



Word Problems: Translating Language into Algebra

Four focused lessons, worked examples, a mistake clinic, and 60 practice problems with full explanations.

In this booster

06.1 Turning Words into Expressions and Equations

06.2 Age, Number, and Consecutive-Integer Problems

06.3 Money, Mixture, and Rate-Time-Distance Problems

06.4 Multi-Step and Work Problems

Every section teaches the idea, walks through the most common problem types, flags the traps students fall into, and ends with fifteen practice problems. Full answers and explanations are at the back.

06.1 Turning Words into Expressions and Equations

Learn the key phrases that turn English sentences into algebra, how to define a variable, and a four-step routine for solving any word problem. Order and parentheses are the keys.

Why This Matters

Math problems in real life do not arrive as neat equations. They arrive as sentences: "The total cost is 53 dollars," or "She has 7 more than her brother." The big skill is translating those sentences into algebra, and it is the step where most students lose points. Once the problem is an equation you already know how to solve it. The hard part is getting there.

Students most often get stuck in three places. They do not know what the unknown is, so they never define a variable. They reverse the order of subtraction in phrases like "5 less than a number." And they try to solve in their heads instead of writing a plan. The routine in this section removes all three problems.

Core Ideas

Think of English as a code for math. Certain words always point to certain operations. Learn the code and a sentence turns into symbols one piece at a time.

Operation	Key phrases	Example	Algebra
Add	sum, plus, more than, increased by, total	7 more than a number	$n + 7$
Subtract	difference, minus, less than, decreased by	5 less than a number	$n - 5$
Multiply	product, times, twice, of	twice a number	$2n$
Divide	quotient, per, divided by, shared equally	a number divided by 4	$\frac{n}{4}$
Equals	is, was, equals, gives, results in	is 20	$= 20$

Order Matters in Subtraction and Division

"a less than b" means $b - a$. Example: "5 less than x" is $x - 5$.

"a decreased by b" or "the difference of a and b" means $a - b$ in the order spoken.

"The quotient of a and b" means $a \div b$.

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Expression versus Equation

An **expression** is a phrase with no equals sign, such as $3n - 9$.

An **equation** is a complete sentence with an equals sign, such as $3n - 9 = 24$. The word "is" or "equals" usually marks where the equals sign goes.

Before writing anything, **define your variable** in a sentence: "Let n be the number of pizzas." This tells you and your teacher exactly what the letter means, and it tells you what units your final answer should have.

The Read, Plan, Solve, Check Routine

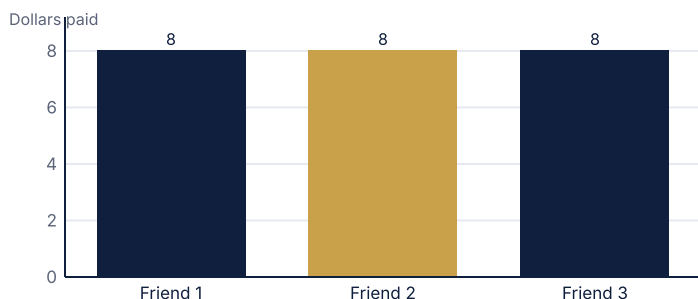
Read the problem twice. Underline the question and circle the numbers.

Plan: define the variable, then write an equation that matches the story.

Solve the equation with inverse operations.

Check your answer in the ORIGINAL words of the story, not just in your equation.

Bar models are great pictures for this. If three friends share a cost of 24 dollars equally, each bar is the same length, and the three bars add to the total.



Three equal shares of 8 dollars make 24 dollars, so $3s = 24$.

A good habit is to read a word problem three times. The first time, just picture the story. The second time, circle the numbers and underline the question. The third time, write one small piece of algebra for each phrase. If any phrase seems confusing, replace the numbers with easy ones. A phrase such as "7 less than a number" is much clearer when you try it with the number 10.

Worked Examples

EXAMPLE 1

Translate into algebra: (a) 7 more than twice a number (b) 5 less than a number (c) the quotient of a number and 4, decreased by 3.

SOLUTION

(a) "Twice a number" is $2n$. "7 more than" that is $2n + 7$.

(b) "5 less than a number" is $n - 5$, NOT $5 - n$.

(c) "The quotient of a number and 4" is $\frac{n}{4}$. "Decreased by 3" gives $\frac{n}{4} - 3$.

EXAMPLE 2

Three times a number, decreased by 9, is 24. Find the number.

SOLUTION

Let n be the number. "Three times a number" is $3n$, "decreased by 9" is $3n - 9$, and "is 24" gives the equation $3n - 9 = 24$.

Add 9: $3n = 33$. Divide by 3: $n = 11$.

Check in words: three times 11 is 33, decreased by 9 is 24. True.

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EXAMPLE 3

For a class party, pizzas cost 12 dollars each and there is a 5 dollar delivery fee. The total was 53 dollars. How many pizzas were ordered?

SOLUTION

Let p be the number of pizzas. The cost is $12p$ plus the fee: $12p + 5 = 53$.

Subtract 5: $12p = 48$. Divide by 12: $p = 4$.

Check: 4 pizzas cost 48 dollars, plus 5 is 53. The class ordered 4 pizzas.

EXAMPLE 4

The sum of a number and its double is 45. What is the number?

SOLUTION

Let n be the number. Its double is $2n$. The sum is $n + 2n$, and it equals 45: $n + 2n = 45$.

Combine like terms: $3n = 45$.

Divide by 3: $n = 15$. Check: $15 + 30 = 45$.

EXAMPLE 5

Leo has 30 dollars and saves 8 dollars each week. Ana has 72 dollars and spends 6 dollars each week. After how many weeks will they have the same amount?

SOLUTION

Let w be the number of weeks. Leo has $30 + 8w$. Ana has $72 - 6w$. Set them equal: $30 + 8w = 72 - 6w$.

Add $6w$ to both sides: $30 + 14w = 72$. Subtract 30: $14w = 42$.

Divide by 14: $w = 3$.

Check: Leo has $30 + 24 = 54$ and Ana has $72 - 18 = 54$. They match after 3 weeks.

EXAMPLE 6

Twice the sum of a number and 3 is 8 more than the number. Find the number.

SOLUTION

Let n be the number. "The sum of a number and 3" is $n + 3$. "Twice" that is $2(n + 3)$.

"Is 8 more than the number" gives $n + 8$. Equation: $2(n + 3) = n + 8$.

Distribute: $2n + 6 = n + 8$. Subtract n : $n + 6 = 8$. Subtract 6: $n = 2$.

Check in words: the sum of 2 and 3 is 5, twice that is 10, and 10 is 8 more than 2. True.

Coach's Playbook

- 1 **Underline the question.** Find out what you are asked for. That is your unknown.
- 2 **Define the variable in a sentence.** Write "Let $x = \dots$ " with units.
- 3 **Translate one chunk at a time.** Work through the sentence left to right, writing a small piece of algebra for each chunk.
- 4 **Find the equals sign.** Words like "is" or "gives" tell you where the two sides meet.
- 5 **Solve, then reread.** Make sure the answer fits the story: no negative people, no half a ticket.

A fast test for "less than": plug in a real number. If a number is 10, then 5 less than it is 5. Write $n - 5$ and check: $10 - 5 = 5$. The wrong order $5 - n$ would give -5 . Plugging in a sample number catches order mistakes in seconds.

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Common traps

1) Writing "5 less than x " as $5 - x$ instead of $x - 5$. 2) Forgetting parentheses: "twice the sum of a number and 3" is $2(n + 3)$, not $2n + 3$. 3) Using two different letters for the same unknown. 4) Giving the value of the variable when the question asks for something else, such as the cost, not the number of items.

Mistake Clinic**Spot the error**

Wrong work: "7 less than a number" is written $7 - n$. Where it went wrong: "less than" reverses the order. You take 7 away FROM the number. Right solution: $n - 7$. Test with $n = 10$: 7 less than 10 is 3, and $10 - 7 = 3$.

Spot the error

Wrong work: "Twice the sum of a number and 3" is written $2n + 3$. Where it went wrong: only the number was doubled. The words say twice the whole SUM. Right solution: $2(n + 3)$. With $n = 4$, twice the sum of 4 and 3 is 14, and $2(4 + 3) = 14$ but $2(4) + 3 = 11$.

Spot the error

Wrong work: A gym costs 20 dollars to join plus 15 dollars per month. The total is 95 dollars. The student writes $35m = 95$. Where it went wrong: the joining fee is paid once, so it is not multiplied by months. Right solution: $15m + 20 = 95$, so $15m = 75$ and $m = 5$ months.

Real-World Connection

Programmers and spreadsheet users translate words into formulas all day. "Total cost is 2 dollars per song plus 1 dollar app fee" becomes the rule $2s + 1$ for s songs.

Challenge Corner

Ana, Ben, and Cy share a bill of 38 dollars. Ana pays 2 dollars more than Ben. Cy pays twice what Ben pays. How much does each person pay? Solution: Let b be Ben's share. Ana pays $b + 2$ and Cy pays $2b$. Equation: $b + (b + 2) + 2b = 38$, so $4b + 2 = 38$ and $4b = 36$, so $b = 9$. Ben pays 9 dollars, Ana pays 11 dollars, and Cy pays 18 dollars. Check: $9 + 11 + 18 = 38$.

Practice**PRACTICE** Turning Words into Expressions and Equations

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- Which expression means "4 less than twice a number"?
 (A) $4 - 2n$ (B) $2n - 4$ (C) $2(n - 4)$ (D) $4n - 2$
- Which expression means "the quotient of a number and 6, increased by 2"?
 (A) $\frac{6}{n} + 2$ (B) $6n + 2$ (C) $\frac{n}{6} + 2$ (D) $\frac{n+2}{6}$
- Which equation matches "Five more than three times a number is 26"?
 (A) $5n + 3 = 26$ (B) $3(n + 5) = 26$ (C) $3n - 5 = 26$ (D) $3n + 5 = 26$

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4. A theater sells adult tickets for 9 dollars and student tickets for 6 dollars. A group buys 3 adult tickets and s student tickets for a total of 45 dollars. How many student tickets did they buy?
 (A) 3 (B) 5 (C) 7 (D) 12
5. Maria is 3 years older than twice her cousin's age. Maria is 17. How old is her cousin?
 (A) 10 (B) 14 (C) 7 (D) 20
6. Which equation correctly says " x is 8 less than y "?
 (A) $x = 8 - y$ (B) $x = y - 8$ (C) $y = x - 8$ (D) $x - y = 8$
7. Which equation matches "The product of 4 and a number, decreased by 7, is 29"?
 (A) $4(n - 7) =$ (B) $4n - 7 = 29$ (C) $7 - 4n = 29$ (D) $4n + 7 = 29$
 29
8. Five times a number minus 12 is 3 more than twice the number. What is the number?
 (A) 3 (B) 15 (C) 5 (D) -3
9. Amir has 40 dollars and spends 5 dollars each day. Ella has 10 dollars and saves 5 dollars each day. After how many days do they have the same amount?
 (A) 6 (B) 8 (C) 30 (D) 3
10. Twice a number plus 9 equals 5 less than 4 times the number. Write an equation and find the number.
11. Four friends split the cost of a video game equally. Then each pays an extra 3 dollars for shipping. Each friend pays 18 dollars in all. What was the price of the game?
12. A trail guide charges a 15 dollar booking fee plus 4 dollars per mile hiked. A group's bill was 51 dollars. How many miles did they hike?
13. The perimeter of a rectangle is 54 cm. The length is 3 cm more than twice the width. Find the width.
14. Four less than three times a number equals twice the number plus 9. Write an equation and solve it.
15. Mr. Lopez buys 5 notebooks and a pen set that costs 4 dollars. The total bill is 19 dollars. How much does one notebook cost?

06.2 Age, Number, and Consecutive-Integer Problems

Set up and solve age, number, and consecutive-integer problems by writing every unknown with one variable. You will also practice checking that an answer makes sense in the story.

Why This Matters

Age, number, and consecutive-integer problems look like riddles, and they are a perfect training ground for algebra. They show up in puzzles, coding challenges, and every math contest, and the same thinking is used when you model any situation where several quantities depend on each other. The big idea is to write ALL the unknown quantities using ONE variable.

Students get stuck because they try to guess, because they mix up "now" and "later" in age problems, or because they forget what "consecutive even" means. After this section you will have a pattern for every type: define one variable, write the others in terms of it, build one equation, and check it against the words.

Core Ideas

When two or more quantities are connected, pick the **smallest or simplest one** to be your variable. Then use the story to write the others with that variable. This is called writing everything in terms of one variable.

Consecutive Integers

Consecutive integers follow each other by 1: $n, n + 1, n + 2, \dots$

Consecutive even integers and **consecutive odd integers** both go up by 2: $n, n + 2, n + 4, \dots$ (for even, start with an even n ; for odd, start with an odd n).



Consecutive integers sit next to each other on the number line, 1 apart.

Notice that consecutive even integers are also 2 apart, such as 24, 26, 28. The reason is that evens come every other number. The same is true for odd numbers: 31, 33, 35.

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Age Problems

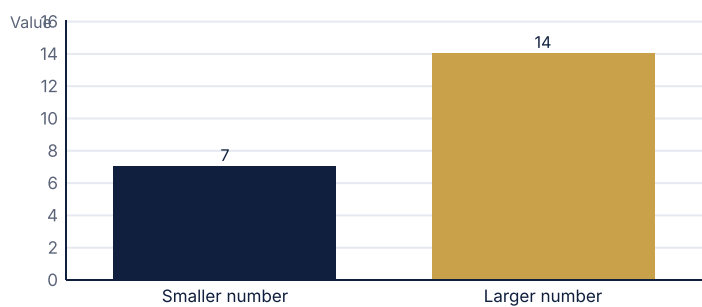
Ages change by the SAME amount for everyone. If someone is a now, they are $a + k$ in k years and $a - k$ was k years ago. Organize facts in a table with columns Now and Later, then write an equation for the relationship the story gives.

Person	Now	In 6 years
Brother	b	$b + 6$
Jada (4 times as old)	$4b$	$4b + 6$

Be careful: "4 times as old" applies to the ages at the time the story says. If Jada is 4 times as old NOW, then Jada is $4b$ now, but in 6 years she is $4b + 6$, not $4(b + 6)$. Adding years never multiplies.

Number Problems

- "A number and another number 7 more" is n and $n + 7$.
- "Twice the sum of a number and 3" means $2(n + 3)$.
- "The sum of two numbers is 40" means one is n and the other is $40 - n$.



The larger number is 7 more than the smaller. Here 7 and 14 solve Example 4.

Always check **reasonableness**. Ages are positive, so a negative age tells you the equation is set up wrong. A problem about "how many people" must have a whole number answer. And if the problem says consecutive odd integers but your answer is even, go back and look for the error.

Another tip: when a problem gives you a total for several consecutive numbers, try writing them centered on the middle one, such as $m - 1, m, m + 1$. The sum is then just $3m$, because the -1 and $+1$ cancel. For three consecutive integers with sum 96, the middle one is $m = 32$. This trick makes some problems almost mental math.

Worked Examples

EXAMPLE 1
The sum of two consecutive integers is 47. Find them.

SOLUTION
Let the smaller be n . The next is $n + 1$.
Equation: $n + (n + 1) = 47$, so $2n + 1 = 47$.
Subtract 1: $2n = 46$. Divide by 2: $n = 23$.
The integers are 23 and 24. Check: $23 + 24 = 47$.

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EXAMPLE 2

The sum of three consecutive even integers is 78. Find them.

SOLUTION

Let the smallest be n . The three are n , $n + 2$, $n + 4$.

Equation: $n + (n + 2) + (n + 4) = 78$, so $3n + 6 = 78$.

Subtract 6: $3n = 72$. Divide by 3: $n = 24$.

The integers are 24, 26, 28. Check: $24 + 26 + 28 = 78$, and all are even.

EXAMPLE 3

Jada is 4 times as old as her brother. In 6 years, the sum of their ages will be 37. How old is each now?

SOLUTION

Let b be the brother's age now. Jada is $4b$ now.

In 6 years: brother $b + 6$ and Jada $4b + 6$. Their sum is 37: $(b + 6) + (4b + 6) = 37$.

Combine: $5b + 12 = 37$. Subtract 12: $5b = 25$. So $b = 5$.

The brother is 5 and Jada is $4 \cdot 5 = 20$. Check: in 6 years they are 11 and 26, and $11 + 26 = 37$.

EXAMPLE 4

One number is 7 more than another. Three times the smaller plus twice the larger is 49. Find the numbers.

SOLUTION

Let the smaller be n . The larger is $n + 7$.

Equation: $3n + 2(n + 7) = 49$. Distribute: $3n + 2n + 14 = 49$.

Combine: $5n + 14 = 49$. Subtract 14: $5n = 35$, so $n = 7$.

The numbers are 7 and 14. Check: $3(7) + 2(14) = 21 + 28 = 49$.

EXAMPLE 5

A father is 36 and his son is 8. In how many years will the father be 3 times as old as the son?

SOLUTION

Let t be the number of years from now. Father: $36 + t$. Son: $8 + t$.

Equation: $36 + t = 3(8 + t)$. Distribute: $36 + t = 24 + 3t$.

Subtract t and subtract 24: $12 = 2t$. So $t = 6$.

Check: in 6 years the father is 42 and the son is 14, and $42 = 3 \cdot 14$.

EXAMPLE 6

The sum of the smallest and largest of three consecutive odd integers is 58. Find all three integers.

SOLUTION

Let the smallest be n . The three odd integers are n , $n + 2$, $n + 4$.

Smallest plus largest: $n + (n + 4) = 58$, so $2n + 4 = 58$.

Subtract 4: $2n = 54$. Divide by 2: $n = 27$.

The integers are 27, 29, 31. Check: $27 + 31 = 58$, and all three are odd.

Coach's Playbook

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- 1 **Pick the base variable.** Choose the smallest or the one other quantities depend on.
- 2 **Express every other quantity with it.** Use the words: "7 more than," "3 times as old," "next even number."
- 3 **Make a table if time is involved.** Rows are people, columns are Now and Later.
- 4 **Write one equation from the relationship.** Find the sentence that says two things are equal or gives a total.
- 5 **Check with the words.** Plug your answers into the original story, not into your equation.

A shortcut for consecutive integers: if you have an ODD number of consecutive integers with a given sum, the middle one is the sum divided by the count. Three consecutive integers adding to 96 have the middle one at $96 \div 3 = 32$, so they are 31, 32, 33. This works because the numbers above and below the middle balance each other out.

Common traps

1) Using n , $n + 1$, $n + 2$ for consecutive even or odd integers; evens and odds jump by 2. 2) Writing $4(b + 6)$ when the story says Jada is 4 times as old NOW. 3) Answering with the variable's value when the question asks for another quantity, such as the larger number. 4) Not checking the sum or the parity of the result.

Mistake Clinic

Spot the error

Wrong work: three consecutive even integers are written n , $n + 1$, $n + 2$. Where it went wrong: adding 1 alternates between even and odd. Evens are 2 apart. Right solution: n , $n + 2$, $n + 4$. For example, 6, 8, 10.

Spot the error

Wrong work: Jada is 4 times as old as her brother now. In 6 years, the student writes her age as $4(b + 6)$. Where it went wrong: that says she will be 4 times as old as her brother in 6 years, but the story said NOW. Right solution: now she is $4b$, so in 6 years she is $4b + 6$.

Spot the error

Wrong work: Two numbers add to 50 and one is 12 more than the other. The student solves $n + (n + 12) = 50$ and gets $n = 19$, then answers "The larger number is 19." Where it went wrong: 19 is the smaller number, which was the variable. Right solution: the larger number is $19 + 12 = 31$. Check: $19 + 31 = 50$.

Real-World Connection

Seat numbers in a row are consecutive integers. If three friends sit in seats whose numbers add to 51, they sit in seats 16, 17, 18, because the middle number is $51 \div 3 = 17$.

Challenge Corner

The sum of the smallest and twice the largest of three consecutive integers is 43. Find the integers. Solution: Let the integers be n , $n + 1$, $n + 2$. Smallest plus twice the largest: $n + 2(n + 2) = 43$. Distribute: $n + 2n + 4 = 43$, so $3n + 4 = 43$. Subtract 4: $3n = 39$, so $n = 13$. The integers are 13, 14, 15. Check: $13 + 2(15) = 43$.

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Practice

PRACTICE Age, Number, and Consecutive-Integer Problems

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- Which list shows three consecutive odd integers, if n is odd?
 (A) $n, n + 1, n + 2$ (B) $n, n + 2, n + 4$ (C) $n, n + 3, n + 6$
 (D) $n, 2n, 3n$
- The sum of two consecutive integers is 61. What is the smaller integer?
 (A) 31 (B) 29 (C) 30 (D) 60
- Four times a number minus 5 equals three times the number plus 9. What is the number?
 (A) 4 (B) -14 (C) 28 (D) 14
- Sara is 5 years older than Tim. The sum of their ages is 31. How old is Sara?
 (A) 18 (B) 13 (C) 26 (D) 15.5
- The sum of three consecutive integers is 96. What is the largest of the three?
 (A) 31 (B) 33 (C) 32 (D) 35
- Nia is 6 years old. In 4 years, her brother Eli will be twice as old as Nia will be then. How old is Eli now?
 (A) 20 (B) 12 (C) 24 (D) 16
- The sum of three consecutive even integers is 66. What is the smallest of the three?
 (A) 20 (B) 22 (C) 24 (D) 18
- Rina is 4 years older than twice her brother's age. Rina is 20. How old is her brother?
 (A) 12 (B) 8 (C) 10 (D) 16
- In 5 years, Omar will be 3 times as old as he was 7 years ago. How old is Omar now?
 (A) 18 (B) 11 (C) 15 (D) 13
- The sum of three consecutive odd integers is 99. Find the integers.
- A mother is 3 times as old as her daughter. In 8 years, the mother will be twice as old as her daughter. How old is the mother now?
- One number is 6 less than another. Twice the larger plus the smaller is 63. Find both numbers.
- Four consecutive integers have a sum equal to 5 times the smallest integer minus 9. Find the four integers.
- Find two consecutive odd integers whose sum is 76.
- Two numbers add to 40. Twice the smaller number is 5 more than the larger number. Find both numbers.

06.3 Money, Mixture, and Rate-Time-Distance Problems

Use tables and simple relationships to solve coin and ticket problems, mixtures, and distance-rate-time problems, including catch-up stories where one traveler leaves later.

Why This Matters

Money, mixtures, and motion problems are the classic word problems because they come straight from life. How many of each coin are in a jar? How much of two juices should be blended? How long until a faster rider catches a slower one? The same patterns appear in chemistry labs, cooking, budgeting, travel planning, and even game design.

Students get stuck because these stories have two unknown quantities and several numbers. The trick is that each type has one simple relationship: value is the number of items times the price of each, amount of substance is the amount of mixture times its strength, and distance is rate times time. Put the relationship in a table and the algebra falls out.

Core Ideas

The Three Big Relationships

Money: total value = (number of items) $\times$ (value of each).

Mixtures: amount of pure substance = (amount of mixture) $\times$ (percent as a decimal).

Motion: distance = rate $\times$ time, or $d = rt$.

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Each type has two unknowns that are connected by a total. For example, if there are 15 coins in all and n of them are nickels, then the number of dimes is $15 - n$. Writing the second unknown as a total minus the first lets you use only one variable.

Coin type	Number	Value each (cents)	Total value (cents)
Nickels	n	5	$5n$
Dimes	$15 - n$	10	$10(15 - n)$
All coins	15		120

This kind of table is the best organizer. Fill in what you know, then write the equation from the total column: $5n + 10(15 - n) = 120$. Tickets work the same way, with prices instead of coin values.

Mixture problems use the idea that the pure part adds up. If you mix 6 liters of 10 percent solution with some liters of 40 percent solution, the pure amounts add: $0.10(6) + 0.40x$ equals the pure amount in the final mix, $0.25(6 + x)$. Always change percents to decimals first.

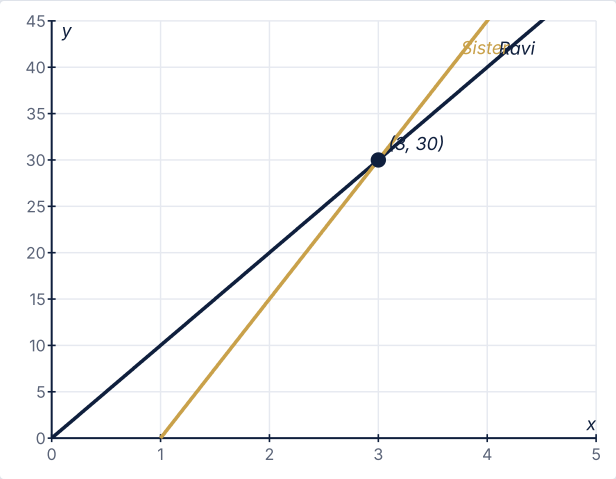
Solution	Liters	Percent	Pure amount
Weak	6	10%	$0.10(6) = 0.6$
Strong	x	40%	$0.40x$
Mix	$6 + x$	25%	$0.25(6 + x)$

Rate, Time, Distance

$d = rt$. Solve for any piece: $r = \frac{d}{t}$ and $t = \frac{d}{r}$. Keep units consistent (miles with miles per hour and hours).

For **catch-up** problems, the two travelers cover the SAME distance when they meet. If one leaves later, her time is shorter by the head start.

The graph shows a catch-up story. Ravi bikes at 10 miles per hour starting at time 0. His sister leaves one hour later at 15 miles per hour. Each line shows distance traveled. Where they cross is when she catches him.



Distance in miles against time in hours. The lines meet at 3 hours and 30 miles.

When you need to compare speeds in different units, convert first. A speed of 30 miles per hour is 0.5 miles per minute. A hike of 45 minutes is 0.75 hours. Doing the conversion before you set up the equation prevents the most common motion-problem error, which is mixing minutes with hours.

Worked Examples

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EXAMPLE 1

A jar has 15 coins, all nickels and dimes, worth 1.20 dollars (120 cents). How many of each?

SOLUTION

Let n be the number of nickels. Then dimes: $15 - n$.

Value in cents: $5n + 10(15 - n) = 120$.

Distribute: $5n + 150 - 10n = 120$, so $-5n + 150 = 120$.

Subtract 150: $-5n = -30$, so $n = 6$.

There are 6 nickels and 9 dimes. Check: $30 + 90 = 120$.

EXAMPLE 2

Tickets for a school play cost 8 dollars for adults and 5 dollars for children. In all, 40 tickets were sold for 281 dollars. How many of each?

SOLUTION

Let a be adult tickets. Children: $40 - a$.

Money: $8a + 5(40 - a) = 281$.

Distribute: $8a + 200 - 5a = 281$, so $3a + 200 = 281$.

Subtract 200: $3a = 81$. So $a = 27$.

27 adult and 13 child tickets. Check: $216 + 65 = 281$.

EXAMPLE 3

How many liters of a 40 percent juice drink must be added to 6 liters of a 10 percent juice drink to make a 25 percent drink?

SOLUTION

Let x be the liters of 40 percent drink.

Pure juice: $0.10(6) + 0.40x = 0.25(6 + x)$.

Distribute: $0.6 + 0.4x = 1.5 + 0.25x$.

Subtract $0.25x$ and subtract 0.6: $0.15x = 0.9$. So $x = 6$.

Add 6 liters. Check: $0.6 + 2.4 = 3.0$ and $0.25(12) = 3.0$.

EXAMPLE 4

A school group travels 150 miles. The bus goes 60 miles per hour for 1.5 hours, then slows to 40 miles per hour for the rest. How long does the second part take?

SOLUTION

First part: $d = rt = 60 \cdot 1.5 = 90$ miles.

Remaining distance: $150 - 90 = 60$ miles.

Time at 40 mph: $t = \frac{d}{r} = \frac{60}{40} = 1.5$ hours.

EXAMPLE 5

Ravi bikes at 10 miles per hour. One hour later his sister leaves from the same place and follows at 15 miles per hour. How long after Ravi starts does she catch him, and how far have they gone?

SOLUTION

Let t be Ravi's time in hours. His sister's time is $t - 1$.

They have gone the same distance: $10t = 15(t - 1)$.

Distribute: $10t = 15t - 15$. Subtract $15t$: $-5t = -15$, so $t = 3$.

She catches Ravi 3 hours after he starts (2 hours after she starts). Distance: $10 \cdot 3 = 30$ miles.

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EXAMPLE 6

Two hikers start 18 miles apart and walk toward each other along the same trail. One walks 2 miles per hour and the other walks 4 miles per hour. How long until they meet?

SOLUTION

Their distances add up to 18 miles. Together they close the gap at $2 + 4 = 6$ miles per hour.

Let t be the hours. Distance: $2t + 4t = 18$, so $6t = 18$.

Divide by 6: $t = 3$.

They meet after 3 hours. Check: one hiker walked $2 \cdot 3 = 6$ miles, the other $4 \cdot 3 = 12$ miles, and $6 + 12 = 18$.

Coach's Playbook

- 1 Name two unknowns with one letter.** If there are 40 tickets, adults are a and children are $40 - a$.
- 2 Build a table.** Use columns such as Number, Value each, and Total. Fill the row for each group.
- 3 Write the equation from the total column.** Add the group totals and set equal to the given total.
- 4 For motion, draw a quick line diagram.** Show start, who goes where, and what is equal at the meeting point.
- 5 Check with the story.** Whole numbers for items, positive times, and the totals must match.

For opposite-direction problems, add the rates: two cars driving apart at 50 and 60 miles per hour are separating at 110 miles per hour. For same-direction chases, subtract the rates to find how fast the gap closes. In Example 5 the gap closes at $15 - 10 = 5$ miles per hour, and the 10 mile head start takes $10 \div 5 = 2$ hours to erase.

Common traps

1) Mixing units: minutes with miles per hour. Convert first. 2) Using 25 instead of 0.25 for a percent. 3) Giving both travelers the same time in a catch-up problem when one started later. 4) Forgetting the question: finding the number of nickels when the problem asks for the value of the dimes.

Mistake Clinic**Spot the error**

Wrong work: 20 coins, all nickels and dimes, worth 1.10 dollars. The student writes $5n + 10(20 - n) = 1.10$. Where it went wrong: the coin values are in cents, but the total is in dollars. Right solution: write the total as 110 cents: $5n + 200 - 10n = 110$, so $-5n = -90$ and $n = 18$. There are 18 nickels and 2 dimes: $90 + 20 = 110$.

Spot the error

Wrong work: To mix 6 liters of 10 percent solution with x liters of 40 percent solution to make 25 percent, the student writes $0.10(6) + 0.40x = 25(6 + x)$. Where it went wrong: 25 percent must be 0.25. Right solution: $0.6 + 0.4x = 0.25(6 + x)$, which gives $x = 6$.

Spot the error

Wrong work: Ravi bikes at 10 mph. His sister leaves one hour later at 15 mph. The student writes $10t = 15t$ and gets $t = 0$. Where it went wrong: the sister has traveled for one hour LESS than Ravi. Right solution: $10t = 15(t - 1)$, so $t = 3$ hours after Ravi starts.

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Real-World Connection

A smoothie shop blends two juices to hit a sugar target. The percent equation is the same one you used to mix 10 percent and 40 percent drinks: the pure amounts always add up.

Challenge Corner

A boat travels 12 miles downstream in 1 hour. It returns the same 12 miles upstream in 1.5 hours. Find the boat's speed in still water and the speed of the current. Solution: Downstream speed is $12 \div 1 = 12$ miles per hour. Upstream speed is $12 \div 1.5 = 8$ miles per hour. The current helps by the same amount it hurts, so the boat's still-water speed is the middle value: $\frac{12+8}{2} = 10$ miles per hour. The current is $\frac{12-8}{2} = 2$ miles per hour. Check: $10 + 2 = 12$ and $10 - 2 = 8$.

Practice**PRACTICE Money, Mixture, and Rate-Time-Distance Problems**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

1. A jar holds 12 coins, all quarters and dimes, worth 2.40 dollars. How many quarters are there?
(A) 4 (B) 8 (C) 12 (D) 6
2. A train travels 180 miles at 45 miles per hour. How many hours does the trip take?
(A) 135 (B) 0.25 (C) 4 (D) 225
3. A theater sells adult tickets at 10 dollars and child tickets at 6 dollars. It sold 30 tickets for 236 dollars. How many adult tickets were sold?
(A) 16 (B) 18 (C) 20 (D) 14
4. Dana rides her bike at 12 miles per hour for 2 hours and then at 8 miles per hour for 3 hours. What is her total distance?
(A) 48 (B) 20 (C) 40 (D) 60
5. How many liters of an 80 percent juice drink must be added to 4 liters of a 20 percent juice drink to make a 40 percent drink?
(A) 4 (B) 1 (C) 2 (D) 8
6. Two cars leave the same spot at the same time and drive in opposite directions at 50 and 60 miles per hour. After how many hours are they 330 miles apart?
(A) 5.5 (B) 3 (C) 6.6 (D) 0.33
7. A jar holds 18 coins, all nickels and quarters, worth 2.90 dollars. How many quarters are in the jar?
(A) 8 (B) 10 (C) 12 (D) 6
8. How many liters of water must be added to 10 liters of a 30 percent solution to make a 20 percent solution?
(A) 3 (B) 10 (C) 5 (D) 2
9. Two cyclists start 60 miles apart and ride toward each other at 12 and 18 miles per hour. After how many hours do they meet?
(A) 5 (B) 3.3 (C) 4 (D) 2
10. Lena has 20 coins, all dimes and quarters, worth 3.80 dollars. How many quarters does she have?
11. A bus leaves at noon traveling 40 miles per hour. A car leaves the same place one hour later traveling 60 miles per hour. How many hours after the car leaves does it catch the bus?
12. A scientist has 12 liters of a 25 percent salt solution. How many liters of plain water must be added to make a 15 percent solution?
13. Maya jogs to the park at 6 miles per hour and walks home along the same path at 3 miles per hour. Her total time is 1.5 hours. How far away is the park, in miles?
14. A theater sells adult tickets for 12 dollars and student tickets for 7 dollars. It sold 25 tickets for 255 dollars. How many adult tickets were sold?
15. Cara leaves home driving 45 miles per hour. Two hours later, Dev leaves from the same place on the same road driving 60 miles per hour. How many hours after Dev leaves does he catch Cara?

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06.4

Multi-Step and Work Problems

Tackle percent problems, work-rate problems, and multi-stage stories by using multipliers, rates, and diagrams. You will learn to break a long problem into clear stages and check the result.

Why This Matters

Some of the most useful math you will ever do happens in multi-step word problems. A store takes 25 percent off and then adds sales tax. Two friends paint a room together in less time than either would need alone. A person spends a fraction of their money, then a fixed amount, and wonders what they started with. Each situation has several stages, and the answer comes from handling the stages one at a time.

Students struggle here because the problem is long, because percents feel slippery, and because work problems seem to break the rule that bigger helpers should mean bigger times. In this section you will learn three tools: percent multipliers, work rates, and diagrams that break a story into stages. These tools are used in finance, science labs, construction planning, and every later algebra course.

Core Ideas

Percent Multipliers

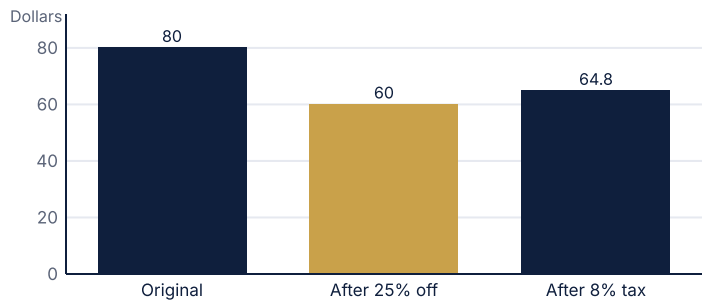
Percent means "out of 100". To find r percent of an amount, multiply by $\frac{r}{100}$.

A discount of r percent leaves $(100 - r)$ percent: multiply by $1 - \frac{r}{100}$. An increase of r percent multiplies by $1 + \frac{r}{100}$.

Percent change = $\frac{\text{new} - \text{old}}{\text{old}} \times 100$ percent.

Situation	Multiplier	Example on 80 dollars
25% off	0.75	$80 \times 0.75 = 60$
8% tax added	1.08	$60 \times 1.08 = 64.80$
40% off	0.60	$80 \times 0.60 = 48$
20% increase	1.20	$80 \times 1.20 = 96$

Multipliers let you do two steps in one move. A 25 percent discount followed by 8 percent tax is the same as multiplying by 0.75×1.08 . The bar chart shows the price of a jacket at each stage.



A jacket priced at 80 dollars is 60 dollars after the discount and 64.80 dollars after tax.

Work Rate Problems

If a job takes t hours, the **rate** is $\frac{1}{t}$ of the job per hour.

When people work together, **add their rates**. If one takes a hours and the other b hours, the combined rate is $\frac{1}{a} + \frac{1}{b}$ and the time together is $\frac{1}{\frac{1}{a} + \frac{1}{b}}$.

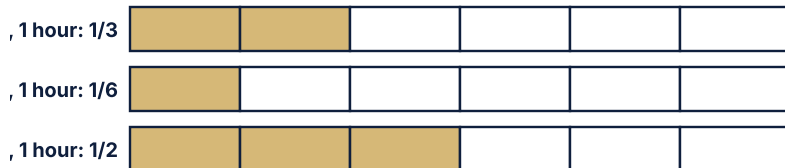
A helper who undoes work, like a drain, has a **NEGATIVE** rate.

Why add rates? Ana paints a fence in 3 hours, so in one hour she completes $\frac{1}{3}$ of it. Ben takes 6 hours, so in one hour he completes $\frac{1}{6}$. Together in one hour they complete $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$ of the fence, so the whole fence takes 2 hours.

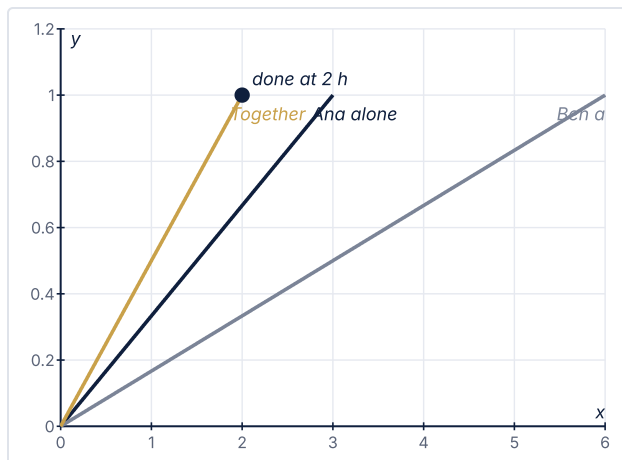
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In one hour Ana paints 2 of 6 equal parts and Ben paints 1 of 6, so together they paint 3 of 6, which is one half.



Fraction of the fence painted over time. The steepest line, together, reaches 1 (the whole job) first, at 2 hours.

Multi-Stage Problems

Break the story into stages. Make a **diagram or table** showing the amount at the start, what happens in each stage, and the amount at the end.

Work forward if you know the start. Work **backward** (undo each stage with the inverse operation) if you know the end, or write an equation for the whole chain.

A diagram makes a long story easy to follow. Draw a bar for the starting amount. Mark off the portion spent first, then the next portion, and label what is left. If you are working backward from the ending amount, reverse each stage: undo a subtraction by adding and undo a fraction taken away by dividing by what is left.

Worked Examples

EXAMPLE 1

A jacket costs 80 dollars. It is on sale for 25 percent off, and then 8 percent sales tax is added to the sale price. What is the final cost?

SOLUTION

Sale price: $80 \times 0.75 = 60$ dollars.

Add tax to the SALE price: $60 \times 1.08 = 64.80$.

The final cost is 64.80 dollars.

EXAMPLE 2

After a 40 percent discount, a pair of shoes costs 45 dollars. What was the original price?

SOLUTION

The shoes cost 60 percent of the original, so $0.60p = 45$.

Divide by 0.60: $p = \frac{45}{0.60} = 75$.

The original price was 75 dollars. Check: 40 percent of 75 is 30, and $75 - 30 = 45$.

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EXAMPLE 3

A school club grew from 40 members to 52 members. What is the percent increase?

SOLUTION

Change: $52 - 40 = 12$.

Percent change: $\frac{12}{40} = 0.30$, which is 30 percent.

The club grew by 30 percent. Always divide by the OLD amount.

EXAMPLE 4

Ana can paint a fence in 3 hours. Ben can paint it in 6 hours. How long will it take them working together?

SOLUTION

Ana's rate is $\frac{1}{3}$ and Ben's is $\frac{1}{6}$ of the fence per hour.

Combined rate: $\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{1}{2}$.

Time = $1 \div \frac{1}{2} = 2$ hours.

This is less than either person alone, as it should be.

EXAMPLE 5

A pipe fills a tank in 4 hours. A drain empties a full tank in 12 hours. If both are open, how long will it take to fill an empty tank?

SOLUTION

Fill rate: $\frac{1}{4}$ per hour. Drain rate: $-\frac{1}{12}$ per hour.

Net rate: $\frac{1}{4} - \frac{1}{12} = \frac{3}{12} - \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$.

Time = 6 hours. It is slower than the pipe alone, which makes sense because the drain works against it.

EXAMPLE 6

Mia spent one fourth of her money on a ticket, then 12 dollars on snacks. She has 33 dollars left. How much did she start with?

SOLUTION

Let m be her starting money. After the ticket she has $m - \frac{m}{4}$.

After snacks: $m - \frac{m}{4} - 12 = 33$. Combine: $\frac{3m}{4} - 12 = 33$.

Add 12: $\frac{3m}{4} = 45$. Multiply by $\frac{4}{3}$: $m = 60$.

Check backward: 33 plus 12 is 45, and 45 is three fourths of 60. She started with 60 dollars.

Coach's Playbook

- 1 Read once for the story.** Picture the stages. Who has what at the start, in the middle, and at the end?
- 2 Decide the base for each percent.** A percent is always "of" something. In a discount-then-tax story, the tax base is the sale price, not the original.
- 3 Turn percents into multipliers.** Use 0.75 for 25 percent off and 1.08 for 8 percent more. Multiply stage by stage.
- 4 Use rates for work.** Write each worker as $\frac{1}{\text{time}}$, add them, and flip the result to get the time.
- 5 Sketch it.** A tiny bar diagram, table, or timeline often shows the equation instantly.
- 6 Check the answer for sense.** Working together must be faster than either alone. A sale price must be below the original.

A neat shortcut for work problems with two workers is $\frac{ab}{a+b}$. For 3 hours and 6 hours, the time together is $\frac{3 \cdot 6}{3+6} = \frac{18}{9} = 2$ hours. It is the same calculation in a compact form. The shortcut only works when exactly two workers are helping each other; for three or more, add the rates.

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Common traps

1) Subtracting the percent as if it were dollars: 25 percent off 80 is not $80 - 25$. 2) Using the new amount as the base for percent change instead of the old amount. 3) Averaging work times: workers who take 3 and 6 hours do NOT take 4.5 hours together. 4) Adding percents from different bases: a 20 percent increase followed by a 20 percent decrease does not return to the start.

Mistake Clinic**Spot the error**

Wrong work: A jacket costs 80 dollars and is 25 percent off. The student computes $80 - 25 = 55$. Where it went wrong: 25 percent means 25 out of every 100, so it is a fraction of the price, not 25 dollars. Right solution: 25 percent of 80 is $0.25 \cdot 80 = 20$, so the sale price is $80 - 20 = 60$ dollars.

Spot the error

Wrong work: Ana paints a fence in 3 hours and Ben in 6 hours. The student averages: $\frac{3+6}{2} = 4.5$ hours together. Where it went wrong: working together should be FASTER than the faster worker alone, but 4.5 hours is slower than Ana's 3 hours. Right solution: add the rates, $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$, so the time is 2 hours.

Spot the error

Wrong work: A price rises 20 percent and then falls 20 percent, so it must be back to the original. Where it went wrong: the second percent is taken of a larger amount. Right solution: start at 100 dollars. After a 20 percent rise: $100 \cdot 1.2 = 120$. After a 20 percent drop: $120 \cdot 0.8 = 96$. The final price is 4 percent lower than the start.

Real-World Connection

Restaurants, stores, and phone plans all stack percents. A bill of 40 dollars with a 15 percent tip added is $40 \cdot 1.15 = 46$ dollars. Knowing the multiplier lets you check a receipt in your head.

Challenge Corner

Ana can paint a room in 6 hours and Ben can paint it in 12 hours. Ana works alone for 2 hours, and then Ben joins her. How much longer does it take to finish? Solution: In 2 hours Ana paints $2 \cdot \frac{1}{6} = \frac{1}{3}$ of the room, so $\frac{2}{3}$ remains. Together their rate is $\frac{1}{6} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$ of the room per hour. Time = $\frac{2/3}{1/4} = \frac{8}{3}$ hours, which is 2 hours and 40 minutes. Check: $\frac{1}{4} \cdot \frac{8}{3} = \frac{2}{3}$.

Practice**PRACTICE Multi-Step and Work Problems**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What is 15 percent of 80?
(A) 15 (B) 12 (C) 65 (D) 1.2
- A shirt costs 40 dollars and is on sale for 30 percent off. What is the sale price?
(A) 12 (B) 70 (C) 28 (D) 10
- A price rises from 50 dollars to 65 dollars. What is the percent increase?
(A) 30% (B) 23% (C) 15% (D) 130%

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4. Rena can read a book in 6 hours and Sam can read the same book in 3 hours. How long will it take them to read it if they work together?
(A) 4.5 hours (B) 9 hours (C) 3 hours (D) 2 hours
5. A jacket is on sale for 72 dollars after a 20 percent discount. What was the original price?
(A) 92 dollars (B) 90 dollars (C) 86.40 dollars
(D) 57.60 dollars
6. One pipe can fill a tank in 10 hours and another can fill it in 15 hours. How long will it take both pipes together?
(A) 6 hours (B) 12.5 hours (C) 25 hours
(D) 5 hours
7. Jo spent one third of her money on a game and then 15 dollars on lunch. She has 25 dollars left. How much did she start with?
(A) 40 dollars (B) 45 dollars (C) 60 dollars
(D) 120 dollars
8. A store sells a bike helmet for 91 dollars, which is a 40 percent markup on the store's cost. What was the cost?
(A) 54.60 dollars (B) 50.96 dollars (C) 126.40 dollars
(D) 65 dollars
9. A bike costs 200 dollars. The price is raised 10 percent and then lowered 10 percent. What is the final price?
(A) 200 dollars (B) 198 dollars (C) 180 dollars
(D) 220 dollars
10. What is 35 percent of 240?
11. A restaurant bill is 48 dollars. A tip of 15 percent of the bill and sales tax of 8 percent of the bill are both added. What is the total?
12. Printer A can print a report in 20 minutes and printer B can print it in 30 minutes. How many minutes will it take them working together?
13. A town's population fell from 8,000 to 6,800. What was the percent decrease?
14. Eli spent 40 percent of his savings on a tablet, then spent half of what was left on gifts. He has 36 dollars left. How much were his savings at the start?
15. A large pipe fills a pool in 6 hours, a small pipe fills it in 12 hours, and a drain empties a full pool in 9 hours. If all three are open, how many hours does it take to fill an empty pool? Give your answer as a decimal.

Answers**Answers and Explanations**

Complete solutions for every practice problem, numbered to match. Read the explanation even when your answer is right.

06.1 Turning Words into Expressions and Equations**PRACTICE · TURNING WORDS INTO EXPRESSIONS AND EQUATIONS**

1. B. "Twice a number" is $2n$, and "4 less than" subtracts 4 from it: $2n - 4$. Choice A reverses the order of subtraction.
2. C. "The quotient of a number and 6" is $\frac{n}{6}$, and "increased by 2" adds 2 afterward: $\frac{n}{6} + 2$. Choice A puts 6 on top.
3. D. "Three times a number" is $3n$. "Five more than" that is $3n + 5$. "Is 26" gives $3n + 5 = 26$. Choice A swaps the 3 and the 5.
4. A. The equation is $27 + 6s = 45$. Subtract 27: $6s = 18$. So $s = 3$. Choice B divides 45 by 9 and ignores the student tickets.
5. C. Let c be the cousin's age: $2c + 3 = 17$. Subtract 3: $2c = 14$. So $c = 7$. Choice B forgets to divide by 2.
6. B. "8 less than y " means $y - 8$, so $x = y - 8$. Test: if $y = 10$, then $x = 2$. Choice A gives $x = -2$ instead.
7. B. "The product of 4 and a number" is $4n$. "Decreased by 7" subtracts 7 from that: $4n - 7$. Choice A subtracts before multiplying.
8. C. Equation: $5n - 12 = 2n + 3$. Subtract $2n$: $3n - 12 = 3$. Add 12: $3n = 15$. So $n = 5$. Choice B (15) is the value of $3n$, not n .
9. D. Amir has $40 - 5d$ and Ella has $10 + 5d$. Set equal: $40 - 5d = 10 + 5d$, so $30 = 10d$ and $d = 3$. Check: both have 25 dollars. Choice C (30) is only an intermediate number.
10. 7. Equation: $2n + 9 = 4n - 5$. Subtract $2n$: $9 = 2n - 5$. Add 5: $14 = 2n$. So $n = 7$. Check: $14 + 9 = 23$ and $28 - 5 = 23$.
11. 60 dollars. Let g be the price: $\frac{g}{4} + 3 = 18$. Subtract 3: $\frac{g}{4} = 15$. Multiply by 4: $g = 60$. Check: $60 \div 4 = 15$, and $15 + 3 = 18$.
12. 9 miles. Equation: $15 + 4m = 51$. Subtract 15: $4m = 36$. Divide by 4: $m = 9$.
13. 8 cm. Let w be the width. The length is $2w + 3$. Perimeter: $2(w + 2w + 3) = 54$, so $6w + 6 = 54$. Then $6w = 48$ and $w = 8$. Check: length is 19 and $2(8 + 19) = 54$.
14. 13. Equation: $3n - 4 = 2n + 9$. Subtract $2n$: $n - 4 = 9$. Add 4: $n = 13$. Check: $39 - 4 = 35$ and $26 + 9 = 35$.
15. 3 dollars. Let x be the cost of one notebook: $5x + 4 = 19$. Subtract 4: $5x = 15$. Divide by 5: $x = 3$.

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06.2 Age, Number, and Consecutive-Integer Problems**PRACTICE · AGE, NUMBER, AND CONSECUTIVE-INTEGER PROBLEMS**

1. B. Odd numbers are 2 apart, so the next two are $n + 2$ and $n + 4$. Choice A lists consecutive integers, which alternate between odd and even.
2. C. Let the smaller be n : $n + (n + 1) = 61$, so $2n = 60$ and $n = 30$. Choice A is the larger integer (31), which is a tempting wrong answer.
3. D. Equation: $4n - 5 = 3n + 9$. Subtract $3n$: $n - 5 = 9$. Add 5: $n = 14$. Choice B has the wrong sign.
4. A. Let Tim be t and Sara $t + 5$: $2t + 5 = 31$, so $t = 13$. Sara is $13 + 5 = 18$. Choice B is Tim's age, not Sara's.
5. B. Let the integers be n , $n + 1$, $n + 2$: $3n + 3 = 96$, so $n = 31$. The largest is $31 + 2 = 33$. Choice C is the middle number.
6. D. In 4 years Nia is 10, so Eli will be 20. Eli is 4 years younger than that now: $20 - 4 = 16$. Choice A forgets to subtract the 4 years.
7. A. Let the integers be n , $n + 2$, $n + 4$: $3n + 6 = 66$, so $3n = 60$ and $n = 20$. They are 20, 22, 24. Choice B is the middle number.
8. B. Let the brother's age be b : $2b + 4 = 20$. Subtract 4: $2b = 16$, so $b = 8$. Choice D (16) forgets to divide by 2, and choice C divides 20 by 2 without subtracting 4.
9. D. Let Omar's age be o : $o + 5 = 3(o - 7)$. Distribute: $o + 5 = 3o - 21$. So $26 = 2o$ and $o = 13$. Check: in 5 years he is 18, and 7 years ago he was 6, and $18 = 3 \cdot 6$. Choice A (18) is his age in 5 years.
10. 31, 33, 35. Let the integers be n , $n + 2$, $n + 4$. Then $3n + 6 = 99$, so $3n = 93$ and $n = 31$. Check: $31 + 33 + 35 = 99$.
11. 24 years old. Let the daughter be d and the mother $3d$. In 8 years: $3d + 8 = 2(d + 8)$. Distribute: $3d + 8 = 2d + 16$. So $d = 8$ and the mother is 24. Check: in 8 years they are 16 and 32.
12. 23 and 17. Let the larger be n and the smaller $n - 6$. Equation: $2n + (n - 6) = 63$, so $3n = 69$ and $n = 23$. The smaller is 17. Check: $46 + 17 = 63$.
13. 15, 16, 17, 18. Let the smallest be n : $n + (n + 1) + (n + 2) + (n + 3) = 5n - 9$, so $4n + 6 = 5n - 9$. Then $n = 15$. Check: the sum is 66 and $5(15) - 9 = 66$.
14. 37 and 39. Let the smaller be n and the larger $n + 2$. Then $2n + 2 = 76$, so $n = 37$. Check: $37 + 39 = 76$, and both are odd.
15. 15 and 25. Let the smaller be s , so the larger is $40 - s$. Equation: $2s = (40 - s) + 5$. Add s : $3s = 45$, so $s = 15$ and the larger is 25. Check: $2 \cdot 15 = 30 = 25 + 5$.

06.3 Money, Mixture, and Rate-Time-Distance Problems**PRACTICE · MONEY, MIXTURE, AND RATE-TIME-DISTANCE PROBLEMS**

1. B. Let q be quarters: $25q + 10(12 - q) = 240$. So $15q + 120 = 240$, $15q = 120$, and $q = 8$. Choice A is the number of dimes.
2. C. Time is distance divided by rate: $\frac{180}{45} = 4$ hours. Choice A multiplies the wrong way around or subtracts, and choice B divides the rate by the distance.
3. D. Let a be adult tickets: $10a + 6(30 - a) = 236$, so $4a + 180 = 236$ and $a = 14$. Choice A (16) is the number of child tickets.
4. A. Distance is rate times time for each part: $12 \cdot 2 = 24$ and $8 \cdot 3 = 24$. The total is 48 miles. Choice C uses an average speed of 10 for 4 hours, which is not the situation.
5. C. Pure juice: $0.2(4) + 0.8x = 0.4(4 + x)$. So $0.8 + 0.8x = 1.6 + 0.4x$, and $0.4x = 0.8$, giving $x = 2$. Choice A assumes the midpoint, but 40 percent is not halfway between 20 and 80.
6. B. The cars separate at $50 + 60 = 110$ miles per hour, so $110t = 330$ and $t = 3$. Choice A divides by only one speed.
7. B. Let n be nickels: $5n + 25(18 - n) = 290$. So $450 - 20n = 290$, $n = 8$. Quarters are $18 - 8 = 10$. Choice A is the number of nickels.
8. C. The pure part stays the same: $0.30(10) = 0.20(10 + w)$. So $3 = 2 + 0.2w$, $w = 5$. Choice A (3) is the amount of pure solution, not the water.
9. D. They approach each other at $12 + 18 = 30$ miles per hour: $30t = 60$, so $t = 2$. Choices A and B divide by only one cyclist's speed.
10. 12 quarters. Let q be quarters: $25q + 10(20 - q) = 380$. So $15q + 200 = 380$, $15q = 180$, and $q = 12$. There are 8 dimes: $300 + 80 = 380$.
11. 2 hours. Let t be the car's travel time. The bus has traveled $t + 1$ hours. Same distance: $40(t + 1) = 60t$, so $40 = 20t$ and $t = 2$. Check: bus $40 \cdot 3 = 120$, car $60 \cdot 2 = 120$.
12. 8 liters. Pure salt stays the same: $0.25(12) = 0.15(12 + w)$. So $3 = 1.8 + 0.15w$, $1.2 = 0.15w$, and $w = 8$.
13. 3 miles. Let d be the distance. Time going plus time returning: $\frac{d}{6} + \frac{d}{3} = 1.5$. Multiply by 6: $d + 2d = 9$, so $d = 3$. Check: 0.5 hour plus 1 hour is 1.5 hours.
14. 16 adult tickets. Let a be adult tickets: $12a + 7(25 - a) = 255$, so $5a + 175 = 255$ and $a = 16$. There are 9 student tickets: $192 + 63 = 255$.
15. 6 hours. Let t be Dev's hours. Cara's hours are $t + 2$. Equal distances: $45(t + 2) = 60t$, so $90 = 15t$ and $t = 6$. Check: Cara $45 \cdot 8 = 360$, Dev $60 \cdot 6 = 360$.

06.4 Multi-Step and Work Problems**PRACTICE · MULTI-STEP AND WORK PROBLEMS**

1. B. $0.15 \times 80 = 12$. Choice A uses the percent number itself, 15, as the answer. Choice D misplaces the decimal.
2. C. 30 percent of 40 is 12, which is the discount. The sale price is $40 - 12 = 28$. Choice A is the discount amount, not the price to pay.
3. A. Change is $65 - 50 = 15$. Percent increase is $\frac{15}{50} = 0.30$, or 30 percent. Choice B divides by the new price (65) instead of the old.
4. D. Rates: $\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$ book per hour, so the time is 2 hours. Choice A averages the two times, which cannot be right because the team is faster than Sam alone.
5. B. The sale price is 80 percent of the original: $0.80p = 72$, so $p = 90$. Choice A adds 20 dollars, and choice C multiplies 72 by 1.20.

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6. A. Rates: $\frac{1}{10} + \frac{1}{15} = \frac{3}{30} + \frac{2}{30} = \frac{5}{30} = \frac{1}{6}$, so the time is 6 hours. Choice B averages the times, which is wrong.
7. C. Let m be the start: $m - \frac{m}{3} - 15 = 25$. So $\frac{2m}{3} = 40$ and $m = 60$. Choice A adds just 25 and 15 and forgets the fraction she spent.
8. D. The selling price is 140 percent of the cost: $1.40c = 91$, so $c = 65$. Choice A takes 40 percent off 91, which uses the wrong base.
9. B. After a 10 percent rise: $200 \cdot 1.10 = 220$. After a 10 percent drop: $220 \cdot 0.90 = 198$. Choice A wrongly assumes the changes cancel, but the second change is on a larger amount.
10. 84. $0.35 \times 240 = 84$. Check: 10 percent is 24, so 30 percent is 72 and 5 percent is 12, total 84.
11. 59.04 dollars. Total percent added is $15 + 8 = 23$ percent, so the multiplier is 1.23. $48 \times 1.23 = 59.04$.
12. 12 minutes. Rates: $\frac{1}{20} + \frac{1}{30} = \frac{3}{60} + \frac{2}{60} = \frac{5}{60} = \frac{1}{12}$ report per minute, so the time is 12 minutes. The shortcut gives $\frac{20 \cdot 30}{50} = 12$.
13. 15 percent. Change is $8000 - 6800 = 1200$. Divide by the old value: $\frac{1200}{8000} = 0.15$, so a 15 percent decrease.
14. 120 dollars. After the tablet, 60 percent is left. After gifts, half of that remains: $0.60 \cdot 0.50 \cdot s = 36$, so $0.30s = 36$ and $s = 120$. Check: 40 percent of 120 is 48, leaving 72; half of 72 is 36.
15. 7.2 hours. Net rate: $\frac{1}{6} + \frac{1}{12} - \frac{1}{9} = \frac{6}{36} + \frac{3}{36} - \frac{4}{36} = \frac{5}{36}$. Time = $\frac{36}{5} = 7.2$ hours.

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