



Integers, Order of Operations, and Number Sense

Four focused lessons, worked examples, a mistake clinic, and 60 practice problems with full explanations.

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03.1 Negative Numbers and Integer Operations

03.2 Order of Operations

03.3 Factors, Multiples, GCF, LCM, and Primes

03.4 Mental Math and Estimation

Every section teaches the idea, walks through the most common problem types, flags the traps students fall into, and ends with fifteen practice problems. Full answers and explanations are at the back.

03.1 Negative Numbers and Integer Operations

Learn to place integers on a number line and add, subtract, multiply, and divide them with confidence. You will also use absolute value as a distance from zero.

Why This Matters

Negative numbers show up everywhere: temperatures below zero, a bank account that is overdrawn, a diver below sea level, a football team losing yards. Yet many students feel nervous the moment a minus sign appears. The reason is simple: one little symbol, the minus sign, does three different jobs. It can mean **subtract**, it can mean **negative**, and it can mean **the opposite of**. Mixing those jobs up is the number one source of mistakes.

This skill is the foundation for everything that follows in algebra, from solving equations to graphing on a coordinate plane. Students most often get stuck on three things: subtracting a negative, deciding the sign of a product, and confusing absolute value with the number itself. In this section you will build a picture in your mind of the number line so the rules stop feeling like magic.

Core Ideas

Integers are the whole numbers and their opposites: $\dots, -3, -2, -1, 0, 1, 2, 3, \dots$. On a horizontal number line, numbers to the right are larger. So -2 is greater than -9 , even though 9 looks bigger than 2. Think of temperature: -2 degrees is warmer than -9 degrees.



Integers on a number line. Larger numbers are farther right: $A < B < O < C$.

Opposites and Absolute Value

The **opposite** of n is $-n$. It sits the same distance from 0 on the other side. The opposite of -7 is 7.

The **absolute value** $|n|$ is the distance from n to 0, so it is never negative: $|-7| = 7$ and $|7| = 7$.

Adding is moving on the number line. Start at the first number. Add a positive: move right. Add a negative: move left. For example, $-3 + 5$ starts at -3 and moves 5 to the right, landing at 2.

Another helpful picture is the **zero pair**. Imagine a positive chip and a negative chip. Together they cancel to 0, like a dollar earned and a dollar spent. To compute $-7 + 3$, start with seven negative chips and three positive chips. Three zero pairs cancel, and four negative chips are left, so the answer is -4 . This works for any sum of integers with different signs: the cancelling is the 'subtract the sizes' step, and the leftover chips tell you the sign.

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Rules for Adding and Subtracting Integers

Same signs when adding: add the absolute values and keep the sign. $-4 + (-6) = -10$.

Different signs when adding: subtract the smaller absolute value from the larger and keep the sign of the number with the larger absolute value. $-9 + 4 = -5$.

Subtracting means adding the opposite: $a - b = a + (-b)$. So $5 - (-3) = 5 + 3 = 8$.

Operation	Same signs	Different signs
Multiply	positive	negative
Divide	positive	negative
Example	$(-4)(-5) = 20$	$(-4)(5) = -20$

Rules for Multiplying and Dividing Integers

Two numbers with the **same sign** give a **positive** result. Two numbers with **different signs** give a **negative** result.

With many factors, count the negatives: an **even** count gives a positive product, an **odd** count gives a negative product. Division follows the same sign rules, and you can never divide by 0.

Absolute value also measures **distance between two numbers**. The distance from -3 to 5 is $|5 - (-3)| = |8| = 8$ units. Notice that you subtract, and the bars make sure the answer is not negative. Distance is the one place where a negative answer would not make sense, so the bars are doing real work.

Finally, remember that 0 is neither positive nor negative, and that multiplying by 0 always gives 0. Dividing 0 by a nonzero number gives 0, but dividing by 0 is undefined, because no number multiplied by 0 can give a nonzero answer.

Worked Examples

EXAMPLE 1: ORDERING INTEGERS

Write -7 , 3 , -2 , 0 , -10 from least to greatest.

SOLUTION

Picture the number line. The farther left a number is, the smaller it is.

The negatives, from farthest left: -10 , then -7 , then -2 .

Then 0, then 3.

Answer: -10 , -7 , -2 , 0 , 3 .



Example 1 on the number line, least to greatest from left to right.

EXAMPLE 2: ADDING AND SUBTRACTING

Find (a) $-8 + 5$, (b) $4 - (-6)$, (c) $-3 - 9$.

SOLUTION

(a) Different signs. $8 - 5 = 3$, and 8 has the larger absolute value and is negative, so $-8 + 5 = -3$.

(b) Subtracting a negative means adding: $4 - (-6) = 4 + 6 = 10$.

(c) Rewrite as $-3 + (-9)$. Same signs, so add and keep the sign: -12 .

EXAMPLE 3: MULTIPLYING AND DIVIDING

Find (a) $(-6)(7)$, (b) $(-48) \div (-8)$, (c) $(-2)(-3)(-4)$.

SOLUTION

(a) Different signs, so negative: -42 .

(b) Same signs, so positive: 6.

(c) Three negatives is an odd count, so the answer is negative. $2 \cdot 3 \cdot 4 = 24$, so the answer is -24 .

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EXAMPLE 4: ABSOLUTE VALUE

Evaluate $|-9| - |4 - 10|$.

SOLUTION

Work inside the bars first: $4 - 10 = -6$.

Take absolute values: $|-9| = 9$ and $|-6| = 6$.

Subtract: $9 - 6 = 3$.

EXAMPLE 5: A WORD PROBLEM

At dawn the temperature was -4 degrees. It dropped 11 degrees by noon, then rose 7 degrees by evening. What was the evening temperature?

SOLUTION

Write one expression: $-4 - 11 + 7$.

$-4 - 11 = -15$ (same direction, move left again).

$-15 + 7 = -8$.

The evening temperature was -8 degrees.

EXAMPLE 6: A MULTI-STEP EXPRESSION

Evaluate $(-4)(3 - 8) - (-18) \div 6$.

SOLUTION

Parentheses first: $3 - 8 = -5$.

Multiply and divide next: $(-4)(-5) = 20$ and $(-18) \div 6 = -3$.

Subtract: $20 - (-3) = 20 + 3 = 23$.

Check the signs: two negatives multiplied gave a positive, and subtracting a negative added 3.

Coach's Playbook

- 1 Name the minus sign.** Ask: is this subtraction, a negative number, or an opposite? Circle the job before you calculate.
 - 2 Turn subtraction into addition.** Rewrite $a - b$ as $a + (-b)$. Now you only have addition to think about.
 - 3 Sign first, size second.** For multiplying and dividing, decide the sign using the same-or-different rule, then multiply the sizes.
 - 4 Check with the number line.** Quick sketch: right for positive moves, left for negative moves. Does your answer land where the picture says?
- ◆ Say it in words. Read $-3 - (-5)$ as 'negative three minus negative five' and then as 'negative three plus five'. Hearing it helps.
 - ◆ Use money. Owing 8 dollars and then earning 5 dollars leaves you owing 3 dollars: $-8 + 5 = -3$.
 - ◆ Estimate the sign first. If the bigger-sized number is negative, a sum of different-sign numbers will be negative.
 - ◆ Parentheses around negatives are your friend. Write $7 + (-2)$ instead of $7 + -2$ so the two signs never crash into each other.
 - ◆ For a long product, cross out pairs of negatives. Whatever is left (zero or one negative) decides the sign.

Common traps

1. Writing $5 - (-3) = 2$. Subtracting a negative moves you right, so the answer is 8.
2. Thinking $-9 > -2$ because 9 is bigger than 2. On the number line -9 is far to the left.
3. Saying $(-4)(-5) = -20$. Two negatives multiply to a positive.
4. Treating $|-7|$ as -7 . Absolute value is a distance and is never negative.

Mistake Clinic

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Spot the error

A student writes: $-7 - 3 = 4$. **What went wrong:** the student treated the minus sign as if it cancelled the negative sign. Here -7 is already left of zero and subtracting 3 moves you 3 more steps left. **Right solution:** $-7 - 3 = -7 + (-3) = -10$.

Spot the error

A student writes: $-12 \div (-4) = -3$. **What went wrong:** the signs are the same, so the quotient must be positive. The student kept a negative sign out of habit. **Right solution:** $12 \div 4 = 3$, so $-12 \div (-4) = 3$.

Spot the error

A student writes: $-|-6| + 9 = 15$. **What went wrong:** absolute value only changes the number inside the bars. The negative sign outside the bars stays. **Right solution:** $|-6| = 6$, so $-|-6| = -6$, and $-6 + 9 = 3$.

Real-World Connection

A scuba diver at -12 meters swims up 5 meters, so she is now at $-12 + 5 = -7$ meters. Elevation, money owed, and temperature below zero all use the same rules.

Challenge Corner

Start at -3 . Repeat this rule four times: multiply by -2 , then add 5. What number do you end with? **Solution:** Step 1: $(-3)(-2) + 5 = 6 + 5 = 11$. Step 2: $11(-2) + 5 = -22 + 5 = -17$. Step 3: $(-17)(-2) + 5 = 34 + 5 = 39$. Step 4: $39(-2) + 5 = -78 + 5 = -73$. The final number is -73 . Notice the sign flips every time you multiply by -2 .

Practice**PRACTICE Negative Numbers and Integer Operations**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- Which list is in order from least to greatest?
 (A) $-1, -5, 0, 4$ (B) $-5, -1, 0, 4$ (C) $0, -1, -5, 4$
 (D) $4, 0, -1, -5$
- What is $-15 + (-9)$?
 (A) -6 (B) 6 (C) -24 (D) 24
- What is $12 - 20$?
 (A) -8 (B) 8 (C) 32 (D) -32
- What is $9 - (-5)$?
 (A) 4 (B) -4 (C) -14 (D) 14
- What is $(-5)(-8)$?
 (A) -40 (B) 40 (C) -13 (D) 13
- What is the value of $|-3| + |-11|$?
 (A) -14 (B) -8 (C) 8 (D) 14
- What is $-6 - (-10)$?
 (A) -16 (B) -4 (C) 4 (D) 16
- Which expression has the greatest value?
 (A) $(-2)(-4)$ (B) $-2 + 8$ (C) $-2 - 8$ (D) $|-2| - 8$

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9. What is the distance between -9 and 4 on a number line?
(A) 5 (B) 13 (C) -13 (D) 36
10. Evaluate $72 \div (-9)$.
11. Evaluate $6 - (-3)(2)$.
12. A submarine is at -80 meters. It rises 35 meters, then dives 50 meters. What is its final depth as an integer (use a negative number for below sea level)?
13. The product of three integers is -60 . Two of them are -3 and 4 . What is the third integer?
14. At 6 a.m. the temperature is -9 degrees. It rises 4 degrees each hour until 10 a.m., then falls 3 degrees each hour for 2 hours. What is the final temperature?
15. Two integers have a sum of -3 and a product of -40 . Find both integers.

03.2

Order of Operations

Learn the priority rules that give every expression exactly one value, including grouping symbols, fraction bars, exponents, and negative bases, and how to evaluate expressions with letters.

Why This Matters

Imagine two friends compute $2 + 3 \cdot 4$. One gets 20 and the other gets 14. Both did honest arithmetic, so who is right? Without an agreed order, math would be chaos, and calculators, spreadsheets, and computer programs would all give different answers. The **order of operations** is the shared rule book that makes every expression have exactly one value. Students get stuck because they memorize a word like PEMDAS and then apply it too literally. They multiply before dividing, or add before subtracting, because M comes before D and A before S in the letters. They also lose track of what a fraction bar or an exponent is grouping. This skill is used in every formula you will ever evaluate: area, interest, speed, and spreadsheet cells.

Core Ideas

An expression is a mix of numbers and operations. To find its value, follow the priority levels below, from top to bottom. Operations on the same level are done **left to right**.

Level	What to do	Example
1	Grouping: parentheses, brackets, and inside a fraction bar, absolute value bars	$(3 + 4) \cdot 2$
2	Exponents and roots	5^2
3	Multiply and divide, left to right	$12 \div 4 \cdot 3$
4	Add and subtract, left to right	$10 - 3 + 2$

Order of Operations

1. Grouping symbols first (innermost first).

2. Exponents.

3. Multiplication and division together, from left to right.

4. Addition and subtraction together, from left to right.

PEMDAS is only a memory aid. Multiplication and division share a level, and so do addition and subtraction.

Look at $20 - 12 \div 3 + 2$. Division comes first: $12 \div 3 = 4$. Now only subtraction and addition remain, so go left to right: $20 - 4 = 16$, then $16 + 2 = 18$. If you added first you would get $20 - 12 \div 5 = 17.6$, which is wrong.

Exponents and Negatives

An exponent applies only to what is directly touching it. So $-3^2 = -(3 \cdot 3) = -9$, but $(-3)^2 = (-3)(-3) = 9$.

A **fraction bar** is a grouping symbol: $\frac{18 + 6}{2 + 4}$ means $(18 + 6) \div (2 + 4)$. Work the top and bottom separately, then divide.

Step	Work	Result
Top	$18 + 6$	24
Bottom	$2 + 4$	6
Divide	$24 \div 6$	4

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When you see grouping symbols inside other grouping symbols, start with the innermost pair, like peeling an onion. In $3[2 + (10 - 4)]$, the innermost group is $(10 - 4) = 6$, then $2 + 6 = 8$, then $3 \cdot 8 = 24$. To **evaluate an expression** with letters, replace each letter by its value inside parentheses, then follow the same order.

Expression	Value	Why
$2 + 3 \cdot 4$	14	multiply first
$(2 + 3) \cdot 4$	20	parentheses first
$2 \cdot 3^2$	18	exponent only touches 3
$(2 \cdot 3)^2$	36	parentheses first

Worked Examples

EXAMPLE 1: MULTIPLY BEFORE ADD

Evaluate $8 + 3 \cdot 4$.

SOLUTION

Multiplication is higher priority: $3 \cdot 4 = 12$.

Then add: $8 + 12 = 20$.

EXAMPLE 2: LEFT TO RIGHT

Evaluate (a) $6 \div 2 \cdot 3$ and (b) $20 - 12 \div 3 + 2$.

SOLUTION

(a) Division and multiplication share a level, so go left to right: $6 \div 2 = 3$, then $3 \cdot 3 = 9$. It is not 1.

(b) Divide first: $12 \div 3 = 4$. Then $20 - 4 = 16$ and $16 + 2 = 18$.

EXAMPLE 3: GROUPING AND EXPONENTS

Evaluate $5(7 - 2)^2 - 10$ and $3[2 + (10 - 4)]$.

SOLUTION

First: parentheses give $7 - 2 = 5$. Exponent: $5^2 = 25$. Multiply: $5 \cdot 25 = 125$. Subtract: $125 - 10 = 115$.

Second: innermost $(10 - 4) = 6$. Then $2 + 6 = 8$. Then $3 \cdot 8 = 24$.

EXAMPLE 4: FRACTION BARS AND NEGATIVE BASES

Evaluate (a) $\frac{18 + 6}{2 + 4}$ and (b) -3^2 compared with $(-3)^2$.

SOLUTION

(a) Top: $18 + 6 = 24$. Bottom: $2 + 4 = 6$. Divide: $24 \div 6 = 4$.

(b) -3^2 means the opposite of 3^2 , which is -9 . But $(-3)^2 = 9$. They are different.

EXAMPLE 5: EVALUATE WITH VALUES

Find the value of $3a^2 - 4b + |a - b|$ when $a = -2$ and $b = 5$.

SOLUTION

Substitute with parentheses: $3(-2)^2 - 4(5) + |-2 - 5|$.

Exponent: $(-2)^2 = 4$. Bars: $|-7| = 7$.

Now $3 \cdot 4 - 4 \cdot 5 + 7 = 12 - 20 + 7$.

Left to right: $12 - 20 = -8$, then $-8 + 7 = -1$.

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EXAMPLE 6: A FRACTION WITH POWERS

Evaluate $\frac{2^3 + 4 \cdot 5}{(7 - 3)^2 - 12}$.

SOLUTION

Treat the fraction bar as grouping. Top: $2^3 = 8$, then $4 \cdot 5 = 20$, so $8 + 20 = 28$.

Bottom: $(7 - 3)^2 = 4^2 = 16$, then $16 - 12 = 4$.

Divide: $28 \div 4 = 7$.

Coach's Playbook

- 1 Scan first.** Before calculating, look for grouping symbols, exponents, and fraction bars. Underline the part that goes first.
 - 2 Rewrite one step at a time.** After each operation, copy the whole expression with that part replaced by its value. Never do two levels in your head at once.
 - 3 Go left to right on ties.** When you see only multiplication and division (or only addition and subtraction), read like a sentence from left to right.
 - 4 Check with a different path.** Estimate roughly: does $4 + 6 \cdot (5 - 2)^2 \div 9$ look like about 10? A quick estimate catches a skipped step.
- ◆ Treat a fraction bar like invisible parentheses around the top and the bottom.
 - ◆ In a calculator, type parentheses around the top and bottom of a fraction so the machine follows the same grouping you do.
 - ◆ Use a different symbol for each nesting level, such as () inside [], to keep your place.

Common traps

1. Doing all multiplication before all division (or all addition before subtraction). They tie, so go left to right. 2. Writing $-3^2 = 9$. The exponent touches only the 3, so it is -9 . 3. Adding before multiplying, as in $2 + 3 \cdot 4 = 20$. The correct value is 14. 4. Ignoring a fraction bar, for example computing $18 + \frac{6}{2} + 4$ for $\frac{18+6}{2+4}$.

Mistake Clinic**Spot the error**

A student writes: $12 - 8 \div 4 + 2 = 4 \div 4 + 2 = 1 + 2 = 3$. **What went wrong:** the student subtracted first because it came first on the page. Division has higher priority than subtraction. **Right solution:** $8 \div 4 = 2$, then $12 - 2 + 2 = 10 + 2 = 12$.

Spot the error

A student evaluates $3x^2$ for $x = -4$ and writes $3(-4)^2 = -48$. **What went wrong:** the exponent applies to -4 as a whole because of the parentheses, so the square is positive. **Right solution:** $(-4)^2 = 16$, then $3 \cdot 16 = 48$.

Spot the error

A student writes: $5 - 2(4 - 7) = (5 - 2)(4 - 7) = -9$. **What went wrong:** the student subtracted $5 - 2$ first. The 2 is multiplying the parentheses, and multiplication comes before subtraction. **Right solution:** $4 - 7 = -3$, then $2(-3) = -6$, and $5 - (-6) = 11$.

Real-World Connection

Three friends each buy a ticket for 12 dollars with a 2 dollar coupon, and they pay a 4 dollar service fee once. The total is $3(12 - 2) + 4 = 34$ dollars. If you forgot the parentheses, you would get a very different bill.

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Challenge Corner

Insert exactly one pair of parentheses in $2 + 3 \cdot 4 + 5$ (anywhere). How many different values can you get, and what is the greatest? **Solution:** With no parentheses the value is $2 + 12 + 5 = 19$. Try $(2 + 3) \cdot 4 + 5 = 20 + 5 = 25$. Try $2 + 3 \cdot (4 + 5) = 2 + 27 = 29$. Other placements, such as $2 + (3 \cdot 4) + 5$ or $(2 + 3 \cdot 4) + 5$, just repeat 19. So there are three different values, 19, 25, and 29, and the greatest is 29. Parentheses can only change the order when they group a lower-priority operation.

Practice**PRACTICE Order of Operations**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What is the value of $7 + 2 \cdot 5$?
(A) 45 (B) 17 (C) 35 (D) 27
- What is the value of $24 \div 4 \cdot 3$?
(A) 2 (B) 9 (C) 18 (D) 72
- What is the value of -4^2 ?
(A) 16 (B) -8 (C) 8 (D) -16
- What is the value of $2(9 - 3)^2 \div 6$?
(A) 12 (B) 6 (C) 24 (D) 2
- What is the value of $\frac{30 - 6}{2 + 4} + 7$?
(A) 23 (B) 14 (C) 11 (D) 7
- What is the value of $x^2 + 2x$ when $x = -3$?
(A) -15 (B) 3 (C) 15 (D) -3
- What is $15 - 3 \cdot (2 + 1)$?
(A) 36 (B) 10 (C) 6 (D) 9
- What is $(-2)^3 + 5 \cdot 2^2$?
(A) 12 (B) 14 (C) -28 (D) 92
- Which expression equals 11?
(A) $2 \cdot (4 - 3)$ (B) $(3 + 4) \cdot 2$ (C) $3 \cdot 4 + 2$ (D) $3 + 4 \cdot 2$
- Evaluate $4 + 6 \cdot (5 - 2)^2 \div 9$.
- Find the value of $3(2a - b) + |b - a|$ when $a = 4$ and $b = -2$.
- Evaluate $2^3 \cdot 3 - (18 - 3 \cdot 4)^2 \div 4$.
- A baker makes 4 trays of 12 muffins and sells 30 of them. The rest are packed into boxes of 6. Write one expression with parentheses and find the number of boxes.
- Evaluate $\frac{5^2 - 1}{2 \cdot 3} + \left(\frac{12}{4}\right)^2$.
- Mia buys 3 notebooks at 4 dollars each and 2 pens at 1.50 dollars each. She uses a 2 dollar coupon and splits the total cost equally with a friend. Write one expression and find what Mia pays, in dollars.

03.3 Factors, Multiples, GCF, LCM, and Primes

Understand factors, multiples, and primes, use divisibility tests and factor trees, and solve real problems with the greatest common factor and least common multiple.

Why This Matters

Factors and multiples are the hidden machinery behind fractions, ratios, and scheduling. When you simplify $\frac{18}{24}$ you divide by a common factor. When you add $\frac{1}{6} + \frac{1}{4}$ you look for a common multiple. When two buses leave the station on different schedules and you want to know when they leave together again, you are finding a least common multiple.

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Students get stuck because the vocabulary is crowded: factor, multiple, prime, composite, GCF, LCM. They mix up GCF with LCM, forget that 1 is not prime, or stop factoring too early. Learn the meaning of each word and the picture behind it, and the problems become routine.

Core Ideas

Key Vocabulary

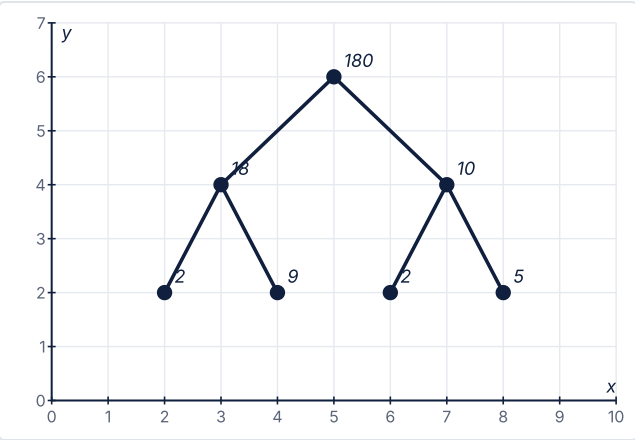
- A **factor** of n divides n with no remainder. The factors of 12 are 1, 2, 3, 4, 6, 12.
- A **multiple** of n is n times a whole number. Multiples of 12 are 12, 24, 36, and so on.
- A **prime** number has exactly two factors, 1 and itself (2, 3, 5, 7, 11, ...). A **composite** number has more than two factors. The number 1 is neither prime nor composite.

Factors come in pairs. To list every factor of 36, test 1, 2, 3, 4, 5, 6 and write each partner: 1×36 , 2×18 , 3×12 , 4×9 , 6×6 . You only need to test up to the square root of the number, because after that the pairs repeat.

Divisible by	Quick test	Example
2	last digit is 0, 2, 4, 6, or 8	3,418
3	digit sum is divisible by 3	417: $4 + 1 + 7 = 12$
4	last two digits divisible by 4	3,512: 12 works
5	last digit 0 or 5	865
6	divisible by both 2 and 3	522
9	digit sum divisible by 9	738: $7 + 3 + 8 = 18$
10	last digit 0	140

Multiples of	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
12	12	24	36	48	60	72	84	96	108
18	18	36	54	72	90	108	126	144	162

A **factor tree** breaks a number into primes. Split into any two factors, then keep splitting until every branch ends in a prime. The result is the **prime factorization**, and it is unique for every number (apart from the order). It is usually written with exponents: $180 = 2^2 \cdot 3^2 \cdot 5$.



Factor tree for 180: $180 = 18 \times 10$, $18 = 2 \times 9$, $10 = 2 \times 5$. Keep splitting 9 into 3×3 to finish: $2 \times 2 \times 3 \times 3 \times 5$.

GCF and LCM

- The **greatest common factor (GCF)** is the largest number that divides both numbers. From prime factorizations, take each shared prime to the **smaller** power.
- The **least common multiple (LCM)** is the smallest positive number that both divide. From prime factorizations, take every prime that appears to the **larger** power.
- For two numbers: $GCF \times LCM = \text{product of the two numbers}$.

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Here is a way to remember which is which. GCF problems are about **splitting into equal groups** (the answer is no bigger than the smaller number). LCM problems are about **things repeating at the same time** (the answer is at least as big as the larger number).

Worked Examples

EXAMPLE 1: DIVISIBILITY TESTS

Is 342 divisible by 2, 3, 4, 5, 6, or 9?

SOLUTION

It ends in 2, so it is divisible by 2. The digit sum is $3 + 4 + 2 = 9$, so it is divisible by 3 and by 9.

Divisible by 2 and 3 means divisible by 6.

Last two digits 42 is not divisible by 4, and the last digit is not 0 or 5.

So 342 is divisible by 2, 3, 6, and 9, but not by 4 or 5.

EXAMPLE 2: PRIME FACTORIZATION

Write 180 as a product of primes.

SOLUTION

Split: $180 = 18 \cdot 10$. Then $18 = 2 \cdot 9 = 2 \cdot 3 \cdot 3$ and $10 = 2 \cdot 5$.

Collect: $2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 2^2 \cdot 3^2 \cdot 5$.

Check: $4 \cdot 9 \cdot 5 = 180$.

EXAMPLE 3: GCF AND LCM

Find the GCF and LCM of 36 and 48.

SOLUTION

Prime forms: $36 = 2^2 \cdot 3^2$ and $48 = 2^4 \cdot 3$.

GCF uses the smaller powers: $2^2 \cdot 3 = 12$.

LCM uses the larger powers: $2^4 \cdot 3^2 = 144$.

Check: $12 \cdot 144 = 1728 = 36 \cdot 48$.

EXAMPLE 4: A GCF WORD PROBLEM

A teacher has 24 pencils and 36 erasers. She wants identical supply bags with nothing left over. What is the greatest number of bags, and what is in each?

SOLUTION

The number of bags must divide both 24 and 36, and we want the greatest, so find the GCF.

$24 = 2^3 \cdot 3$ and $36 = 2^2 \cdot 3^2$. GCF = $2^2 \cdot 3 = 12$.

12 bags, each with $24 \div 12 = 2$ pencils and $36 \div 12 = 3$ erasers.

EXAMPLE 5: AN LCM WORD PROBLEM

A bus comes every 12 minutes and a train every 18 minutes. Both arrive at 8:00 a.m. When do they next arrive together?

SOLUTION

This is a repeating-together problem, so find the LCM.

$12 = 2^2 \cdot 3$ and $18 = 2 \cdot 3^2$. LCM = $2^2 \cdot 3^2 = 36$.

They arrive together again 36 minutes later, at 8:36 a.m.

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EXAMPLE 6: ADDING FRACTIONS WITH THE LCMFind $\frac{5}{12} + \frac{7}{18}$.**SOLUTION**The common denominator should be the LCM of 12 and 18. $12 = 2^2 \cdot 3$ and $18 = 2 \cdot 3^2$, so $\text{LCM} = 2^2 \cdot 3^2 = 36$.Rewrite: $\frac{5}{12} = \frac{15}{36}$ and $\frac{7}{18} = \frac{14}{36}$.Add: $\frac{15+14}{36} = \frac{29}{36}$.

29 and 36 share no factor except 1, so the answer is already in lowest terms.

Coach's Playbook

- 1 Decide the question type.** Equal groups or splitting up: GCF. Repeating or lining up again: LCM.
 - 2 Factor into primes.** Use a factor tree and write the result with exponents.
 - 3 Compare exponents.** GCF takes the smaller exponent of shared primes only. LCM takes the larger exponent of all primes.
 - 4 Check reasonableness.** The GCF must divide both numbers. The LCM must be divisible by both.
- ◆ For small numbers, list multiples of the larger number until the smaller number divides one of them.
 - ◆ Test primes in order (2, 3, 5, 7) when deciding if a number is prime. You only need primes up to its square root.
 - ◆ If one number is a multiple of the other, the smaller is the GCF and the larger is the LCM.

Common traps

1. Calling 1 a prime number. It has only one factor. 2. Mixing up GCF and LCM. If your GCF is larger than one of the numbers, something is wrong. 3. Stopping a factor tree at 9 or 10, which are not prime. 4. Adding the digits for a divisibility test by 4 or 5. Those tests use the last digits.

Mistake Clinic**Spot the error**

A student is asked for the GCF of 12 and 18 and writes 36. **What went wrong:** 36 is a multiple of both numbers, so it is the LCM. A GCF can never be larger than the smaller number. **Right solution:** $12 = 2^2 \cdot 3$ and $18 = 2 \cdot 3^2$. Shared primes: $2 \cdot 3 = 6$, so the GCF is 6.

Spot the error

A student writes the prime factorization of 48 as $6 \cdot 8$. **What went wrong:** 6 and 8 are composite, so the factoring is not finished. **Right solution:** $6 = 2 \cdot 3$ and $8 = 2^3$, so $48 = 2^4 \cdot 3$.

Spot the error

A student says 735 is divisible by 6 because the digit sum is $7 + 3 + 5 = 15$, which is divisible by 3. **What went wrong:** divisibility by 6 needs divisibility by both 2 and 3. 735 ends in 5, so it is odd. **Right solution:** 735 is divisible by 3 but not by 2, so it is not divisible by 6.

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Real-World Connection

Music rhythms use LCMs: a drum that hits every 3 beats and a bell every 4 beats line up every 12 beats, and the pattern repeats.

Challenge Corner

Find the smallest positive whole number that is divisible by 7 and leaves a remainder of 1 when divided by 2, by 3, and by 4. **Solution:** If the number leaves remainder 1 for 2, 3, and 4, then the number minus 1 is a multiple of $\text{LCM}(2, 3, 4) = 12$. So the number is $12k + 1$: 13, 25, 37, 49, and so on. Test each for divisibility by 7. 13, 25, and 37 fail, but $49 = 7 \cdot 7$ works. Check: $49 = 2 \cdot 24 + 1 = 3 \cdot 16 + 1 = 4 \cdot 12 + 1$. The answer is 49.

Practice**PRACTICE Factors, Multiples, GCF, LCM, and Primes**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- Which number is prime?
(A) 51 (B) 57 (C) 63 (D) 59
- What is the prime factorization of 84?
(A) $2^2 \cdot 3 \cdot 7$ (B) $2 \cdot 6 \cdot 7$ (C) $4 \cdot 21$ (D) $2 \cdot 3^2 \cdot 7$
- What is the GCF of 18 and 30?
(A) 3 (B) 90 (C) 6 (D) 2
- What is the LCM of 6 and 10?
(A) 60 (B) 30 (C) 2 (D) 16
- Which number is divisible by 9?
(A) 4, 614 (B) 4, 620 (C) 4, 632 (D) 4, 617
- What is the GCF of $2^3 \cdot 3^2 \cdot 5$ and $2^2 \cdot 3^4 \cdot 7$?
(A) 36 (B) 12 (C) 5, 040 (D) 180
- How many positive factors does 36 have?
(A) 6 (B) 8 (C) 9 (D) 12
- What is the LCM of 9 and 12?
(A) 3 (B) 108 (C) 21 (D) 36
- A florist has 30 roses and 45 tulips. She makes identical bouquets with no flowers left over. What is the greatest number of bouquets she can make?
(A) 5 (B) 15 (C) 90 (D) 75
- Find the LCM of 8, 12, and 15.
- Find the GCF of 84 and 126.
- Two lights start blinking together. One blinks every 15 seconds and the other every 20 seconds. Not counting the start, how many times do they blink together in the next 5 minutes?
- What is the smallest whole number greater than 1 that leaves a remainder of 1 when divided by 4, by 5, and by 6?
- Is 1,001 prime? If not, write its prime factorization.
- Two whole numbers have GCF 6 and LCM 180. One of them is 36. Find the other.

03.4 Mental Math and Estimation

Build a toolbox of mental strategies: compensation, breaking numbers apart, benchmark fractions, and quick percents, and learn to check any answer with a reasonableness estimate.

Why This Matters

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Mental math is not a party trick. It is how you decide whether a store discount is a good deal, whether your calculator answer makes sense, and how you keep moving on a test when the numbers get messy. People who do mental math well are not 'faster brains'. They have a toolbox of strategies and they pick the right one for the numbers in front of them.

Students get stuck by trying to picture the written algorithm (carrying and borrowing) in their head, which overloads memory. The fix is to **change the numbers into friendlier ones**, then adjust. The most common struggles are forgetting to adjust back, losing a part when breaking numbers apart, and not checking whether the final answer is reasonable.

Core Ideas

Friendly numbers are tens, hundreds, halves, and other numbers you already know well. Every strategy below is about turning an awkward problem into a friendly one without changing the answer, or changing it by an amount you can track.

Compensation

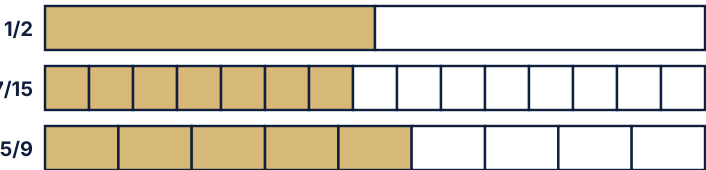
Nudge one number to a friendly one, then undo the nudge.
 $59 + 38 = 60 + 38 - 1 = 97$. You added 1 extra, so subtract 1.
 $83 - 29 = 83 - 30 + 1 = 54$. You subtracted 1 too many, so add 1 back.
Another version: change both numbers the same way. $83 - 29 = 84 - 30 = 54$ (adding 1 to both keeps the difference).

Breaking Numbers Apart

Split a number into easy parts (tens and ones, or a round number minus a small one), do each part, and combine.
 $7 \times 48 = 7 \times 50 - 7 \times 2 = 350 - 14 = 336$.
 $23 \times 12 = 23 \times 10 + 23 \times 2 = 230 + 46 = 276$. This uses the distributive property: $a(b + c) = ab + ac$.

Problem	Friendly version	Answer
$98 + 47$	$100 + 47 - 2$	145
7×99	$7 \times 100 - 7$	693
25×36	100×9 (since $25 \times 4 = 100$)	900
$1,002 - 485$	$1,000 - 485 + 2$	517

Benchmark fractions are 0 , $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1 . To compare fractions quickly, see which side of a benchmark each lies on. For $\frac{7}{15}$, half of 15 is 7.5, and 7 is less than that, so $\frac{7}{15}$ is just under one half. For $\frac{5}{9}$, half of 9 is 4.5, and 5 is more, so it is just over.



$7/15$ falls just short of the half-way mark and $5/9$ goes just past it, so $5/9$ is larger.

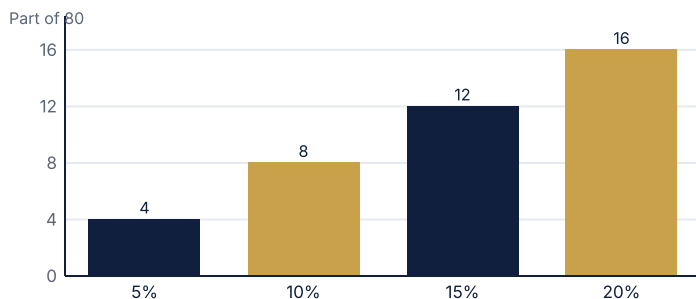
Quick Percent Tricks

10% of a number: move the decimal one place left. 1%: two places left.
5% is half of 10%. 15% is 10% plus 5%. 20% is double 10%. 25% is one quarter.
Percents commute: 15% of 80 equals 80% of 15, whichever is easier.

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Percent parts of 80: build 15% from 10% and 5%, and 20% by doubling 10%.

Estimation means rounding to friendly numbers to get an answer that is close. It is also your best error detector. If an exact calculation gives 2,530 but your estimate says about 250, you are off by a factor of ten, probably a misplaced decimal point.

Worked Examples

EXAMPLE 1: COMPENSATION

Find $59 + 38$ and $83 - 29$ mentally.

SOLUTION

$59 + 38$: round 59 up to 60 (added 1). $60 + 38 = 98$. Subtract the extra 1: 97.

$83 - 29$: round 29 up to 30 (subtracted 1 too many). $83 - 30 = 53$. Add 1 back: 54.

EXAMPLE 2: BREAKING APART

Find 7×48 and 23×12 mentally.

SOLUTION

48 is 2 less than 50. $7 \times 50 = 350$, and $7 \times 2 = 14$. $350 - 14 = 336$.

$12 = 10 + 2$. $23 \times 10 = 230$ and $23 \times 2 = 46$. Add: 276.

EXAMPLE 3: BENCHMARK FRACTIONS

Which is larger, $\frac{5}{9}$ or $\frac{7}{15}$? Use benchmarks.

SOLUTION

Half of 9 is 4.5, and 5 is more than 4.5, so $\frac{5}{9} > \frac{1}{2}$.

Half of 15 is 7.5, and 7 is less than 7.5, so $\frac{7}{15} < \frac{1}{2}$.

Therefore $\frac{5}{9}$ is larger.

EXAMPLE 4: QUICK PERCENTS

Find 15% of 80, and find a 15% tip on a 46 dollar bill.

SOLUTION

10% of 80 is 8. 5% is half of that, 4. So 15% is $8 + 4 = 12$.

10% of 46 is 4.60. 5% is 2.30. Tip: $4.60 + 2.30 = 6.90$ dollars.

EXAMPLE 5: ESTIMATION AND REASONABLENESS

A student computes 48.7×5.2 and writes 2,532.4. Is that reasonable? What is the correct value?

SOLUTION

Round to friendly numbers: $50 \times 5 = 250$.

2,532.4 is about ten times too large, so the decimal point is misplaced.

Exact: $48.7 \times 5.2 = 253.24$, which agrees with the estimate.

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EXAMPLE 6: MULTIPLYING AND DIVIDING BY 25

Find 25×48 and $1,500 \div 25$ mentally.

SOLUTION

25 is a quarter of 100. So $25 \times 48 = 48 \div 4 \times 100 = 12 \times 100 = 1,200$.

To divide by 25, multiply by 4 and divide by 100: $1,500 \times 4 = 6,000$, then $6,000 \div 100 = 60$.

Check with money: 25 cents goes into 15 dollars exactly 60 times. Yes.

Coach's Playbook

- 1 Look at the numbers first.** Are any close to a ten, a hundred, or a half? Choose the strategy that uses that.
 - 2 Change, then track.** Say out loud what you changed ('I added 1') so you remember to undo it.
 - 3 Do the easy parts left to right.** Working from the big place values first (hundreds, then tens) keeps your estimate close even if you slip later.
 - 4 Do a reasonableness check.** Round everything and compare. If your answer and your estimate disagree by a lot, find the mistake.
- ◆ Multiply by 5 by multiplying by 10 and halving. Multiply by 25 by dividing by 4 and multiplying by 100.
 - ◆ To divide by 5, double the number and divide by 10.
 - ◆ Use known facts: $15 \times 4 = 60$ so $15 \times 8 = 120$ by doubling.

Common traps

1. Compensating the wrong way: $59 + 38 = 60 + 38 = 98$ and forgetting to subtract 1. 2. Breaking apart but dropping a part, such as $7 \times 48 = 7 \times 50 = 350$. 3. Taking 15% to be 10% twice. 4. Trusting an answer without comparing to an estimate, which lets a misplaced decimal point slip by.

Mistake Clinic**Spot the error**

A student finds $83 - 29$ by changing it to $83 - 30 = 53$ and stops. **What went wrong:** the student subtracted 1 too many (30 instead of 29) and never corrected it. **Right solution:** $83 - 30 = 53$, then add the 1 back: $53 + 1 = 54$.

Spot the error

A student finds 15% of 60 as $10\% + 5 = 6 + 5 = 11$. **What went wrong:** 5 is not 5% of 60. The student added the number 5 instead of 5% of 60. **Right solution:** 10% of 60 is 6, 5% is half of that, 3, so 15% is $6 + 3 = 9$.

Spot the error

A student finds 6×98 as $6 \times 100 + 6 \times 2 = 612$. **What went wrong:** 98 is 2 less than 100, so you must subtract the extra 6×2 . **Right solution:** $6 \times 100 - 6 \times 2 = 600 - 12 = 588$.

Real-World Connection

A pair of shoes costs 59.99 dollars and you have a coupon for 20% off. Round to 60 dollars: 10% is 6, 20% is 12, so you pay about 48 dollars, before tax.

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Challenge Corner

A jacket costs 79.99 dollars. It is 30% off, and then 10% sales tax is added to the sale price. First estimate the final cost using friendly numbers, then find it exactly to the nearest cent. **Solution:** Estimate: $80 \times 0.70 = 56$ dollars after the discount. Tax is 10% of 56, which is 5.60, so the total is about 61.60 dollars. Exact: $79.99 \times 0.70 = 55.993$, and $55.993 \times 1.10 = 61.5923$, which rounds to 61.59 dollars. The estimate and the exact answer agree, so the answer is reasonable. Notice that a 30% discount followed by 10% tax is not the same as 20% off.

Practice**PRACTICE Mental Math and Estimation**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What is $98 + 47$?
(A) 135 (B) 155 (C) 149 (D) 145
- What is 25×36 ?
(A) 900 (B) 600 (C) 750 (D) 1,000
- What is 15% of 60?
(A) 6 (B) 12 (C) 9 (D) 15
- Which is closest to 49×21 ?
(A) 100 (B) 500 (C) 10,000 (D) 1,000
- Which fraction is greater than $\frac{1}{2}$?
(A) $\frac{11}{23}$ (B) $\frac{17}{35}$ (C) $\frac{21}{41}$ (D) $\frac{9}{19}$
- What is 7×99 ?
(A) 693 (B) 700 (C) 707 (D) 6,993
- What is 20% of 45?
(A) 4.5 (B) 9 (C) 15 (D) 90
- What is 64×25 ?
(A) 1,200 (B) 1,280 (C) 1,500 (D) 1,600
- Which is the best estimate for $312 + 489 + 198$?
(A) 1,000 (B) 800 (C) 1,200 (D) 900
- Find 8×47 mentally.
- Find 35% of 80 mentally.
- Find $1,002 - 485$ by changing the problem to a friendlier one.
- A store sells folders for 4.95 dollars each. Use a friendly number to find the exact cost of 18 folders in dollars.
- Find 6×197 mentally.
- A restaurant bill is 64 dollars. Find a 15% tip mentally, then the total with the tip.

Answers**Answers and Explanations**

Complete solutions for every practice problem, numbered to match. Read the explanation even when your answer is right.

03.1 Negative Numbers and Integer Operations**PRACTICE · NEGATIVE NUMBERS AND INTEGER OPERATIONS**

- B. -5 is farthest left, then -1 , then 0, then 4. Choice A puts -1 before -5 , but -1 is greater.
- C. Same signs, so add 15 and 9 to get 24 and keep the negative sign. Choice A subtracts instead.
- A. Rewrite as $12 + (-20)$. Different signs, so subtract $20 - 12 = 8$ and keep the sign of the larger absolute value: -8 . Choice B drops the sign.
- D. Subtracting a negative means adding the opposite: $9 + 5 = 14$. Choice A treats it as $9 - 5$.
- B. The signs are the same, so the product is positive: $5 \cdot 8 = 40$. Choice A forgets that two negatives make a positive.
- D. $|-3| = 3$ and $|-11| = 11$, and $3 + 11 = 14$. Choice C subtracts the two distances, which is not what the problem asks.
- C. Subtracting a negative means adding the opposite: $-6 + 10 = 4$. Choice B subtracts the absolute values but keeps a negative sign.
- A. The values are 8, 6, -10 , and -6 . Two negatives multiply to a positive, and 8 beats 6.

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9. B. Distance is $|4 - (-9)| = |13| = 13$. Choice A subtracts the absolute values, which only works when both numbers are on the same side of 0.
10. -8. The signs differ, so the quotient is negative, and $72 \div 9 = 8$.
11. 12. Multiply first: $(-3)(2) = -6$. Then $6 - (-6) = 6 + 6 = 12$.
12. -95. Compute $-80 + 35 - 50$: $-80 + 35 = -45$, then $-45 - 50 = -95$.
13. 5. $(-3)(4) = -12$, and $-12 \cdot x = -60$ gives $x = 5$ (negative divided by negative is positive).
14. 1 degree. From 6 a.m. to 10 a.m. is 4 hours, so the rise is $4 \cdot 4 = 16$ and $-9 + 16 = 7$. Then the fall is $3 \cdot 2 = 6$, and $7 - 6 = 1$.
15. 5 and -8. The product is negative, so the signs differ. The pair 5, -8 has product -40 and sum $5 + (-8) = -3$. The pair -5, 8 has sum 3, so it fails.

03.2 Order of Operations

PRACTICE · ORDER OF OPERATIONS

1. B. Multiply first: $2 \cdot 5 = 10$, then $7 + 10 = 17$. Choice A adds first and gets $9 \cdot 5$.
2. C. Same level, so go left to right: $24 \div 4 = 6$, then $6 \cdot 3 = 18$. Choice A divides by $4 \cdot 3$ first.
3. D. The exponent touches only the 4, so $-4^2 = -(16) = -16$. Choice A would be $(-4)^2$.
4. A. Parentheses: 6. Exponent: 36. Then $2 \cdot 36 = 72$, and $72 \div 6 = 12$. Choice B forgets to multiply by 2.
5. C. Top is 24, bottom is 6, so the fraction is 4 and $4 + 7 = 11$. Choice A divides only by 2 and ignores the grouping.
6. B. Substitute: $(-3)^2 + 2(-3) = 9 - 6 = 3$. Choice A uses $-3^2 = -9$ by mistake.
7. C. Parentheses: $2 + 1 = 3$. Multiply: $3 \cdot 3 = 9$. Subtract: $15 - 9 = 6$. Choice A subtracts $15 - 3$ first.
8. A. $(-2)^3 = -8$ and $5 \cdot 2^2 = 5 \cdot 4 = 20$, so $-8 + 20 = 12$. Choice D computes $5 \cdot 2 = 10$ and then squares it wrongly to get 100.
9. D. $3 + 4 \cdot 2 = 3 + 8 = 11$. Choice B adds first and gets 14, and choice C gets 14 as well.
10. 10. Parentheses: 3. Exponent: 9. Then $6 \cdot 9 = 54$, $54 \div 9 = 6$, and $4 + 6 = 10$.
11. 36. $2a - b = 8 - (-2) = 10$, so $3 \cdot 10 = 30$. Then $|-2 - 4| = |-6| = 6$. Total $30 + 6 = 36$.
12. 15. $2^3 \cdot 3 = 24$. Inside: $18 - 12 = 6$, and $6^2 = 36$, $36 \div 4 = 9$. Then $24 - 9 = 15$.
13. $(4 \cdot 12 - 30) \div 6 = 3$ boxes. $4 \cdot 12 = 48$, $48 - 30 = 18$, and $18 \div 6 = 3$.
14. 13. Top: $25 - 1 = 24$. Bottom: 6. The fraction is 4. Then $(\frac{12}{4})^2 = 3^2 = 9$, and $4 + 9 = 13$.
15. $(3 \cdot 4 + 2 \cdot 1.5 - 2) \div 2 = 6.5$, so Mia pays 6.50 dollars. Inside: $12 + 3 - 2 = 13$. Then $13 \div 2 = 6.5$.

03.3 Factors, Multiples, GCF, LCM, and Primes

PRACTICE · FACTORS, MULTIPLES, GCF, LCM, AND PRIMES

1. D. $51 = 3 \cdot 17$, $57 = 3 \cdot 19$, and $63 = 7 \cdot 9$ are composite. Only 59 has no factor besides 1 and itself.
2. A. $84 = 4 \cdot 21 = 2^2 \cdot 3 \cdot 7$. Choices B and C contain composite numbers (6, 4, 21), so they are not prime factorizations.
3. C. $18 = 2 \cdot 3^2$ and $30 = 2 \cdot 3 \cdot 5$. Shared: $2 \cdot 3 = 6$. Choice B is the LCM, not the GCF.
4. B. $6 = 2 \cdot 3$ and $10 = 2 \cdot 5$, so $\text{LCM} = 2 \cdot 3 \cdot 5 = 30$. Choice A is the product $6 \cdot 10$, which counts the shared factor 2 twice.
5. D. The digit sum of 4,617 is $4 + 6 + 1 + 7 = 18$, so it is divisible by 9. Choice A has digit sum 15, which works for 3 but not 9.
6. A. Take shared primes at the smaller power: $2^2 \cdot 3^2 = 36$. Choice C multiplies bigger powers, which is an LCM idea, and 5 and 7 are not shared.
7. C. The factors are 1, 2, 3, 4, 6, 9, 12, 18, 36, which is 9. Choice A counts only the factor pairs and misses that $6 \cdot 6$ gives a single factor, 6.
8. D. $9 = 3^2$ and $12 = 2^2 \cdot 3$, so $\text{LCM} = 2^2 \cdot 3^2 = 36$. Choice B is the product, which counts the shared factor 3 twice. Choice A is the GCF.
9. B. This is a GCF problem. $30 = 2 \cdot 3 \cdot 5$ and $45 = 3^2 \cdot 5$, so $\text{GCF} = 15$. Each bouquet has 2 roses and 3 tulips. Choice C is the LCM.
10. 120. $8 = 2^3$, $12 = 2^2 \cdot 3$, $15 = 3 \cdot 5$. Larger powers: $2^3 \cdot 3 \cdot 5 = 120$.
11. 42. $84 = 2^2 \cdot 3 \cdot 7$ and $126 = 2 \cdot 3^2 \cdot 7$. Smaller powers: $2 \cdot 3 \cdot 7 = 42$.
12. 5 times. $\text{LCM}(15, 20) = 60$ seconds. In 300 seconds they line up at 60, 120, 180, 240, and 300.
13. 61. The number minus 1 must be a multiple of 4, 5, and 6. $\text{LCM} = 60$, so the number is $60 + 1 = 61$.
14. Not prime. $1001 = 7 \cdot 11 \cdot 13$. Test small primes: 2, 3, and 5 fail, but $1001 \div 7 = 143$, and $143 = 11 \cdot 13$.
15. 30. For two numbers, GCF times LCM equals the product, so $6 \cdot 180 = 1080$. Then $1080 \div 36 = 30$. Check: $\text{GCF}(36, 30) = 6$ and $\text{LCM}(36, 30) = 180$.

03.4 Mental Math and Estimation

PRACTICE · MENTAL MATH AND ESTIMATION

1. D. $98 + 47 = 100 + 47 - 2 = 145$. Choice B forgets to subtract the extra 2 and choice C is a careless slip.
2. A. $25 \times 4 = 100$, so $25 \times 36 = 25 \times 4 \times 9 = 900$. Choice A is correct because 36 is 4 times 9.
3. C. 10% of 60 is 6 and 5% is 3, so 15% is $6 + 3 = 9$. Choice A is only 10%.
4. D. Round to $50 \times 20 = 1,000$. The exact value is 1,029. Choice B would be about 25×20 .
5. C. Half of 41 is 20.5, and 21 is more, so $\frac{21}{41} > \frac{1}{2}$. For the others, the numerator is just below half the denominator ($11 < 11.5$, $17 < 17.5$, $9 < 9.5$).
6. A. $7 \times 99 = 7 \times 100 - 7 = 693$. Choice B forgets to subtract the 7.
7. B. 10% of 45 is 4.5, so 20% is double that, 9. Choice A is only 10%.
8. D. $64 \div 4 = 16$, and $16 \times 100 = 1,600$. Choice B comes from 64×20 .
9. A. Round to $300 + 500 + 200 = 1,000$. The exact sum is 999. Choice C overshoots by rounding every number up too far.
10. 376. $8 \times 50 = 400$ and $8 \times 3 = 24$, so $400 - 24 = 376$.

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11. 28. 10% is 8, so 30% is 24, and 5% is 4. $24 + 4 = 28$.
12. 517. Subtract from 1,000 instead: $1,000 - 485 = 515$, then add the 2 back to get 517.
13. 89.10 dollars. $18 \times 5 = 90$, and each folder was 5 cents less, so subtract $18 \times 0.05 = 0.90$. $90 - 0.90 = 89.10$.
14. 1,182. $197 = 200 - 3$, so $6 \times 200 - 6 \times 3 = 1,200 - 18 = 1,182$.
15. Tip: 9.60 dollars; total: 73.60 dollars. 10% of 64 is 6.40 and 5% is 3.20, so 15% is $6.40 + 3.20 = 9.60$. Then $64 + 9.60 = 73.60$.

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