$, outward. At 5.0 cm the enclosed charge is -3.0 nC (sphere + shell = $2.0 - 5.0$): $E = 1.1 \times 10^4 \text{ N/C}$, directed inward.
 (c) Shell: $V = k(-3.0 \text{ nC})/0.040 = -675 \text{ V}$. Sphere: $V = V_{\text{shell}} + kq(1/0.010 - 1/0.030) = -675 + 18(66.7) = +525 \text{ V}$.
 (d) The sphere is at the higher potential, so positive charge flows through the wire from the sphere to the shell (equivalently the $+2.0 \text{ nC}$ of the sphere cancels the -2.0 nC on the inner wall). Afterward all -3.0 nC sits on the outer surface, the field in the gap is zero, and both conductors are at -675 V .

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Unit 11: Electric Circuits

Exam weighting: about 15-25% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 9 practice items, all with full explanations.

11 Electric Circuits

Current and current density, resistivity by integration, power, series-parallel and Kirchhoff analysis, and the exponential charging and discharging of RC circuits derived from a differential equation.

Why This Matters

Electric circuits carry 15-25% of the multiple-choice section, tied with Unit 8 for the largest share, and circuit ideas show up in free-response questions as resistivity labs, capacitor-charging derivations, and Kirchhoff analyses. The calculus content is a first-order differential equation: you should be able to derive the exponential charging and discharging laws, not just quote them. Students lose points by treating a capacitor as an ordinary resistor, by making sign errors in the loop rule, and by plotting a linearized graph with the wrong axes.

Core Ideas

11.1 Electric Current

Current is the rate at which charge crosses a cross-section, $I = dQ/dt$, and charge transferred is the area under the current graph, $Q = \int I dt$. By convention the current points the way positive charges would drift, opposite to the electron drift in a metal. If the current is spread unevenly over a cross-section, define the **current density** $\vec{J}$ (current per area, in A/m²) and add up the pieces.

Current and current density

$$I = \frac{dQ}{dt} \text{ and } \Delta Q = \int I dt.$$

$I = \int \vec{J} \cdot d\vec{A}$; for uniform J , $I = JA$. In terms of carriers, $I = nqv_d A$ where n is carrier density and v_d the drift speed (typically a tenth of a millimeter per second or less).

In an ohmic material $\vec{J} = \sigma \vec{E} = \vec{E}/\rho$.

11.2 Simple Circuits

A source of **emf** $\mathcal{E}$ does work per unit charge, raising the charge's potential from the negative to the positive terminal. A real battery with internal resistance r delivers a terminal voltage $V = \mathcal{E} - Ir$ when it supplies a current I . Treat wires, switches, ammeters and (unless told otherwise) batteries as ideal. An ideal **ammeter** has zero resistance and is placed in series; an ideal **voltmeter** has infinite resistance and is placed in parallel across the element.

11.3 Resistance, Resistivity, and Ohm's Law

For a uniform conductor of length L and cross-section A , the resistance is $R = \rho L/A$, where the resistivity ρ is a property of the material. Ohm's law, $V = IR$, describes ohmic elements whose R stays constant; a bulb or diode that does not obey it is non-ohmic. When area or resistivity changes along the length, slice the conductor into pieces in series and integrate.

Resistance by integration

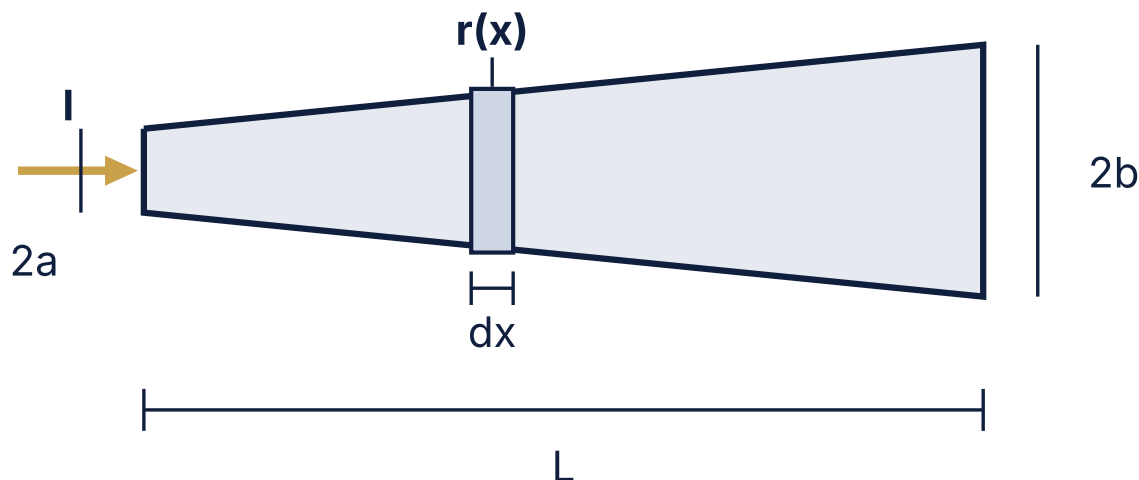
$$R = \int \frac{\rho(x) dx}{A(x)} \text{ along the current direction (slices in series add).}$$

Quick checks: doubling the length doubles R ; doubling the diameter divides R by 4.

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Conductor whose radius grows linearly from a to b . A thin slice of thickness dx has resistance $dR = \rho dx / (\pi r(x)^2)$.

11.4 Electric Power

A circuit element with voltage V across it and current I through it transfers energy at the rate $P = IV$. For a resistor this is $P = I^2 R = V^2/R$, dissipated as heat; for a battery, $P = \mathcal{E}I$ delivered to the circuit. Total energy over an interval is $\int P dt$. For a fixed battery voltage, a smaller resistance dissipates more power, but for a fixed current, a larger resistance does.

11.5 Compound Direct Current Circuits

Series elements carry the same current; their resistances add, $R_s = R_1 + R_2 + \dots$. **Parallel** elements share the same voltage; $1/R_p = 1/R_1 + 1/R_2 + \dots$, and the equivalent is always smaller than the smallest branch. To reduce a network, collapse the innermost series or parallel pair first, solve for the source current, then work backward, applying $V = IR$ to each branch. Identical bulbs in parallel each get the full source voltage; adding a bulb in parallel lowers the total resistance and raises the source current.

11.6 Kirchhoff's Loop Rule and 11.7 Kirchhoff's Junction Rule

Some networks cannot be reduced by series and parallel rules. Kirchhoff's rules always work. They express conservation of energy and of charge.

Kirchhoff's rules

Loop rule: the sum of potential changes around any closed loop is zero, $\sum \Delta V = 0$.

Sign convention: crossing a battery from $-$ to $+$ is $+\mathcal{E}$; crossing a resistor in the direction of the assumed current is $-IR$ (and $+IR$ against it); crossing a capacitor from its $+$ plate to its $-$ plate is $-q/C$.

Junction rule: the current entering a junction equals the current leaving, $\sum I_{\text{in}} = \sum I_{\text{out}}$.

Procedure: label each branch current with an assumed direction, mark the junctions, write the junction equation, then write loop equations until you have as many independent equations as unknowns. A negative answer only means the real current opposes your arrow. For a circuit with n unknown branch currents you need n independent equations; the outer loop of a two-loop circuit adds no new information. Example 2 below applies the procedure to a two-loop circuit.

11.8 Resistor-Capacitor (RC) Circuits

A capacitor in a circuit stores charge q while the resistor limits the current. Write the loop rule for the charging circuit, with $I = dq/dt$, and the differential equation is separable.

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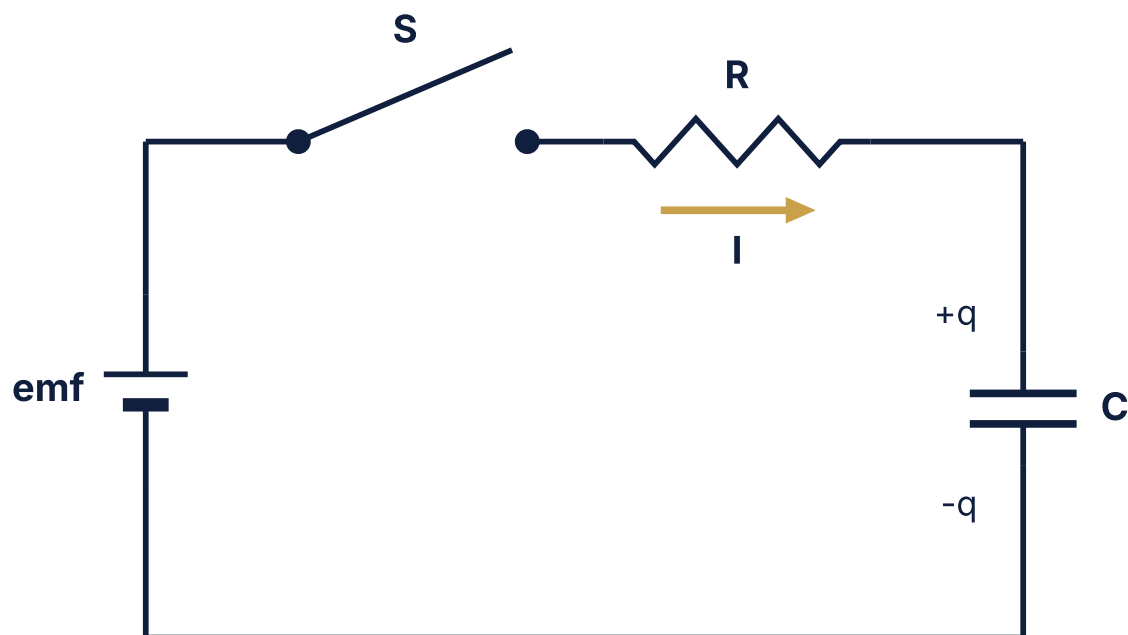
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Charging a capacitor through R

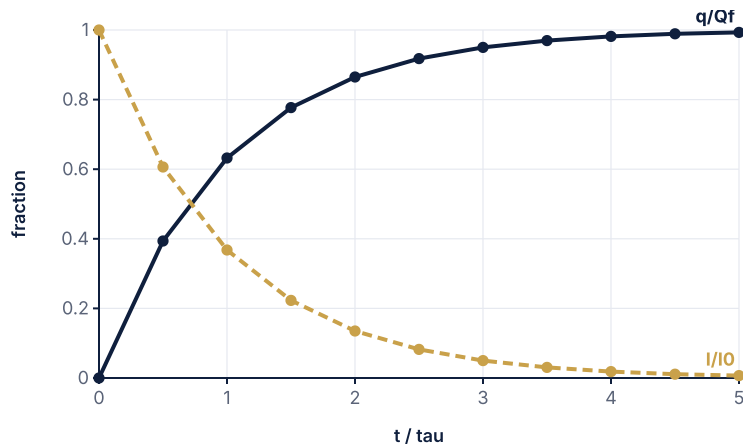
$$\varepsilon - R \frac{dq}{dt} - \frac{q}{C} = 0 \Rightarrow \int_0^q \frac{dq'}{C\varepsilon - q'} = \int_0^t \frac{dt'}{RC}$$

so $q(t) = C\varepsilon(1 - e^{-t/\tau})$, $I(t) = \frac{\varepsilon}{R}e^{-t/\tau}$, with **time constant** $\tau = RC$.

Discharging (no source): $R dq/dt = -q/C$ gives $q = Q_0 e^{-t/\tau}$ and $|I| = (Q_0/\tau)e^{-t/\tau}$.



Series RC circuit. When S closes the emf drives current $I = dq/dt$ onto the capacitor through R .



Charging a capacitor: charge (solid) rises toward its final value while the current (dashed) decays. A discharge looks like the dashed curve for both q and I .

At $t = \tau$ the capacitor has reached $1 - e^{-1} = 63\%$ of its final charge (or fallen to 37% in a discharge); the half-life is $t_{1/2} = \tau \ln 2 \approx 0.69\tau$. Two limiting behaviors let you analyze any RC circuit quickly. **Just after a switch changes**, an uncharged capacitor acts like a wire (the voltage across a capacitor cannot jump). **A long time later**, the current through a capacitor branch is zero and it acts like a gap. For a network with several resistors, find the Thevenin-style resistance the capacitor sees with the sources replaced by wires; that resistance times C is τ .

Energy bookkeeping: the battery supplies $\varepsilon Q_f = C\varepsilon^2$; the capacitor stores $\frac{1}{2}C\varepsilon^2$; the other half is dissipated in the resistor, since $\int_0^\infty I^2 R dt = \frac{1}{2}C\varepsilon^2$ independent of R . In the lab, discharge data are linearized: $\ln V = \ln V_0 - t/RC$, so a plot of $\ln V$ against t is a line with slope $-1/RC$.

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Worked Examples

EXAMPLE 1: RESISTANCE OF A TAPERED CONDUCTOR

A conducting rod of length $L = 0.30$ m has circular cross-section whose radius grows linearly from $a = 1.0$ mm to $b = 3.0$ mm. Its resistivity is $\rho = 2.0 \times 10^{-6} \Omega \cdot \text{m}$. Find its resistance between the ends.

SOLUTION

Radius at position x : $r(x) = a + (b - a)x/L$. A slice dx has $dR = \rho dx / (\pi r^2)$, and the slices are in series.

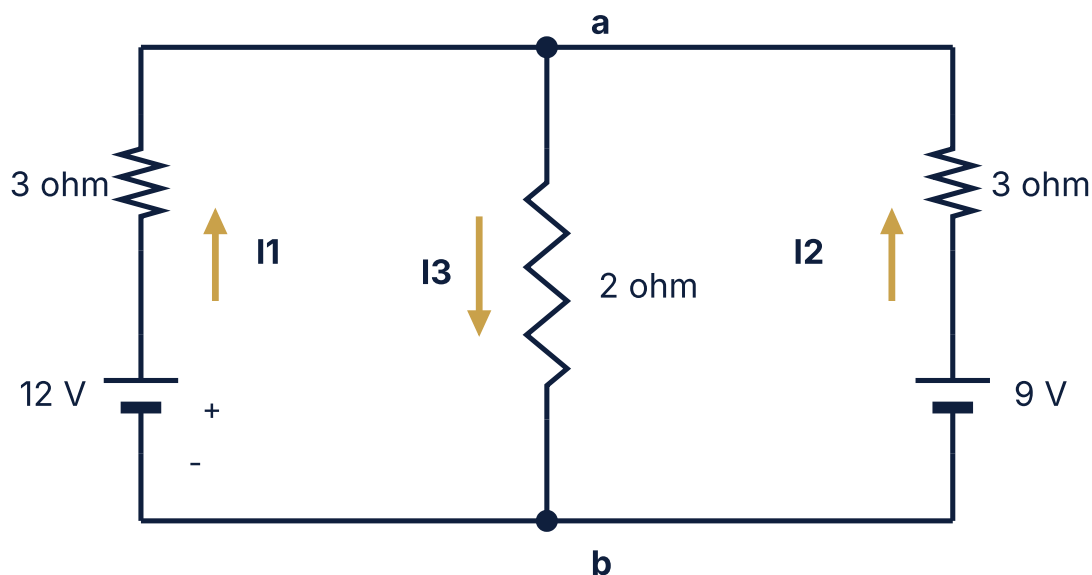
$$R = \frac{\rho}{\pi} \int_0^L \frac{dx}{[a + (b - a)x/L]^2} = \frac{\rho}{\pi} \frac{L}{b - a} \left[\frac{-1}{r} \right] \text{ evaluated between } r = a \text{ and } r = b, \text{ which equals } \frac{\rho L}{\pi} \frac{1}{b - a} \left(\frac{1}{a} - \frac{1}{b} \right) = \frac{\rho L}{\pi ab}.$$

$$\text{Numerically } R = \frac{(2.0 \times 10^{-6})(0.30)}{\pi(1.0 \times 10^{-3})(3.0 \times 10^{-3})} = 0.064 \Omega.$$

Check: a uniform rod of radius a would have 0.19Ω and one of radius b would have 0.021Ω ; the answer lies between them, as it must.

EXAMPLE 2: KIRCHHOFF'S RULES

In the circuit shown, find I_1 , I_2 , I_3 , the power delivered by each battery, and the total power dissipated.



Two-battery circuit. Both batteries have their positive terminals on the top rail. Currents are drawn in assumed directions; a negative result means the opposite direction.

SOLUTION

Junction a: $I_1 + I_2 = I_3$.

Left loop (up through the 12 V battery and down through the middle resistor): $12 - 3I_1 - 2I_3 = 0$. Right loop: $9 - 3I_2 - 2I_3 = 0$.

Substitute $I_3 = I_1 + I_2$: $12 = 5I_1 + 2I_2$ and $9 = 2I_1 + 5I_2$. Subtracting $2 \times$ the second from $5 \times$ the first leaves $42 = 21I_1$, so $I_1 = 2.0$ A, then $I_2 = 1.0$ A and $I_3 = 3.0$ A. All are positive, so the assumed directions were right.

Battery power: $12(2.0) = 24$ W and $9(1.0) = 9$ W, total 33 W. Dissipated: $(2.0)^2(3) + (1.0)^2(3) + (3.0)^2(2) = 12 + 3 + 18 = 33$ W. Energy is conserved.

$V_{ab} = I_3(2) = 6.0$ V, with the top node at the higher potential.

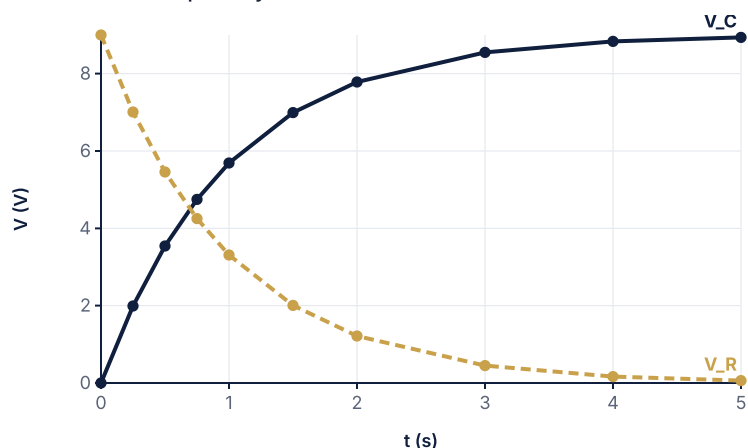
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EXAMPLE 3: CHARGING A CAPACITOR

An initially uncharged $5.0\ \mu\text{F}$ capacitor is connected through a $200\ \text{k}\Omega$ resistor to a $9.0\ \text{V}$ battery at $t = 0$. (a) Find τ and the initial current. (b) At what time does the capacitor voltage reach 80% of $9.0\ \text{V}$, and what is the current then? (c) How much energy does the resistor dissipate by $t \rightarrow \infty$?



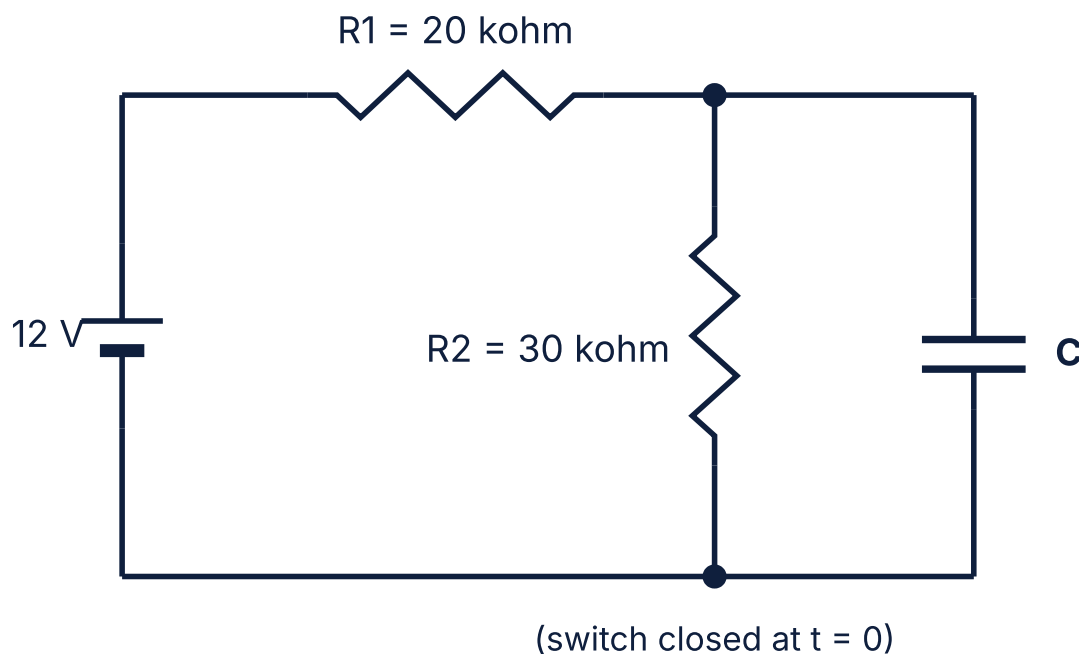
Capacitor voltage (solid) and resistor voltage (dashed) always add to $9.0\ \text{V}$.

SOLUTION

(a) $\tau = RC = (2.0 \times 10^5)(5.0 \times 10^{-6}) = 1.0\ \text{s}$. At $t = 0$ the capacitor has no voltage, so $I_0 = \mathcal{E}/R = 9.0/(2.0 \times 10^5) = 45\ \mu\text{A}$.

(b) $V_C = \mathcal{E}(1 - e^{-t/\tau}) = 0.80\mathcal{E}$ gives $e^{-t/\tau} = 0.20$, so $t = \tau \ln 5 = 1.6\ \text{s}$. The current then is $I = I_0(0.20) = 9.0\ \mu\text{A}$ (check: $V_R/R = (0.20)(9.0)/(2.0 \times 10^5)$).

(c) The capacitor ends with $\frac{1}{2}C\mathcal{E}^2 = 2.0 \times 10^{-4}\ \text{J}$ ($202\ \mu\text{J}$). The battery supplied $C\mathcal{E}^2 = 405\ \mu\text{J}$, so the resistor dissipates the other $202\ \mu\text{J}$. The graph shows the two voltages crossing at $4.5\ \text{V}$ when $t = \tau \ln 2 = 0.69\ \text{s}$.



Capacitor C in parallel with R2, fed through R1 from a $12\ \text{V}$ source.

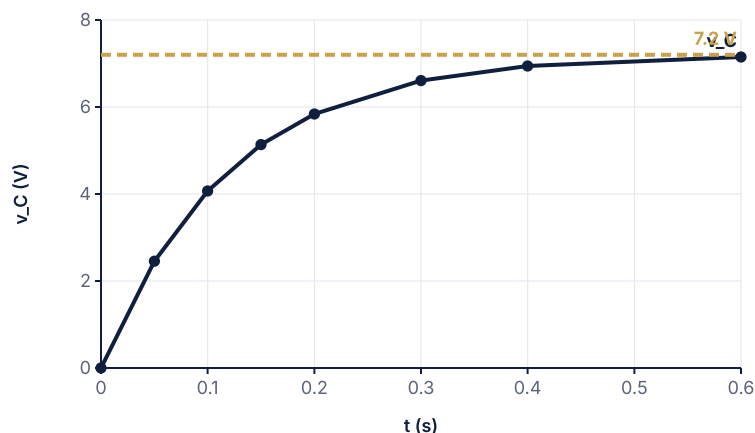
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EXAMPLE 4: CAPACITOR WITH A PARALLEL RESISTOR

In the circuit shown, the capacitor is uncharged at $t = 0$ when the switch closes. $\varepsilon = 12 \text{ V}$, $R_1 = 20 \text{ k}\Omega$, $R_2 = 30 \text{ k}\Omega$, $C = 10 \text{ }\mu\text{F}$. Find (a) the initial current in R_1 , (b) the final capacitor voltage, (c) $v_C(t)$ by solving the differential equation, and (d) the time for v_C to reach 5.0 V .



Capacitor voltage approaches 7.2 V , not 12 V .

SOLUTION

(a) At $t = 0$ the uncharged capacitor acts like a wire across R_2 , so all the current goes through R_1 into the capacitor: $I = 12/20\,000 = 0.60 \text{ mA}$.

(b) At $t \rightarrow \infty$ no current enters the capacitor, so R_1 and R_2 form a divider: $v_C = 12(30)/(20 + 30) = 7.2 \text{ V}$.

(c) Junction rule at the top node: current through R_1 equals the current into C plus the current in R_2 , so $\frac{\varepsilon - v}{R_1} = C \frac{dv}{dt} + \frac{v}{R_2}$.

Rearranged, $\frac{dv}{dt} = \frac{1}{R_p C} (V_\infty - v)$ with $R_p = \frac{R_1 R_2}{R_1 + R_2} = 12 \text{ k}\Omega$ and $V_\infty = \varepsilon R_2 / (R_1 + R_2) = 7.2 \text{ V}$. This is the charging equation with $\tau = R_p C = 0.12 \text{ s}$, so $v_C = 7.2 (1 - e^{-t/0.12}) \text{ V}$.

(d) $5.0 = 7.2(1 - e^{-t/0.12})$ gives $e^{-t/0.12} = 0.306$ and $t = -0.12 \ln(0.306) = 0.14 \text{ s}$.

The time constant uses the resistance the capacitor sees with the source replaced by a wire, which here is R_1 and R_2 in parallel, not R_1 alone.

Write It Like a 5**Model Response**

Prompt: An uncharged capacitor C is connected through a resistor R to a battery of emf ε at $t = 0$. (a) Derive $q(t)$. (b) Find the time at which the voltage across the capacitor equals the voltage across the resistor. (a) Kirchhoff's loop rule with $I = dq/dt$: $\varepsilon - R dq/dt - q/C = 0$. Separate variables: $\frac{dq}{C\varepsilon - q} = \frac{dt}{RC}$. Integrate from $q = 0$ at $t = 0$: $-\ln\left(\frac{C\varepsilon - q}{C\varepsilon}\right) = \frac{t}{RC}$, so $q(t) = C\varepsilon(1 - e^{-t/RC})$. (b) $v_C = \varepsilon(1 - e^{-t/RC})$ and $v_R = IR = \varepsilon e^{-t/RC}$. Setting them equal gives $e^{-t/RC} = 1/2$, so $t = RC \ln 2$ (about $0.69RC$), when each equals $\varepsilon/2$.

What the grader looks for

(1) Loop rule written with a correct sign for each term. (2) $I = dq/dt$ substituted. (3) Separation of variables and integration with correct limits shown. (4) Exponential result with $\tau = RC$. (5) Expressions for both voltages in part (b), then solved. Skipping the integration steps and quoting the result earns only the final-answer point.

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Coach's Playbook

- 1 Redraw and label.** Mark every node, each element's value, and an assumed direction for each current.
- 2 Reduce or write equations.** Use series and parallel rules if the network allows; otherwise use one junction equation and enough loop equations.
- 3 Check the two limits.** For RC circuits state the behavior at $t = 0$ and $t \rightarrow \infty$ before doing any algebra.
- 4 Verify with power.** Power supplied by sources must equal power dissipated; it catches sign errors.
- 5 Label graphs.** State the quantity and unit on each axis, mark τ or the final value, and justify a linear fit with the equation it comes from.

Common traps

(1) Treating an uncharged capacitor as an open circuit at $t = 0$ (it is a wire at that instant). (2) Putting a voltmeter in series or an ammeter in parallel. (3) Taking $\tau = R_1 C$ when other resistors act on the capacitor, or writing $\tau = R/C$. (4) Computing a slope from one data point and the origin on a $\ln V$ versus t graph without showing the line passes through the origin; the intercept is $\ln V_0$, not zero.

Quick Check

Q1: What fraction of the final charge is on a charging capacitor after one time constant? A: about 63%. Q2: A junction has 5.0 A entering and one wire leaving with 2.0 A; what is in the third wire? A: 3.0 A leaving. Q3: A resistor's voltage is doubled; what happens to its power? A: it quadruples (for fixed R).

Challenge Corner

In the charging circuit of Example 3, at what time is the power delivered to the capacitor greatest, and what is it?

Solution: $P_C = v_C I = \frac{\varepsilon^2}{R} (e^{-t/\tau} - e^{-2t/\tau})$. Set $dP_C/dt = 0$: $-e^{-t/\tau} + 2e^{-2t/\tau} = 0$, so $e^{-t/\tau} = 1/2$ and $t = \tau \ln 2 = 0.69$ s. Then $P_C = \frac{\varepsilon^2}{R} (\frac{1}{2} - \frac{1}{4}) = \frac{\varepsilon^2}{4R} = \frac{81}{4(2.0 \times 10^5)} = 1.0 \times 10^{-4}$ W, a quarter of the maximum a resistor alone could take from the battery.

Practice

PRACTICE Unit 11 Practice

Show your work. A calculator is allowed. Justify conclusions.

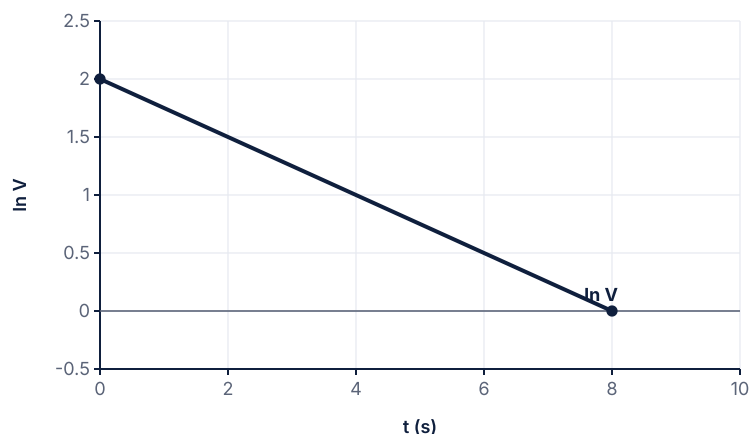
1. A copper-like wire of diameter 1.0 mm carries 2.0 A. The free-carrier density is $8.5 \times 10^{28} \text{ m}^{-3}$ and each carrier has charge magnitude $1.6 \times 10^{-19} \text{ C}$. What is the drift speed?
 (A) $4.7 \times 10^{-5} \text{ m/s}$ (B) $1.5 \times 10^{-4} \text{ m/s}$ (C) $1.9 \times 10^{-4} \text{ m/s}$
 (D) $7.5 \times 10^{-4} \text{ m/s}$
2. An ideal battery is connected to bulb A in series with two identical bulbs B and C that are in parallel with each other. All bulbs are identical and ohmic. Bulb C burns out and becomes an open circuit. What happens to the brightness of A and B?
 (A) A dims and B brightens. (B) A brightens and B dims. (C) Both dim.
 (D) Both are unchanged.
3. A charged capacitor discharges through a resistor. After 3.0 s its charge is 25% of the initial value. What is the time constant?
 (A) 0.75 s (B) 4.2 s (C) 3.0 s (D) 2.2 s
4. An uncharged capacitor C is connected in series with a resistor R and an ideal battery of emf ε . At the instant the switch is closed, the current and the capacitor voltage are, respectively,
 (A) 0 and ε (B) ε/R and 0 (C) ε/R and ε (D) 0 and 0

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5. A capacitor discharges through a $2.0\text{ M}\Omega$ resistor. A student plots the natural log of the capacitor voltage (in volts) against time, getting the straight line shown. What is the capacitance?



Discharge data, linearized.

- (A) $0.13\text{ }\mu\text{F}$ (B) $0.50\text{ }\mu\text{F}$ (C) $2.0\text{ }\mu\text{F}$ (D) $8.0\text{ }\mu\text{F}$
6. A 12 V battery, a $2.0\text{ }\Omega$ resistor, a $4.0\text{ }\Omega$ resistor and a 3.0 V battery are connected in a single series loop, with the 3.0 V battery opposing the 12 V battery. What is the potential difference across the $4.0\text{ }\Omega$ resistor?
- (A) 4.5 V (B) 8.0 V (C) 10 V (D) 6.0 V
7. A capacitor $C = 4.0\text{ }\mu\text{F}$ charged to $V_0 = 10\text{ V}$ is connected at $t = 0$ across a resistor $R = 250\text{ k}\Omega$.
- Starting from the loop rule, derive $q(t)$ for the discharge.
 - Sketch $q(t)$ and $I(t)$ on the same time axis, marking τ and the initial values.
 - Find the time for the voltage to fall to 5.0 V .
 - Integrate I^2R from 0 to infinity and show the energy dissipated equals the initial stored energy. Evaluate it.
8. A student discharges a capacitor through a resistor of known resistance $R = 47\text{ k}\Omega$ and records the voltage across the capacitor with a voltmeter.
- $t\text{ (s): } 0, 3, 6, 9, 12, 15$
 $V\text{ (V): } 8.00, 4.23, 2.23, 1.18, 0.62, 0.33$
- Describe how to run the experiment so the voltmeter does not disturb the circuit, and how to time the readings accurately.
 - State what to plot to obtain a straight line and what the slope represents.
 - Use the first and last points to find the slope, the time constant, and the capacitance.
 - The student repeats the experiment with R doubled. Predict the new slope and explain.
9. A graded-resistivity rod of length $L = 0.40\text{ m}$ and constant cross-section $A = 2.0 \times 10^{-6}\text{ m}^2$ has resistivity $\rho(x) = \rho_0(1 + 2x/L)$, with $\rho_0 = 5.0 \times 10^{-5}\text{ }\Omega\text{ m}$ and x measured from one end. A 6.0 V battery is connected across the ends.
- Derive and evaluate the resistance of the rod.
 - Find the current and the power dissipated.
 - Find the electric field inside the rod at $x = 0$ and $x = L$.
 - Sketch E versus x and show that the area under the graph equals 6.0 V .

Answers Unit 11: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

11 Electric Circuits

PRACTICE · UNIT 11 PRACTICE

- $C. v_d = I/(nqA)$ with $A = \pi(0.50 \times 10^{-3}\text{ m})^2 = 7.9 \times 10^{-7}\text{ m}^2$, giving $1.9 \times 10^{-4}\text{ m/s}$. Choice A comes from using the diameter as the radius in the area; the drift speed is tiny even though the signal travels almost instantly.
- A. Before the failure the current in A splits equally between B and C. After C opens, the load resistance rises from $1.5R$ to $2R$, so the source current through A falls and A dims, while all of that smaller current now goes through B, which is more than the half it had before, so B brightens. Choice D wrongly assumes that parallel bulbs are independent of the rest of the circuit.
- D. $q/Q_0 = e^{-t/\tau} = 0.25$ gives $\tau = t/\ln 4 = 3.0/1.386 = 2.2\text{ s}$. Choice B multiplies by $\ln 4$ instead of dividing; choice A assumes the decay is linear.

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4. B. The capacitor voltage cannot change instantly, so it starts at zero and the full emf appears across R , giving $I = \varepsilon/R$. Choice A describes the situation a long time later.
5. C. The slope is $-0.25 \text{ s}^{-1} = -1/RC$, so $\tau = 4.0 \text{ s}$ and $C = \tau/R = 4.0/(2.0 \times 10^6) = 2.0 \mu\text{F}$. Choice A uses the slope itself as τ ; choice D uses the 8 s intercept as τ .
6. D. The net emf is $12 - 3.0 = 9.0 \text{ V}$, so $I = 9.0/6.0 = 1.5 \text{ A}$ and $V = (1.5)(4.0) = 6.0 \text{ V}$. Choice B ignores the opposing battery ($I = 2.0 \text{ A}$); choice C adds the two emfs.
7. (a) Loop rule with the capacitor as the only source: $q/C - IR = 0$ with $I = -dq/dt$ (charge decreases), so $dq/dt = -q/RC$. Separate: $\int_{Q_0}^q dq'/q' = -\int_0^t dt'/RC$, giving $q = Q_0 e^{-t/RC}$ with $Q_0 = CV_0 = 40 \mu\text{C}$ and $\tau = RC = 1.0 \text{ s}$.
 (b) Both curves are decaying exponentials with the same τ . q starts at $40 \mu\text{C}$, I starts at $V_0/R = 40 \mu\text{A}$; each is 37% of its start at $t = 1.0 \text{ s}$ and falls toward zero without reaching it. Axes labeled with quantity and unit.
 (c) $V = V_0 e^{-t/\tau} = 5.0$ gives $t = \tau \ln 2 = 0.69 \text{ s}$.
 (d) $\int_0^\infty I^2 R dt = R(V_0/R)^2 \int_0^\infty e^{-2t/\tau} dt = \frac{V_0^2}{R} \cdot \frac{\tau}{2} = \frac{1}{2} CV_0^2 = 2.0 \times 10^{-4} \text{ J}$ ($200 \mu\text{J}$), independent of R .
8. (a) Use a voltmeter whose resistance is much larger than R (so little current is diverted), connect it in parallel across the capacitor, discharge the capacitor fully before each run, and record the readings from video of the meter and a stopwatch (or use a data logger) so the times are not subject to reaction delay. Repeat the run and average.
 (b) Plot $\ln V$ versus t . From $V = V_0 e^{-t/RC}$, $\ln V = \ln V_0 - t/RC$: the slope is $-1/RC$ and the intercept is $\ln V_0$.
 (c) $\ln(8.00) = 2.079$ and $\ln(0.33) = -1.109$, so slope $= (-1.109 - 2.079)/15 \text{ s} = -0.213 \text{ s}^{-1}$. Then $\tau = 1/0.213 = 4.7 \text{ s}$ and $C = \tau/R = 4.7/(4.7 \times 10^4) = 1.0 \times 10^{-4} \text{ F}$ ($100 \mu\text{F}$). The line should be shown to pass through the points over the whole range rather than only the two used.
 (d) τ doubles, so the slope magnitude is halved to about -0.106 s^{-1} ; the line starts at the same intercept but falls more slowly.
9. (a) Slices in series: $R = \int_0^L \frac{\rho_0(1+2x/L)}{A} dx = \frac{\rho_0}{A} [L + L] = \frac{2\rho_0 L}{A} = \frac{2(5.0 \times 10^{-5})(0.40)}{2.0 \times 10^{-6}} = 20 \Omega$.
 (b) $I = V/R = 6.0/20 = 0.30 \text{ A}$; $P = IV = 1.8 \text{ W}$.
 (c) The current density is uniform, $J = I/A = 1.5 \times 10^5 \text{ A/m}^2$, and $E = \rho J$: at $x = 0$, $E = 7.5 \text{ V/m}$; at $x = L$, $\rho = 3\rho_0$, so $E = 22.5 \text{ V/m}$.
 (d) The graph is a straight line from 7.5 V/m at $x = 0$ to 22.5 V/m at $x = 0.40 \text{ m}$ (rising, not constant). The area is a trapezoid, $\frac{1}{2}(7.5 + 22.5)(0.40) = 6.0 \text{ V}$, equal to the battery voltage as $\int E dx$ must.

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Unit 12: Magnetic Fields and Electromagnetism

Exam weighting: about 10-20% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

12 Magnetic Fields and Electromagnetism

Magnetic force on charges and currents, circular motion, the Biot-Savart law for wires and loops, forces between wires, and Ampere's law with uniform and nonuniform current density.

Why This Matters

Magnetic fields and electromagnetism carry 10-20% of the multiple-choice section and have anchored recent free-response questions: Ampere's law for a wire with nonuniform current density, superposition of wire fields, and the Lorentz force on a charge moving past two wires. Students lose points by confusing the direction of a field with the direction of the force it causes, by writing an incorrect enclosed current, and by using a Biot-Savart integral when the question only wanted a force comparison.

Core Ideas

12.1 Magnetic Fields

Moving charges (currents) and magnetic materials produce **magnetic fields**, measured in tesla. Field lines form closed loops because isolated magnetic poles have not been observed; the net magnetic flux through any closed surface is zero. The field direction at a point is the direction a compass needle's north end points. For a long straight wire, the field circles the wire: point the right thumb along the current and the fingers curl in the direction of $\vec{B}$. Fields from several sources add as vectors, so the net field is the vector sum of each source's field. In diagrams a dot means out of the page and a cross means into the page. For scale, Earth's field is about 5×10^{-5} T.

12.2 Magnetism and Moving Charges

Magnetic force

On a charge: $\vec{F} = q\vec{v} \times \vec{B}$, magnitude $qvB \sin \theta$, perpendicular to both $\vec{v}$ and $\vec{B}$. Right-hand rule: fingers along $\vec{v}$, curl toward $\vec{B}$, thumb gives the force on a positive charge (reverse for negative).

On a current element: $d\vec{F} = I d\vec{\ell} \times \vec{B}$. For a straight wire of length L in a uniform field, $F = ILB \sin \theta$; for a curved wire in a uniform field, replace $\vec{L}$ by the straight vector from start to end.

The magnetic force does no work, so it changes the direction of $\vec{v}$ but not the speed.

The force is perpendicular to the velocity, so a charge moving perpendicular to a uniform field travels in a circle. Setting $qvB = mv^2/r$ gives the radius, and the period follows.

Circular motion in a uniform field

$r = \frac{mv}{qB}$, $T = \frac{2\pi m}{qB}$, $\omega = \frac{qB}{m}$. The period does not depend on the speed.

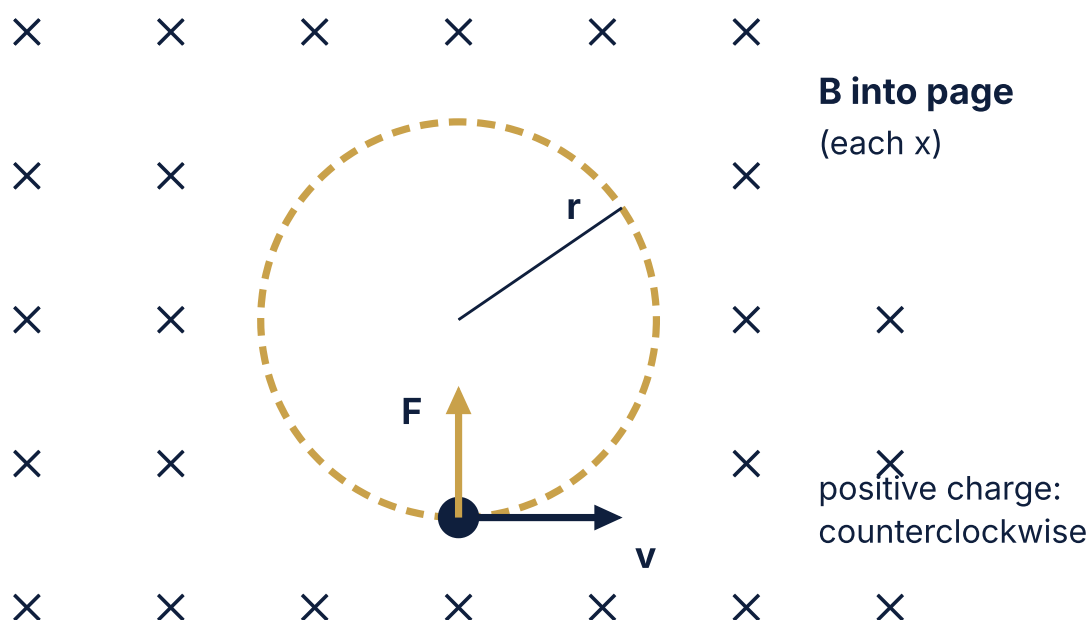
If $\vec{v}$ has a component along $\vec{B}$, that component is unaffected and the path is a helix. A charge moving along $\vec{B}$ feels no force.

Crossed fields: a charge moves in a straight line when $qE = qvB$, so $v = E/B$ for any charge or mass (a velocity selector).

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A positive charge moving at speed v perpendicular to a uniform field into the page. The force is always toward the center, so the path is a circle with $r = mv/(qB)$.

12.3 Magnetic Fields of Current-Carrying Wires and the Biot-Savart Law

The **Biot-Savart law** gives the field of any current as a sum over small elements. It is the magnetic counterpart of Coulomb's law, but with a cross product, so the direction needs the right-hand rule at each element.

Biot-Savart law

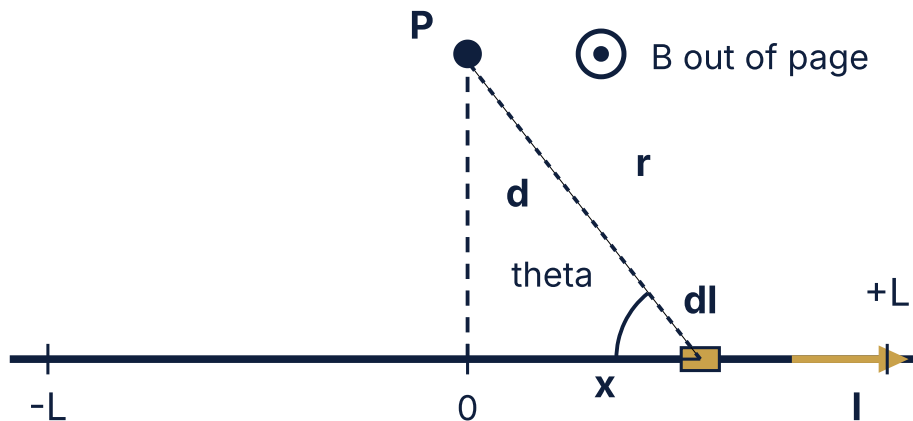
$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2}$, with $\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$ and $\hat{r}$ pointing from the element to the field point.

Procedure: draw $d\vec{\ell}$, $\vec{r}$ and the angle between them; write $|d\vec{\ell} \times \hat{r}| = d\ell \sin \theta$; express every variable in terms of one integration variable; integrate, keeping only components that survive by symmetry.

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Straight wire from $x = -L$ to $+L$. Each element dl contributes $dB = (\mu_0 I / 4\pi) dl \sin(\theta) / r^2$, all directed out of the page at P.

Applying this to common shapes gives the results below. They are worth deriving once and recognizing in forms on the exam: for the straight wire, $\sin \theta = d/r$ and $r = \sqrt{x^2 + d^2}$, so $B = \frac{\mu_0 I d}{4\pi} \int_{-L}^{+L} \frac{dx}{(x^2 + d^2)^{3/2}}$.

Source	Field magnitude
Straight wire, ends at $x = \pm L$, point on the perpendicular bisector at distance d	$\frac{\mu_0 I}{4\pi d} \frac{2L}{\sqrt{L^2 + d^2}}$
Infinite straight wire ($L \rightarrow \infty$)	$\frac{\mu_0 I}{2\pi d}$
Circular arc of angle ϕ (radians) at its center	$\frac{\mu_0 I \phi}{4\pi R}$
Circular loop, on the axis at distance z	$\frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}}$ (center: $\mu_0 I / 2R$)

Force between parallel wires. The field of wire 1 at wire 2 is $\mu_0 I_1 / (2\pi d)$, so wire 2 feels a force per length $F/L = \mu_0 I_1 I_2 / (2\pi d)$. Currents in the same direction attract; opposite currents repel. The two forces are equal and opposite. To find the direction you need the right-hand rule twice: once for the field of one wire at the location of the other, and once for the force on a current in that field. The force direction is not the field direction.

12.4 Ampere's Law

Ampere's law (steady currents)

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$, where $I_{\text{enc}} = \int \vec{J} \cdot d\vec{A}$ over any surface bounded by the loop.

Use it when symmetry makes B constant in magnitude and tangent (or perpendicular) along the loop, so $\oint \vec{B} \cdot d\vec{\ell} = B \ell_{\text{loop}}$.

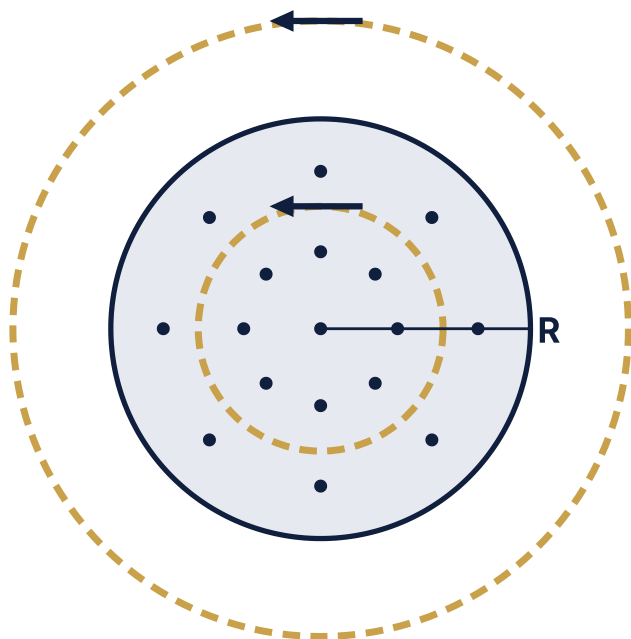
Otherwise use Biot-Savart.

Results: long straight wire $B = \frac{\mu_0 I}{2\pi r}$; ideal solenoid $B = \mu_0 n I$ inside (zero outside); toroid with N turns $B = \frac{\mu_0 N I}{2\pi r}$.

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**Amperian loops:**

$r < R$ encloses
part of the current

$r > R$ encloses
all of it

dots: I out of page
 B counterclockwise

Cylindrical conductor, current out of the page. Concentric circles are Amperian loops on which B is constant and tangential.

For a long cylindrical wire of radius R with **uniform** current density, $I_{\text{enc}} = I r^2 / R^2$ for $r < R$, so $B = \mu_0 I r / (2\pi R^2)$ rises linearly inside and falls as $1/r$ outside. If the density varies with r , compute the enclosed current with an integral over thin rings of area $dA = 2\pi r dr$: $I_{\text{enc}} = \int_0^r J(r') 2\pi r' dr'$. For the loop, the circumference is $2\pi r$ using the loop's own radius, and the enclosed current stops growing once r reaches R .

Worked Examples**EXAMPLE 1: A PROTON IN A MAGNETIC FIELD**

A proton ($m = 1.67 \times 10^{-27}$ kg, $q = 1.60 \times 10^{-19}$ C) is accelerated from rest through 2.0 kV and enters a uniform field $B = 0.050$ T directed into the page, moving to the right. Find its speed, the radius and period of its orbit, and the direction of the force when it enters.

SOLUTION

Energy: $qV = \frac{1}{2}mv^2$, so $v = \sqrt{2qV/m} = \sqrt{2(1.60 \times 10^{-19})(2000)/(1.67 \times 10^{-27})} = 6.2 \times 10^5$ m/s.

Radius: $r = mv/(qB) = (1.67 \times 10^{-27})(6.19 \times 10^5)/[(1.60 \times 10^{-19})(0.050)] = 0.13$ m.

Period: $T = 2\pi m/(qB) = 1.3 \times 10^{-6}$ s, the same for any entry speed.

Direction: $\hat{x} \times (-\hat{z}) = +\hat{y}$, so the force on the positive charge points up the page, and the proton circulates counterclockwise, as in the diagram. The force is $qvB = 5.0 \times 10^{-15}$ N, perpendicular to the velocity, so the work done by the magnetic force is zero and the speed stays 6.2×10^5 m/s.

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EXAMPLE 2: FIELD OF A FINITE STRAIGHT WIRE

A straight wire carries $I = 5.0$ A along the x axis from $x = -0.20$ m to $x = +0.20$ m. Use the Biot-Savart law to find B at the point P on the y axis, 0.10 m from the origin, and compare it with the field of an infinite wire.

SOLUTION

An element dx at position x is at distance $r = \sqrt{x^2 + d^2}$ from P , and $|d\vec{\ell} \times \hat{r}| = dx \sin \theta = dx d/r$. Every element's field points out of the page at P , so the magnitudes add.

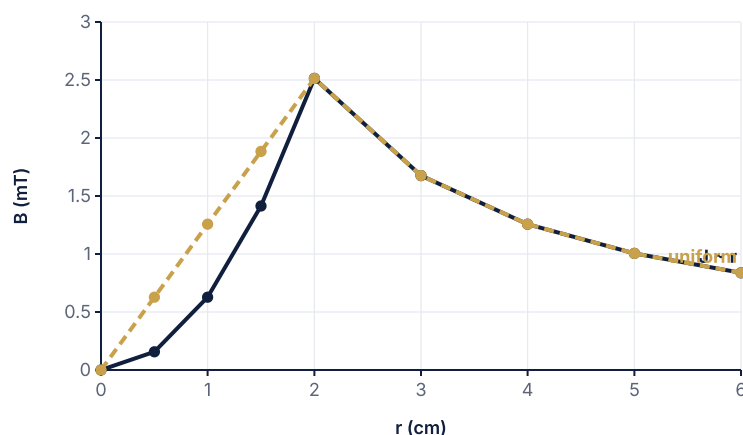
$$B = \frac{\mu_0 I}{4\pi} \int_{-L}^L \frac{d dx}{(x^2 + d^2)^{3/2}} = \frac{\mu_0 I}{4\pi} \left[\frac{x}{d\sqrt{x^2 + d^2}} \right]_{-L}^L = \frac{\mu_0 I}{4\pi d} \frac{2L}{\sqrt{L^2 + d^2}} \quad (\text{check by differentiating the bracket}).$$

$$\text{Numerically: } B = \frac{(1.0 \times 10^{-7})(5.0)}{0.10} \cdot \frac{0.40}{\sqrt{0.20^2 + 0.10^2}} = 8.9 \times 10^{-6} \text{ T, directed out of the page.}$$

Infinite wire: $\mu_0 I / (2\pi d) = 1.0 \times 10^{-5}$ T. The finite wire gives 89% of that; as $L \rightarrow \infty$ the factor $L/\sqrt{L^2 + d^2} \rightarrow 1$.

EXAMPLE 3: AMPERE'S LAW WITH NONUNIFORM CURRENT DENSITY

A long cylindrical conductor of radius $R = 2.0$ cm carries a current along its axis with density $J(r) = J_0 r/R$, where $J_0 = 3.0 \times 10^5$ A/m². Find (a) the total current, (b) B at $r = R/2$, and (c) B at $r = 2R$. (d) Compare with a uniform distribution carrying the same total current.



Same total current (251 A): both curves agree outside the wire but differ inside.

SOLUTION

(a) $I_{\text{enc}}(r) = \int_0^r J_0 \frac{r'}{R} 2\pi r' dr' = \frac{2\pi J_0 r^3}{3R}$. At $r = R$: $I = 2\pi J_0 R^2/3 = 2\pi(3.0 \times 10^5)(0.020)^2/3 = 251$ A.

(b) Loop of radius r : $B(2\pi r) = \mu_0 I_{\text{enc}}$, so $B = \mu_0 J_0 r^2 / (3R)$ for $r \leq R$. At $r = R/2$: $B = (4\pi \times 10^{-7})(3.0 \times 10^5)(0.010)^2 / (3 \times 0.020) = 6.3 \times 10^{-4}$ T.

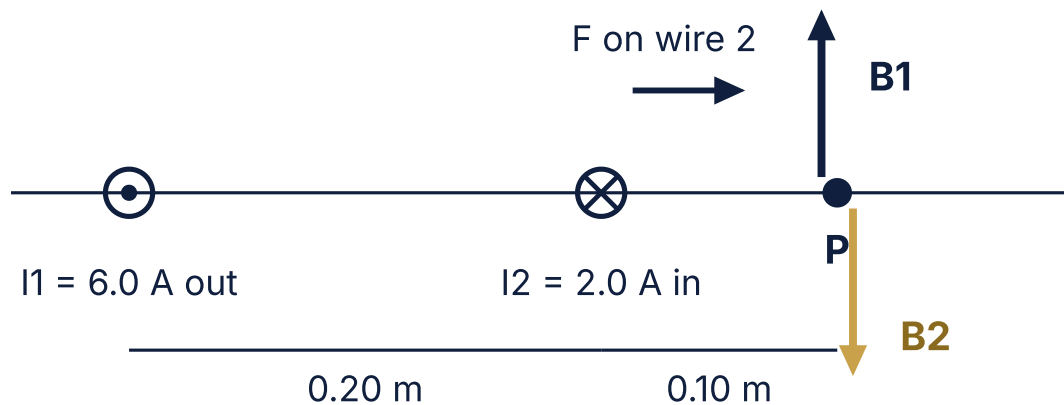
(c) Outside, all 251 A is enclosed: $B = \mu_0 I / (2\pi r) = (2 \times 10^{-7})(251)/0.040 = 1.3 \times 10^{-3}$ T. At the surface the field is 2.5×10^{-3} T, so the graph peaks at $r = R$ and falls as $1/r$.

(d) A uniform distribution has $B(R/2) = \mu_0 I(R/2) / (2\pi R^2) = 1.3 \times 10^{-3}$ T, twice as large. The nonuniform wire has its current concentrated near the surface, so a smaller fraction of it is inside the loop at $R/2$.

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Cross-section of two long parallel wires with opposite currents. At P (0.30 m from wire 1 and 0.10 m from wire 2) the two fields have equal magnitude and opposite direction.

EXAMPLE 4: TWO PARALLEL WIRES

Wire 1 carries 6.0 A out of the page at $x = 0$; wire 2 carries 2.0 A into the page at $x = 0.20$ m. (a) Find the force per unit length on wire 2, with direction. (b) Find the point on the x axis where the net field is zero. (c) Find the net field at $x = 0.10$ m.

SOLUTION

(a) $F/L = \mu_0 I_1 I_2 / (2\pi d) = (2 \times 10^{-7})(6.0)(2.0)/0.20 = 1.2 \times 10^{-5}$ N/m. Wire 1 produces a field at wire 2 that points $+y$ (counterclockwise around an out-of-page current). Then $\vec{F} = I\vec{L} \times \vec{B}$ with $\vec{L}$ into the page gives $(-\hat{z}) \times \hat{y} = +\hat{x}$: away from wire 1. Opposite currents repel.

(b) Between the wires both fields point $+y$ and add, so a zero can only lie outside, nearer the smaller current, at $x > 0.20$ m. There the fields point in opposite directions: $\frac{\mu_0(6.0)}{2\pi x} = \frac{\mu_0(2.0)}{2\pi(x - 0.20)}$, so $6.0(x - 0.20) = 2.0x$ and $x = 0.30$ m, which is the point P in the figure.

(c) At $x = 0.10$ m: $B_1 = (2 \times 10^{-7})(6.0)/0.10 = 1.2 \times 10^{-5}$ T and $B_2 = (2 \times 10^{-7})(2.0)/0.10 = 4.0 \times 10^{-6}$ T, both in $+y$, so $B_{\text{net}} = 1.6 \times 10^{-5}$ T in the $+y$ direction (a vector sum, not a sum of currents).

Write It Like a 5

Model Response

Prompt: A long solid cylindrical wire of radius R carries a total current I with uniform current density. Derive B for $r < R$ and $r > R$ and sketch B versus r . By symmetry the field lines are circles around the axis, with constant magnitude on a circle of radius r . Apply Ampere's law on such a loop: $\oint \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 I_{\text{enc}}$. For $r > R$ the loop encloses all the current: $B = \mu_0 I / (2\pi r)$. For $r < R$ the uniform density is $J = I / (\pi R^2)$, so $I_{\text{enc}} = J\pi r^2 = Ir^2 / R^2$ and $B = \mu_0 I r / (2\pi R^2)$. Graph: a straight line from the origin to the maximum $\mu_0 I / (2\pi R)$ at $r = R$ (the two expressions agree there), then a $1/r$ curve decreasing for $r > R$.

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What the grader looks for

(1) A symmetry statement that justifies a loop of constant B . (2) Ampere's law written with the left side evaluated as $B(2\pi r)$. (3) The correct enclosed current, with the area ratio r^2/R^2 shown for the interior. (4) Both results. (5) A graph that is linear inside, continuous at R , and $1/r$ outside, with labeled axes. A graph that rises as r^2 , or one that drops to zero at $r = R$, loses the graph points.

Coach's Playbook

- 1 **Decide the tool.** Symmetric current with a known pattern means Ampere's law; a finite or curved shape means Biot-Savart; a moving charge means $q\vec{v} \times \vec{B}$.
- 2 **Draw the vectors.** Sketch $\vec{B}$ from each source at the point of interest before computing anything, then add as vectors.
- 3 **Apply the right-hand rule twice.** Once for a field from a current, once for the force on a current or charge in that field.
- 4 **State enclosed current carefully.** If the loop radius is less than R , integrate J over the loop's area only.
- 5 **Check limits and units.** Fields should be continuous at a boundary and $\mu_0 I/r$ must come out in tesla.

Common traps

(1) Taking the force direction to be the field direction. (2) Using the full current or the wrong circumference in Ampere's law ($2\pi R$ instead of $2\pi r$). (3) Adding currents instead of adding field vectors from each wire. (4) Saying the magnetic force speeds a particle up, or does work; it only turns it.

Quick Check

Q1: What is the force on a charge moving parallel to a magnetic field? A: zero. Q2: Two parallel wires carry currents in the same direction; attract or repel? A: attract. Q3: Inside an ideal solenoid, how does B change if the current doubles and turns per meter is halved? A: unchanged.

Challenge Corner

A circular loop of radius $R = 5.0$ cm carries $I = 3.0$ A. Using Biot-Savart, derive the on-axis field and find where it has fallen to half its center value. Solution: each element has $d\vec{\ell} \perp \hat{r}$ and $r^2 = R^2 + z^2$. Components perpendicular to the axis cancel in pairs; the axial component carries a factor R/r . $B_z = \frac{\mu_0 I}{4\pi} \frac{R}{r^3} \cdot 2\pi R = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}}$. At the center this is $\mu_0 I/(2R) = 3.8 \times 10^{-5}$ T. Half the center value requires $(1 + z^2/R^2)^{3/2} = 2$, so $z = R\sqrt{2^{2/3} - 1} = 0.77R = 3.8$ cm.

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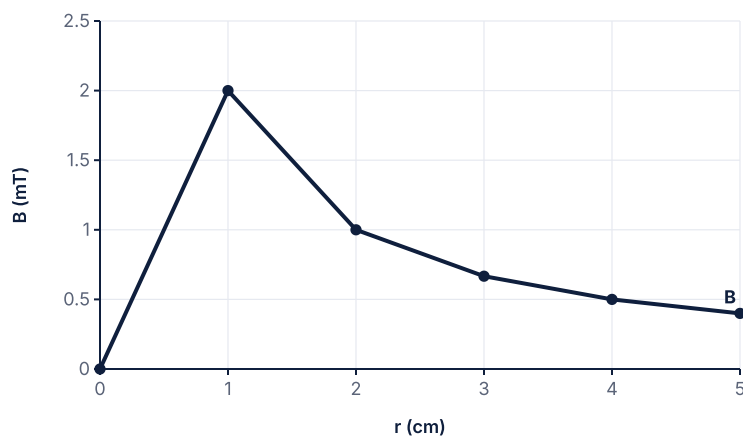
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Practice

PRACTICE Unit 12 Practice

Show your work. A calculator is allowed. Justify conclusions.

- A proton moves in the $+x$ direction through a uniform magnetic field pointing into the page ($-z$ direction). The initial magnetic force on the proton is directed
 - toward $-y$ (down the page)
 - out of the page
 - along $+x$
 - toward $+y$ (up the page)
- Two particles carry the same charge and have masses m and $4m$. They are accelerated through the same potential difference and then enter the same uniform magnetic field perpendicular to their velocities. What is the ratio of the radius of the heavier particle's orbit to the lighter one's?
 - 1
 - 2
 - 4
 - 16
- Two long parallel wires carry currents I and $3I$ in the same direction. The force per unit length on the wire carrying I has magnitude f . The force per unit length on the other wire has magnitude and direction
 - f , toward the first wire
 - $3f$, toward the first wire
 - f , away from the first wire
 - $f/3$, toward the first wire
- A long straight cylinder carries uniform current density. The magnetic field magnitude at the surface ($r = R$) is B_0 . What is the field magnitude at $r = R/2$ inside?
 - $B_0/4$
 - B_0
 - $B_0/2$
 - $2B_0$
- The magnetic field of a long wire carrying current along its axis is measured at several distances r from the axis (shown). The field is 2.0 mT at $r = 1.0$ cm, the wire's surface. What is B at $r = 4.0$ cm?



Field versus distance from the axis.

- 8.0 mT
 - 1.0 mT
 - 0.125 mT
 - 0.50 mT
- A velocity selector has a uniform electric field of 4.0×10^4 V/m perpendicular to a uniform magnetic field of 0.20 T. Charged particles pass through undeflected only if their speed is
 - 8.0×10^3 m/s
 - 2.0×10^5 m/s
 - 5.0×10^{-6} m/s
 - 4.0×10^4 m/s
 - A long cylindrical wire of radius R carries a current along its axis with density $J(r) = J_0(1 - r/R)$ for $r \leq R$.
 - Derive the total current I in terms of J_0 and R .
 - Use Ampere's law to derive $B(r)$ for $r \leq R$ and for $r \geq R$.
 - Show that B inside the wire is greatest at $r = 3R/4$, and find that maximum in terms of μ_0 , J_0 , R .
 - For $J_0 = 2.0 \times 10^5$ A/m² and $R = 1.0$ cm evaluate I , $B(R)$ and the maximum field.

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8. A student measures the magnetic field near a long straight wire carrying a steady 10.0 A with a field probe held perpendicular to the wire. The probe also records a constant background field in the same direction as the wire's field.
 r (cm): 2.0, 3.0, 4.0, 5.0, 6.0
 B (microtesla): 120, 86.7, 70.0, 60.0, 53.3
 (a) Describe a procedure that minimizes the uncertainty in r and accounts for the background field.
 (b) State what to plot to obtain a straight line, and what the slope and intercept represent.
 (c) Use the data to find the slope, then the value of μ_0 implied, and the background field.
 (d) Explain how reversing the current direction could be used to remove the background field.
9. Two long straight wires pass through the page at $x = -d$ and $x = +d$ (separation $2d = 0.10$ m). The left wire carries $I = 10$ A out of the page and the right wire carries 10 A into the page. A proton is at the origin.
 (a) Determine the direction of the net magnetic field at the origin and justify it using the field of each wire separately. Calculate its magnitude.
 (b) The proton moves with speed 3.0×10^5 m/s in the $+y$ direction. State the magnetic force and justify.
 (c) The proton instead moves in the $+x$ direction at the same speed. Give the direction and magnitude of the force.
 (d) Find the force per unit length between the two wires and state whether it is attractive or repulsive.
10. A circular loop of radius $R = 5.0$ cm carries current $I = 3.0$ A. The z axis passes through the center of the loop perpendicular to its plane.
 (a) Use the Biot-Savart law to derive $B_z(z)$ on the axis.
 (b) Evaluate B at the center and at $z = 12$ cm.
 (c) Show that for $z \gg R$ the field approaches $\mu_0 I R^2 / (2z^3)$, and compare this approximation with the exact value at $z = 30$ cm.

Answers Unit 12: Answers and Explanations

Read every explanation, including the ones for problems you answered correctly.

12 Magnetic Fields and Electromagnetism

PRACTICE · UNIT 12 PRACTICE

1. $\vec{F} = q\vec{v} \times \vec{B} = q\hat{x} \times (-\hat{z}) = +\hat{y}$ for a positive charge. Choice A is the answer for an electron or for $\vec{B}$ out of the page. The force is never along $\vec{v}$ or out of the page here, since it is perpendicular to both $\vec{v}$ and $\vec{B}$.
2. B. After acceleration $qV = \frac{1}{2}mv^2$, so $v \propto 1/\sqrt{m}$ and $r = mv/(qB) \propto \sqrt{m}$; the ratio is $\sqrt{4} = 2$. Choice C assumes equal speeds, which would give $r \propto m$ but ignores that the heavier particle is slower.
3. A. By Newton's third law the forces are equal and opposite. Parallel currents attract, so each force points toward the other wire. Choice B treats the force as proportional to the receiving wire's current only, forgetting that the field it sits in is proportional to the other wire's current.
4. C. Inside, $I_{\text{enc}} \propto r^2$ and the loop length is $\propto r$, so $B \propto r$ and $B(R/2) = B_0/2$. Choice A uses r^2 , forgetting that the loop circumference also grows with r .
5. D. Outside the wire $B \propto 1/r$, so $B(4.0) = 2.0 \text{ mT} \times (1.0/4.0) = 0.50 \text{ mT}$. Choice C uses $1/r^2$ as for a point charge; choice A extends the interior linear rise beyond the surface.
6. B. Undelected motion requires $qE = qvB$, so $v = E/B = 4.0 \times 10^4 / 0.20 = 2.0 \times 10^5$ m/s, independent of q and m . Choice A multiplies E and B ; choice C inverts the ratio.
7. (a) $I = \int_0^R J_0(1 - r/R) 2\pi r dr = 2\pi J_0 \left(\frac{R^2}{2} - \frac{R^2}{3} \right) = \frac{\pi J_0 R^2}{3}$.
 (b) $I_{\text{enc}}(r) = 2\pi J_0 \left(\frac{r^2}{2} - \frac{r^3}{3R} \right)$ and $B(2\pi r) = \mu_0 I_{\text{enc}}$ give $B = \mu_0 J_0 \left(\frac{r}{2} - \frac{r^2}{3R} \right)$ for $r \leq R$. For $r \geq R$: $B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 J_0 R^2}{6r}$. At $r = R$ both give $\mu_0 J_0 R/6$.
 (c) $dB/dr = \mu_0 J_0 \left(\frac{1}{2} - \frac{2r}{3R} \right) = 0$ at $r = 3R/4$, and the second derivative is negative, so this is a maximum. $B_{\text{max}} = \mu_0 J_0 \left(\frac{3R}{8} - \frac{3R}{16} \right) = \frac{3\mu_0 J_0 R}{16}$.
 (d) $I = \pi(2.0 \times 10^5)(0.010)^2/3 = 21$ A. $B(R) = \mu_0 J_0 R/6 = 4.2 \times 10^{-4}$ T. $B_{\text{max}} = 3\mu_0 J_0 R/16 = 4.7 \times 10^{-4}$ T, which exceeds the field at the surface because the current density is largest near the axis.
8. (a) Mount the probe on a ruler or rail with its sensing axis perpendicular to the wire; measure r from the wire's center line to the probe's sensor (not to its housing); keep other wires and magnets far away; take several readings at each r ; and measure the background with the current off.
 (b) Plot B versus $1/r$. From $B = \mu_0 I/(2\pi r) + B_{\text{bg}}$, the slope is $\mu_0 I/2\pi$ and the intercept is B_{bg} .
 (c) $1/r$ values: 50, 33.3, 25, 20, 16.7 m^{-1} . Slope from the end points is $(120 - 53.3) \mu\text{T}/(50 - 16.7) \text{m}^{-1} = 2.0 \mu\text{T m}$. Then $\mu_0 = 2\pi(\text{slope})/I = 2\pi(2.0 \times 10^{-6})/10.0 = 1.26 \times 10^{-6}$ T m/A. The intercept is $120 - 2.0(50) = 20 \mu\text{T}$ of background field.
 (d) Reversing the current flips the wire's contribution but not the background: $B_+ = B_w + B_{\text{bg}}$ and $B_- = -B_w + B_{\text{bg}}$. Half the difference, $(B_+ - B_-)/2 = B_w$, is the wire's field alone.

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9. (a) At the origin, the left wire (out of the page) produces a field circling counterclockwise, which at a point to its right points $+y$. The right wire (into the page) circulates clockwise; at a point to its left the field also points $+y$. The fields add: $B = 2 \times \mu_0 I / (2\pi d) = 2(2 \times 10^{-7})(10)/0.050 = 8.0 \times 10^{-5}$ T in the $+y$ direction.
- (b) The velocity is parallel to $\vec{B}$, so $\vec{v} \times \vec{B} = 0$ and the magnetic force is zero.
- (c) $\vec{v} = +x$ and $\vec{B} = +y$: $\hat{x} \times \hat{y} = +\hat{z}$, so the force on the positive charge is out of the page, with magnitude $qvB = (1.6 \times 10^{-19})(3.0 \times 10^5)(8.0 \times 10^{-5}) = 3.8 \times 10^{-18}$ N.
- (d) $F/L = \mu_0 I^2 / (2\pi(2d)) = (2 \times 10^{-7})(100)/0.10 = 2.0 \times 10^{-4}$ N/m. The currents are opposite, so the wires repel.
10. (a) Each element has $d\vec{\ell} \perp \hat{r}$, with $r^2 = R^2 + z^2$, so $dB = \frac{\mu_0 I d\ell}{4\pi r^2}$. Components perpendicular to the axis cancel in pairs; the axial component is $dB \cos \alpha$ with $\cos \alpha = R/r$. Integrating $d\ell$ around the loop gives $2\pi R$: $B_z = \frac{\mu_0 I R}{4\pi r^3} (2\pi R) = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}}$.
- (b) Center: $\mu_0 I / (2R) = (4\pi \times 10^{-7})(3.0)/(0.10) = 3.8 \times 10^{-5}$ T. At $z = 0.12$ m: $\frac{(4\pi \times 10^{-7})(3.0)(0.050)^2}{2(0.0025 + 0.0144)^{3/2}} = 2.1 \times 10^{-6}$ T.
- (c) For $z \gg R$, $(R^2 + z^2)^{3/2} \rightarrow z^3$, so $B \rightarrow \mu_0 I R^2 / (2z^3)$. At $z = 0.30$ m the approximation gives 1.75×10^{-7} T and the exact expression gives 1.68×10^{-7} T, a difference of about 4%.

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Unit 13: Electromagnetic Induction

Exam weighting: about 10-20% of the multiple-choice section. Concept review, worked examples, exam-style writing, and 10 practice items, all with full explanations.

13 Electromagnetic Induction

Flux, Faraday's and Lenz's laws, motional emf, inductors, LR and LC circuits, and the differential equations behind each. Practice builds directions, derivations and exponentials.

Why This Matters

Induction ties together fields, flux, circuits and calculus, and carries 10 to 20 percent of Section I. Recent free-response items have used rotating loops and time-varying fields. Students lose points by confusing emf with current, by guessing Lenz-law directions without reasoning about flux, and by quoting RL results without the differential equation.

Core Ideas

Magnetic Flux

Flux counts how much field passes through a surface, using the field component along the surface normal.

Magnetic flux

$\Phi_B = \int \vec{B} \cdot d\vec{A}$, which is $BA \cos \theta$ when B is uniform; θ is the angle between $\vec{B}$ and the area normal.

Unit: weber, $1 \text{ Wb} = 1 \text{ T m}^2$. Flux is a scalar with a sign set by the chosen normal.

Flux changes when B changes, the area changes, or the angle changes.

When B varies over the loop, integrate. A long wire carrying current I makes a field $\mu_0 I / 2\pi r$ that circles the wire. For a rectangle of length ℓ parallel to the wire, with near edge at distance a and far edge at $a + b$, take strips $dA = \ell dr$: $\Phi = \frac{\mu_0 I \ell}{2\pi} \ln \frac{a+b}{a}$. Flux is zero when $\vec{B}$ lies in the loop's plane.

Electromagnetic Induction

A changing flux through a loop produces an emf around it. The sign comes from Lenz's law, which is energy conservation in disguise: the induced current opposes the change in flux, not the flux itself.

Faraday's law and Lenz's law

$\mathcal{E} = -N \frac{d\Phi_B}{dt}$; the magnitude is the rate of flux change, the minus sign records opposition.

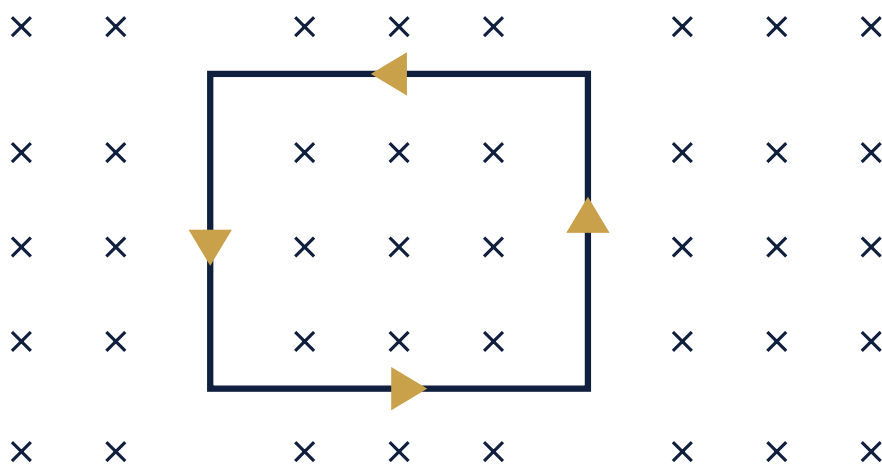
Direction: (1) flux direction; (2) growing or shrinking; (3) growing flux gives an opposite induced field, shrinking gives a parallel one; (4) right-hand rule for the current.

Charge passed: $q = N \Delta\Phi / R$, independent of how fast the change happens.

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B into page and increasing; induced current is counterclockwise

Loop in an increasing field. The induced current (gold arrows) makes a field out of the page inside the loop, opposing the increase.

Motional emf. A rod of length ℓ moving at speed v perpendicular to B sweeps area at rate ℓv , so Φ changes at $B\ell v$ and $\mathcal{E} = B\ell v$. Equivalently, $q\vec{v} \times \vec{B}$ pushes charges along the rod. An open circuit has an emf but no current.

Rotating loop. With $\theta = \omega t$, $\Phi = BA \cos \omega t$ and $\mathcal{E} = NBA\omega \sin \omega t$. The emf is zero when the flux is largest and largest when the flux is zero, crossing zero twice per revolution. Power $\mathcal{E}^2/R$ varies as $\sin^2 \omega t$, so it never goes negative and repeats at twice the rotation frequency.

Induced electric fields. A changing flux creates an electric field with $\oint \vec{E} \cdot d\vec{\ell} = -d\Phi_B/dt$. It is non-conservative, so it has no potential in the electrostatic sense.

Induced Currents and Magnetic Forces

An induced current in a field feels a magnetic force, and that force always opposes the motion that created it. For the rod on rails, $I = B\ell v/R$ and the retarding force is $F = I\ell B = B^2\ell^2 v/R$. Mechanical power Fv equals thermal power $I^2 R$. Newton's second law then becomes a differential equation: for a rod with no applied force, $m dv/dt = -B^2\ell^2 v/R$, so $v = v_0 e^{-t/\tau}$ with $\tau = mR/(B^2\ell^2)$. A falling rod on vertical rails approaches terminal speed $v_t = mgR/(B^2\ell^2)$.

Inductance

A coil that carries a changing current produces a changing flux through itself and so induces an emf in itself. Self-inductance L measures how much flux the coil links per unit current.

Inductor relations

$L = N\Phi_B/I$ (unit henry, H). Voltage magnitude across an ideal inductor: $|V_L| = L|dI/dt|$, directed to oppose the change in current.

Ideal solenoid: $L = \mu_0 N^2 A/\ell$, with N total turns, area A , length ℓ .

Energy stored: $U = \frac{1}{2}LI^2$, from $P = LI dI/dt$ integrated from 0 to I . Field energy density: $B^2/2\mu_0$.

Inductor current cannot jump (that would need infinite voltage). Right after a switch changes, the inductor keeps its old current: it acts like an open circuit if that current was zero. With steady current, $V_L = 0$ and an ideal inductor acts like a wire.

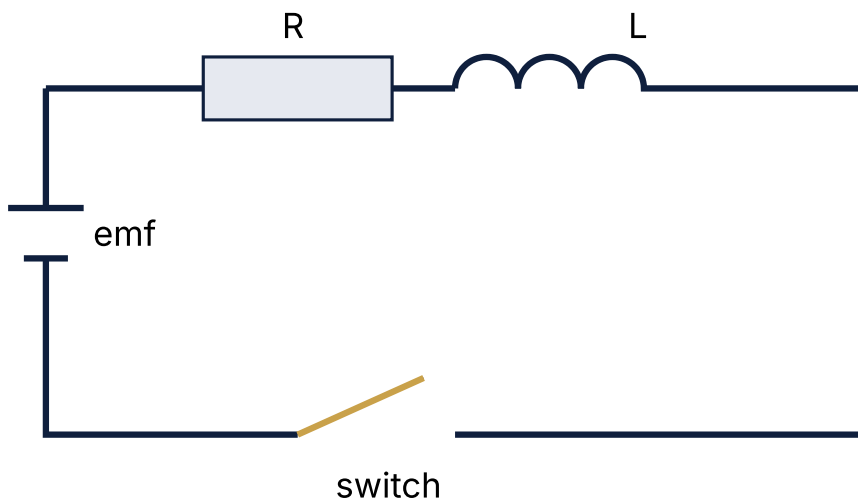
Circuits with Resistors and Inductors (LR Circuits)

Close the switch in the series circuit below at $t = 0$ with $I(0) = 0$. The loop rule gives $\mathcal{E} - IR - L dI/dt = 0$. Separate variables: $\frac{dI}{\mathcal{E}/R - I} = \frac{R}{L} dt$. Integrating from 0 and exponentiating gives the rise; removing the battery (loop rule $L dI/dt + IR = 0$) gives the decay.

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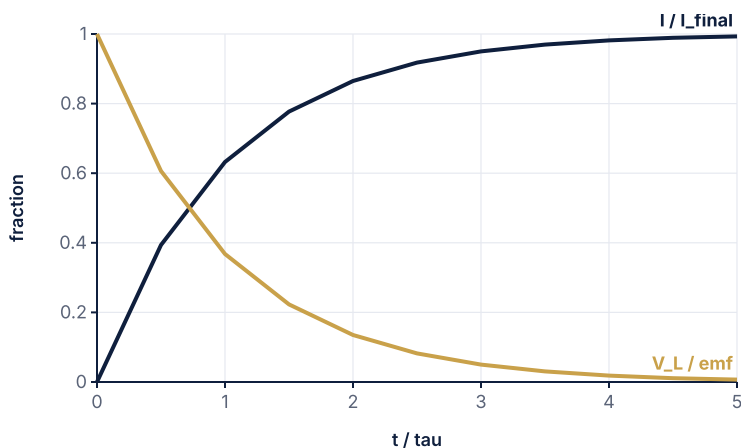
Series RL circuit: battery, resistor, inductor and switch. The loop rule gives $\text{emf} - IR - L \, dI/dt = 0$ after the switch closes.

LR solutions

Rise: $I = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau})$, $V_L = \mathcal{E}e^{-t/\tau}$. Decay from I_0 : $I = I_0 e^{-t/\tau}$.

Time constant $\tau = L/R$ (larger L slows the change, larger R speeds it up). At $t = \tau$ the current has covered 63% of the way to its final value; at 5τ , over 99%.

Check: $V_R + V_L = \mathcal{E}$ always; initial slope is $\mathcal{E}/L$.



Current in a series RL circuit after the switch closes: current rises, inductor voltage falls. Both use the same time constant.

Circuits with Capacitors and Inductors (LC Circuits)

Charged capacitor plus inductor: $\frac{q}{C} + L \frac{d^2q}{dt^2} = 0$, the same equation as a mass on a spring with $q \leftrightarrow x$, $L \leftrightarrow m$, $1/C \leftrightarrow k$. Hence $q = Q \cos(\omega t + \phi)$ with $\omega = 1/\sqrt{LC}$, current amplitude ωQ , and total energy $Q^2/2C$ that moves back and forth between electric and magnetic form. Current is largest when the charge is zero.

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Worked Examples

CHANGING FIELD THROUGH A COIL

A flat coil has $N = 50$ turns, each of area $4.0 \times 10^{-3} \text{ m}^2$, with its plane perpendicular to a field into the page. The field is $B = 0.20 + 0.30t^2$ in tesla (t in s). The coil resistance is 6.0Ω . Find (a) the emf at $t = 2.0$ s, (b) the current and its direction, (c) the charge that flows from $t = 0$ to 2.0 s.

SOLUTION

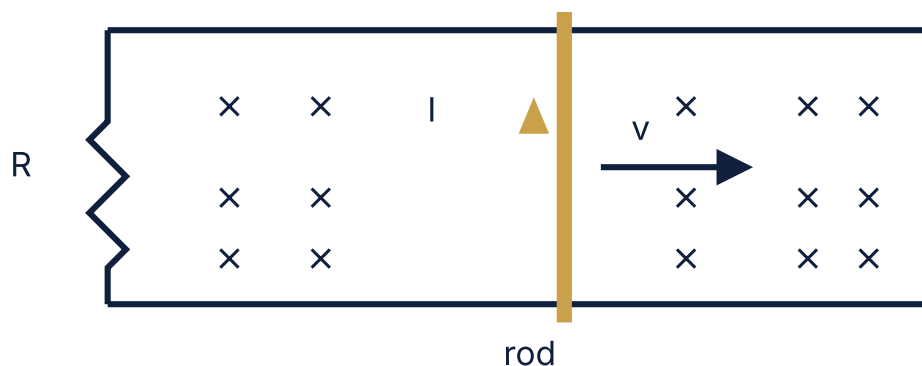
(a) $\Phi = NAB$, so $\mathcal{E} = NAdB/dt = (50)(4.0 \times 10^{-3})(0.60t) = 0.12t$ V. At 2.0 s, $\mathcal{E} = 0.24$ V.

(b) $I = \mathcal{E}/R = 0.24/6.0 = 0.040$ A. Flux into the page is increasing, so the induced field points out of the page; the right-hand rule gives a counterclockwise current.

(c) $q = N\Delta\Phi/R = (50)(4.0 \times 10^{-3})(0.30 \cdot 4.0)/6.0 = 0.040$ C. Check: $\int_0^2 (0.12t/6) dt = 0.010t^2 \Big|_0^2 = 0.040$ C.

ROD ON RAILS

In the figure, $B = 0.25$ T, the rod has length $\ell = 0.40$ m, speed 3.0 m/s, and the circuit has total resistance 2.0Ω . Find the emf, the current and its direction, the force an agent must apply, and the power it delivers.



B into page; flux grows, so I is counterclockwise

Rod sliding on rails. The magnetic force on the upward current points left, opposing the motion.

SOLUTION

$\mathcal{E} = B\ell v = (0.25)(0.40)(3.0) = 0.30$ V; $I = 0.30/2.0 = 0.15$ A.

The enclosed flux into the page grows, so the current is counterclockwise: upward in the rod.

The force on the rod is $I\ell B = (0.15)(0.40)(0.25) = 0.015$ N, directed left, so the agent pushes right with 0.015 N at constant speed.

Power = $Fv = 0.045$ W, matching $I^2R = (0.15)^2(2.0) = 0.045$ W. All of it becomes heat.

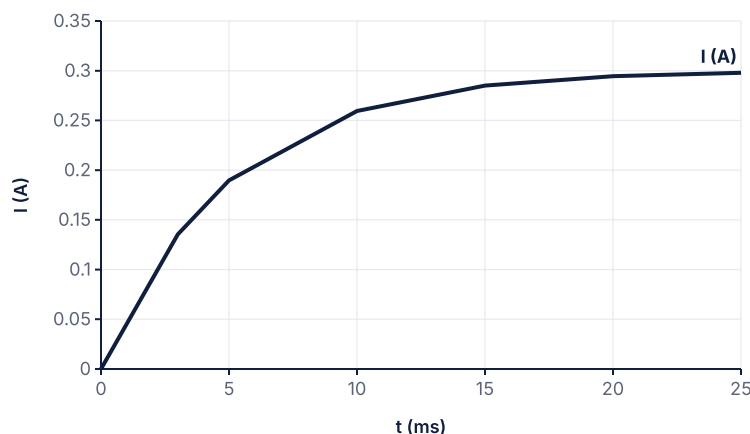
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SWITCHING ON AN LR CIRCUIT

A 12 V battery of negligible resistance is connected at $t = 0$ to a $40\ \Omega$ resistor in series with a $0.20\ \text{H}$ inductor. Find (a) the time constant and final current, (b) the current at $3.0\ \text{ms}$, (c) the time when the inductor voltage equals the resistor voltage and the energy then stored.



$I(t)$ for Example 3; $I = 0.19\ \text{A}$ at one time constant.

SOLUTION

(a) $\tau = L/R = 0.20/40 = 5.0\ \text{ms}$; $I_f = 12/40 = 0.30\ \text{A}$.

(b) $I = 0.30(1 - e^{-3.0/5.0}) = 0.30(1 - 0.549) = 0.135\ \text{A}$.

(c) $V_L = V_R$ means each is $6.0\ \text{V}$, so $I = 0.15\ \text{A}$, half of final. Then $e^{-t/\tau} = 1/2$ and $t = \tau \ln 2 = 3.5\ \text{ms}$. Energy: $U = \frac{1}{2}(0.20)(0.15)^2 = 2.3 \times 10^{-3}\ \text{J}$.

ROTATING COIL GENERATOR

A coil of 200 turns and area $0.025\ \text{m}^2$ rotates at 30 revolutions per second in a uniform $0.080\ \text{T}$ field. It is connected to a $15\ \Omega$ resistor. Find the peak emf, peak current, average power, and the emf at $t = 5.0\ \text{ms}$ if the plane is perpendicular to B at $t = 0$.

SOLUTION

$\omega = 2\pi(30) = 188.5\ \text{rad/s}$. $\Phi = NBA \cos \omega t$, so $\mathcal{E} = NBA\omega \sin \omega t$ and $\mathcal{E}_{\max} = (200)(0.080)(0.025)(188.5) = 75.4\ \text{V}$.

$I_{\max} = 75.4/15 = 5.03\ \text{A}$. Power $P = \mathcal{E}^2/R$ averages half its peak: $\bar{P} = \frac{1}{2}I_{\max}^2 R = \frac{1}{2}(5.03)^2(15) = 190\ \text{W}$.

At $5.0\ \text{ms}$, $\omega t = 0.942\ \text{rad}$, so $\mathcal{E} = 75.4 \sin 0.942 = 61.0\ \text{V}$.

Write It Like a 5**Model Response**

Prompt: A long straight wire carries current $I = (40\ \text{A/s})t$ upward. A rectangular loop of resistance $0.020\ \Omega$ lies in the plane of the wire, with two sides of length $\ell = 0.50\ \text{m}$ parallel to the wire at distances $a = 0.040\ \text{m}$ and $a + b = 0.100\ \text{m}$. Derive an expression for the emf and find its value and the current direction. Response: With $B = \mu_0 I / 2\pi r$ into the page, a strip of width dr has area ℓdr , so $\Phi = \int_a^{a+b} \frac{\mu_0 I}{2\pi r} \ell dr = \frac{\mu_0 I \ell}{2\pi} \ln \frac{a+b}{a}$. Faraday's law gives $|\mathcal{E}| = \frac{d\Phi}{dt} = \frac{\mu_0 \ell}{2\pi} \ln \frac{a+b}{a} \frac{dI}{dt} = (2 \times 10^{-7})(0.50) \ln(2.5)(40) = 3.7 \times 10^{-6}\ \text{V}$. The current is $I_{\text{loop}} = \mathcal{E}/R = 1.8 \times 10^{-4}\ \text{A}$. To the right of an upward current the wire's field points into the page; it is growing, so the loop's induced field must point out of the page, which requires a counterclockwise current.

What the grader looks for

Wire field stated; integral set up with limits before substituting; derivative with respect to time (only I varies); value with units; direction justified by growing flux and the opposing induced field; emf and current kept distinct.

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Coach's Playbook

- 1 Name what changes (B, area, angle) and write flux as a function of time before differentiating.
- 2 For direction, state flux direction, grow or shrink, induced field, then current.
- 3 For RL problems write the loop rule, then the equation, then the solution; check $t = 0$ and $t = \infty$.
- 4 Cross-check with energy: mechanical power in equals $I^2 R$ out.

Common traps

1) Lenz's law opposes the CHANGE in flux; the induced field is not always opposite to B. 2) Zero emf does not mean zero flux (rotating loop: emf is zero when flux is largest). 3) Inductor current is continuous but its voltage can jump. 4) The time constant is L/R , not LR or R/L .

Quick Check

Q1: In an RL circuit, which is continuous when the switch closes, current or inductor voltage? A: current. Q2: Doubling a rod's speed changes the retarding force by what factor? A: 2 (power rises by 4). Q3: Henry in circuit units? A: V s/A, or ohm s.

Challenge Corner

A rod of mass 0.020 kg and length 0.50 m slides without friction down vertical rails connected by a resistance of $0.10\ \Omega$ in a horizontal 0.40 T field (into the page). Find the terminal speed and the time to reach 90 percent of it, starting from rest. Solution: $m dv/dt = mg - B^2 \ell^2 v/R$. With $k = B^2 \ell^2 = 0.040$, $v_t = mgR/k = (0.020)(9.8)(0.10)/0.040 = 0.49$ m/s and $\tau = mR/k = 0.050$ s. The solution is $v = v_t(1 - e^{-t/\tau})$. For 90 percent, $e^{-t/\tau} = 0.10$, so $t = \tau \ln 10 = 0.115$ s.

Practice

PRACTICE Unit 13 Practice

Show your work. A calculator is allowed. Justify conclusions.

1. A flat loop of area $0.040\ \text{m}^2$ sits in a uniform 0.50 T field. The normal to the loop makes 60° with the field. What is the magnetic flux through the loop?
 (A) 1.0×10^{-2} Wb (B) 1.7×10^{-2} Wb (C) 2.0×10^{-2} Wb
 (D) zero
2. The north pole of a bar magnet is pushed straight down toward a horizontal loop. Viewed from above, the induced current in the loop is
 (A) clockwise, because the field is downward
 (B) clockwise, because the induced field should point down
 (C) counterclockwise, because the induced field points up and opposes the growing downward flux
 (D) zero, because the magnet does not touch the loop
3. A rod slides on rails in a uniform field at constant speed. If the rod's speed is doubled, the rate at which the agent does work on the rod
 (A) doubles (B) quadruples
 (C) stays the same (D) increases by a factor of 8
4. The current in an ideal 0.50 H inductor rises uniformly from 2.0 A to 6.0 A in 0.20 s. The magnitude of the voltage across it is
 (A) 40 V (B) 15 V (C) 0.40 V (D) 10 V

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5. A battery of 10 V is connected at $t = 0$ to a $20\ \Omega$ resistor in series with a $0.10\ \text{H}$ inductor. The current reaches half its final value at about
 (A) 2.5 ms (B) 5.0 ms (C) 3.5 ms (D) 10 ms
6. In an ideal LC circuit, $L = 0.10\ \text{H}$ and $C = 1.0\ \mu\text{F}$. The angular frequency of the oscillation is
 (A) $1.0 \times 10^7\ \text{rad/s}$ (B) $5.0 \times 10^2\ \text{rad/s}$ (C) $3.2 \times 10^{-4}\ \text{rad/s}$
 (D) $3.2 \times 10^3\ \text{rad/s}$
7. A coil is built to measure an unknown inductance. It is connected in series with a total resistance (coil plus resistor) of 100 ohms, a 6.0 V battery, and a switch. After closing the switch, a current sensor records:
 t (ms): 1, 2, 3, 4, 5, 6
 I (mA): 17.0, 29.2, 37.9, 44.2, 48.7, 51.9
 (a) Write the differential equation and the solution for $I(t)$. [2 points]
 (b) Explain how to linearize the data, and state what the slope represents. [2 points]
 (c) Using a pair of points from the linearized data, determine the inductance. [3 points]
 (d) State one source of systematic error and its effect on the result. [1 point]
8. A square coil of N turns, side s , rotates with angular speed ω about an axis in its plane, perpendicular to a uniform field B . At $t = 0$ the plane is perpendicular to B . The coil is connected to a resistance R .
 (a) Derive the emf and the power dissipated as functions of time.
 (b) Sketch the emf and the power versus time for two periods on the same time axis. Mark where each is zero.
 (c) By what factor does the maximum power change if ω is doubled? Justify with your derivation.
9. A rod of mass $m = 0.15\ \text{kg}$ and length $\ell = 0.30\ \text{m}$ slides on horizontal frictionless rails connected through a resistor $R = 0.60\ \Omega$ in a vertical field $B = 0.50\ \text{T}$. It is given an initial speed $v_0 = 4.0\ \text{m/s}$ and then released.
 (a) Derive the differential equation for $v(t)$ and solve it. [3 points]
 (b) Find the time constant and the distance traveled before stopping. [3 points]
 (c) Show that the thermal energy dissipated equals the initial kinetic energy. [2 points]
10. A 12 V ideal battery, a switch S, a 60 ohm resistor R2, and a branch containing a 20 ohm resistor R1 in series with a 0.40 H ideal inductor are arranged so that the two branches are in parallel across the battery through S. S has been closed for a long time.
 (a) Find the current in each branch and the energy stored in the inductor. [3 points]
 (b) S is now opened. Find the current through R2 and its direction relative to before, the initial voltage across R2, and the time constant of the decay. [4 points]
 (c) Explain why the voltage across R2 can exceed the battery emf even though no battery is attached. [2 points]

Answers**Unit 13: Answers and Explanations**

Read every explanation, including the ones for problems you answered correctly.

13 Electromagnetic Induction**PRACTICE · UNIT 13 PRACTICE**

1. A. $\Phi = BA \cos \theta = (0.50)(0.040) \cos 60^\circ = 1.0 \times 10^{-2}\ \text{Wb}$. Choice B uses the sine, which would be the flux if the angle were measured from the plane of the loop.
2. C. The downward flux grows, so the induced field points up, requiring counterclockwise current seen from above. Choice A confuses the field with the induced field; choice B says the induced field matches the original direction, but Lenz's law opposes the change.
3. B. $F = B^2 \ell^2 v / R$ doubles, and power Fv therefore quadruples (equivalently $I^2 R$ with I doubled). Choice A forgets that both force and speed double.
4. D. $|V| = L \Delta I / \Delta t = (0.50)(4.0) / (0.20) = 10\ \text{V}$. Choice A comes from dividing by L instead of multiplying, and choice B uses only the final current.
5. C. $\tau = L / R = 5.0\ \text{ms}$, and half of final current is reached at $\tau \ln 2 = 3.5\ \text{ms}$. Choice B reports the time constant itself, which is only the 63 percent point.
6. D. $\omega = 1 / \sqrt{LC} = 1 / \sqrt{(0.10)(1.0 \times 10^{-6})} = 3.2 \times 10^3\ \text{rad/s}$. Choice A forgets the square root (that is $1/LC$), and choice B is near the 503 Hz ordinary frequency, which is omega divided by 2 pi.
7. (a) 2 points: loop rule $\mathcal{E} - IR - L dI/dt = 0$ [1]; $I = (\mathcal{E}/R)(1 - e^{-t/\tau})$ with $\tau = L/R$ [1]. (b) 2 points: $I_f = \mathcal{E}/R = 60\ \text{mA}$; $\ln(I_f - I) = \ln I_f - t/\tau$, so plot $\ln(I_f - I)$ against t [1], a straight line with slope $-1/\tau = -R/L$ [1]. (c) 3 points: $\ln(60 - I)$ is 3.76, 3.43, 3.10, 2.76, 2.42, 2.09; between $t = 1$ and 6 ms the slope is $(2.09 - 3.76)/5.0 = -0.334\ \text{ms}^{-1}$ [1: slope uses $\ln$ of I_f minus I] [1-2 for valid computation from two points far apart]; $\tau = 2.99\ \text{ms}$; $L = R\tau = (100)(2.99 \times 10^{-3}) = 0.30\ \text{H}$ [1]. (d) 1 point: for example, an unmeasured battery resistance makes the true R larger than 100 ohms, so $L = R\tau$ is underestimated. Any reasonable source with its direction of effect.

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8. (a) 4 points: flux $\Phi = Ns^2B \cos \omega t$ [1]; $\mathcal{E} = -d\Phi/dt = N Bs^2 \omega \sin \omega t$ [1]; $P = \mathcal{E}^2/R = N^2 B^2 s^4 \omega^2 \sin^2 \omega t/R$ [2 with the square carried]. (b) 3 points: emf is a sine starting at zero with zeros at $t = 0, T/2, T, 3T/2, 2T$ [1]; power starts at zero, never negative, with period $T/2$ [1]; power has two maxima per period of the emf, at the times when the emf is extreme, and the same zeros as the emf [1]. (c) 2 points: $P_{max} \propto \omega^2$, so doubling ω gives a factor of 4 [2: the dependence from the derivation is shown].
9. (a) $m dv/dt = -IB\ell = -B^2 \ell^2 v/R$ [1]; separate: $dv/v = -(B^2 \ell^2/mR)dt$ [1]; $v = v_0 e^{-t/\tau}$ [1]. (b) $B^2 \ell^2 = (0.25)(0.090) = 0.0225$; $\tau = mR/B^2 \ell^2 = (0.15)(0.60)/0.0225 = 4.0$ s [1]; distance $= \int_0^\infty v dt = v_0 \tau = 16$ m [2]. (c) Heat $= \int I^2 R dt = \int_0^\infty (B\ell v_0/R)^2 R e^{-2t/\tau} dt = \frac{B^2 \ell^2 v_0^2}{R} \frac{\tau}{2} = \frac{1}{2} m v_0^2 = 1.2$ J [2].
10. (a) 3 points: at steady state $V_L = 0$, so $I_1 = 12/20 = 0.60$ A [1], $I_2 = 12/60 = 0.20$ A [1], $U = \frac{1}{2}(0.40)(0.60)^2 = 0.072$ J [1]. (b) 4 points: the inductor keeps 0.60 A, which now flows through R2 [1]; this is reversed relative to the 0.20 A before, since the loop of the two branches is now closed [1]; $V_2 = (0.60)(60) = 36$ V [1]; $\tau = L/(R_1 + R_2) = 0.40/80 = 5.0$ ms [1]. (c) 2 points: the inductor opposes the fall of current by producing an emf $L |dI/dt|$ [1]; right after opening, this emf drives 0.60 A through the total 80 ohms, so it is 48 V, which exceeds 12 V; the energy comes from the magnetic field (0.072 J, of which 0.054 J heats R2) [1].

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Workshop: Experimental Design, Graphs, and Statistics

Skills that earn points on every free-response section.

W Workshop: Calculus Skills, Symmetry, and Lab Analysis

The calculus, symmetry reasoning and graph skills that every E&M unit assumes: charge elements, integrals along axes, Gaussian and Amperian paths, separable circuit equations, linearizing lab data, and writing derivations that earn credit.

Why This Matters

Calculus is not a separate topic in this course; it is the language every unit is written in. The Mathematical Routines free-response question, usually worth 10 points, is a derivation. The lab question rewards linearization and slope analysis. Even the qualitative question is easier if you can see a $1/r$ or an exponential behind a sentence. Students rarely lose points because integrals are hard. They lose them because the setup is vague, a symmetry argument is missing, or a graph is not tied back to the physics.

Core Ideas

Building the Charge Element

Every continuous-charge problem starts with one sentence: what small piece of charge am I adding up? Match the element to the geometry and the density you are given. Be strict about which density you have, since mixing them is a classic error.

Geometry	Density	Element	Total charge
Line, rod, arc	λ (C/m)	$dq = \lambda dx$ or $\lambda R d\theta$	$\int \lambda dx$
Surface, disk, sheet	σ (C/m ²)	$dq = \sigma dA$, ring: $\sigma 2\pi r dr$	$\int \sigma dA$
Volume, sphere, cylinder	ρ (C/m ³)	$dq = \rho dV$, shell: $\rho 4\pi r^2 dr$	$\int \rho dV$

When the density varies, it must stay inside the integral. If a rod has $\lambda = Ax$ then $dq = Ax dx$ and the total charge is $AL^2/2$, not AL . A quick unit check catches most slips: λdx must come out in coulombs.

Setting up a charge integral

1. Sketch the object, choose an origin and an integration variable (x , r , or θ).
2. Write dq in terms of that variable, keeping any variable density inside.
3. Write the distance from dq to the field point in terms of the same variable.
4. Write the contribution (dE or dV) and decide which components survive.
5. Integrate with limits that cover the whole object, then substitute numbers last.

Integrating Along an Axis

Choose the variable that makes the distance simple. For a point on the line of a rod, the distance is a difference of coordinates and the integrand is a power of one variable. For a point off the rod, the distance is $\sqrt{x^2 + y^2}$ and the angle often makes the integral easier. Potential is a scalar, so $dV = k dq/r$ adds with no components; that is why potential integrals are usually shorter than field integrals for the same object. Write the integral with its limits before you substitute, because the exam rewards the visible step. Check the result against a limit: far from a finite rod of total charge Q the answer must look like a point charge, and in the limit of a very long rod it must approach the infinite-line result.

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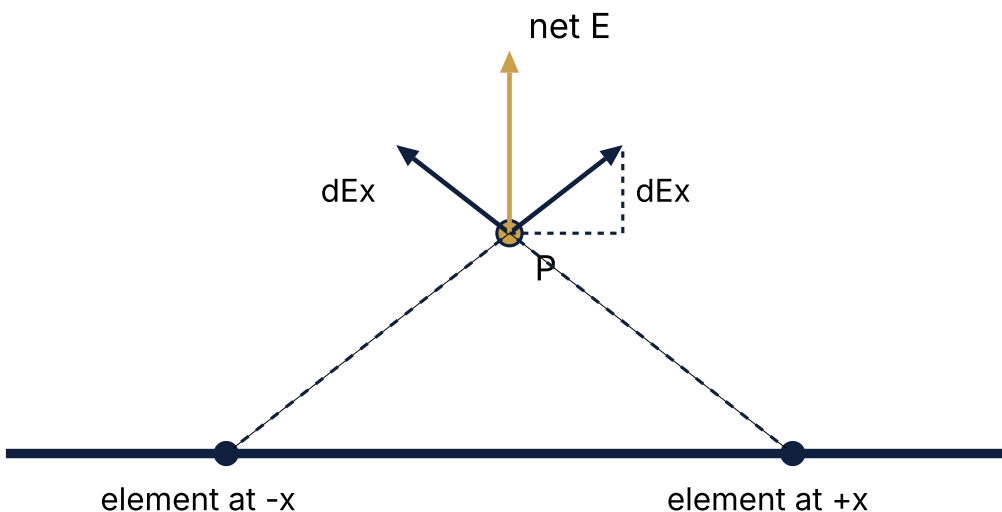
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Symmetry Before Algebra

Symmetry decides which components exist before any integral is written. Pairs of elements placed mirror-fashion about the field point cancel the components along the symmetry direction and add the rest. Say this in words; a bare claim that the x-component is zero earns little.

Equal elements at $+x$ and $-x$: dE_x cancel, dE_y add



Symmetry argument for a point on the perpendicular bisector of a uniformly charged rod (positive charge).

- ◆ Rod or arc, point on the bisector: components parallel to the rod cancel in pairs.
- ◆ Ring, point on the axis: components perpendicular to the axis cancel, leaving $\cos \phi = z/r$ in every term.
- ◆ Spherical symmetry: the field points radially and depends only on r .
- ◆ Cylindrical symmetry (long): the field points radially away from the axis and depends only on distance from it.
- ◆ Planar symmetry: the field is perpendicular to the plane and has equal magnitude at equal distance.

Choosing Gaussian and Amperian Paths

Gauss's law and Ampere's law are exact for any surface or loop, but they only deliver the field when symmetry lets you pull it out of the integral. Choose a surface on which the field is either constant and parallel to $d\vec{A}$, or zero or perpendicular to it. Then the integral collapses to a product, and the whole problem moves into the enclosed charge or current, which is where nonuniform density shows up.

Symmetry	Surface or loop	Area or length that counts	Enclosed amount
Sphere	Concentric sphere	$4\pi r^2$	$\int \rho 4\pi r^2 dr$
Long cylinder	Coaxial cylinder, length ℓ	Curved side $2\pi r\ell$ only	$\int \rho 2\pi r\ell dr$
Sheet	Pillbox through the sheet	Two faces, each A	σA
Long wire (Ampere)	Circle of radius r about the axis	Circumference $2\pi r$	$\int J 2\pi r dr$

Enclosed amounts with variable density

Charge: $Q_{enc} = \int \rho dV$ over the region inside the surface only.

Current: $I_{enc} = \int \vec{J} \cdot d\vec{A}$ over the disk bounded by the loop only.

Outside the charged region the integral stops at the outer edge; the field then looks like that of the total.

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Separable Differential Equations for RC and RL

The circuit equations in this course are first order and separable: put every V (or I) on one side, every t on the other, and integrate. For a discharging capacitor, the loop rule gives $q/C + R dq/dt = 0$, so $dq/q = -dt/(RC)$. Integrating both sides from the initial value q_0 to q and from 0 to t gives $\ln(q/q_0) = -t/RC$. The time constant $\tau = RC$ is the time for the quantity to fall to $1/e$. The same pattern solves a charging circuit if you first define the variable that decays, such as $\mathcal{E} - V_C$, and an LR circuit with $\tau = L/R$.

Solving a separable equation

1. Write the loop rule and replace each quantity by one variable and its derivative.
2. Separate: all of one variable with its differential on each side.
3. Integrate both sides with matching limits (initial condition to final value).
4. Exponentiate, then state the time constant and check the limits $t = 0$ and t very large.

Linearizing Data and Reading Slope and Intercept

A straight line is the only curve the eye and a ruler can judge, so lab questions ask you to rearrange a law into $y = mx + b$ where y and x are quantities you measure or calculate. Decide what to plot, predict the slope and intercept in symbols, then state what each means physically. Both axes need a quantity and a unit, and the scales must be uniform.

Law	Plot y vs x	Slope	Intercept
$V = V_0 e^{-t/RC}$	$\ln V$ vs t	$-1/RC$	$\ln V_0$
$R = \rho L/A$	R vs L	ρ/A	contact resistance
$T = 2\pi\sqrt{L(C + C_0)}$	T^2 vs C	$4\pi^2 L$	$4\pi^2 LC_0$
$P = V^2/R$	P vs V^2	$1/R$	0
$q = CV$	q vs V	C	0

Compute slope from two points on the best-fit line that are far apart, never from data points and never from a point and the origin unless the line is shown or argued to pass through it. If the intercept is predicted to be zero and the fitted line misses it, that is evidence of a systematic error, and you should say so.

Translating Between Graphs and Equations

Going from an equation to a sketch, read the functional form: a power law gives a curve whose steepness changes, an exponential has a constant ratio per equal time step, and $1/r$ is steeper than a line but flatter than $1/r^2$. Going the other way, identify the form from the shape, then ask what the area or slope means. The slope of q versus t is current; the slope of flux versus time is the induced emf; the area under I versus t is charge. Mark intercepts, asymptotes and points where the shape changes, for example at the surface of a charged sphere.

Writing Derivations That Earn Credit

Readers score what is visible. Reports on recent exams describe the same losses year after year: a surface integral treated like a one-dimensional integral, the wrong surface used in Gauss's law, charge densities mixed up, and answers that skip from a law to a result.

- ◆ Start from a law written in general form, then show the substitution that specializes it.
- ◆ Name the surface or loop and the symmetry that justifies it.
- ◆ Show each integral with limits before substituting; do not jump to an antiderivative.
- ◆ Define any new symbol, such as λ or A , that the problem did not give you.
- ◆ Keep a partial result; a correct later step that uses an earlier wrong value can still earn credit.
- ◆ End with the requested form, and check units and limiting cases.

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Worked Examples

ROD WITH A VARYING CHARGE DENSITY

A thin rod lies on the x-axis from $x = 0$ to $x = L = 0.40$ m. Its linear density is $\lambda = Ax$ with $A = 2.0 \times 10^{-8}$ C/m². (a) Find the total charge. (b) Find the electric field at the point $x = 0.50$ m, which is $d = 0.10$ m beyond the end.

SOLUTION

(a) $dq = Ax dx$, so $Q = \int_0^L Ax dx = AL^2/2 = (2.0 \times 10^{-8})(0.40)^2/2 = 1.6 \times 10^{-9}$ C. Units: C/m² times m² gives C.

(b) The point lies on the line of the rod, so every dE points along +x with no components to cancel. The distance from the element at x to the point is $L + d - x$.

$$E = \int_0^L \frac{kAx dx}{(L + d - x)^2} = kA \left[\frac{L + d}{L + d - x} + \ln(L + d - x) \right]_0^L = kA \left[\frac{L}{d} - \ln \frac{L + d}{d} \right] \text{ after substituting } u = L + d - x.$$

Numbers: $L/d = 4.0$ and $\ln 5 = 1.609$, so $E = (8.99 \times 10^9)(2.0 \times 10^{-8})(2.391) = 4.3 \times 10^2$ N/C in the +x direction.

Check: a point charge of 1.6 nC at 0.50 m would give 58 N/C, but the charge sits mostly near the far end, only 0.10 m away, so a larger value is reasonable.

PERPENDICULAR BISECTOR OF A FINITE ROD

A uniformly charged rod of length $L = 0.60$ m carries $\lambda = 4.0$ nC/m. Find the field at a point $y = 0.20$ m above its midpoint.

SOLUTION

Symmetry: pair each element at $+x$ with the one at $-x$. Their x-components cancel and their y-components add, so $\vec{E}$ points along +y (see figure above).

$$dE_y = k\lambda dx \frac{y}{(x^2 + y^2)^{3/2}}, \text{ because } \cos \phi = y/r.$$

$$E = k\lambda y \int_{-L/2}^{L/2} \frac{dx}{(x^2 + y^2)^{3/2}} = k\lambda y \left[\frac{x}{y^2 \sqrt{x^2 + y^2}} \right]_{-L/2}^{L/2} = \frac{k\lambda L}{y \sqrt{y^2 + L^2/4}}.$$

Numbers: $\sqrt{0.04 + 0.09} = 0.3606$ m, so $E = \frac{(8.99 \times 10^9)(4.0 \times 10^{-9})(0.60)}{(0.20)(0.3606)} = 3.0 \times 10^2$ N/C along +y.

Limits: for $y \gg L$ the result becomes kQ/y^2 with $Q = \lambda L$, and for $L \gg y$ it becomes $2k\lambda/y$. Both are correct.

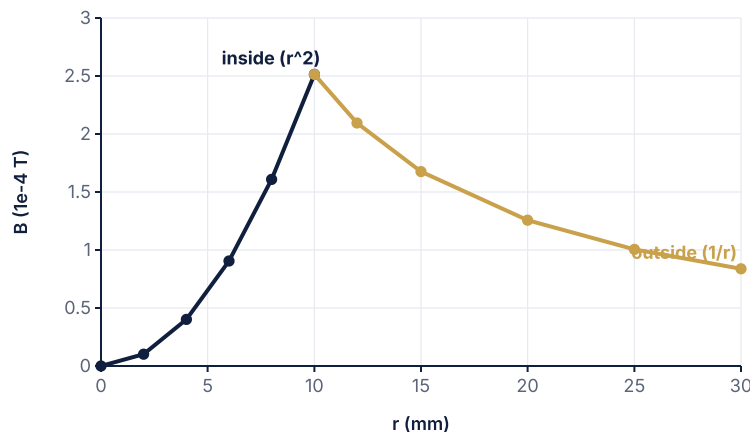
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AMPERE'S LAW WITH NONUNIFORM CURRENT DENSITY

A long straight wire of radius $R = 1.0$ cm carries current along its axis with density $J = J_0 r/R$, where $J_0 = 6.0 \times 10^4$ A/m² and r is the distance from the axis. Find B at $r = R/2$ and at $r = R$, and sketch B versus r .



B versus r for J proportional to r .

SOLUTION

Symmetry: B circles the axis and depends only on r . Use a circular loop of radius r coaxial with the wire, so $\oint \vec{B} \cdot d\vec{\ell} = B(2\pi r)$.

Enclosed current: the element is a ring of area $2\pi r' dr'$, so $I_{enc} = \int_0^r J_0 \frac{r'}{R} 2\pi r' dr' = \frac{2\pi J_0 r^3}{3R}$.

Therefore $B = \frac{\mu_0 I_{enc}}{2\pi r} = \frac{\mu_0 J_0 r^2}{3R}$ inside, a parabola. At $r = R/2$: $I_{enc} = 1.57$ A and $B = 6.3 \times 10^{-5}$ T.

At $r = R$: $I = 2\pi J_0 R^2/3 = 12.6$ A and $B = 2.5 \times 10^{-4}$ T. Outside, I_{enc} stays 12.6 A, so $B \propto 1/r$.

The graph rises as r^2 , peaks at the surface, then falls as $1/r$. Both pieces match at $r = R$.

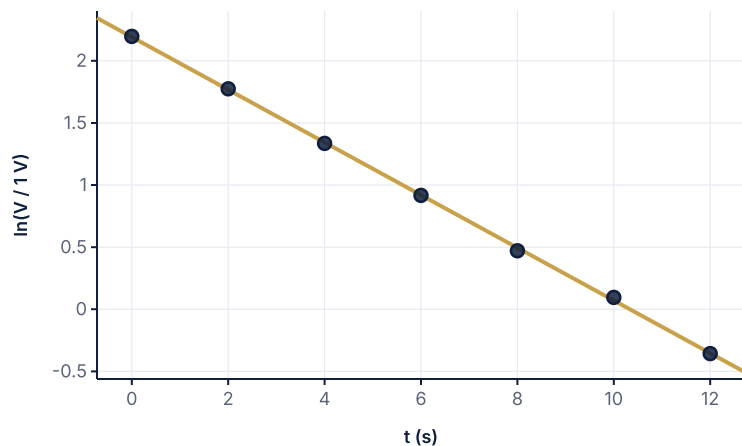
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TIME CONSTANT FROM A DISCHARGE

A capacitor discharges through a resistor of $47\text{ k}\Omega$. A meter records the voltage, listed below. (a) Derive the equation that makes the data linear. (b) Use the line to find τ and C .



Linearized discharge data with best-fit line.

t (s)	0	2	4	6	8	10	12
V (V)	9.0	5.9	3.8	2.5	1.6	1.1	0.70

SOLUTION

(a) From $q/C + R dq/dt = 0$: $dq/q = -dt/RC$, and since $V = q/C$ the same holds for V . Integrating, $\ln V = \ln V_0 - t/RC$. So plot $\ln V$ against t : slope $-1/RC$, intercept $\ln V_0$.

(b) The fitted line is $\ln V = 2.19 - 0.212 t$. Slope from two far-apart points on the line, $(0, 2.19)$ and $(12, -0.35)$: $m = -2.54/12 = -0.212\text{ s}^{-1}$.

$\tau = 1/0.212 = 4.7\text{ s}$, so $C = \tau/R = 4.7/47000 = 1.0 \times 10^{-4}\text{ F} = 100\text{ }\mu\text{F}$.

The intercept gives $V_0 = e^{2.19} = 8.9\text{ V}$, which agrees with the first reading, 9.0 V . That agreement supports the exponential model.

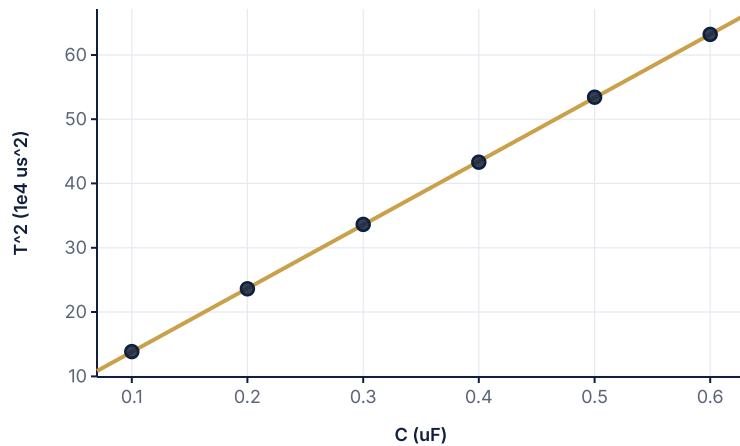
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INTERCEPT WITH MEANING: AN LC LAB

A coil is connected to a series of capacitors and the oscillation period T is measured. A stray capacitance C_0 adds to every capacitor, so $T = 2\pi\sqrt{L(C + C_0)}$. Find L and C_0 from the graph of T^2 against C .



T^2 versus C with best-fit line; the intercept is not zero.

C (uF)	0.10	0.20	0.30	0.40	0.50	0.60
T (us)	372	486	580	658	731	795
T ² (1e4 us ²)	13.8	23.6	33.6	43.3	53.4	63.2

SOLUTION

Square the law: $T^2 = 4\pi^2 L C + 4\pi^2 L C_0$. This is $y = mx + b$ with $m = 4\pi^2 L$ and $b = 4\pi^2 L C_0$.

Best-fit line from the data: $T^2 = 3.9 \times 10^4 \mu\text{s}^2 + (9.88 \times 10^5 \mu\text{s}^2/\mu\text{F}) C$. Convert units: $\mu\text{s}^2/\mu\text{F} = 10^{-6} \text{ s}^2/\text{F}$, so $m = 0.988 \text{ s}^2/\text{F}$.

$L = m/4\pi^2 = 0.988/39.48 = 0.025 \text{ H} = 25 \text{ mH}$.

$C_0 = b/m = (3.9 \times 10^4 \mu\text{s}^2)/(9.88 \times 10^5 \mu\text{s}^2/\mu\text{F}) = 0.040 \mu\text{F}$.

Plotting T against C directly would give a curve that cannot be judged by eye; plotting T^2 turns the unknowns into slope and intercept. Graph axis unit: T^2 in units of $10^4 \mu\text{s}^2$.

Write It Like a 5**Model Response**

Prompt: A rod of length L lies on the x -axis from $x = 0$ to $x = L$ with uniform linear density λ . Derive an expression for the electric potential at the point P on the x -axis a distance d beyond the end of the rod, with $V = 0$ at infinity. Response: Potential is a scalar, so I add contributions from each element with no components. Take an element of length dx at position x . It carries $dq = \lambda dx$ and is a distance $(L + d - x)$ from P . Its contribution is $dV = k dq / r = k \lambda dx / (L + d - x)$. Summing over the whole rod, $V = k\lambda \int_0^L \frac{dx}{L + d - x}$. With $u = L + d - x$, $du = -dx$, the limits become $u = L + d$ at $x = 0$ and $u = d$ at $x = L$, so $V = k\lambda \int_{L+d}^d \frac{du}{u} = k\lambda \ln \frac{L+d}{d}$. Check: for d much larger than L , $\ln(1 + L/d)$ is approximately L/d , so V approaches $k \lambda L / d = kQ/d$, the point-charge result, as required. Why it scores: the general law is stated, the element and its distance are defined, limits are shown before the antiderivative, the substitution is explicit, and a limiting case confirms the answer.

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Common traps

1. Using the area of the whole cylinder, including end caps, for the flux when the field is radial; only the curved side counts. 2. Writing $dq = \rho dV$ when the density given is linear, or integrating a surface integral as if it were a one-dimensional integral. 3. Taking the slope from the origin and one data point, or reading the slope off a plot of the wrong variable (for example T instead of T^2). 4. Skipping the symmetry argument: a statement such as 'the x components cancel' needs the reason, namely equal and opposite elements at mirror positions.

Coach's Playbook

- ◆ Before you integrate, write what the answer must reduce to in a limit; compare after.
- ◆ Keep SI units until the end. A field in N/C and a slope in s^{-1} must carry the correct powers of ten.
- ◆ When asked for 'justify', use the form: law, condition that makes it applicable, substitution, conclusion.
- ◆ On a lab question, name every quantity you measure and every quantity you calculate, and say how.

Practice**PRACTICE Workshop Practice**

Show your work. A calculator is allowed. Justify conclusions.

1. A ring of radius a carries total charge Q spread uniformly. For a point on the axis a distance z from its center, which expression gives the magnitude of the field?

(A) $\frac{kQ}{z^2 + a^2}$

(B) $\frac{kQz}{(z^2 + a^2)^{3/2}}$

(C) $\frac{kQ}{z^2}$

(D) $\frac{kQz}{(z^2 + a^2)^{1/2}}$

2. In a charging RC circuit with battery emf $\mathcal{E}$, a student measures V_C against time. Which plot gives a straight line whose slope is $-1/RC$?

(A) V_C vs t

(B) $\ln V_C$ vs t

(C) $\ln(\mathcal{E} - V_C)$ vs t

(D) $1/V_C$ vs t

3. A long solid cylinder of radius R has charge density $\rho = br$, where r is the distance from its axis. What is the field magnitude at $r < R$?

(A) $\frac{br^2}{2\epsilon_0}$

(B) $\frac{br}{\epsilon_0}$

(C) $\frac{br^3}{3\epsilon_0}$

(D) $\frac{br^2}{3\epsilon_0}$

4. The resistance of a wire of diameter 0.40 mm is measured for several lengths. The graph of R against L is a line with slope 0.80 ohm/m and a positive intercept of 0.30 ohm. What is the resistivity of the wire?

(A) $1.0 \times 10^{-7} \Omega \text{ m}$

(B) $4.0 \times 10^{-7} \Omega \text{ m}$

(C) $1.3 \times 10^{-7} \Omega \text{ m}$

(D) $3.8 \times 10^{-8} \Omega \text{ m}$

5. A thin semicircular arc of radius $R = 0.15$ m carries a total charge $Q = 3.0$ nC distributed uniformly. (a) Use a symmetry argument to state the direction of the field at the center of the circle. (b) Derive an expression for the magnitude of that field in terms of Q , R and k , showing the integral. (c) Calculate its value.
6. A solid sphere of radius $R = 0.30$ m has charge density $\rho = \rho_0 r/R$ with $\rho_0 = 2.0 \times 10^{-6}$ C/m³ and r measured from its center. (a) Derive the total charge. (b) Derive the field magnitude for $r < R$ and for $r > R$, naming your Gaussian surface. (c) Calculate E at $r = R/2$ and at $r = 2R$, and sketch E versus r from 0 to $3R$.
7. A student discharges a capacitor through a 33 k Ω resistor and records the voltage: $t = 0, 1, 2, 3, 4, 5, 6$ s gives $V = 6.0, 3.6, 2.2, 1.3, 0.80, 0.49, 0.29$ V. (a) Derive the relationship and state what to plot so that the graph is a straight line. (b) Describe how to extract the capacitance from the graph and calculate it from the data. (c) Explain one change in the procedure that would reduce the uncertainty in the time constant.
8. An ideal inductor $L = 0.60$ H is connected in series with a resistor $R = 15 \Omega$ and a battery of emf $\mathcal{E}$ at $t = 0$. (a) Derive $I(t)$ by separating variables, starting from the loop rule. (b) Calculate the time at which the current reaches 90 percent of its final value. (c) A student writes 'the current jumps to $\mathcal{E}/R$ at once because the battery supplies the emf'. Identify the flaw and correct it.

Answers Workshop: Answers and Explanations

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W Workshop: Calculus Skills, Symmetry, and Lab Analysis

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- B. Each element contributes $dE = k dq / (z^2 + a^2)$, but only the axial component $\cos \phi = z / \sqrt{z^2 + a^2}$ survives because perpendicular components cancel in opposite pairs. A, the result without the cosine, is the tempting error.
- C. Charging obeys $V_C = \mathcal{E}(1 - e^{-t/RC})$, so $\mathcal{E} - V_C = \mathcal{E}e^{-t/RC}$ and its logarithm is linear in t with slope $-1/RC$. B fails because V_C itself is not exponentially decaying; it grows toward $\mathcal{E}$.
- D. Gauss's law over a coaxial cylinder of length ℓ : $Q_{enc} = \int_0^r br' 2\pi r' \ell dr' = 2\pi b\ell r^3/3$ and the flux is $E 2\pi r \ell$, so $E = br^2/(3\epsilon_0)$. A uses $r^2/2$, which is what you would get by integrating br rather than $br \cdot r$; the extra factor of r from the area element changes the exponent.
- A. The slope is ρ/A , so $\rho = \text{slope} \times A$ with $A = \pi(d/2)^2 = \pi(2.0 \times 10^{-4})^2 = 1.26 \times 10^{-7} \text{ m}^2$, giving $1.0 \times 10^{-7} \Omega\text{m}$. B uses the diameter as if it were the radius, C uses the diameter squared with no factor of $\pi/4$, and the intercept is contact resistance and does not enter.
- (a) Elements at equal angles above and below the symmetry axis have equal dE , so components along the diameter that joins the arc ends cancel; the net field points along the axis of symmetry, away from the arc if $Q > 0$. (1 point)
 (b) $dq = \lambda R d\theta$ with $\lambda = Q/(\pi R)$, and $dE = k dq/R^2$. The surviving component is $dE \sin \theta$ with θ measured from the line joining the arc ends: $E = \int_0^\pi \frac{k\lambda R d\theta}{R^2} \sin \theta = \frac{k\lambda}{R} [-\cos \theta]_0^\pi = \frac{2k\lambda}{R} = \frac{2kQ}{\pi R^2}$. (3 points)
 (c) $E = 2(8.99 \times 10^9)(3.0 \times 10^{-9})/(\pi \cdot 0.0225) = 7.6 \times 10^2 \text{ N/C}$. (1 point)
- (a) $Q = \int_0^R \rho_0(r/R) 4\pi r^2 dr = \pi \rho_0 R^3 = \pi(2.0 \times 10^{-6})(0.027) = 1.7 \times 10^{-7} \text{ C}$.
 (b) Spherical symmetry makes E radial and constant on a concentric sphere of radius r , so $\oint \vec{E} \cdot d\vec{A} = E 4\pi r^2$. For $r < R$: $Q_{enc} = \pi \rho_0 r^4/R$, so $E = \rho_0 r^2/(4\epsilon_0 R)$. For $r > R$: $Q_{enc} = \pi \rho_0 R^3$, so $E = \rho_0 R^3/(4\epsilon_0 r^2)$.
 (c) At $r = R/2$: $E = \rho_0 R/(16\epsilon_0) = 4.2 \times 10^3 \text{ N/C}$. At $r = 2R$: $E = \rho_0 R/(16\epsilon_0) = 4.2 \times 10^3 \text{ N/C}$, the same value. The sketch rises parabolically from zero to a peak $\rho_0 R/(4\epsilon_0) = 1.7 \times 10^4 \text{ N/C}$ at $r = R$, then falls as $1/r^2$ with matching value at $r = R$.
- (a) The loop rule gives $q/C + R dq/dt = 0$, so $dq/q = -dt/RC$ and $\ln V = \ln V_0 - t/RC$. Plot $\ln V$ (vertical, unit-free after dividing by 1 V) against t in seconds with uniform scales.
 (b) The slope is $-1/RC$. A best-fit line through the data gives slope about -0.50 s^{-1} (using two far-apart points on the line, not data points), so $RC = 2.0 \text{ s}$ and $C = 2.0/33000 = 6.0 \times 10^{-5} \text{ F} = 60 \mu\text{F}$. The intercept, $\ln 6.0 = 1.79$, checks the initial voltage.
 (c) Take readings over a longer interval (several time constants) or at more points, or use a meter with larger input resistance so it does not draw current that shortens the apparent time constant; repeated trials let you average timing error.
- (a) Loop rule: $\mathcal{E} - IR - L dI/dt = 0$. Rearranged, $\frac{dI}{\mathcal{E}/R - I} = \frac{R}{L} dt$. Integrating from $I = 0$ at $t = 0$: $-\ln \frac{\mathcal{E}/R - I}{\mathcal{E}/R} = \frac{R}{L} t$, so $I = \frac{\mathcal{E}}{R} (1 - e^{-Rt/L})$.
 (b) $0.90 = 1 - e^{-t/\tau}$ gives $t = \tau \ln 10$ with $\tau = L/R = 0.040 \text{ s}$, so $t = 0.092 \text{ s}$.
 (c) The inductor opposes the change in current by an induced emf $L dI/dt$. At $t = 0$ the current is zero and this emf equals the battery emf; the current rises with time constant L/R , so it cannot jump.

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Mixed Practice Test

A mixed set of multiple-choice and free-response questions across all units, with answers, explanations, and scoring guidance.

T Mixed Practice Test

A full-range check of all six Physics C: E&M units, weighted like the exam. Twelve multiple-choice questions, then three free-response questions.

Instructions and timing (about 50 minutes): Part A, 12 multiple-choice questions (about 20 minutes); Part B, three free-response questions worth 26 points (about 30 minutes). Constants: $k = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$, $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N m}^2)$, $\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$, $e = 1.602 \times 10^{-19} \text{ C}$, electron mass $9.11 \times 10^{-31} \text{ kg}$, proton mass $1.673 \times 10^{-27} \text{ kg}$. Weighting: Unit 8 two items, Unit 9 two, Unit 10 one, Unit 11 three, Unit 12 two, Unit 13 two. Score with the guide that follows.

PRACTICE TEST Mixed Practice Test

Part A: choose the best answer for items 1 to 12. Part B: items 13 to 15 are free response; show derivations step by step and justify claims.

- A very long straight wire carries a uniform charge of $2.0 \mu\text{C}$ per meter. The magnitude of the electric field 0.40 m from the wire is closest to
(A) $4.5 \times 10^4 \text{ N/C}$ (B) $9.0 \times 10^4 \text{ N/C}$ (C) $1.1 \times 10^5 \text{ N/C}$
(D) $1.8 \times 10^5 \text{ N/C}$
- A solid insulating sphere of radius R carries charge Q spread uniformly through its volume. The field magnitude at $r = R/2$, compared with the field magnitude at the surface, is
(A) one-half as large (B) one-fourth as large (C) equal
(D) zero
- In a region the field is $E_x = 6x^2$ (SI units, in the $+x$ direction). The potential at $x = 2.0 \text{ m}$ minus the potential at $x = 0$ is
(A) $+16 \text{ V}$ (B) -48 V (C) -16 V (D) -24 V
- An electron starts from rest and is accelerated through a potential difference of 250 V . Its final speed is closest to
(A) $6.6 \times 10^6 \text{ m/s}$ (B) $8.8 \times 10^{13} \text{ m/s}$ (C) $4.4 \times 10^{13} \text{ m/s}$
(D) $9.4 \times 10^6 \text{ m/s}$
- An isolated, charged parallel-plate capacitor stores energy U . A dielectric with constant 4 is inserted to fill the gap. The stored energy becomes
(A) $4U$ (B) $U/4$ (C) U (D) $U/16$
- A 12 V ideal battery drives a 4Ω resistor in series with a parallel pair of 6Ω and 12Ω resistors. The power dissipated in the 6Ω resistor is
(A) 3.0 W (B) 13.5 W (C) 6.0 W (D) 24 W
- A $2.0 \text{ M}\Omega$ resistor charges an initially empty $5.0 \mu\text{F}$ capacitor from a 9.0 V battery. The capacitor voltage reaches 6.0 V after about
(A) 11 s (B) 10 s (C) 6.9 s (D) 4.1 s
- A wire is replaced by one of the same material that is twice as long and has twice the radius. Its resistance becomes
(A) unchanged (B) twice as large (C) one-fourth as large
(D) half as large
- A proton moves at $2.0 \times 10^6 \text{ m/s}$ perpendicular to a uniform 0.30 T field. The radius of its circular path is closest to
(A) 3.5 cm (B) 14 cm (C) 7.0 cm (D) 0.70 m

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10. Two long parallel wires 0.10 m apart carry 4.0 A and 6.0 A in the same direction. The force per unit length on each wire is
 (A) $4.8\text{e-}5$ N/m, repulsive (B) $4.8\text{e-}5$ N/m, attractive (C) $2.4\text{e-}5$ N/m, attractive
 (D) $9.6\text{e-}5$ N/m, repulsive
11. A square loop of side 0.10 m and resistance $0.25\ \Omega$ is pulled at constant 2.0 m/s out of a 0.50 T field that is perpendicular to its plane. The force needed while one side is still in the field is
 (A) 0.10 N (B) 0.40 N (C) 0.0050 N (D) 0.020 N
12. An ideal 0.30 H inductor and a $60\ \Omega$ resistor are connected in series to an 18 V battery at $t = 0$. At $t = L/R = 5.0$ ms the voltage across the inductor is closest to
 (A) 6.6 V (B) 11 V (C) 18 V (D) zero
13. [10 points] A very long insulating cylinder of radius R carries a volume charge density $\rho(r) = \rho_0 r/R$ for $r < R$, where ρ_0 is a constant, and no charge elsewhere.
 (a) Show that the charge per unit length within radius $r < R$ is $\lambda(r) = 2\pi\rho_0 r^3/(3R)$. [2 points]
 (b) Use Gauss's law to find $E(r)$ for $r < R$ and for $r > R$. [3 points]
 (c) Sketch E versus r from 0 to $3R$, labeling the value at $r = R$. [2 points]
 (d) Derive the potential difference between the axis and the surface and evaluate it for $\rho_0 = 3.0 \times 10^{-6}\text{ C/m}^3$ and $R = 2.0$ cm. [3 points]
14. [8 points] A $500\ \mu\text{F}$ capacitor is charged to 12 V and then connected across a $2.0\text{ k}\Omega$ resistor at $t = 0$.
 (a) Write expressions for the capacitor charge and the current as functions of time, starting from the loop rule. [3 points]
 (b) Calculate the total thermal energy dissipated and the time at which the capacitor's stored energy has fallen to half its initial value. [3 points]
 (c) Sketch the power dissipated in the resistor versus time and state how its decay compares with the decay of the current. [2 points]
15. [8 points] A square loop of uniform wire has side 0.10 m and resistance $0.50\ \Omega$. It lies in a field perpendicular to its plane, directed into the page, of magnitude $B = 0.80 - 0.20t$ (tesla, t in s).
 (a) State the direction of the induced current and justify it. [2 points]
 (b) Calculate the emf and the current. [2 points]
 (c) Calculate the thermal energy dissipated from $t = 0$ to $t = 2.0$ s. [2 points]
 (d) The loop is replaced by a square of side 0.20 m made of the same wire. Does the current increase, decrease or stay the same, and by what factor? Justify. [2 points]

Key**Mixed Practice Test: Answers, Explanations, and Scoring****T Mixed Practice Test****PRACTICE TEST · MIXED PRACTICE TEST**

1. B. Gauss's law with a cylindrical surface gives $E = \lambda/2\pi\epsilon_0 r = 2k\lambda/r = 9.0 \times 10^4$ N/C. Choice A drops the factor of 2, and choice C uses a point-charge $1/r^2$ form that does not apply to a line (Unit 8)
2. A. Only the charge inside the Gaussian sphere counts: $Q(r/R)^3$ over r^2 gives E proportional to r inside, so the field at $R/2$ is half the surface value. Choice B applies the outside $1/r^2$ law inside, and choice D confuses the solid sphere with a conducting shell (Unit 8)
3. C. $V(2) - V(0) = -\text{the integral of } E \text{ dx from } 0 \text{ to } 2 = -(2x^3) \text{ evaluated} = -16$ V. Choice A drops the minus sign; choice B multiplies $E(2) = 24$ by the distance instead of integrating, and choice D uses $E(2)$ alone (Unit 9)
4. D. Energy conservation: $(1/2)mv^2 = eV$, so $v = \sqrt{2eV/m} = 9.4\text{e}6$ m/s. Choice A omits the factor of 2 under the root; the two big values stop before taking the square root (Unit 9)
5. B. With no battery the charge is fixed, capacitance rises by 4, and $U = Q^2/2C$ falls to $U/4$. Choice A ($4U$) would hold only if a battery kept the voltage fixed (Unit 10)
6. C. The parallel pair is 4 ohms, total 8 ohms, $I = 1.5$ A, and the pair has 6.0 V across it, so $P = 6.0^2/6 = 6.0$ W. Choice B uses the total current in the 6 ohm resistor; choice D puts the whole 12 V across it (Unit 11)
7. A. $V = 9(1 - e^{-t/RC})$ with $RC = 10$ s gives $t = 10 \ln 3 = 11$ s. Choice B quotes the time constant itself, which is the 63 percent point (5.7 V) (Unit 11)
8. D. $R = \rho L / A$; L doubles and A quadruples, so R is halved. Choice A lets area merely double, and choice B ignores the radius (Unit 11)
9. C. $r = mv/(qB) = (1.673\text{e-}27)(2.0\text{e}6)/((1.602\text{e-}19)(0.30)) = 0.070$ m. Choice B doubles the result and choice A halves it, as if $mv/(2qB)$ were used (Unit 12)
10. B. $F/L = \mu_0 I_1 I_2 / 2 \pi d = 2\text{e-}7 (24)/0.10 = 4.8\text{e-}5$ N/m. Parallel currents attract. Choice A has the right size but the wrong direction (Unit 12)
11. D. $\text{emf} = Blv = 0.10$ V, $I = 0.40$ A, $F = IIB = 0.020$ N. Choices A and B report the emf and the current as if they were a force (Unit 13)
12. A. $V_L = \text{emf } e^{-t/\tau} = 18/e = 6.6$ V at $t = \tau$. Choice B is the resistor's voltage, and choice C describes the instant just after closing, not a time constant later (Unit 13)

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13. (a) 2 points: $\lambda = \int_0^r \rho \, 2\pi r' \, dr' [1] = \frac{2\pi\rho_0}{R} \int_0^r r'^2 dr' = \frac{2\pi\rho_0 r^3}{3R} [1]$. (b) 3 points: Gaussian cylinder of length ℓ with only the curved side contributing: $E \, 2\pi r \ell = \lambda \ell / \epsilon_0 [1]$; inside $E = \rho_0 r^2 / (3\epsilon_0 R) [1]$; outside $\lambda_{tot} = 2\pi\rho_0 R^2 / 3$ so $E = \rho_0 R^2 / (3\epsilon_0 r) [1]$. (c) 2 points: parabola rising from zero to $\rho_0 R / 3\epsilon_0$ at $r = R [1]$, then a $1/r$ decrease, continuous at $R [1]$. (d) 3 points: $V(0) - V(R) = \int_0^R E \, dr [1] = \rho_0 R^2 / (9\epsilon_0) [1] = (3.0 \times 10^{-6})(4.0 \times 10^{-4}) / (9 \cdot 8.85 \times 10^{-12}) = 15 \, \text{V} [1]$. Axis is at higher potential for positive charge.
Model response (final results): $\lambda = 2\pi\rho_0 r^3 / (3R)$; $E = \rho_0 r^2 / (3\epsilon_0 R)$ inside, $\rho_0 R^2 / (3\epsilon_0 r)$ outside; $V(0) - V(R) = \rho_0 R^2 / (9\epsilon_0) = 15 \, \text{V}$.
14. (a) 3 points: loop rule $q/C + iR = 0$ with $i = -dq/dt [1]$; $q = CV_0 e^{-t/RC} [1]$ and $i = (V_0/R)e^{-t/RC}$ with $RC = 1.0 \, \text{s}$, $i_0 = 6.0 \, \text{mA} [1]$. (b) 3 points: total heat $= \frac{1}{2} CV_0^2 = 0.036 \, \text{J} [1]$; $U \propto e^{-2t/RC} [1]$; half when $t = (RC/2) \ln 2 = 0.35 \, \text{s} [1]$. (c) 2 points: $P = i^2 R = (0.072 \, \text{W})e^{-2t/RC}$, starting at $72 \, \text{mW}$ and falling exponentially $[1]$; it decays twice as fast (time constant $RC/2 = 0.5 \, \text{s}$) as the current $[1]$.
Model response (final results): $q = 6.0 \times 10^{-3} \, \text{C} e^{-t}$, $i = 6.0 \, \text{mA} e^{-t}$, heat $0.036 \, \text{J}$, $t = 0.35 \, \text{s}$, $P = 72 \, \text{mW} e^{-2t}$ with t in seconds.
15. (a) 2 points: clockwise $[1]$; the inward flux is decreasing, so the induced field must also point into the page, which needs a clockwise current $[1]$. (b) 2 points: $\mathcal{E} = A |dB/dt| = (0.010)(0.20) = 2.0 \, \text{mV} [1]$; $I = \mathcal{E}/R = 4.0 \, \text{mA} [1]$. (c) 2 points: $P = I^2 R = 8.0 \, \mu\text{W} [1]$; energy $= P\Delta t = 1.6 \times 10^{-5} \, \text{J} [1]$. (d) 2 points: current increases by a factor of 2 $[1]$; area (hence emf) is 4 times larger, resistance 2 times larger (twice the wire length), so $I = \mathcal{E}/R$ doubles $[1]$.
Model response (final results): clockwise, $2.0 \, \text{mV}$, $4.0 \, \text{mA}$, $1.6 \times 10^{-5} \, \text{J}$, current doubles.

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Free-Response Clinic

Exam-style free-response questions with point-by-point scoring guides and full-credit model responses.

X Free-Response Clinic

Four free-response questions in the exam's styles, plus one extra derivation, with practice point values and point-by-point scoring guides.

How Section II Works

Section II has 4 questions in 95 minutes, worth 50 percent of the score. The question types come in a fixed order. Budget roughly 2.4 minutes per point, and use the last few minutes to check units, labels and signs. Question 5 below is an extra in the style of a derivation with a differential equation. The point values are practice values that follow the exam pattern; score yourself with the guides, never above what the work shows.

Question	Style	Points	Typical verbs
1	Mathematical Routines: derivation, may end in a number	10	derive, calculate, justify
2	Translation Between Representations: graphs and equations	12	derive, sketch, compare
3	Experimental Design and Analysis	10	describe, linearize, determine
4	Qualitative/Quantitative Translation	8	justify, compare, calculate
5 (extra)	Mathematical Routines with a differential equation	10	derive, find the maximum

Plan Your Attack

- ◆ Read all parts first. Early parts often supply an expression you need later, and a wrong early answer can still earn credit when carried forward correctly.
- ◆ In derivation questions, write the law, the geometry and symmetry argument, and the integral before any numbers. Graders look for the multistep path.
- ◆ In graph questions, mark scale values, starting values and zero crossings; a graph with the right shape but no numbers loses points.
- ◆ In experiment questions, name what is measured and what is calculated, say which quantity is varied and which are held fixed, and link the slope to the physical constant.
- ◆ In qualitative questions, argue from fields and symmetry, state directions with the right-hand rule, and avoid unnecessary calculation.

Scoring habits

Write the starting law, then the algebra, then the number with units. Carry an earlier answer forward even if you doubt it. Show integrals before substituting limits. Use words like 'net field', 'enclosed current' and 'rate of change of flux' exactly.

After You Score

Sort every lost point into one of three bins: setup (wrong law or surface), algebra and calculus (integral or limits), or communication (missing justification, unit or label). Redo the question in the bin where you lost the most. Then rewrite one model response in your own words from memory, because the exam rewards clear reasoning more than recalled formulas. Space your practice: attempt a question, wait two days, and try it again cold.

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PRACTICE Free-Response Clinic

Write full answers, then score each part with the guide. A calculator is allowed. Show every step of each derivation and justify claims in words.

1. [10 points] Question 1. A long straight wire of radius $R = 3.0$ mm carries a current whose density depends on the distance r from the axis: $J(r) = J_0(1 - r^2/R^2)$ for $r < R$, with $J_0 = 2.0 \times 10^6$ A/m² and the current along the axis.
 - (a) Derive the current enclosed within radius $r < R$ and the total current. [3 points]
 - (b) Use Ampere's law to derive $B(r)$ for $r < R$ and for $r > R$. [3 points]
 - (c) Find the radius at which B inside the wire is greatest and evaluate that field. [3 points]
 - (d) Is B continuous at $r = R$? Check with your expressions. [1 point]
2. [12 points] Question 2. A coil of 100 turns, each of area 0.020 m², rotates at constant angular speed $\omega = 100$ rad/s in a uniform 0.30 T field about an axis perpendicular to the field. The coil is connected to a $12\ \Omega$ resistor. At $t = 0$ the plane of the coil is perpendicular to the field.
 - (a) Derive the emf as a function of time. [3 points]
 - (b) On one set of axes sketch the emf for two full revolutions. Label the maximum and the first three zero crossings with numerical values. [3 points]
 - (c) On a second set of axes, aligned in time, sketch the power dissipated in the resistor. Label the maximum, the average, and the period. [3 points]
 - (d) The rotation speed is halved. State the new maximum emf and maximum power and explain the change in the shape of both graphs. [3 points]
3. [10 points] Question 3. A student measures the inductance of a coil using an LC oscillator. The coil is connected to a charged capacitor, and an oscilloscope records the oscillation period T for five values of C :
 C (microfarads): 0.10, 0.20, 0.30, 0.40, 0.50
 T (milliseconds): 0.444, 0.628, 0.770, 0.889, 0.993
 - (a) Describe a procedure to take the data, including how the period is read accurately. [2 points]
 - (b) Starting from the circuit equation, show that plotting T^2 versus C should give a straight line, and state what the slope represents. [3 points]
 - (c) Calculate T^2 for each point, plot the graph with labeled axes and units, draw a best-fit line, and use its slope to find L . [4 points]
 - (d) State one change that would reduce the uncertainty in L . [1 point]
4. [8 points] Question 4. Two long straight parallel wires, X and Y, are 0.080 m apart and carry equal 5.0 A currents in opposite directions: X out of the page, Y into the page, X on the left. Point P is midway between them.
 - (a) Determine the direction of the net magnetic field at P and justify using each wire's field. [2 points]
 - (b) A positive charge moves out of the page through P at 2.0×10^5 m/s with charge 1.6×10^{-19} C. State the direction of the magnetic force and calculate its magnitude. [3 points]
 - (c) Y's current is now reversed so both currents are out of the page. Give the net field at P and the force on the moving charge, with a reason. [1 point]
 - (d) With the original currents, find the force per unit length on wire X and state whether it is attractive or repulsive. [2 points]
5. [10 points] Question 5 (extra). A 24 V battery of negligible internal resistance is connected at $t = 0$ to a $30\ \Omega$ resistor in series with a 0.60 H inductor.
 - (a) Starting from the loop rule, derive $I(t)$. [3 points]
 - (b) Derive an expression for the rate at which energy is stored in the inductor and find the time at which this rate is greatest. [3 points]
 - (c) Calculate the maximum rate and the energy stored at that time. [2 points]
 - (d) After a long time the battery is removed and the circuit is closed on itself. Find the total thermal energy dissipated afterward. [2 points]

Key Free-Response Clinic: Scoring Guides and Model Responses**X** Free-Response Clinic

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1. (a) 3 points: $I_{enc} = \int_0^r J 2\pi r' dr' [1] = 2\pi J_0 \left(\frac{r^2}{2} - \frac{r^4}{4R^2} \right) [1]$; at $r = R$, $I = \pi J_0 R^2/2 = 28 \text{ A} [1]$. (b) 3 points: $B 2\pi r = \mu_0 I_{enc} [1]$; inside $B = \frac{\mu_0 J_0}{2} \left(r - \frac{r^3}{2R^2} \right) [1]$; outside $B = \mu_0 J_0 R^2/(4r) [1]$. (c) 3 points: $dB/dr = 0$ gives $1 - 3r^2/(2R^2) = 0 [1]$, $r = R\sqrt{2/3} = 2.4 \text{ mm} [1]$, $B = \frac{\sqrt{6}}{9} \mu_0 J_0 R = 2.1 \text{ mT} [1]$. (d) 1 point: yes; at $r = R$ the inside form gives $\mu_0 J_0 R/4$ and the outside form gives the same, 1.9 mT.

Model response: I sum current over thin rings of area $2\pi r' dr'$ because J varies with radius. Taking a circular Amperian loop of radius r around the axis, symmetry makes B tangential and constant on the loop, so $B(2\pi r) = \mu_0 I_{enc}$. Inside, $B = \frac{\mu_0 J_0}{2} \left(r - \frac{r^3}{2R^2} \right)$, which rises linearly near the axis and then flattens. Outside, all 28 A is enclosed and B falls as $1/r$. Setting the derivative to zero locates the maximum at $r = R\sqrt{2/3}$, inside the wire, which makes sense because the current density is lowest near the surface. Both formulas give 1.9 mT at the surface, so the field is continuous, as it must be for a finite current density.

2. (a) 3 points: $\Phi = NBA \cos \omega t [1]$; $\mathcal{E} = -d\Phi/dt = NBA\omega \sin \omega t [1]$; $\mathcal{E}_{max} = (100)(0.30)(0.020)(100) = 60 \text{ V} [1]$. (b) 3 points: sine starting at zero with peaks of 60 V [1]; period $T = 2\pi/\omega = 62.8 \text{ ms}$ and zeros at 0, 31.4, 62.8 ms [1]; two full cycles drawn to 125.7 ms [1]. (c) 3 points: $P = \mathcal{E}^2/R = 300 \sin^2 \omega t \text{ W}$, never negative [1]; maximum 300 W and average 150 W labeled [1]; period 31.4 ms, half the emf's, same zeros [1]. (d) 3 points: $\mathcal{E}_{max} = 30 \text{ V} [1]$; $P_{max} = 75 \text{ W}$, one fourth, since $P \propto \omega^2 [1]$; periods double, so both graphs stretch horizontally and the amplitudes shrink [1].

Model response: The flux through the coil is $NBA \cos \omega t$, so Faraday's law gives $\mathcal{E} = NBA\omega \sin \omega t$, a sine with amplitude 60 V. It is zero when the plane is perpendicular to the field, at 0, 31 ms and 63 ms, and extreme when the plane is parallel to the field. The current is $\mathcal{E}/R$, so the power is $\mathcal{E}^2/R = 300 \sin^2 \omega t \text{ W}$: a curve that touches zero at every zero of the emf, peaks at 300 W, averages 150 W, and repeats every 31 ms, twice per emf cycle. Halving ω halves the emf amplitude because the flux changes more slowly, and quarters the power, and both curves stretch to twice the period.

3. (a) 2 points: charge the capacitor with a source, then let it oscillate through the coil [1]; record voltage across C on the oscilloscope and time several periods (for example ten) and divide, repeating at each C [1]. (b) 3 points: $L d^2 q/dt^2 + q/C = 0 [1]$; $\omega = 1/\sqrt{LC}$ so $T = 2\pi\sqrt{LC} [1]$; $T^2 = 4\pi^2 LC$, a line through the origin with slope $4\pi^2 L [1]$. (c) 4 points: T^2 in ms^2 is 0.197, 0.394, 0.593, 0.790, 0.986 [1]; axes labeled with quantity and units and a uniform scale [1]; line drawn through the points [1]; slope = $1.97 \text{ ms}^2/\mu\text{F} = 1.97 \text{ s}^2/\text{F}$, so $L = 1.97/(4\pi^2) = 0.050 \text{ H} [1]$. (d) 1 point: for example, measure many periods and average, or use more values of C over a wider range.

Model response: With the capacitor charged, I close the circuit and record the voltage oscillation, timing ten periods and dividing by ten at each capacitance. The loop equation $L\ddot{q} + q/C = 0$ has angular frequency $1/\sqrt{LC}$, so $T^2 = 4\pi^2 LC$. The graph of T^2 against C is a straight line through the origin with slope $4\pi^2 L$. My computed T^2 values rise by about 0.197 ms^2 for each 0.10 microfarad, so the best-fit slope is $1.97 \text{ s}^2/\text{F}$ (taken from the line, not from a single point), which gives $L = 0.050 \text{ H}$. The intercept is within the scatter of zero, which supports the model. Averaging more periods would reduce the timing uncertainty.

4. (a) 2 points: the net field points up the page [1]; X (current out) gives a field at P pointing up and Y (current in) also gives a field at P pointing up, so they add [1]. (b) 3 points: $\vec{F} = q\vec{v} \times \vec{B}$ with v out and B up gives a force toward the left, toward wire X [1]; $B = 2\mu_0 I/(2\pi d)$ with $d = 0.040 \text{ m}$, so $B = 5.0 \times 10^{-5} \text{ T} [1]$; $F = qvB = 1.6 \times 10^{-18} \text{ N} [1]$. (c) 1 point: the fields now cancel, $B_{net} = 0$, so the force is zero [1]. (d) 2 points: $F/\ell = \mu_0 I^2/(2\pi D) = 6.3 \times 10^{-5} \text{ N/m} [1]$; opposite currents repel [1].

Model response: By the right-hand rule, the current out of the page in X makes counterclockwise field circles, which at P (to the right of X) point up the page; the current into the page in Y makes clockwise circles, which at P (to the left of Y) also point up. The two fields add: $B = 2 \times \frac{\mu_0 I}{2\pi(0.040)} = 5.0 \times 10^{-5} \text{ T}$. For a charge moving out of the page, $\vec{v} \times \vec{B}$ points to the left, so the force of $qvB = 1.6 \times 10^{-18} \text{ N}$ pushes it toward X. If the currents are both out of the page, the fields at P point in opposite directions and have equal size, so the net is zero and so is the force. Opposite currents repel, with $F/\ell = 6.3 \times 10^{-5} \text{ N/m}$ on X, directed away from Y.

5. (a) 3 points: $\mathcal{E} - IR - L dI/dt = 0 [1]$; separate and integrate with $I(0) = 0 [1]$; $I = 0.80(1 - e^{-t/\tau}) \text{ A}$, $\tau = L/R = 20 \text{ ms} [1]$. (b) 3 points: $dU/dt = LI dI/dt = IV_L = \frac{\mathcal{E}^2}{R} (e^{-t/\tau} - e^{-2t/\tau}) [1]$; set the derivative to zero: $e^{-t/\tau} = 1/2 [1]$; $t = \tau \ln 2 = 13.9 \text{ ms} [1]$. (c) 2 points: $P_{max} = \mathcal{E}^2/4R = 4.8 \text{ W} [1]$; there $I = 0.40 \text{ A}$ and $U = \frac{1}{2}(0.60)(0.40)^2 = 0.048 \text{ J} [1]$. (d) 2 points: all stored energy is dissipated in R [1]; $\frac{1}{2}LI_f^2 = \frac{1}{2}(0.60)(0.80)^2 = 0.19 \text{ J} [1]$.

Model response: After the switch closes, $\mathcal{E} = IR + L dI/dt$. Separating variables, $\int_0^I \frac{dI'}{0.80 - I'} = \int_0^t dt'/\tau$ gives $I = 0.80(1 - e^{-t/\tau}) \text{ A}$. The stored-energy rate is $LI dI/dt = IV_L$; with $V_L = 24e^{-t/\tau}$ this is $19.2(e^{-t/\tau} - e^{-2t/\tau}) \text{ W}$, largest when $e^{-t/\tau} = 1/2$, at 13.9 ms and equal to 4.8 W. At that moment the current is half of its final value, 0.40 A, and the inductor holds 0.048 J, only a quarter of its eventual 0.19 J. When the battery is removed, the inductor current decays through R and all 0.19 J becomes heat.

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Reference: Formulas, Skills Checklist, and Glossary

Quick-reference pages to review the night before the exam.

R1 Equations and Constants Reference

A working summary of the relationships and constants used in each unit, written in our own words. On exam day use the official reference information provided, and learn its layout in advance.

This page lists what you need to recall or rebuild, organized by unit. The exam supplies a reference sheet, so the goal is fluency: know which relation applies, when it is valid, and how to derive it from calculus. Symbols: ϵ_0 is vacuum permittivity, μ_0 vacuum permeability, κ dielectric constant, ρ resistivity (or charge density by context), λ , σ charge per length and per area.

Conventions and Habits

- ◆ Current direction means the direction positive charge would drift. Potential is zero at infinity for isolated point charges unless a problem sets another reference.
- ◆ Ideal wires, batteries, meters and inductors are assumed unless resistance or internal resistance is given. Capacitors are air-filled unless a dielectric is stated.
- ◆ A conductor in equilibrium has no field inside it, and its surface is an equipotential.
- ◆ Check every result: units, limiting cases (r very large, $t = 0$, t very large), and how the answer changes when a parameter doubles.

Constants

Quantity	Value
Coulomb constant $k = 1/4\pi\epsilon_0$	$8.99 \times 10^9 \text{ N m}^2/\text{C}^2$
Vacuum permittivity ϵ_0	$8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2$
Vacuum permeability μ_0	$4\pi \times 10^{-7} \text{ T m/A}$
Elementary charge e	$1.60 \times 10^{-19} \text{ C}$
Electron mass	$9.11 \times 10^{-31} \text{ kg}$
Proton mass	$1.67 \times 10^{-27} \text{ kg}$
Speed of light c	$3.00 \times 10^8 \text{ m/s}$

Unit 8: Electric Charges, Fields, and Gauss's Law

Relationship	What it tells you
$F = kq_1q_2/r^2$; $\vec{E} = \vec{F}/q$	Force between point charges; field as force per unit test charge
$E = kq/r^2$ (point), $E = \lambda/2\pi\epsilon_0 r$ (line), $\sigma/2\epsilon_0$ (sheet)	Fields of standard sources; integrate dE for rods, rings, arcs
$\Phi_E = \oint \vec{E} \cdot d\vec{A} = Q_{enc}/\epsilon_0$	Gauss's law; choose a surface with spherical, cylindrical or planar symmetry

Unit 9: Electric Potential

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Relationship	What it tells you
$\Delta V = -\int \vec{E} \cdot d\vec{\ell}$; $E_x = -dV/dx$	Potential from field and back; field points toward lower potential
$V = kq/r$; $U = qV = kq_1q_2/r$	Point-charge potential with zero at infinity; pair energy
$K + qV$ is conserved	Energy conservation for charges in a static field

Unit 10: Conductors and Capacitors

Relationship	What it tells you
$C = Q/V$; $C = \kappa\epsilon_0 A/d$ (plates)	Definition; parallel plates (not for cylinders or spheres)
Parallel: $C_1 + C_2$; series: $1/C_1 + 1/C_2$	Combining capacitors
$U = \frac{1}{2}CV^2 = Q^2/2C$; $u = \frac{1}{2}\epsilon_0 E^2$	Stored energy and energy density

In electrostatic equilibrium a conductor has $E = 0$ inside, constant potential throughout, and charge on its surface; connected conductors share the same potential.

Unit 11: Electric Circuits

Relationship	What it tells you
$I = dQ/dt$; $R = \rho\ell/A$; $V = IR$; $P = IV$	Current, resistance geometry, Ohm's law, power
Series: $R_1 + R_2$; parallel: $1/R_1 + 1/R_2$	Equivalent resistance
$\sum \Delta V = 0$ around a loop; $\sum I_{in} = \sum I_{out}$	Loop and junction rules
$q = Q_f(1 - e^{-t/RC})$ charging; $q = Q_0 e^{-t/RC}$ discharging	RC circuits, time constant $\tau = RC$

Unit 12: Magnetic Fields and Electromagnetism

Relationship	What it tells you
$\vec{F} = q\vec{v} \times \vec{B}$; $r = mv/qB$	Force on a moving charge; circular path
$\vec{F} = I\vec{\ell} \times \vec{B}$	Force on a current segment
$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2}$	Biot-Savart; wire $\mu_0 I/2\pi r$, loop center $\mu_0 I/2R$
$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$; solenoid $B = \mu_0 nI$	Ampere's law with symmetry

Unit 13: Electromagnetic Induction

Relationship	What it tells you
$\Phi_B = \int \vec{B} \cdot d\vec{A}$	Magnetic flux
$\mathcal{E} = -N d\Phi_B/dt$; motional $\mathcal{E} = B\ell v$	Faraday's law; minus sign from Lenz's law
$V_L = L dI/dt$; $U = \frac{1}{2}LI^2$; solenoid $L = \mu_0 N^2 A/\ell$	Inductor voltage, energy, geometry
$I = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau})$, $\tau = L/R$	LR circuit after the switch closes
$\omega = 1/\sqrt{LC}$	Natural frequency of an LC circuit

R2 Skills Checklist and Glossary

Check yourself against the nine exam skills, then use the glossary to confirm every term has a precise meaning you can state in a sentence.

Skills Checklist

Tick each statement only when you can do it without notes, on an unfamiliar problem.

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- ◆ Create representations: sketch field lines, equipotentials and circuit diagrams; sketch $E(r)$ and $V(r)$ with correct shapes ($1/r$, $1/r^2$, linear).
- ◆ Create graphs: label quantity and units, use uniform scales, and mark intercepts, zero crossings and asymptotes.
- ◆ Derive symbolically: set up and evaluate charge, flux or current integrals, and show the integral before substituting limits.
- ◆ Calculate with units: carry a symbolic result to a number, and sanity-check magnitude and sign.
- ◆ Compare and predict: state how an answer scales when a radius, length, speed or frequency changes.
- ◆ Design experiments: name measured and calculated quantities, controlled variables, and the linearized plot whose slope gives the constant.
- ◆ Argue from principles: apply Gauss, Ampere, Faraday and Lenz with the stated conditions, and justify directions in words.
- ◆ Differential equations: write the loop rule, solve RC and LR in separable form, and explain the time constant.
- ◆ Right-hand rules: use them twice when a field produces a force.

Glossary

Term	Meaning
Charge quantization	Charge comes in whole multiples of e
Conservation of charge	Net charge of an isolated system is constant
Electric field	Force per unit positive test charge
Superposition	Net field or potential is the sum from each source
Linear charge density	Charge per unit length, λ
Electric flux	Surface integral of $\mathbf{E} \cdot d\mathbf{A}$
Gaussian surface	Closed imagined surface used with Gauss's law
Enclosed charge	Charge inside the Gaussian surface
Electric potential energy	Energy of a charge configuration
Electric potential	Potential energy per unit charge
Equipotential	Surface of constant potential, perpendicular to $\mathbf{E}$
Electron volt	Energy gained by e through 1 V
Electrostatic equilibrium	No net motion of charge in a conductor
Induced charge	Charge shifted on a conductor by a nearby field
Capacitance	Charge stored per volt, $C = Q/V$
Dielectric	Insulator that raises capacitance by κ
Dielectric constant	Factor κ by which capacitance rises
Current	Rate of charge flow, dQ/dt
Current density	Current per unit area, $\mathbf{J}$
Resistivity	Material property in $R = \rho L/A$
Ohmic	Obeying $V = IR$ with constant R
emf	Energy per charge supplied by a source
Loop rule	Potential changes around a closed path sum to zero
Junction rule	Current into a node equals current out
Time constant	Characteristic time for exponential change
Steady state	Quantities no longer changing with time
Magnetic field	Field that exerts force on moving charges
Tesla	SI unit of B , N per A m
Lorentz force	Total electric plus magnetic force on a charge

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Term	Meaning
Cyclotron radius	Radius of a charge's circular path in B
Right-hand rule	Direction convention for cross products
Biot-Savart law	Field from each current element
Amperian loop	Closed path used with Ampere's law
Enclosed current	Current through the loop's surface
Solenoid	Tightly wound coil with nearly uniform interior B
Magnetic flux	Surface integral of $\mathbf{B} \cdot d\mathbf{A}$
Weber	Unit of flux, T m squared
Faraday's law	Induced emf equals rate of flux change
Lenz's law	Induced effect opposes the flux change
Motional emf	emf from a conductor moving through B
Induced electric field	Non-conservative field from changing flux
Self-inductance	Flux linkage per unit current, L
Henry	Unit of inductance, V s per A
LR circuit	Resistor and inductor in one loop
LC circuit	Capacitor and inductor exchanging energy
Eddy current	Circulating induced current in bulk metal
Back emf	Induced voltage opposing the current change
Flux linkage	N times the flux through one turn
Test charge	Small probe charge that does not disturb the field
Potential difference	Work per unit charge between two points
Permittivity	Constant setting electric field strength per charge
Permeability	Constant setting magnetic field strength per current
Energy density	Stored energy per unit volume of field
Equivalent resistance	Single resistor that replaces a network
Net field	Vector sum of fields from all sources

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