



Ratios, Rates, and Proportional Reasoning

Four focused lessons, worked examples, a mistake clinic, and 60 practice problems with full explanations.

In this booster

02.1 Ratios and Equivalent Ratios

02.2 Unit Rates and Unit Pricing

02.3 Solving Proportions

02.4 Unit Conversion and Dimensional Analysis

Every section teaches the idea, walks through the most common problem types, flags the traps students fall into, and ends with fifteen practice problems. Full answers and explanations are at the back.

02.1 Ratios and Equivalent Ratios

Learn to write and simplify ratios, tell part-to-part from part-to-whole, build equivalent ratios with ratio tables and double number lines, and share a total in a given ratio.

Why This Matters

A ratio compares two quantities, and comparing is something we do all day: the number of players on each team, cups of water to cups of rice, miles walked to minutes spent. Ratios drive recipes, maps, medicine doses, paint mixes, screen sizes, and game scores. They are also the foundation of rates, percents, similar figures, slope, and probability.

Students most often get stuck in three places: mixing up the order of a ratio (boys to girls is not the same as girls to boys), confusing part-to-part with part-to-whole, and adding instead of multiplying when making an equivalent ratio. This section gives you tools to avoid those errors: ratio tables, tape diagrams, and the simple idea of a ratio as a "recipe" you can repeat.

Core Ideas

A **ratio** compares two quantities. There are three ways to write the ratio of 12 boys to 18 girls: with the word "to" (12 to 18), with a colon (12:18), or as a fraction ($\frac{12}{18}$). Order matters, so the first number goes with the first thing named. The ratio of girls to boys is 18:12, which is a different ratio.

Part-to-Part and Part-to-Whole Ratios

A **part-to-part** ratio compares one group to another group: boys to girls is 12:18.

A **part-to-whole** ratio compares one group to the total: boys to all students is 12:30, because $12 + 18 = 30$.

To turn a part-to-part ratio $a : b$ into part-to-whole ratios, the whole has $a + b$ parts, so the ratios are $a : (a + b)$ and $b : (a + b)$.

girls = 2 : 3

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ents = 2 : 5

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A class has 5 equal parts: 2 parts are boys and 3 parts are girls. Boys to girls is 2:3 (part-to-part). Boys to the whole class is 2:5 (part-to-whole).

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Equivalent Ratios

Two ratios are **equivalent** when one can be turned into the other by multiplying or dividing BOTH numbers by the same nonzero number:

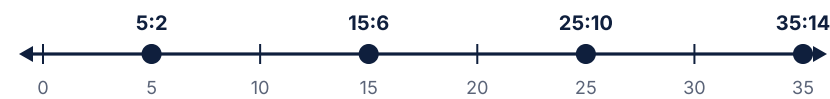
$$a : b = (a \cdot n) : (b \cdot n).$$

To **simplify** a ratio, divide both numbers by their greatest common factor. Ratios with whole numbers are in simplest form when the only common factor is 1.

A **ratio table** is a neat way to list equivalent ratios. Each column is one version of the same recipe. If a recipe uses 2 cups of rice for every 5 cups of water, then doubling, tripling, and so on give more columns. You can also go backwards by dividing.

Cups of rice	2	4	6	10	14
Cups of water	5	10	15	25	35

Look across any column and you will see the same ratio: 2 : 5, 4 : 10, 6 : 15, 10 : 25, 14 : 35. Each water amount is 2.5 times the rice amount. A **double number line** shows the same idea: two number lines side by side, lined up so that matching values stack above each other. When you move to a new spot on the top line, the bottom line moves by the matching amount.



The line shows cups of water. Each label gives water:rice, so 5 cups of water goes with 2 cups of rice, and 35 cups of water goes with 14 cups of rice.

Worked Examples

EXAMPLE 1: WRITING AND SIMPLIFYING RATIOS

A class has 12 boys and 18 girls. Write (a) the ratio of boys to girls and (b) the ratio of boys to all students, both in simplest form.

SOLUTION

(a) Boys to girls is 12 : 18. The GCF of 12 and 18 is 6, so divide both by 6: 2 : 3.

(b) The class has $12 + 18 = 30$ students. Boys to all students is 12 : 30. The GCF is 6, so the ratio is 2 : 5.

Notice that (a) is part-to-part and (b) is part-to-whole.

EXAMPLE 2: FINDING AN EQUIVALENT RATIO

Find the missing number: $3 : 5 = 12 : ?$

SOLUTION

Compare the first numbers: $3 \rightarrow 12$ means multiplying by 4.

Multiply the second number by the same amount: $5 \cdot 4 = 20$.

So $3 : 5 = 12 : 20$.

Check by simplifying: 12 : 20 divided by 4 gives 3 : 5.

EXAMPLE 3: USING A RATIO TABLE

A rice recipe uses 2 cups of rice for every 5 cups of water. (a) How much water goes with 10 cups of rice? (b) How much rice goes with 35 cups of water?

SOLUTION

(a) From 2 to 10 is multiplying by 5, so water is $5 \cdot 5 = 25$ cups.

(b) From 5 to 35 is multiplying by 7, so rice is $2 \cdot 7 = 14$ cups.

Check against the table: the pairs (10, 25) and (14, 35) are both in it.

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EXAMPLE 4: SHARING A TOTAL IN A RATIO

Concrete is made by mixing sand and cement in a ratio of 3:2 by weight. How many kilograms of each are needed to make 40 kilograms of concrete?

SOLUTION

Add the ratio parts: $3 + 2 = 5$ parts in all.

One part weighs $40 \div 5 = 8$ kilograms.

Sand: $3 \cdot 8 = 24$ kg. Cement: $2 \cdot 8 = 16$ kg.

Check: $24 + 16 = 40$, and $24 : 16$ simplifies to $3 : 2$.

EXAMPLE 5: COMPARING TWO MIXES

Orange drink A uses 3 cups of juice with 5 cups of water. Drink B uses 4 cups of juice with 7 cups of water. Which drink tastes more strongly of juice?

SOLUTION

Compare juice to the whole drink, not juice to water, so that the comparison is fair. Drink A has $3 + 5 = 8$ cups in all, and drink B has $4 + 7 = 11$ cups in all.

Juice fractions: A is $\frac{3}{8}$ and B is $\frac{4}{11}$.

Common denominator 88: $\frac{3}{8} = \frac{33}{88}$ and $\frac{4}{11} = \frac{32}{88}$.

$33 > 32$, so drink A tastes more strongly of juice, but only slightly.

EXAMPLE 6: COMPARING TWO AMOUNTS IN A RATIO

At an animal shelter, the ratio of cats to dogs is 3:4. There are 56 cats and dogs in all. How many more dogs than cats are there?

SOLUTION

Add the ratio parts: $3 + 4 = 7$ parts.

One part is $56 \div 7 = 8$ animals.

Cats: $3 \cdot 8 = 24$. Dogs: $4 \cdot 8 = 32$.

Difference: $32 - 24 = 8$ more dogs. (Shortcut: the difference is 1 part, and one part is 8.)

Check: $24 + 32 = 56$, and $24 : 32$ simplifies to $3 : 4$.

Coach's Playbook

- 1 Label the order.** Write what each number is next to it, for example "boys 12 : girls 18." Keep that order for the whole problem.
 - 2 Decide part-to-part or part-to-whole.** If a total is given or asked for, add the ratio parts.
 - 3 Find the multiplier.** Look at the number you know, and ask what you multiply by to get from the ratio to the real amount.
 - 4 Apply it to every part.** Multiply every number in the ratio by that same multiplier.
 - 5 Check by simplifying back.** Your answers must simplify to the original ratio, and the parts must add to the total if there is one.
- ◆ Draw a **tape diagram**: a rectangle cut into equal parts, one rectangle per ratio unit. It makes "3 parts to 2 parts" visible.
 - ◆ If a ratio has decimals or fractions, scale it to whole numbers first. For example $0.5 : 2$ becomes $1 : 4$ when you multiply by 2.
 - ◆ Ratios compare in the same units. Convert first if one amount is in minutes and the other in hours.

Common traps

(1) Adding instead of multiplying: going from $3 : 5$ to $6 : 8$ by adding 3 to each part gives a different ratio. (2) Reversing the order, such as writing girls to boys when the question asks for boys to girls. (3) Treating a part-to-part ratio like part-to-whole, so that in a 3:2 mix you divide the total by 3 instead of by 5. (4) Not simplifying all the way, or simplifying only one number.

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Mistake Clinic

Spot the error

Problem: Simplify $18 : 24$. **Student's work:** Divide the first number by 2 to get 9, and leave the second number alone: $9 : 24$. **What went wrong:** The student divided only the first number. A ratio changes only if you divide (or multiply) BOTH numbers by the same amount. Dividing one number changes the comparison. **Correct solution:** The GCF of 18 and 24 is 6. Divide both: $18 \div 6 = 3$ and $24 \div 6 = 4$. The simplified ratio is $3 : 4$.

Spot the error

Problem: Find an equivalent ratio to $3 : 5$ that starts with 6. **Student's work:** The first number went up by 3, so add 3 to the second number: $6 : 8$. **What went wrong:** The student added instead of multiplying. Ratios are scaled by a multiplier, not by an addition. The ratio $3 : 5$ is about 0.6, but $6 : 8$ is 0.75, so they are different. **Correct solution:** The first number was multiplied by 2 ($3 \cdot 2 = 6$), so multiply the second number by 2 as well: $5 \cdot 2 = 10$. The equivalent ratio is $6 : 10$.

Spot the error

Problem: Red and blue marbles are in the ratio $2:3$. There are 25 marbles in all. How many are red? **Student's work:** $25 \div 2 = 12.5$ red marbles. **What went wrong:** The student divided the total by the red part only. The total is split into $2 + 3 = 5$ equal parts, not 2. **Correct solution:** One part is $25 \div 5 = 5$ marbles. Red is 2 parts: $2 \cdot 5 = 10$ marbles. Blue is 3 parts: 15 marbles. Check: $10 + 15 = 25$.

Real-World Connection

A music teacher mixes a school marching band with 3 woodwinds for every 2 brass players. For 40 players, the ratio tells her to expect 24 woodwinds and 16 brass. The same thinking helps coaches plan teams and cooks scale recipes.

Challenge Corner

The ratio of A to B is $2 : 3$, and the ratio of B to C is $4 : 5$. Find the ratio $A : B : C$ using whole numbers. **Solution:** B appears in both ratios, but with different values (3 and 4). Make B the same in both by using the least common multiple of 3 and 4, which is 12. Scale the first ratio by 4: $2 : 3 = 8 : 12$. Scale the second ratio by 3: $4 : 5 = 12 : 15$. Now B is 12 in both, so $A : B : C = 8 : 12 : 15$. Check: $\frac{8}{12} = \frac{2}{3}$ and $\frac{12}{15} = \frac{4}{5}$.

Practice

PRACTICE Ratios and Equivalent Ratios

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- A shelter has 15 cats and 25 dogs. What is the ratio of cats to dogs in simplest form?
(A) 15:25 (B) 3:5 (C) 5:3 (D) 3:8
- In a class of 24 students, 9 are in the band. What is the ratio of students in the band to students not in the band, in simplest form?
(A) 3:8 (B) 9:24 (C) 5:3 (D) 3:5
- Which ratio is equivalent to $4 : 7$?
(A) 20:35 (B) 8:11 (C) 12:15 (D) 4:10
- The ratio of blue paint to yellow paint is $5 : 3$. A mix has 15 liters of blue. How many liters of yellow are in it?
(A) 25 (B) 5 (C) 9 (D) 24

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5. A team has girls and boys in the ratio 4 : 5. There are 36 players. How many are girls?
(A) 20 (B) 16 (C) 32 (D) 9
6. Ana trains by walking, biking, and running in the ratio 2 : 3 : 5. Her workout lasts 60 minutes. How many minutes does she run?
(A) 12 (B) 18 (C) 20 (D) 30
7. Which ratio is NOT equivalent to 6 : 15?
(A) 10:20 (B) 2:5 (C) 12:30 (D) 18:45
8. The ratio of mushroom slices to pepper slices on a pizza is 3 : 7. There are 21 pepper slices. How many mushroom slices are there?
(A) 49 (B) 18 (C) 9 (D) 24
9. At a school, the ratio of fourth graders to fifth graders is 5 : 6. There are 330 students in those two grades. How many more fifth graders than fourth graders are there?
(A) 150 (B) 30 (C) 180 (D) 55
10. Write 48 : 84 in simplest form.
11. A smoothie uses 3 cups of fruit for every 2 cups of yogurt. How many cups of fruit go with 12 cups of yogurt?
12. Paint X has red and white in the ratio 5:3. Paint Y has red and white in the ratio 7:4. Which paint is redder (has the larger fraction of red)? Answer X or Y.
13. Sam and Ava share 84 dollars in the ratio 4:3 (Sam:Ava). How many dollars must Sam give Ava so that they have the same amount?
14. Write the ratio of 36 minutes to 2 hours in simplest form.
15. A 24-liter mixture has water and syrup in the ratio 5:3. How many liters of water must be added so that the ratio of water to syrup becomes 2:1?

02.2 Unit Rates and Unit Pricing

Find unit rates and speeds, compare prices to find the best buy, use distance, rate, and time, and spot the constant of proportionality in tables, graphs, and equations.

Why This Matters

A **rate** compares two quantities with different units, such as miles per hour, dollars per pound, or words per minute. Rates tell you how fast, how much for how many, and which choice is the better deal. Every time you compare prices in a store, check your phone data plan, or judge a runner's pace, you are using unit rates.

Students get stuck because they divide in the wrong order (hours divided by miles instead of miles divided by hours), compare totals instead of unit prices, and forget what the units mean. This section shows how to find a unit rate, how to build the habit of reading units, and how to recognize a proportional relationship in a table or a graph.

Core Ideas

A **rate** is a ratio of two quantities with different units, such as 168 miles in 3 hours. A **unit rate** is a rate whose second quantity is 1: it tells you how much of the first quantity there is for ONE of the second. To find a unit rate, divide the first quantity by the second quantity.

Unit Rate

$$\text{unit rate} = \frac{\text{amount of the first quantity}}{\text{amount of the second quantity}}$$

The word "per" means "for each one." Miles per hour means miles for each hour, so divide miles by hours: 168 miles ÷ 3 hours = 56 miles per hour.

Speed, distance, and time. Speed is a rate. The three quantities are linked by one formula that you can rearrange: distance equals speed times time. If you know any two, you can find the third.

Distance, Rate, and Time

$$d = r \cdot t \quad r = \frac{d}{t} \quad t = \frac{d}{r}$$

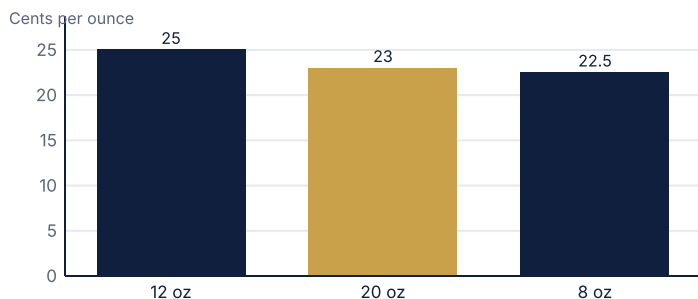
Keep units consistent. If the speed is in kilometers per hour, then the time must be in hours.

Unit pricing. The unit price is the cost of one unit (one ounce, one pound, one item). The **best buy** is the choice with the lowest unit price. A bigger package is not always cheaper per unit, so always divide to check.

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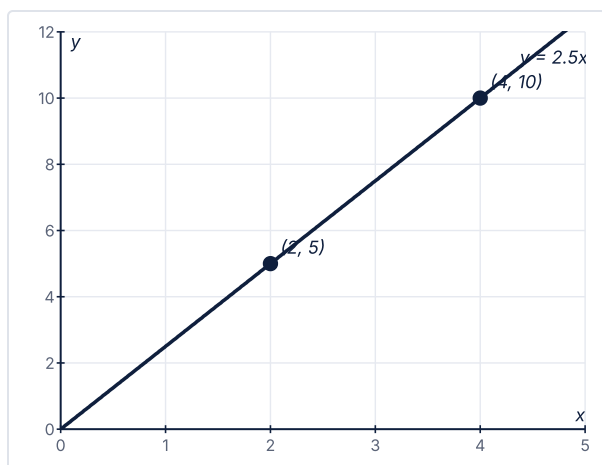
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Unit prices for three sizes of the same snack. The smallest package has the lowest price per ounce.

Proportional relationships. Two quantities are in a **proportional relationship** when their ratio is always the same. That constant ratio $k = \frac{y}{x}$ is the **constant of proportionality**, and it is the unit rate. The equation is $y = kx$. In a table, $\frac{y}{x}$ is the same in every column. On a graph, the points lie on a straight line that passes through the origin $(0, 0)$.



A proportional relationship is a line through the origin. The point $(1, 2.5)$ tells you the unit rate: 2.5 for each 1.

Worked Examples

EXAMPLE 1: FINDING A UNIT RATE

(a) A bus travels 168 miles in 3 hours. Find its speed. (b) Five pounds of apples cost 7.50 dollars. Find the price per pound.

SOLUTION

(a) Miles per hour: $168 \div 3 = 56$ miles per hour.

(b) Dollars per pound: $7.50 \div 5 = 1.50$ dollars per pound.

Check by multiplying back: $56 \times 3 = 168$ and $1.50 \times 5 = 7.50$.

EXAMPLE 2: THE BEST BUY

A snack comes in three sizes: 12 ounces for 3.00 dollars, 20 ounces for 4.60 dollars, and 8 ounces for 1.80 dollars. Which is the best buy?

SOLUTION

Find the price per ounce for each size.

12 oz: $3.00 \div 12 = 0.25$ dollars (25 cents).

20 oz: $4.60 \div 20 = 0.23$ dollars (23 cents).

8 oz: $1.80 \div 8 = 0.225$ dollars (22.5 cents).

The lowest unit price is 22.5 cents, so the 8 ounce size is the best buy, even though it is the smallest.

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EXAMPLE 3: DISTANCE, RATE, AND TIME

A cyclist rides at a steady 15 kilometers per hour. (a) How far does she ride in 2.4 hours? (b) How long does a 27 kilometer ride take?

SOLUTION

(a) $d = r \cdot t = 15 \times 2.4 = 36$ kilometers.

(b) $t = \frac{d}{r} = \frac{27}{15} = 1.8$ hours.

1.8 hours is 1 hour and 48 minutes, since $0.8 \times 60 = 48$.

EXAMPLE 4: IS THE TABLE PROPORTIONAL?

Table A shows $x = 2, 5, 8$ and $y = 7, 17.5, 28$. Table B shows $x = 2, 5, 8$ and $y = 7, 13, 19$. Which table is proportional, and what is its constant of proportionality?

SOLUTION

Table A: $\frac{7}{2} = 3.5$, $\frac{17.5}{5} = 3.5$, $\frac{28}{8} = 3.5$. All the ratios match.

Table B: $\frac{7}{2} = 3.5$ but $\frac{13}{5} = 2.6$. The ratios are not equal.

Table A is proportional with $k = 3.5$, so $y = 3.5x$.

Table B has a constant increase of 2 per unit but it does not pass through the origin, so it is not proportional.

EXAMPLE 5: READING A GRAPH

The graph above shows the cost y (in dollars) of x pounds of cherries. What is the constant of proportionality, what does it mean, and how much do 6 pounds cost?

SOLUTION

Pick a point on the line: $(2, 5)$. Then $k = \frac{y}{x} = \frac{5}{2} = 2.5$. The point $(4, 10)$ gives the same $\frac{10}{4} = 2.5$.

The number 2.5 means that cherries cost 2.5 dollars for each pound.

The equation is $y = 2.5x$, so 6 pounds cost $2.5 \times 6 = 15$ dollars.

EXAMPLE 6: COMPARING TWO SPEEDS

Pia runs 2.4 kilometers in 18 minutes. Quinn runs 3.5 kilometers in 28 minutes. Who is running faster, and what is each speed in kilometers per hour?

SOLUTION

Find each speed in kilometers per minute, then convert to per hour (multiply by 60).

Pia: $2.4 \div 18 = 0.1\overline{3}$ km per minute. In one hour: $0.1\overline{3} \times 60 = 8$ km per hour.

Quinn: $3.5 \div 28 = 0.125$ km per minute. In one hour: $0.125 \times 60 = 7.5$ km per hour.

Since $8 > 7.5$, Pia is running faster.

Another way: compare minutes per kilometer. Pia takes $18 \div 2.4 = 7.5$ minutes per km and Quinn takes $28 \div 3.5 = 8$ minutes per km. A smaller pace number means faster.

Coch's Playbook

- 1 Say it with units.** Write the rate as " per ." The thing after "per" goes on the bottom of the fraction.
- 2 Divide to get one.** Divide the total amount by the number of units to get the amount for one unit.
- 3 Compare like with like.** To compare deals, convert each to a price per ounce, per pound, or per item.
- 4 Convert units first.** If one time is in minutes and the other in hours, change one before dividing.
- 5 Check with multiplication.** Multiply the unit rate by the number of units. You should get back the total you started with.

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- ◆ A bigger package is not always the best buy. Divide and check.
- ◆ To compare unit prices with different units, change to the same unit first (for example, ounces to pounds, 16 ounces = 1 pound).
- ◆ For an "equation form" answer in a proportional relationship, write $y = kx$ and put in the number for k .

Common traps

(1) Dividing in the wrong order, such as $3 \div 168$ instead of $168 \div 3$, and then reading the wrong units. (2) Choosing the lowest total price instead of the lowest unit price. (3) Assuming a table is proportional just because the numbers go up by the same amount each step; the ratio y over x must stay the same. (4) Mixing units, like using minutes with a speed measured in miles per hour.

Mistake Clinic

Spot the error

Problem: A car goes 150 miles in 6 hours. What is its speed? **Student's work:** $6 \div 150 = 0.04$ miles per hour. **What went wrong:** The student divided in the wrong order. Miles per hour means miles divided by hours. The answer 0.04 would mean the car crawls along at far less than one mile per hour, which cannot be right for a car that covered 150 miles. **Correct solution:** $150 \div 6 = 25$ miles per hour.

Spot the error

Problem: Which is the better buy: 3 pens for 2.40 dollars or 5 pens for 3.75 dollars? **Student's work:** 2.40 is less than 3.75, so 3 pens for 2.40 dollars is cheaper. **What went wrong:** The student compared the total prices, but the two packs have different numbers of pens. The fair comparison is the price of one pen. **Correct solution:** 3 pens: $2.40 \div 3 = 0.80$ dollars each. 5 pens: $3.75 \div 5 = 0.75$ dollars each. The 5-pen pack costs less per pen, so it is the better buy.

Spot the error

Problem: A table shows $x = 1, 2, 3$ and $y = 5, 7, 9$. The student says it is proportional with $k = 2$ because y goes up by 2 each time. **Student's work:** y increases by the same amount (2) each step, so it is proportional. **What went wrong:** A proportional relationship needs $\frac{y}{x}$ to be constant, not the difference. Here $\frac{5}{1} = 5$, $\frac{7}{2} = 3.5$, and $\frac{9}{3} = 3$, which are all different. **Correct solution:** The relationship is $y = 2x + 3$. It is linear but not proportional, because when $x = 0$ the value of y is 3, not 0. The graph does not pass through the origin.

Real-World Connection

Runners use pace (minutes per mile) and drivers use speed (miles per hour). Both are unit rates. A runner who covers 3.5 miles in 28 minutes has a pace of 8 minutes per mile.

Challenge Corner

A cyclist rides 30 kilometers to a lake at 20 kilometers per hour and returns along the same route at 15 kilometers per hour. What is her average speed for the whole round trip? **Solution:** Average speed is total distance divided by total time, not the average of the two speeds. Going: $30 \div 20 = 1.5$ hours. Returning: $30 \div 15 = 2$ hours. Total time: 3.5 hours. Total distance: 60 km. Average speed = $60 \div 3.5 = \frac{120}{7} \approx 17.1$ km per hour. The simple average, 17.5, is wrong because she spends more time at the slower speed.

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Practice

PRACTICE Unit Rates and Unit Pricing

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

1. A typist types 240 words in 8 minutes. What is the rate in words per minute?
(A) 32 (B) 1,920 (C) 30 (D) 0.033
2. Which package of pencils is the best buy?
(A) 6 for 4.50 dollars (B) 10 for 7.20 dollars (C) 12 for 9.30 dollars
(D) 15 for 11.40 dollars
3. A car travels 150 miles using 6 gallons of gas. At the same rate, how many miles can it travel on 10 gallons?
(A) 60 (B) 25 (C) 900 (D) 250
4. A table shows $x = 3, 6, 9$ and $y = 12, 24, 36$. The relationship is proportional. What is the constant of proportionality k in $y = kx$?
(A) 4 (B) $\frac{1}{4}$ (C) 9 (D) 12
5. Which situation is a proportional relationship?
(A) A gym charges 20 dollars to join plus 15 dollars per month.
(B) Movie tickets cost 7 dollars each, and the total depends on how many are bought.
(C) A plant is 4 centimeters tall and grows 2 centimeters each week.
(D) A phone starts at 100% charge and loses 5% each hour.
6. Maria runs 3.5 miles in 28 minutes. At the same steady pace, how long does it take her to run 5 miles?
(A) 20 minutes (B) 35 minutes (C) 32 minutes
(D) 40 minutes
7. A hose fills a 45-liter tub in 6 minutes. At the same rate, how many minutes does it take to fill a 75-liter tub?
(A) 10 minutes (B) 7.5 minutes (C) 12.5 minutes
(D) 16 minutes
8. A proportional relationship passes through the points $(0, 0)$ and $(4, 10)$. Which equation describes it?
(A) $y = 0.4x$ (B) $y = 2.5x$ (C) $y = x + 6$ (D) $y = 4x$
9. Which is the fastest speed?
(A) 90 km in 1.5 hours (B) 200 km in 3.5 hours (C) 130 km in 2 hours
(D) 48 km in 0.75 hour
10. A faucet fills 18 liters in 4 minutes. How many liters per minute is that?
11. A train travels 252 kilometers in 3.5 hours at a steady speed. How many hours does it take to travel 396 kilometers at the same speed?
12. Store A sells an 18 ounce box of cereal for 4.14 dollars. Store B sells a 24 ounce box for 5.28 dollars. Which store has the lower price per ounce, and by how many cents per ounce? Answer in the form "Store __, by __ cents."
13. The amount y is proportional to x . When $x = 4.5$, $y = 12.6$. What is y when $x = 7$?
14. Cheese costs 8.40 dollars for 1.5 pounds. At this price, how much do 4 pounds cost, in dollars?
15. Tap A fills 12 liters in 5 minutes. Tap B fills 7.5 liters in 3 minutes. If both run together for 10 minutes, how many liters do they fill in all?

02.3 Solving Proportions

Set up and solve proportions with cross multiplication and scale factors, then use them for maps, scale drawings, similar figures, and recipe scaling.

Why This Matters

A **proportion** says that two ratios are equal. Whenever one quantity scales with another, a proportion lets you find the missing amount: enlarging a recipe, reading a map, finding the height of a tree from its shadow, or converting between currencies. Proportions are a core tool in science, engineering, art, and design.

Students get stuck when they set up the proportion with mismatched units (putting miles on top in one ratio and inches on top in the other), or when they multiply the wrong numbers together. The cure is to label every number and to keep the same order on both sides. Once the setup is right, solving is straightforward.

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Core Ideas

Proportion and Cross Multiplication

A **proportion** is an equation stating that two ratios are equal:

$$\frac{a}{b} = \frac{c}{d}$$

Cross multiplication: if $\frac{a}{b} = \frac{c}{d}$ (with b and d not zero), then $a \cdot d = b \cdot c$.

Use it to find a missing value: multiply diagonally, then divide by the number that is multiplied by the unknown.

Here is why cross multiplication works. Multiply both sides of $\frac{a}{b} = \frac{c}{d}$ by $b \cdot d$. The left side becomes $a \cdot d$ and the right side becomes $c \cdot b$. So the diagonal products are equal. Another way to solve a proportion is the **scale factor** method: notice how you get from one denominator to the other, and apply the same multiplier to the numerator.

Setting Up a Proportion

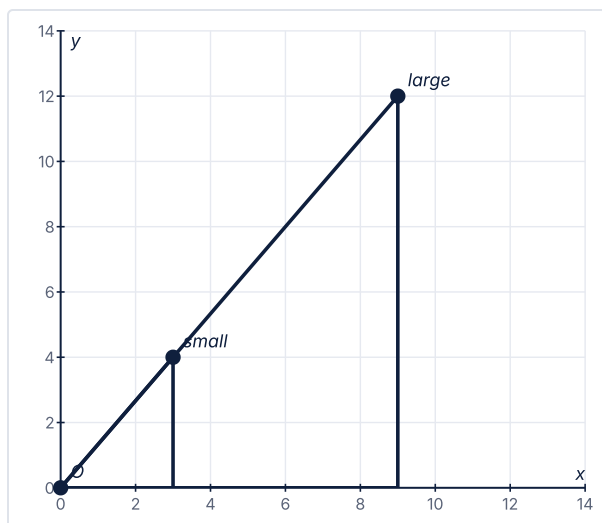
Write the ratios as fractions with the same units in the same positions:

$$\frac{\text{miles}}{\text{hours}} = \frac{\text{miles}}{\text{hours}} \quad \text{or} \quad \frac{\text{map cm}}{\text{real km}} = \frac{\text{map cm}}{\text{real km}}$$

Put the unknown in a spot with a variable like x , solve, and check that your answer is reasonable.

Scale drawings and maps. A scale such as "1 cm = 25 km" is a ratio. Every length on the drawing is the same fraction of the real length. To use it, write a proportion with drawing length over real length on both sides.

Similar figures. Two figures are **similar** if they have the same shape but maybe different sizes. Their angles are equal and their matching sides are in proportion. If one triangle has sides 3, 4, 5 and a larger similar triangle has a shortest side of 9, the scale factor is $9 \div 3 = 3$, and the other sides are $3 \cdot 4 = 12$ and $3 \cdot 5 = 15$.



Two similar right triangles that share a corner at the origin. Legs 3 and 4 (hypotenuse 5) scale by 3 to legs 9 and 12 (hypotenuse 15).

Scale	Meaning	Example
1 cm : 25 km	1 cm on the map is 25 km real	6.4 cm is 160 km
1 : 50	1 unit on the drawing is 50 of the same unit	16 cm is 800 cm, or 8 m
3 in = 120 miles	3 inches show 120 miles	1 inch shows 40 miles

Two ways to look at a proportion. There are two equally good ways to read $\frac{3}{4} = \frac{x}{12}$. The first is **between** the ratios: the numerators and denominators scale by the same factor, $4 \rightarrow 12$ is times 3, so $x = 3 \cdot 3 = 9$. The second is **within** each ratio: the numerator is always $\frac{3}{4}$ of the denominator, so $x = \frac{3}{4} \cdot 12 = 9$. That second number, $\frac{3}{4}$, is a unit rate. Use whichever view makes the numbers easiest.

Check with a unit rate. If 5 notebooks cost 12.50 dollars, then each costs $12.50 \div 5 = 2.50$ dollars. To find the cost of 8 notebooks, multiply $8 \cdot 2.50 = 20$ dollars. You get the same answer as the proportion $\frac{5}{12.50} = \frac{8}{x}$. When a proportion is messy, finding the unit rate first is often the cleaner path.

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Know when a proportion does NOT apply. A proportion works only when the two quantities scale together at a constant rate. Your height at age 10 compared with age 20 is not proportional, and a taxi fare with a starting fee is not proportional. Before you set up a proportion, ask yourself: if the first quantity doubles, does the second one double too?

Worked Examples

EXAMPLE 1: A BASIC PROPORTION

Solve $\frac{x}{12} = \frac{15}{20}$.

SOLUTION

Cross multiply: $20x = 12 \cdot 15 = 180$.

Divide by 20: $x = 9$.

Check: $\frac{9}{12} = \frac{3}{4}$ and $\frac{15}{20} = \frac{3}{4}$. They match.

EXAMPLE 2: THE UNKNOWN INSIDE AN EXPRESSION

Solve $\frac{3}{x-1} = \frac{6}{10}$.

SOLUTION

Cross multiply: $3 \cdot 10 = 6 \cdot (x - 1)$, so $30 = 6(x - 1)$.

Divide both sides by 6: $5 = x - 1$.

Add 1: $x = 6$.

Check: $\frac{3}{6-1} = \frac{3}{5}$ and $\frac{6}{10} = \frac{3}{5}$. Yes.

EXAMPLE 3: A MAP SCALE

On a map, 1 cm stands for 25 km. (a) Two towns are 6.4 cm apart on the map. How far apart are they in real life? (b) Two cities are 350 km apart in real life. How far apart are they on the map?

SOLUTION

(a) Set up $\frac{1 \text{ cm}}{25 \text{ km}} = \frac{6.4 \text{ cm}}{x \text{ km}}$. Then $x = 25 \cdot 6.4 = 160$ km.

(b) Set up $\frac{1}{25} = \frac{y}{350}$. Then $25y = 350$, so $y = 14$ cm.

Both answers have the right units and sizes: a real distance is much bigger than its map length.

EXAMPLE 4: SIMILAR TRIANGLES

A right triangle has legs 3 and 4 and hypotenuse 5. A similar triangle has a shorter leg of 9. Find the other leg and the hypotenuse of the larger triangle.

SOLUTION

The shorter legs match, so the scale factor is $9 \div 3 = 3$.

Other leg: $4 \cdot 3 = 12$. Hypotenuse: $5 \cdot 3 = 15$.

Check with the proportion $\frac{3}{9} = \frac{4}{12} = \frac{5}{15}$. All three equal $\frac{1}{3}$.

Check with the Pythagorean theorem: $9^2 + 12^2 = 81 + 144 = 225 = 15^2$.

EXAMPLE 5: SCALING A RECIPE

A recipe for 4 servings uses $\frac{3}{4}$ cup oats, $1\frac{1}{2}$ cups milk, and 2 eggs. How much of each ingredient is needed for 10 servings?

SOLUTION

The scale factor is $10 \div 4 = \frac{5}{2} = 2.5$.

Oats: $\frac{3}{4} \cdot \frac{5}{2} = \frac{15}{8} = 1\frac{7}{8}$ cups.

Milk: $\frac{3}{2} \cdot \frac{5}{2} = \frac{15}{4} = 3\frac{3}{4}$ cups.

Eggs: $2 \cdot \frac{5}{2} = 5$ eggs.

Multiply every ingredient by the same scale factor so the recipe keeps its taste.

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EXAMPLE 6: PREDICTING FROM A SAMPLE

A quality inspector tests 40 light bulbs from a large batch and finds that 3 are defective. At that rate, about how many defective bulbs should be expected in a batch of 2,000?

SOLUTION

The sample is representative, so the batch should have the same ratio of defective bulbs: $\frac{3}{40} = \frac{x}{2000}$.

Cross multiply: $40x = 3 \cdot 2000 = 6000$.

Divide: $x = 150$.

About 150 defective bulbs are expected. Because it is a prediction from a sample, the real number will probably be close to 150, not exactly 150.

Coach's Playbook

- 1 Label first.** Write the units next to every number, and decide which quantity goes on top.
 - 2 Keep the same order.** If "inches" is on top on the left, it must be on top on the right.
 - 3 Cross multiply.** Multiply the diagonal numbers and set them equal.
 - 4 Isolate the unknown.** Divide (or add and subtract, if the unknown is inside an expression) to get the variable by itself.
 - 5 Check reasonableness.** Substitute back, and ask whether the answer is the right size and has the right units.
- ◆ If the multiplier between two denominators is easy to see (like $12 \rightarrow 20$ or $4 \rightarrow 10$), use it directly without cross multiplying.
 - ◆ For similar figures, match corresponding sides: the shortest side goes with the shortest side, the longest with the longest.
 - ◆ When a scale is written as a ratio without units, like 1:50, the two sides use the same unit.

Common traps

(1) Putting units in a different order on each side, such as inches over miles on one side and miles over inches on the other. (2) Cross multiplying and then dividing by the wrong number. (3) Adding the same amount to both sides of a scale instead of multiplying, such as thinking 4 to 10 means each side grows by 6. (4) Forgetting that the unknown is inside an expression like $x - 1$ and must be solved in two steps.

Mistake Clinic**Spot the error**

Problem: Solve $\frac{5}{8} = \frac{x}{20}$. **Student's work:** $5x = 8 \cdot 20 = 160$, so $x = 32$. **What went wrong:** The student cross multiplied correctly but attached the products to the wrong variable. The diagonal products are $5 \cdot 20$ and $8 \cdot x$. The answer 32 is also greater than 20, but $\frac{5}{8}$ is less than 1, so x must be less than 20. **Correct solution:** Cross multiply: $8x = 5 \cdot 20 = 100$. Then $x = 100 \div 8 = 12.5$. Check: $\frac{5}{8} = 0.625$ and $\frac{12.5}{20} = 0.625$.

Spot the error

Problem: A map scale says 1 inch = 40 miles. Two towns are 150 miles apart. How far apart are they on the map?
Student's work: $150 \times 40 = 6000$ inches. **What went wrong:** The student multiplied when the real distance should be divided by the scale. A map is smaller than real life, so the map distance must be much smaller than 150, not 6,000.
Correct solution: Set up $\frac{1}{40} = \frac{m}{150}$. Cross multiply: $40m = 150$, so $m = 3.75$ inches.

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Spot the error

Problem: A muffin recipe uses 3 cups of flour for 12 muffins. How much flour is needed for 18 muffins? **Student's work:** 18 is 6 more than 12, so add 6 cups: 9 cups of flour. **What went wrong:** The student added the same amount to both quantities. Recipes scale by multiplying. Nine cups of flour for 18 muffins is three times as much flour per muffin as the original. **Correct solution:** The scale factor is $18 \div 12 = 1.5$. Flour: $3 \times 1.5 = 4.5$ cups.

Real-World Connection

A 6 foot pole casts a 4 foot shadow. At the same time a tree casts a 22 foot shadow. The sun makes similar triangles, so $\frac{6}{4} = \frac{h}{22}$ and the tree is 33 feet tall.

Challenge Corner

At a school the ratio of girls to boys is 7:5. After 12 more boys join, the ratio of girls to boys becomes 7:6. How many girls are at the school? **Solution:** Let the original numbers be $7k$ girls and $5k$ boys. After 12 boys join, the boys number $5k + 12$. Set up the new ratio: $\frac{7k}{5k+12} = \frac{7}{6}$. Cross multiply: $42k = 7(5k + 12) = 35k + 84$. Subtract $35k$: $7k = 84$, so $k = 12$. Girls: $7 \cdot 12 = 84$. Check: boys were 60, then 72, and $\frac{84}{72} = \frac{7}{6}$.

Practice**PRACTICE Solving Proportions**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- Solve $\frac{x}{8} = \frac{15}{24}$.
(A) 5 (B) 3 (C) 45 (D) 12
- Solve $\frac{9}{x} = \frac{6}{14}$.
(A) 12 (B) 6 (C) 21 (D) 9
- A map uses a scale of 1 inch = 40 miles. Two cities are 3.5 inches apart on the map. What is the actual distance between them?
(A) 11.4 miles (B) 140 miles (C) 43.5 miles (D) 14 miles
- A 4 inch by 6 inch photo is enlarged so that the shorter side is 10 inches. The photo stays similar. What is the longer side?
(A) 12 inches (B) 14 inches (C) 16 inches (D) 15 inches
- Twelve cookies use 2 cups of flour. How many cups of flour are needed for 30 cookies?
(A) 5 (B) 4 (C) 6.5 (D) 72
- Solve $\frac{3}{x+1} = \frac{9}{x+7}$.
(A) 1 (B) 3 (C) 2 (D) 4
- Solve $\frac{7}{12} = \frac{x}{60}$.
(A) 5 (B) 420 (C) 72 (D) 35
- A model car is built at a scale of 1:24. The real car is 4.8 meters long. How long is the model?
(A) 115.2 centimeters (B) 20 centimeters (C) 2 centimeters (D) 200 centimeters
- A rectangle is 5 inches by 8 inches. A similar rectangle has a perimeter of 52 inches. What is the length of its longer side?
(A) 13 (B) 20 (C) 16 (D) 26
- Solve $\frac{2.4}{x} = \frac{6}{15}$.
- A blueprint uses a scale of 1:50. A room is 8 meters long. How long is the room on the blueprint, in centimeters?
- A 6-foot pole casts a 4-foot shadow. At the same time, a tree casts a 22-foot shadow. How tall is the tree, in feet?
- A soup recipe makes 8 servings with $3\frac{1}{2}$ cups of broth. How many servings can you make with $10\frac{1}{2}$ cups of broth?
- Solve $\frac{x+3}{4} = \frac{x-1}{2}$.

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15. 3.5 gallons of paint cover 1,120 square feet. How many gallons are needed to cover a wall area of 2,560 square feet?

02.4 Unit Conversion and Dimensional Analysis

Convert metric and customary units with conversion factors, chain several conversions in one line, change rates like miles per hour, and handle squared and cubed units correctly.

Why This Matters

Measurements only make sense with units, and the units are not always the ones you need. A recipe might use cups while your measuring jug shows milliliters. A car's speed is in miles per hour while a physics problem wants meters per second. Scientists, nurses, builders, pilots, and engineers convert units every day, and one wrong conversion can ruin a project or even cause a safety problem.

Students get stuck for two reasons. First, they forget whether to multiply or divide, and end up with a number that is the wrong size. Second, when units are squared or cubed (like square feet or cubic centimeters), they use the plain conversion factor and forget to square or cube it. This section gives you one reliable method, dimensional analysis, which handles every case by treating units like numbers that can cancel.

Core Ideas

A **conversion factor** is a fraction that equals 1 because the top and bottom are the same amount in different units. For example, $1\text{ m} = 100\text{ cm}$, so $\frac{100\text{ cm}}{1\text{ m}} = 1$ and $\frac{1\text{ m}}{100\text{ cm}} = 1$. Multiplying by 1 never changes the amount, only the unit you describe it in.

Dimensional Analysis

To convert, start with the given measurement and multiply by conversion factors, each written so that the unit you want to remove is on the **bottom** and the unit you want to reach is on the **top**:

$$3.4\text{ km} \times \frac{1000\text{ m}}{1\text{ km}} = 3400\text{ m}.$$

The "km" on top and bottom cancel. If the units that remain are the ones you wanted, the setup is right.

Measure	Metric	Customary
Length	1 km = 1000 m; 1 m = 100 cm; 1 cm = 10 mm	1 mi = 5280 ft; 1 yd = 3 ft; 1 ft = 12 in
Mass or weight	1 kg = 1000 g; 1 g = 1000 mg	1 lb = 16 oz
Volume of liquid	1 L = 1000 mL	1 gal = 4 qt; 1 qt = 2 pt; 1 pt = 2 cups; 1 cup = 8 fl oz
Time	1 h = 60 min; 1 min = 60 s	same

The metric system is built on powers of ten, so converting means moving the decimal point. Going to a smaller unit (km to m) gives a bigger number, so you multiply. Going to a bigger unit (cm to m) gives a smaller number, so you divide. The customary system uses different numbers (12, 3, 5280, 16), so you need the table or your memory.

Squared and Cubed Units

When you convert area or volume, the conversion factor must be squared or cubed too.

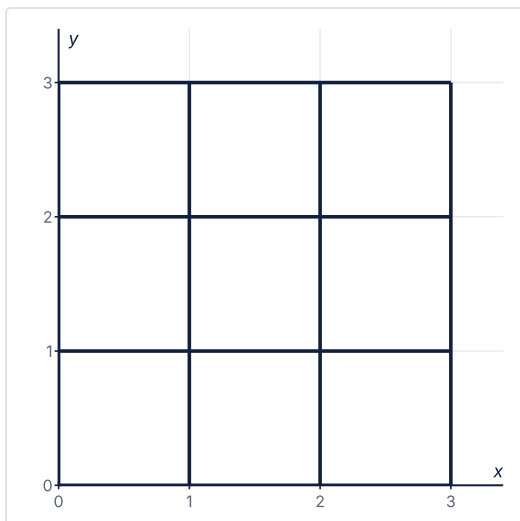
If $1\text{ yd} = 3\text{ ft}$, then $1\text{ yd}^2 = 3^2 = 9\text{ ft}^2$ and $1\text{ yd}^3 = 3^3 = 27\text{ ft}^3$.

If $1\text{ m} = 100\text{ cm}$, then $1\text{ m}^2 = 10,000\text{ cm}^2$ and $1\text{ m}^3 = 1,000,000\text{ cm}^3$.

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One square yard (3 ft by 3 ft) is cut into 1-foot squares. There are 9 of them, so 1 square yard equals 9 square feet, not 3.

Rates with units. A rate such as 72 kilometers per hour has two units, so you may need two conversion factors: one for the top unit and one for the bottom unit. The bottom unit works the opposite way, so write the factor so that the old bottom unit lands on top and cancels. Chain the factors one after another in a single line, and cross out units as you go.

Worked Examples

EXAMPLE 1: ONE-STEP METRIC AND CUSTOMARY

(a) Convert 3.4 km to meters. (b) Convert 275 cm to meters. (c) Convert 5 pounds to ounces.

SOLUTION

(a) $3.4 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} = 3400 \text{ m}$. Meters are smaller than kilometers, so the number gets bigger.

(b) $275 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}} = 2.75 \text{ m}$. Meters are bigger than centimeters, so the number gets smaller.

(c) $5 \text{ lb} \times \frac{16 \text{ oz}}{1 \text{ lb}} = 80 \text{ oz}$.

EXAMPLE 2: CUSTOMARY CONVERSIONS

(a) How many feet is 54 inches? (b) How many cups are in $3\frac{1}{2}$ gallons?

SOLUTION

(a) $54 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} = 4.5 \text{ ft}$, which is 4 feet 6 inches.

(b) Gallons to quarts to pints to cups, in one chain: $3.5 \text{ gal} \times \frac{4 \text{ qt}}{1 \text{ gal}} \times \frac{2 \text{ pt}}{1 \text{ qt}} \times \frac{2 \text{ cups}}{1 \text{ pt}}$.

Multiply the numbers: $3.5 \times 4 \times 2 \times 2 = 56$ cups.

Every unit cancels except cups. Check: 1 gallon is 16 cups, and $3.5 \times 16 = 56$.

EXAMPLE 3: CONVERTING A RATE

A drone flies at 72 kilometers per hour. What is its speed in meters per second?

SOLUTION

Set up two factors. Kilometers to meters on top, hours to seconds on the bottom: $\frac{72 \text{ km}}{1 \text{ h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}}$.

Cancel km and h. Multiply the numbers: $\frac{72 \times 1000}{3600} = \frac{72000}{3600} = 20$.

The speed is 20 meters per second.

Shortcut to remember: divide km per hour by 3.6 to get meters per second.

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EXAMPLE 4: MILES PER HOUR TO FEET PER SECOND

A train travels at 45 miles per hour. Express its speed in feet per second.

SOLUTION

$$\frac{45 \text{ mi}}{1 \text{ h}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ h}}{3600 \text{ s}}$$

Multiply: $45 \times 5280 = 237,600$. Divide by 3600: $237,600 \div 3600 = 66$.

The speed is 66 feet per second.

Check: 60 mph is 88 ft/s, so 45 mph (three fourths of 60) should be $0.75 \times 88 = 66$. Yes.

EXAMPLE 5: SQUARE UNITS

A rectangular room is 4 yards by 5 yards. What is its area in square yards and in square feet?

SOLUTION

Area in square yards: $4 \times 5 = 20 \text{ yd}^2$.

Square feet: $20 \text{ yd}^2 \times \frac{9 \text{ ft}^2}{1 \text{ yd}^2} = 180 \text{ ft}^2$.

Check by converting the sides first: 4 yd is 12 ft and 5 yd is 15 ft, and $12 \times 15 = 180$. Both ways agree.

Using 3 instead of 9 would give 60 square feet, which is far too small for a 12 foot by 15 foot room.

EXAMPLE 6: CUBIC UNITS AND CAPACITY

A fish tank measures 60 cm long, 30 cm wide, and 40 cm high. Find its volume in cubic centimeters and in liters. (1 liter equals 1000 cubic centimeters.)

SOLUTION

Volume in cubic centimeters: $60 \times 30 \times 40 = 72,000 \text{ cm}^3$.

Convert to liters: $72,000 \text{ cm}^3 \times \frac{1 \text{ L}}{1000 \text{ cm}^3} = 72 \text{ L}$.

The tank holds 72 liters.

Check the size: a 1-liter bottle fits about 72 times in a tank this large. That seems reasonable.

Coach's Playbook

- 1 Write what you start with.** Include the number and the unit, for example 72 km/h.
 - 2 Decide the goal unit.** Say it aloud: "I want meters per second."
 - 3 Chain conversion factors.** Write each factor as a fraction with the unwanted unit on the bottom and the wanted unit on the top.
 - 4 Cancel units, then multiply numbers.** Cross out matching units diagonally. If the units left over are not the goal unit, rethink the setup.
 - 5 Do a size check.** Smaller units mean bigger numbers. If you convert to a smaller unit and the number shrank, you divided when you should have multiplied.
- ◆ **Keep a short list of facts** in your head: 12 in per ft, 3 ft per yd, 5280 ft per mi, 16 oz per lb, 4 qt per gal, 60 min per h, 1000 m per km, 100 cm per m, 1000 mL per L.
 - ◆ **Multiply for smaller units, divide for larger units** (or let the cancellation tell you).
 - ◆ **Convert before you calculate** area and volume if you can: change the side lengths to the same unit first, then multiply. It avoids squared and cubed factors.

Common traps

(1) Multiplying when you should divide, so that 4.5 m becomes 0.045 cm. Meters to centimeters should give a bigger number. (2) Using 3 instead of 9 for square yards to square feet, or 100 instead of 10,000 for square meters to square centimeters. (3) Mixing units inside one calculation, like multiplying 5 feet by 30 inches. (4) Forgetting to convert the time unit in a rate and getting miles per minute instead of miles per hour.

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Mistake Clinic

Spot the error

Problem: Convert 4.5 meters to centimeters. **Student's work:** $4.5 \div 100 = 0.045$ cm. **What went wrong:** The student divided, but a centimeter is smaller than a meter, so there must be MORE centimeters. A 4.5 meter board is not 0.045 centimeters long, which is smaller than a grain of rice. **Correct solution:** $4.5 \text{ m} \times \frac{100 \text{ cm}}{1 \text{ m}} = 450 \text{ cm}$.

Spot the error

Problem: How many square feet are in 1 square yard? **Student's work:** 1 yard is 3 feet, so 1 square yard is 3 square feet. **What went wrong:** The student used the length conversion for an area. A square yard is a square with sides of 3 feet, so it covers 3 times 3 square feet. **Correct solution:** $1 \text{ yd}^2 = (3 \text{ ft})^2 = 9 \text{ ft}^2$. Look at the grid picture above: there are 9 small squares.

Spot the error

Problem: Convert 30 miles per hour to feet per second. **Student's work:** $30 \times 5280 = 158,400$ feet per second. **What went wrong:** The student converted miles to feet but forgot to convert hours to seconds. The result, 158,400 feet per second, would be about 108,000 miles per hour. The time conversion is missing. **Correct solution:** $30 \times \frac{5280}{3600} = 44$ feet per second. Always convert both the top and the bottom unit.

Real-World Connection

A 5-kilometer race is 5000 meters. A runner who holds a steady 4 meters per second needs $5000 \div 4 = 1250$ seconds, which is 20 minutes and 50 seconds. Choosing the right units makes the answer easy to read.

Challenge Corner

A garden hose delivers 12 liters of water per minute. How many hours does it take to fill a swimming pool that holds 9 cubic meters of water? (1 cubic meter holds 1000 liters.) **Solution:** Convert the pool to liters: $9 \text{ m}^3 \times \frac{1000 \text{ L}}{1 \text{ m}^3} = 9000 \text{ L}$. Time in minutes: $9000 \text{ L} \times \frac{1 \text{ min}}{12 \text{ L}} = 750 \text{ min}$. Convert to hours: $750 \text{ min} \times \frac{1 \text{ h}}{60 \text{ min}} = 12.5 \text{ h}$. The pool takes 12.5 hours to fill. In one line: $9 \text{ m}^3 \times \frac{1000 \text{ L}}{1 \text{ m}^3} \times \frac{1 \text{ min}}{12 \text{ L}} \times \frac{1 \text{ h}}{60 \text{ min}} = 12.5 \text{ h}$.

Practice

PRACTICE Unit Conversion and Dimensional Analysis

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What is 3.5 kilograms in grams?
(A) 350 (B) 3,500 (C) 35 (D) 0.0035
- How many feet is 90 inches?
(A) 1,080 (B) 10 (C) 7 (D) 7.5
- A bottle holds 2.4 liters. How many milliliters is that?
(A) 2,400 (B) 24 (C) 240 (D) 0.0024
- How many ounces are in 3.5 pounds?
(A) 19 (B) 0.22 (C) 56 (D) 560
- A scooter travels at 36 kilometers per hour. What is that in meters per second?
(A) 0.6 (B) 10 (C) 100 (D) 129.6

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6. A rug has an area of 5 square yards. What is its area in square feet?
(A) 15 (B) 135 (C) 25 (D) 45
7. A car travels at 60 miles per hour. How many feet per second is that?
(A) 88 (B) 316,800 (C) 1.47 (D) 3,600
8. A storage cube has a volume of 1.5 cubic meters. What is its volume in cubic centimeters?
(A) 150 (B) 15,000 (C) 1,500,000 (D) 150,000
9. Which is the longest length?
(A) 0.8 km (B) 82,000 cm (C) 750 m
(D) 790,000 mm
10. How many seconds are in 2 hours and 45 minutes?
11. A bottle holds 2.5 liters of juice. How many 250-milliliter servings can be poured from it?
12. A cyclist moves at 22 feet per second. What is the speed in miles per hour?
13. A rug is 3 feet by 4 feet. What is its area in square inches?
14. A water tank is 2 meters long, 1.5 meters wide, and 0.8 meter deep. How many liters does it hold when full? (1 cubic meter is 1000 liters.)
15. A medicine label says the dose is 15 milligrams per kilogram of body weight per day, split into 3 equal doses. How many milligrams is each dose for a child who weighs 24 kilograms?

Answers**Answers and Explanations**

Complete solutions for every practice problem, numbered to match. Read the explanation even when your answer is right.

02.1 Ratios and Equivalent Ratios**PRACTICE · RATIOS AND EQUIVALENT RATIOS**

1. B. The GCF of 15 and 25 is 5, so $15 : 25 = 3 : 5$. Choice C reverses the order, and choice A is not simplified.
2. D. Students not in the band: $24 - 9 = 15$. Band to not band is $9 : 15$, which simplifies to $3 : 5$. Choice B compares band to everyone, not to the other group.
3. A. Multiply both numbers by 5: $4 \cdot 5 = 20$ and $7 \cdot 5 = 35$. Choice B adds 4 to both numbers, which does not keep the ratio.
4. C. $15 \div 5 = 3$, so one part is 3 liters. Yellow is 3 parts: $3 \cdot 3 = 9$. Choice A flips the ratio and multiplies 15 by $\frac{5}{3}$.
5. B. There are $4 + 5 = 9$ parts, so one part is $36 \div 9 = 4$ players. Girls are 4 parts: $4 \cdot 4 = 16$. Choice A is the number of boys.
6. D. There are $2 + 3 + 5 = 10$ parts, and one part is $60 \div 10 = 6$ minutes. Running is 5 parts: $5 \cdot 6 = 30$ minutes. Choice C comes from dividing 60 by 3.
7. A. $6 : 15$ simplifies to $2 : 5$. Choices B, C, and D all simplify to $2 : 5$ ($12 : 30$ and $18 : 45$ by dividing by 6 and 9). But $10 : 20$ simplifies to $1 : 2$, so it is not equivalent.
8. C. Peppers are 7 parts, so one part is $21 \div 7 = 3$. Mushrooms are 3 parts: $3 \cdot 3 = 9$. Choice A multiplies 21 by $\frac{7}{3}$, using the ratio upside down.
9. B. There are $5 + 6 = 11$ parts, so one part is $330 \div 11 = 30$. Fourth: 150. Fifth: 180. Difference: $180 - 150 = 30$. Choices A and C are the grade sizes, not the difference.
10. $4 : 7$. The GCF of 48 and 84 is 12. $48 \div 12 = 4$ and $84 \div 12 = 7$.
11. 18 cups. From 2 to 12 is multiplying by 6, so fruit is $3 \cdot 6 = 18$.
12. Y. Red as a fraction of the paint: X has $\frac{5}{8} = 0.625$ and Y has $\frac{7}{11} \approx 0.636$. Y is slightly redder.
13. 6 dollars. Total parts $4 + 3 = 7$, so one part is $84 \div 7 = 12$. Sam has 48 and Ava has 36. To be equal they each need 42, so Sam gives Ava 6.
14. $3 : 10$. Change to the same unit: 2 hours is 120 minutes. Then $36 : 120$ has GCF 12, so it simplifies to $3 : 10$.
15. 3 liters. There are $5 + 3 = 8$ parts, so one part is 3 liters: 15 liters of water and 9 liters of syrup. The syrup stays at 9, so for a 2:1 ratio the water must be 18. Add $18 - 15 = 3$ liters.

02.2 Unit Rates and Unit Pricing**PRACTICE · UNIT RATES AND UNIT PRICING**

1. C. Words per minute: $240 \div 8 = 30$. Choice A divides by the wrong number, and choice B multiplies instead of dividing.
2. B. Unit prices: $4.50 \div 6 = 0.75$, $7.20 \div 10 = 0.72$, $9.30 \div 12 = 0.775$, $11.40 \div 15 = 0.76$ dollars each. The lowest is 0.72 dollars, which is the 10 pack. The largest pack is not the cheapest per pencil.
3. D. Rate: $150 \div 6 = 25$ miles per gallon. For 10 gallons: $25 \times 10 = 250$ miles. Choice B is the unit rate, not the final answer.
4. A. $k = \frac{y}{x} = \frac{12}{3} = 4$. Check: $\frac{24}{6} = 4$ and $\frac{36}{9} = 4$. Choice B is $\frac{x}{y}$, which is the reverse.
5. B. Tickets cost $y = 7x$, a line through the origin: zero tickets cost zero. The other situations each have a starting amount (20 dollars, 4 centimeters, 100%) that is not zero, so they are not proportional.
6. D. Pace: $28 \div 3.5 = 8$ minutes per mile. For 5 miles: $8 \times 5 = 40$ minutes. Choice B ($28 + 7$) simply adds time without using the rate.
7. A. Rate: $45 \div 6 = 7.5$ liters per minute. Time: $75 \div 7.5 = 10$ minutes. Choice B is the rate, not the time.

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8. $B. k = \frac{y}{x} = \frac{10}{4} = 2.5$, so $y = 2.5x$. Choice A uses $\frac{x}{y}$, and choice C is linear but does not pass through the origin.
9. C. Speeds in km per hour: 60, about 57.1, 65, and 64. The fastest is 130 km in 2 hours at 65 km per hour. Choice D is close at 64, so it is worth dividing carefully.
10. 4.5 liters per minute. $18 \div 4 = 4.5$.
11. 5.5 hours. Speed: $252 \div 3.5 = 72$ kilometers per hour. Time: $396 \div 72 = 5.5$ hours.
12. Store B, by 1 cent. Store A: $4.14 \div 18 = 0.23$ dollars per ounce. Store B: $5.28 \div 24 = 0.22$ dollars per ounce. The difference is 0.01 dollars, which is 1 cent.
13. 19.6. The constant is $k = \frac{12.6}{4.5} = 2.8$. Then $y = 2.8 \times 7 = 19.6$.
14. 22.40 dollars. Unit price: $8.40 \div 1.5 = 5.60$ dollars per pound. Cost: $5.60 \times 4 = 22.40$.
15. 49 liters. Tap A: $12 \div 5 = 2.4$ liters per minute. Tap B: $7.5 \div 3 = 2.5$ liters per minute. Together: 4.9 liters per minute. In 10 minutes: $4.9 \times 10 = 49$ liters.

02.3 Solving Proportions

PRACTICE · SOLVING PROPORTIONS

1. A. Cross multiply: $24x = 8 \cdot 15 = 120$, so $x = 5$. Choice C multiplies 15 by 3 instead of using the proportion.
2. C. Cross multiply: $6x = 9 \cdot 14 = 126$, so $x = 21$. Choice A adds 3 to 9 because 6 plus 3 is 9, which does not preserve the ratio.
3. B. Set up $\frac{1}{40} = \frac{3.5}{x}$. Then $x = 40 \cdot 3.5 = 140$ miles. Choice A divides 40 by 3.5, using the scale backwards.
4. D. The scale factor is $10 \div 4 = 2.5$. The longer side is $6 \times 2.5 = 15$ inches. Choice A adds 6 inches (the amount the short side grew) instead of multiplying.
5. A. Scale factor: $30 \div 12 = 2.5$. Flour: $2 \times 2.5 = 5$ cups. Choice D multiplies 12 by 6 and ignores the ratio.
6. C. Cross multiply: $3(x + 7) = 9(x + 1)$. Then $3x + 21 = 9x + 9$, so $12 = 6x$ and $x = 2$. Check: $\frac{3}{3} = 1$ and $\frac{9}{9} = 1$.
7. D. The denominator was multiplied by 5 ($12 \cdot 5 = 60$), so $x = 7 \cdot 5 = 35$. Choice B multiplies 7 by 60 and forgets to divide by 12.
8. B. Change to centimeters first: $4.8 \text{ m} = 480 \text{ cm}$. Then $480 \div 24 = 20 \text{ cm}$. Choice A multiplies by 24 instead of dividing.
9. C. The first rectangle has perimeter $2(5 + 8) = 26$. The scale factor is $52 \div 26 = 2$, so the new sides are 10 and 16. The longer side is 16. Choice D is half the perimeter, not a side.
10. 6. Cross multiply: $6x = 2.4 \cdot 15 = 36$, so $x = 6$.
11. 16 cm. The scale 1:50 means the drawing is $\frac{1}{50}$ of real size. 8 meters is 800 cm, and $800 \div 50 = 16 \text{ cm}$.
12. 33 feet. Set up $\frac{6}{4} = \frac{h}{22}$. Cross multiply: $4h = 132$, so $h = 33$.
13. 24 servings. The scale factor is $10.5 \div 3.5 = 3$, so servings are $8 \times 3 = 24$.
14. $x = 5$. Cross multiply: $2(x + 3) = 4(x - 1)$. Then $2x + 6 = 4x - 4$, so $10 = 2x$ and $x = 5$. Check: $\frac{8}{4} = 2$ and $\frac{4}{2} = 2$.
15. 8 gallons. One gallon covers $1120 \div 3.5 = 320$ square feet. Gallons needed: $2560 \div 320 = 8$.

02.4 Unit Conversion and Dimensional Analysis

PRACTICE · UNIT CONVERSION AND DIMENSIONAL ANALYSIS

1. B. 1 kg is 1000 g, so $3.5 \times 1000 = 3500 \text{ g}$. Choice A multiplies by 100, which would be the factor for centimeters.
2. D. $90 \div 12 = 7.5$ feet. Choice C drops the half foot, and choice A multiplies instead of dividing.
3. A. 1 L is 1000 mL, so $2.4 \times 1000 = 2400 \text{ mL}$. Choice C uses 100 instead of 1000.
4. C. $3.5 \times 16 = 56$ ounces. Choice B divides by 16, which goes the wrong way, and choice A adds 3.5 to 16.
5. B. $\frac{36 \times 1000}{3600} = 10$ meters per second. Choice D multiplies by 3.6 instead of dividing, and choice A divides by 60 only.
6. D. One square yard is 9 square feet, so $5 \times 9 = 45$. Choice A uses 3 (the length factor), and choice B uses 27, which is for cubic units.
7. A. $60 \times \frac{5280}{3600} = 88$ feet per second. Choice B forgets to convert hours to seconds.
8. C. $1 \text{ m}^3 = 100^3 = 1,000,000 \text{ cm}^3$, so $1.5 \times 1,000,000 = 1,500,000$. Choice A uses 100, and choice B uses 100^2 .
9. B. Convert all to meters: 0.8 km is 800 m, 82,000 cm is 820 m, 750 m, and 790,000 mm is 790 m. The longest is 820 m, which is 82,000 cm. Choice A looks small but equals 800 m.
10. 9,900 seconds. $2 \text{ h} = 2 \times 3600 = 7200 \text{ s}$. $45 \text{ min} = 45 \times 60 = 2700 \text{ s}$. Total: $7200 + 2700 = 9900 \text{ s}$.
11. 10 servings. 2.5 L is 2500 mL. Then $2500 \div 250 = 10$.
12. 15 miles per hour. $22 \frac{\text{ft}}{\text{s}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} \times \frac{3600 \text{ s}}{1 \text{ h}} = \frac{22 \times 3600}{5280} = 15$.
13. 1,728 square inches. Area in square feet: $3 \times 4 = 12$. Each square foot is $12^2 = 144$ square inches. So $12 \times 144 = 1728$. (Or convert the sides: 36 in by 48 in gives $36 \times 48 = 1728$.)
14. 2,400 liters. Volume: $2 \times 1.5 \times 0.8 = 2.4 \text{ m}^3$. Then $2.4 \times 1000 = 2400 \text{ L}$.
15. 120 milligrams. Daily total: $15 \frac{\text{mg}}{\text{kg}} \times 24 \text{ kg} = 360 \text{ mg}$. Split into 3 doses: $360 \div 3 = 120 \text{ mg}$ each.

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