



Geometry Basics: Angles, Perimeter, and Area

Four focused lessons, worked examples, a mistake clinic, and 60 practice problems with full explanations.

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08.1 Angles and Angle Relationships

08.2 Perimeter and Area of Polygons

08.3 Circles: Circumference and Area

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Every section teaches the idea, walks through the most common problem types, flags the traps students fall into, and ends with fifteen practice problems. Full answers and explanations are at the back.

08.1 Angles and Angle Relationships

Measure and classify angles, use complementary, supplementary, and vertical angle facts, work with parallel lines and a transversal, and find missing angles in triangles.

Why This Matters

Angles describe turns and corners, and they hide inside almost everything built or drawn: roof lines, bike ramps, pool shots, and the hands of a clock. Students usually get stuck when they cannot tell which angles are equal and which add to 180, and when they set up an equation but pick the wrong relationship.

Angle facts are the first steps toward proof, trigonometry, and design. Carpenters, pilots, and game designers all rely on them. In this section you will learn a small set of facts and a method that turns every angle puzzle into a short equation.

Core Ideas

An **angle** is formed by two rays that share an endpoint called the vertex. We measure angles in degrees. A full turn is 360° and a straight line is 180° . We name angles by size:

Type	Size	Picture
Acute	less than 90°	narrow opening
Right	exactly 90°	square corner
Obtuse	between 90° and 180°	wide opening
Straight	exactly 180°	a straight line

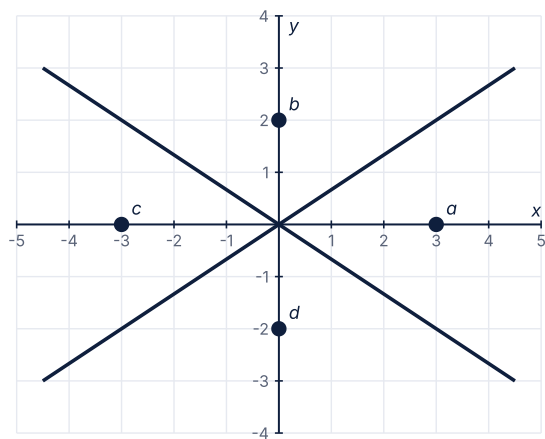
Complementary and supplementary angles

Two angles are **complementary** if their measures add to 90° . Two angles are **supplementary** if their measures add to 180° . Angles that sit side by side on a straight line are supplementary (a **linear pair**).

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Two lines cross. Angles a and c are vertical angles and are equal. Angles b and d are also equal. Angles a and b make a linear pair and add to 180 degrees.

Vertical angles

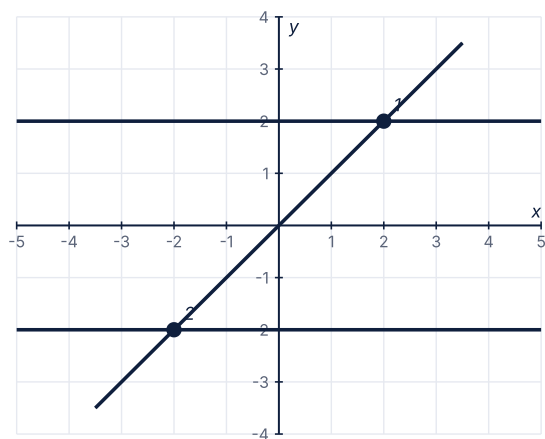
When two lines cross, the angles directly across from each other are **vertical angles**, and they are **equal**.

Each angle plus its neighbor makes a straight line, so neighbors add to 180° .

A **transversal** is a line that crosses two other lines. When the two lines are **parallel**, the transversal makes matching angle pairs with special rules.

Parallel lines and a transversal

If two parallel lines are cut by a transversal: **corresponding** angles are equal, **alternate interior** angles are equal, and **same-side interior** angles add to 180° . Every angle is either equal to or supplementary to any other angle in the figure.



Two parallel lines and a transversal. Equal pairs have the same measure; angles that form a straight line add to 180 degrees.

Triangle angle sum

The three interior angles of any triangle add to 180° .

An **exterior angle** equals the sum of the two remote interior angles.

Worked Examples

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EXAMPLE 1: COMPLEMENT AND SUPPLEMENT

Find the complement and the supplement of 35° .

SOLUTION

Complement: $90 - 35 = 55^\circ$.

Supplement: $180 - 35 = 145^\circ$.

EXAMPLE 2: COMPLEMENTARY ANGLES WITH ALGEBRA

Two complementary angles measure $(2x + 10)^\circ$ and $(3x - 5)^\circ$. Find both angles.

SOLUTION

They add to 90: $(2x + 10) + (3x - 5) = 90$.

$5x + 5 = 90$, so $5x = 85$ and $x = 17$.

Angles: $2(17) + 10 = 44^\circ$ and $3(17) - 5 = 46^\circ$. Check: $44 + 46 = 90$.

EXAMPLE 3: VERTICAL ANGLES

Two lines intersect. One angle is $(5x - 12)^\circ$ and its vertical angle is $(3x + 20)^\circ$. Find the angle and its neighbor.

SOLUTION

Vertical angles are equal: $5x - 12 = 3x + 20$.

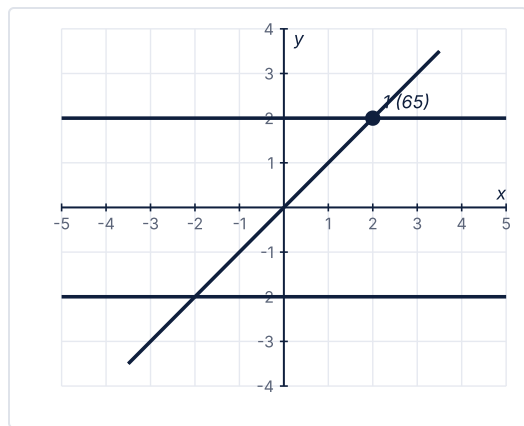
$2x = 32$, so $x = 16$.

The angle is $5(16) - 12 = 68^\circ$.

A neighbor makes a linear pair: $180 - 68 = 112^\circ$.

EXAMPLE 4: PARALLEL LINES

In the figure, the lines are parallel and angle 1 is 65° . Find a corresponding angle, an alternate interior angle, and a same-side interior angle.

**SOLUTION**

Corresponding angle: equal, so 65° .

Alternate interior angle: equal, so 65° .

Same-side interior angle: supplementary, so $180 - 65 = 115^\circ$.

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EXAMPLE 5: TRIANGLE ANGLE SUM

The angles of a triangle are x° , $(2x + 10)^\circ$, and $(3x - 4)^\circ$. Find all three angles.

SOLUTION

Add to 180: $x + (2x + 10) + (3x - 4) = 180$.

$6x + 6 = 180$, so $6x = 174$ and $x = 29$.

Angles: 29° , $2(29) + 10 = 68^\circ$, $3(29) - 4 = 83^\circ$.

Check: $29 + 68 + 83 = 180$.

EXAMPLE 6: EXTERIOR ANGLE OF A TRIANGLE

In a triangle, two interior angles are 48° and 67° . Find the third interior angle and the exterior angle at that vertex.

SOLUTION

Third interior angle: $180 - 48 - 67 = 65^\circ$.

The exterior angle and the interior angle at that vertex form a straight line: $180 - 65 = 115^\circ$.

Shortcut check: an exterior angle equals the sum of the two remote interior angles, $48 + 67 = 115^\circ$.

Coach's Playbook

- 1 Draw it.** Sketch the figure and mark any known angles. Small pictures prevent big mistakes.
- 2 Name the relationship.** Is it complementary (90), supplementary (180), vertical (equal), parallel-line equal, or triangle sum (180)?
- 3 Write the equation.** Equal angles: set them equal. Adding angles: set the sum to 90 or 180.
- 4 Solve and check.** Find x , then find each angle. Make sure the angles add up as the figure says, and that no angle is negative.

Common traps

1. Mixing up complementary (90) and supplementary (180). 2. Stopping at x and forgetting to find the angle the question asks for. 3. Calling same-side interior angles equal. They add to 180. 4. Using the equal-angle rule on two angles that only look equal in the sketch.

Mistake Clinic**Spot the error**

Find the complement of 40° . A student answers 140° . **Where it went wrong:** the student subtracted from 180, which finds the supplement. **Right way:** complementary angles add to 90, so $90 - 40 = 50^\circ$.

Spot the error

Parallel lines are cut by a transversal and one same-side interior angle is 70° . A student says the other is also 70° .

Where it went wrong: the student treated same-side interior angles as equal. **Right way:** they are supplementary, so the other is $180 - 70 = 110^\circ$.

Spot the error

A triangle has angles $3x$, $4x$, and $5x$. A student writes $3x + 4x + 5x = 360$ and gets $x = 30$. **Where it went wrong:** the angles of a triangle add to 180, not 360. **Right way:** $12x = 180$, so $x = 15$ and the angles are 45° , 60° , and 75° .

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Real-World Connection

A skateboard ramp makes an angle with the ground. The angle between the ramp and the vertical wall is its complement. Builders use that fact to check that supports are cut correctly.

Challenge Corner

In triangle ABC , angle A is $2x^\circ$, angle B is $(x + 20)^\circ$, and the exterior angle at C is $(5x - 10)^\circ$. Find all three interior angles. **Solution:** the exterior angle equals the sum of the remote interior angles: $5x - 10 = 2x + (x + 20)$, so $5x - 10 = 3x + 20$ and $x = 15$. Then $A = 30^\circ$, $B = 35^\circ$, and the exterior angle is 65° , so $C = 180 - 65 = 115^\circ$. Check: $30 + 35 + 115 = 180$.

Practice**PRACTICE Angles and Angle Relationships**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What is the supplement of a 112° angle?
(A) 22° (B) 68° (C) 78° (D) 248°
- Two lines cross. One of the angles is 74° . What is the angle directly across from it?
(A) 16° (B) 106° (C) 148° (D) 74°
- Two angles of a triangle measure 52° and 71° . What is the third angle?
(A) 57° (B) 123° (C) 109° (D) 67°
- Two parallel lines are cut by a transversal. A same-side interior angle is 125° . What is the other same-side interior angle?
(A) 125° (B) 35° (C) 55° (D) 65°
- An angle is 4 times the size of its complement. What is the angle?
(A) 18° (B) 72° (C) 36° (D) 60°
- Two alternate interior angles formed by parallel lines are $(3x + 5)^\circ$ and $(5x - 25)^\circ$. What is the measure of each angle?
(A) 50° (B) 15° (C) 130° (D) 100°
- What is the complement of a 63° angle?
(A) 117° (B) 63° (C) 37° (D) 27°
- A triangle has a 90° angle and a 35° angle. What is the third angle?
(A) 145° (B) 125° (C) 55° (D) 35°
- A triangle has interior angles of 48° and 67° at two vertices. What is the exterior angle at the third vertex?
(A) 65° (B) 115° (C) 132° (D) 180°
- The angles of a triangle are $2x^\circ$, $3x^\circ$, and $(4x + 9)^\circ$. What is the largest angle in degrees?
- Two lines intersect so that vertical angles measure $(4x + 8)^\circ$ and $(6x - 12)^\circ$. Find the measure in degrees of an angle adjacent to the $(4x + 8)^\circ$ angle.
- Parallel lines are cut by a transversal. Two same-side interior angles are $(2x + 15)^\circ$ and $(3x + 10)^\circ$. Find x .
- In a right triangle, one acute angle is 6 degrees more than twice the other acute angle. Find both acute angles.
- Two angles form a linear pair and measure $(3x + 20)^\circ$ and $(2x + 10)^\circ$. Find x .
- Two parallel lines are cut by a transversal. Corresponding angles measure $(4x - 5)^\circ$ and $(3x + 18)^\circ$. Find the measure of each angle in degrees.

08.2 Perimeter and Area of Polygons

Find the perimeter and area of rectangles, triangles, parallelograms, trapezoids, and composite figures, and learn which formula to use and why it works.

Why This Matters

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Perimeter and area answer two different questions. Perimeter asks how far it is around a shape, like the fence for a garden. Area asks how much surface it covers, like the paint for a wall or the grass seed for a lawn. Students get stuck when they mix the two, forget that triangle and trapezoid formulas include one half, or use a slanted side where a height belongs.

These skills are used in building, design, farming, and shopping for materials. Once you can break a strange shape into simple pieces, you can find the area of nearly anything.

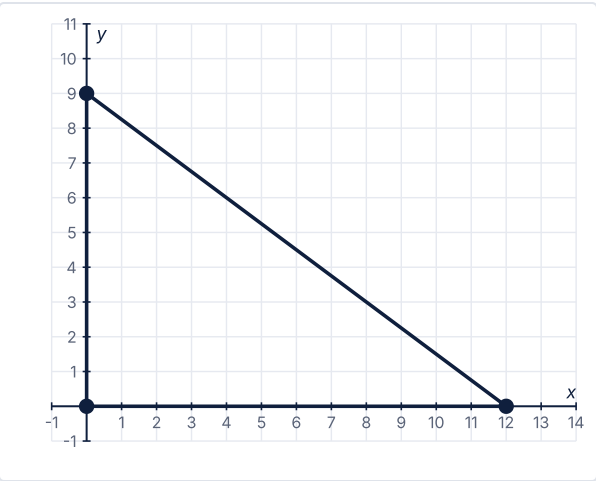
Core Ideas

Perimeter is the total length around a shape, measured in units like meters. **Area** is the amount of flat space inside, measured in square units like square meters. The **height** of a shape is always measured at a right angle to its base, not along a slanted side.

Shape	Area	Perimeter
Rectangle (length l , width w)	$A = lw$	$P = 2l + 2w$
Triangle (base b , height h)	$A = \frac{1}{2}bh$	add the 3 sides
Parallelogram	$A = bh$	add the 4 sides
Trapezoid (bases b_1, b_2)	$A = \frac{1}{2}(b_1 + b_2)h$	add the 4 sides

Area formulas, and why they work

A rectangle has area lw . A parallelogram can be cut and slid into a rectangle, so its area is bh . A triangle is half of a parallelogram, so its area is $\frac{1}{2}bh$. A trapezoid is the average of its two bases times the height: $\frac{1}{2}(b_1 + b_2)h$.



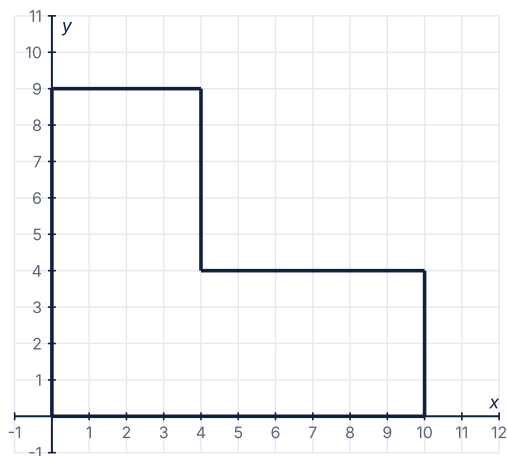
A right triangle with legs 12 and 9. Either leg can be the base, and the other leg is then the height.

Composite figures

To find the area of an odd shape, **split** it into rectangles and triangles and **add** their areas, or **subtract** a missing piece from a big rectangle. For perimeter, add every outside edge once. Do not count the inside lines you drew to split the shape.

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An L-shaped figure. Split it into a 10 by 4 rectangle and a 4 by 5 rectangle.

Worked Examples

EXAMPLE 1: RECTANGLE

A garden is 14 m long and 9 m wide. Find the perimeter and area.

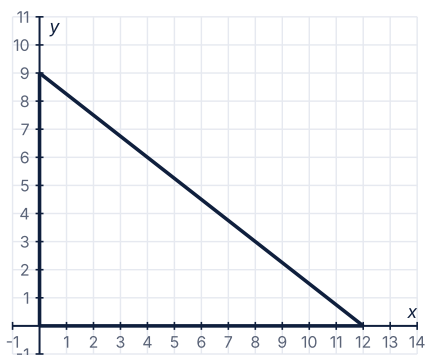
SOLUTION

Perimeter: $2(14) + 2(9) = 28 + 18 = 46$ m.

Area: $14 \times 9 = 126$ square meters.

EXAMPLE 2: TRIANGLE

A right triangle has legs 9 cm and 12 cm and hypotenuse 15 cm. Find the perimeter and area.



SOLUTION

Perimeter: $9 + 12 + 15 = 36$ cm.

Use a leg as base and the other as height: $A = \frac{1}{2}(12)(9) = 54$ square cm.

Do not use 15 as a height. The hypotenuse is slanted.

EXAMPLE 3: PARALLELOGRAM

A parallelogram has base 15 in, slanted side 8 in, and height 6 in. Find its area and perimeter.

SOLUTION

Area: $bh = 15 \times 6 = 90$ square inches.

Perimeter: two sides of 15 and two of 8, so $2(15) + 2(8) = 46$ in.

The slanted side (8) is used for perimeter only. The height (6) is used for area.

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EXAMPLE 4: TRAPEZOID

A trapezoid has parallel sides 10 ft and 16 ft, and the distance between them is 7 ft. Find the area.

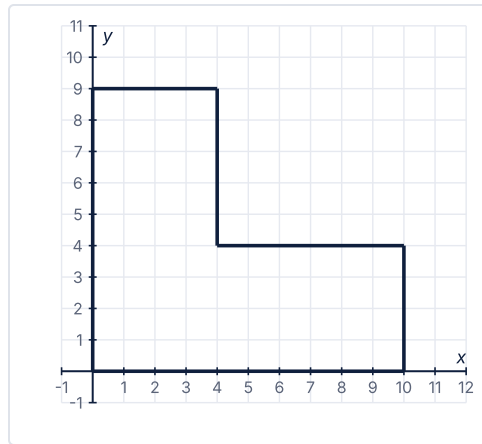
SOLUTION

$$A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(10 + 16)(7).$$

$$= \frac{1}{2}(26)(7) = 13 \times 7 = 91 \text{ square feet.}$$

EXAMPLE 5: COMPOSITE FIGURE

Find the area and perimeter of the L-shape shown in the figure (all corners are right angles).

**SOLUTION**

Area: split into a 10×4 rectangle (40) and a 4×5 rectangle (20). Total 60 square units.

Check by subtraction: a 10×9 rectangle minus the missing 6×5 corner is $90 - 30 = 60$.

Perimeter: add the outside edges $10 + 4 + 6 + 5 + 4 + 9 = 38$ units.

Shortcut: the perimeter equals that of the enclosing 10×9 rectangle, $2(10 + 9) = 38$.

EXAMPLE 6: FINDING A MISSING DIMENSION

A parallelogram has area 96 square cm and base 12 cm. Find its height. If the slanted side is 10 cm, find its perimeter.

SOLUTION

$A = bh$, so $96 = 12h$ and $h = 96 \div 12 = 8$ cm.

Perimeter: two bases and two slanted sides, $2(12) + 2(10) = 44$ cm.

The height (8) is perpendicular to the base, so it is not the slanted side (10).

Coach's Playbook

- 1 Name the shape and its parts.** Mark the base and the matching height at a right angle. Cross out any slanted side you do not need.
- 2 Pick the formula.** Write it down before you plug in numbers.
- 3 Keep the units.** Perimeter uses plain units. Area uses square units. Label your answer.
- 4 Sanity check.** A triangle should have half the area of the rectangle around it. A trapezoid should be between the areas of the two rectangles made from each base.

Common traps

1. Forgetting the $\frac{1}{2}$ in a triangle or trapezoid.
2. Using the slanted side as the height.
3. Adding in the inside cut lines when finding the perimeter of a composite shape.
4. Reporting area in plain units or perimeter in square units.

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Mistake Clinic

Spot the error

Find the area of a triangle with base 10 and height 6. A student writes $10 \times 6 = 60$. **Where it went wrong:** the student forgot the one half. **Right way:** $A = \frac{1}{2} \cdot 10 \cdot 6 = 30$. A triangle is half of the 10×6 rectangle.

Spot the error

A parallelogram has base 9, slanted side 5, and height 4. A student writes $A = 9 \times 5 = 45$. **Where it went wrong:** the slanted side is not the height. **Right way:** $A = bh = 9 \times 4 = 36$ square units.

Spot the error

A corner measuring 3 by 2 is cut from an 8 by 6 rectangle. A student says the new perimeter is $28 - 3 - 2 = 23$. **Where it went wrong:** the cut removes two edges but adds two new edges of the same lengths. **Right way:** the perimeter stays $2(8 + 6) = 28$. Only the area changes, from 48 to 42.

Real-World Connection

A painter must find the wall area, subtract doors and windows, then compare to how much one can of paint covers. That is a composite area problem.

Challenge Corner

A house-shaped figure is a rectangle 6 wide and 8 tall with a triangle on top. The triangle has base 6, height 4, and two slanted sides of 5. Find the area and perimeter. **Solution:** area of the rectangle $= 6 \times 8 = 48$. Area of the triangle $= \frac{1}{2} \cdot 6 \cdot 4 = 12$. Total area $= 60$. For the perimeter, walk around the outside: $8 + 6 + 8 + 5 + 5 = 32$. The top of the rectangle is inside the figure, so it is not counted.

Practice

PRACTICE Perimeter and Area of Polygons

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- A rectangle is 8 cm by 13 cm. What is its area?
 (A) 21 square cm (B) 42 square cm (C) 104 square cm
 (D) 52 square cm
- A triangle has base 14 m and height 9 m. What is its area?
 (A) 63 square m (B) 126 square m (C) 23 square m
 (D) 46 square m
- A parallelogram has base 12 in, side 5 in, and height 4 in. What is its area?
 (A) 60 square in (B) 34 square in (C) 24 square in
 (D) 48 square in
- A trapezoid has parallel sides 7 cm and 11 cm and height 6 cm. What is its area?
 (A) 108 square cm (B) 54 square cm (C) 66 square cm
 (D) 42 square cm
- A rectangle has perimeter 34 ft and length 11 ft. What is its width?
 (A) 23 ft (B) 12 ft (C) 6 ft (D) 3 ft

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6. From a 12 m by 8 m rectangle, a 4 m by 3 m rectangle is cut out of a corner. What is the area of what is left?
 (A) 108 square m (B) 96 square m (C) 72 square m (D) 84 square m
7. A triangle has area 40 square cm and base 10 cm. What is its height?
 (A) 8 cm (B) 4 cm (C) 16 cm (D) 20 cm
8. A rectangle is 9 m by 6 m. What is its perimeter?
 (A) 54 m (B) 15 m (C) 30 m (D) 45 m
9. A trapezoid has area 60 square units and parallel sides 8 and 12. What is its height?
 (A) 3 (B) 6 (C) 5 (D) 12
10. Find the area in square centimeters of a triangle with base 18 cm and height 11 cm.
11. A trapezoid has area 105 square meters and parallel sides 12 m and 18 m. How tall is it?
12. A floor is a 14 ft by 10 ft rectangle with a 5 ft by 4 ft rectangular notch cut out of one corner. Find the perimeter in feet and the area in square feet.
13. A wall is 12 ft by 9 ft. It has a door 3 ft by 7 ft and a window 4 ft by 3 ft. How many square feet need paint?
14. Find the area of a parallelogram with base 15 cm and height 8 cm.
15. A lawn is 30 ft by 20 ft. A patio of 12 ft by 5 ft and a triangular flower bed with base 8 ft and height 6 ft are not grass. How many square feet of grass remain?

08.3 Circles: Circumference and Area

Learn the parts of a circle, what pi means, how to find circumference and area, and how to work with sectors, arcs, rings, and shapes that include circles.

Why This Matters

Wheels, pizzas, clocks, and coins are circles, and circle formulas come up constantly in science and design. Students usually get stuck by mixing up circumference and area, by using the diameter where the formula needs the radius, and by forgetting to square the radius.

Circle ideas lead directly to geometry in later grades, to trigonometry, and to physics topics like rotation and waves. Pay attention to the two key facts: circumference is a length, and area is a surface.

Core Ideas

A circle is all the points at the same distance from a center. The **radius** r goes from the center to the circle. The **diameter** d goes all the way across through the center, so $d = 2r$. The **circumference** C is the distance around the circle.

If you divide the circumference of any circle by its diameter, you always get the same number, called **pi** (π). It is about 3.14 or $\frac{22}{7}$, and it never ends as a decimal. A calculator's π key is more accurate. Unless a problem says to use a decimal, leave answers **in terms of** π , like 12π .

Circumference and area

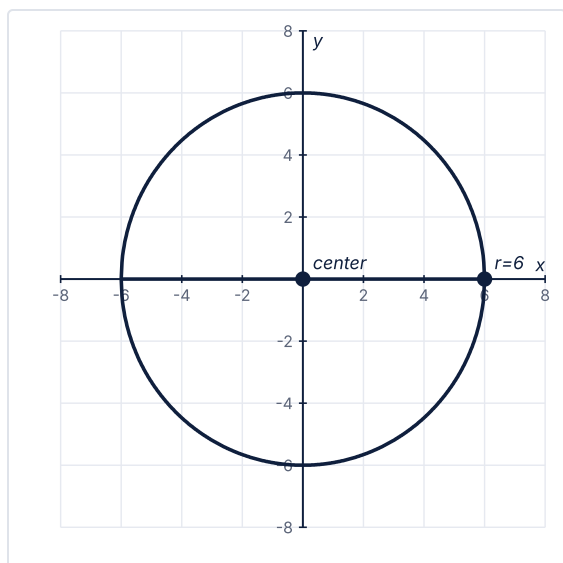
$$C = 2\pi r = \pi d \quad A = \pi r^2$$

Circumference is a length (units). Area is a surface (square units). Always find the radius first.

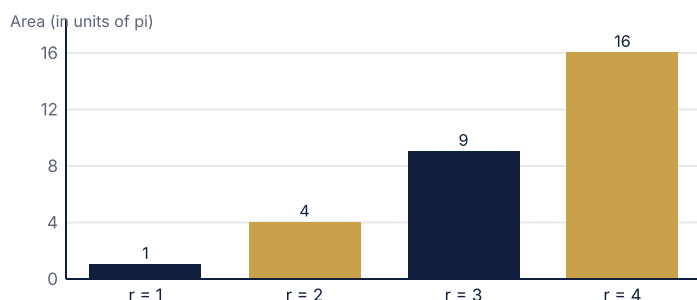
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A circle of radius 6 and diameter 12. Its circumference is 12π and its area is 36π .



Doubling the radius makes the area 4 times as big, because the radius is squared.

Sectors and arcs

A **sector** is a slice of a circle, like a pizza slice, and an **arc** is the curved edge of that slice. For a central angle of θ degrees, the slice is the fraction $\frac{\theta}{360}$ of the whole circle:

$$\text{arc length} = \frac{\theta}{360} \cdot 2\pi r \quad \text{sector area} = \frac{\theta}{360} \cdot \pi r^2.$$

Rings and composite shapes

A **ring** (annulus) between an outer circle of radius R and an inner circle of radius r has area $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$. For a shape built from circles and polygons, split it into pieces and add or subtract areas.

Square each radius first, then subtract. Do not subtract first: $\pi(R - r)^2$ is wrong.

Worked Examples

EXAMPLE 1: CIRCUMFERENCE AND AREA FROM THE RADIUS

A circle has radius 7 cm. Find the circumference and area, first in terms of π , then using $\pi \approx 3.14$.

SOLUTION

$C = 2\pi(7) = 14\pi$ cm. With 3.14: $14 \times 3.14 = 43.96$ cm.

$A = \pi(7)^2 = 49\pi$ square cm. With 3.14: $49 \times 3.14 = 153.86$ square cm.

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EXAMPLE 2: STARTING FROM THE DIAMETER

A round table has a diameter of 18 inches. Find its area.

SOLUTION

The formula needs the radius: $r = 18 \div 2 = 9$.

$$A = \pi(9)^2 = 81\pi \text{ square inches.}$$

Using the diameter directly would give $\pi \cdot 18^2 = 324\pi$, which is four times too large.

EXAMPLE 3: WORKING BACKWARDS

A circle has circumference 30π meters. Find its radius and area.

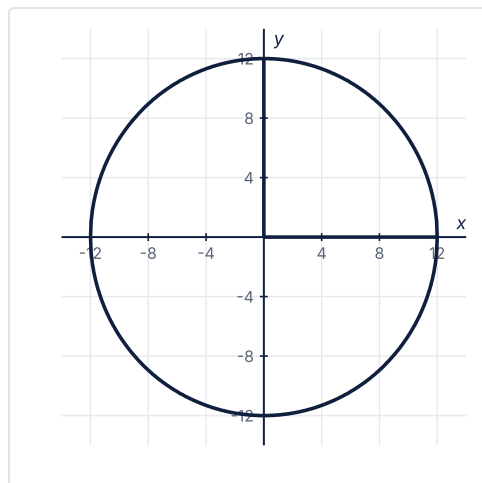
SOLUTION

$$2\pi r = 30\pi. \text{ Divide both sides by } 2\pi: r = 15 \text{ m.}$$

$$A = \pi(15)^2 = 225\pi \text{ square meters.}$$

EXAMPLE 4: SECTOR

A circle has radius 12 cm. Find the arc length and the area of the 90° sector.

**SOLUTION**

90° is $\frac{90}{360} = \frac{1}{4}$ of the circle.

$$\text{Arc length: } \frac{1}{4} \cdot 2\pi(12) = \frac{1}{4} \cdot 24\pi = 6\pi \text{ cm.}$$

$$\text{Sector area: } \frac{1}{4} \cdot \pi(12)^2 = \frac{1}{4} \cdot 144\pi = 36\pi \text{ square cm.}$$

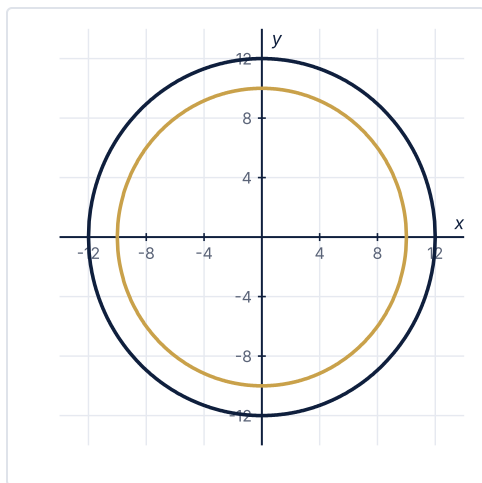
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EXAMPLE 5: RING

A circular pool has radius 10 m. A walkway 2 m wide goes all the way around it. Find the area of the walkway using $\pi \approx 3.14$.

**SOLUTION**

The outer circle has radius $10 + 2 = 12$ m.

Walkway area $= \pi(12^2) - \pi(10^2) = \pi(144 - 100) = 44\pi$.

$44 \times 3.14 = 138.16$ square meters.

EXAMPLE 6: A SHAPE WITH A SEMICIRCLE

A window is a 6 ft wide, 8 ft tall rectangle with a semicircle on top whose diameter is the 6 ft width. Find the area and the outer perimeter. Use $\pi \approx 3.14$ for the decimal.

SOLUTION

Rectangle area: $6 \times 8 = 48$. The semicircle has radius 3, so its area is $\frac{1}{2}\pi(3)^2 = 4.5\pi$.

Total area: $48 + 4.5\pi \approx 48 + 14.13 = 62.13$ square feet.

Perimeter: two sides of 8, the bottom of 6, and the curved top, which is half a circumference: $\frac{1}{2} \cdot 2\pi(3) = 3\pi$.

Total: $8 + 8 + 6 + 3\pi = 22 + 3\pi \approx 31.42$ feet. The flat edge between the rectangle and the semicircle is inside, so it is not counted.

Coach's Playbook

- 1 Find the radius first.** If you are given the diameter, cut it in half. If you are given circumference or area, work backwards to r .
- 2 Choose the formula by the question.** "How far around" is circumference. "How much surface" is area. A slice is a fraction of one of them.
- 3 Keep π until the end.** Leave answers like 36π unless asked for a decimal. Round only at the last step.
- 4 Estimate.** With $\pi \approx 3$, a radius 7 circle has area about $3 \times 49 = 147$. Does your answer match?

Common traps

1. Using the diameter as the radius. 2. Forgetting to square the radius when finding area, or squaring when finding circumference. 3. Subtracting radii before squaring in a ring. 4. Writing the fraction of a sector upside down, such as $\frac{360}{90}$ instead of $\frac{90}{360}$.

Mistake Clinic

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Spot the error

Find the area of a circle with diameter 10. A student writes $A = \pi \cdot 10^2 = 100\pi$. **Where it went wrong:** the student used the diameter in place of the radius. **Right way:** $r = 5$, so $A = \pi \cdot 5^2 = 25\pi$.

Spot the error

Find the circumference of a circle with radius 4. A student writes $\pi \cdot 4^2 = 16\pi$. **Where it went wrong:** that is the area formula. **Right way:** $C = 2\pi r = 8\pi$.

Spot the error

Find the area of a 45° sector of a circle with radius 8. A student writes $\frac{1}{8} \cdot 2\pi \cdot 8 = 2\pi$. **Where it went wrong:** that is the arc length. Area needs πr^2 . **Right way:** $\frac{45}{360} \cdot \pi \cdot 64 = \frac{1}{8} \cdot 64\pi = 8\pi$ square units.

Real-World Connection

A pizza with a 14 inch diameter has area 49π , but two 10 inch pizzas have area 50π . Area grows with the square of the radius, so size matters more than it looks.

Challenge Corner

A circle has circumference 20π . Find the area of its 72° sector. **Solution:** $2\pi r = 20\pi$, so $r = 10$. The full area is $\pi(10)^2 = 100\pi$. The fraction is $\frac{72}{360} = \frac{1}{5}$. Sector area $= \frac{1}{5} \cdot 100\pi = 20\pi$ square units.

Practice**PRACTICE** Circles: Circumference and Area

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What is the circumference of a circle with diameter 10 inches?
(A) 5π in (B) 25π in (C) 100π in (D) 10π in
- What is the area of a circle with radius 6 cm?
(A) 12π square cm (B) 36π square cm (C) 6π square cm
(D) 144π square cm
- A circle has circumference 16π . What is its radius?
(A) 8 (B) 16 (C) 4 (D) 64
- A circle has area 49π . What is its diameter?
(A) 7 (B) 49 (C) 14 (D) 98
- What is the arc length of a 60° sector of a circle with radius 9?
(A) 18π (B) 3π (C) 9π (D) $\frac{27}{2}\pi$
- A ring has outer radius 8 and inner radius 5. What is its area?
(A) 3π (B) 9π (C) 89π (D) 39π
- What is the circumference of a circle with radius 3.5 ft?
(A) 3.5π ft (B) 12.25π ft (C) 7π ft (D) 49π ft
- What is the area of a circle with diameter 12?
(A) 36π (B) 144π (C) 12π (D) 6π
- What is the area of a semicircle with diameter 10?
(A) 25π (B) 50π (C) 10π (D) $\frac{25}{2}\pi$
- Use $\pi \approx 3.14$ to find the area in square inches of a circle with diameter 20 inches.

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- 11. Find the area, in terms of π , of a 120° sector of a circle with radius 6.
- 12. A running track has two straight sides of 20 m and a half circle (diameter 10 m) at each end. Find the length of one lap in terms of π .
- 13. A restaurant sells one pizza with a 14 inch diameter or two pizzas with 10 inch diameters. Which choice has more pizza, and by how many square inches (in terms of π)?
- 14. A circle has circumference 44 inches. Using $\pi \approx \frac{22}{7}$, find its radius.
- 15. Find the arc length of a 150° sector of a circle with radius 12, in terms of π .

08.4 Triangles and Polygons

Classify triangles, use the triangle inequality, find interior and exterior angles of polygons, and work with regular polygons using a few simple formulas.

Why This Matters

Triangles are the strongest shape in building, which is why bridges and roof frames are full of them. Polygons with more sides show up in stop signs, tiles, and game boards. Students get stuck when they must decide whether three lengths can really make a triangle, and when they forget the formula for the angle total of a polygon.

These ideas lead to proofs, trigonometry, and design work. The good news is that only two formulas do most of the work: the triangle inequality and $(n - 2) \cdot 180$.

Core Ideas

A **polygon** is a closed figure made of straight sides. A **regular** polygon has all sides equal and all angles equal. We can sort triangles in two ways.

By sides	Description	By angles	Description
Equilateral	3 equal sides (and 3 equal angles of 60°)	Acute	all angles less than 90°
Isosceles	at least 2 equal sides; angles opposite them are equal	Right	one angle exactly 90°
Scalene	no equal sides	Obtuse	one angle more than 90°

Triangle inequality

Three lengths can form a triangle only if **each side is shorter than the sum of the other two**. It is enough to check that the two shorter sides add to **more than** the longest side.
If two sides are a and b , the third side c must satisfy $|a - b| < c < a + b$.



If two sides of a triangle are 5 and 9, the third side must be strictly between 4 and 14.

Angles in a polygon

The interior angles of a polygon with n sides add to

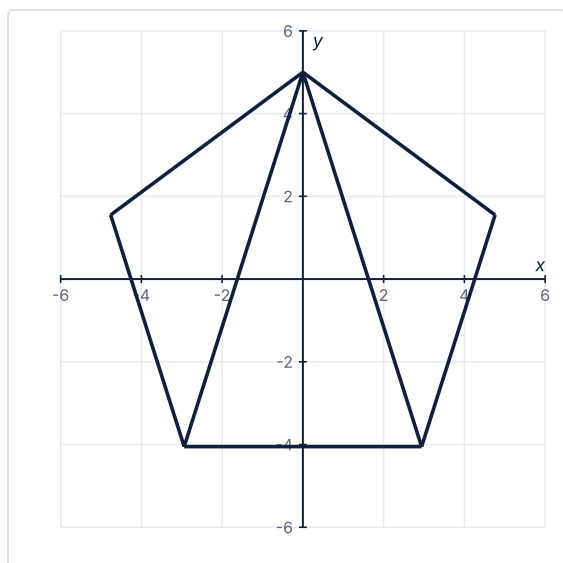
$(n - 2) \cdot 180^\circ$.

In a **regular** polygon, each interior angle is $\frac{(n-2) \cdot 180^\circ}{n}$. The exterior angles of any polygon add to 360° , so each exterior angle of a regular polygon is $\frac{360^\circ}{n}$.

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A pentagon split into 3 triangles from one corner: $(5 - 2) \times 180 = 540$ degrees.

Sides n	3	4	5	6	8	10	12
Name	triangle	quadrilateral	pentagon	hexagon	octagon	decagon	dodecagon
Angle sum	180°	360°	540°	720°	1080°	1440°	1800°

In an isosceles triangle, the two angles opposite the equal sides, called **base angles**, are equal. If the top angle is 40° , each base angle is $\frac{180-40}{2} = 70^\circ$.

Worked Examples

EXAMPLE 1: CLASSIFYING TRIANGLES

Classify each triangle by sides and angles. (a) sides 5, 5, 8 (b) sides 3, 4, 5 (c) angles $40^\circ, 60^\circ, 80^\circ$ (d) angles $30^\circ, 30^\circ, 120^\circ$

SOLUTION

- (a) Two equal sides: **isosceles**.
 (b) No equal sides: scalene. Since $3^2 + 4^2 = 25 = 5^2$, it is a **right** triangle. So scalene right.
 (c) Angles are all different (so scalene sides) and all under 90° : **scalene acute**.
 (d) Two equal angles (so isosceles) and one over 90° : **isosceles obtuse**.

EXAMPLE 2: DO THE LENGTHS MAKE A TRIANGLE?

Can sides of lengths (a) 4, 7, 12 or (b) 6, 8, 13 form a triangle?

SOLUTION

- (a) Add the two shorter: $4 + 7 = 11$, which is **not more than** 12. No triangle. The short sides cannot reach each other.
 (b) $6 + 8 = 14 > 13$. Yes, it forms a triangle.

EXAMPLE 3: RANGE FOR THE THIRD SIDE

Two sides of a triangle are 5 and 9. Describe all possible lengths for the third side c .

SOLUTION

Upper limit: $c < 5 + 9 = 14$.

Lower limit: $c > 9 - 5 = 4$.

So $4 < c < 14$. Open circles on the number line mean 4 and 14 themselves do not work.

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EXAMPLE 4: INTERIOR ANGLE SUMS

(a) Find the angle sum of a hexagon. (b) A quadrilateral has angles 85° , 110° , and 70° . Find the fourth angle.

SOLUTION

(a) $n = 6$, so $(6 - 2) \cdot 180 = 4 \cdot 180 = 720^\circ$.

(b) A quadrilateral totals 360° . The three known angles add to $85 + 110 + 70 = 265$.

Fourth angle: $360 - 265 = 95^\circ$.

EXAMPLE 5: REGULAR POLYGONS

(a) Find each interior angle of a regular octagon. (b) Each interior angle of a regular polygon is 150° . How many sides does it have?

SOLUTION

(a) Sum $= 6 \cdot 180 = 1080$, so each angle is $1080 \div 8 = 135^\circ$. Check: the exterior angle is $360 \div 8 = 45$ and $135 + 45 = 180$.

(b) The exterior angle is $180 - 150 = 30^\circ$.

Then $n = \frac{360}{30} = 12$ sides.

EXAMPLE 6: AN ISOSCELES TRIANGLE

An isosceles triangle has perimeter 34 cm and a base of 10 cm. Find the length of each equal side and check that the triangle exists.

SOLUTION

The two equal sides share what is left: $34 - 10 = 24$, so each is $24 \div 2 = 12$ cm.

Check the triangle inequality with the two shorter sides: $10 + 12 = 22 > 12$. Yes. The triangle exists.

Another check: $12 + 12 = 24 > 10$. Yes.

Coach's Playbook

- 1 Sort first.** For any triangle question, decide whether it is about sides, angles, or both.
- 2 Test the longest side.** For the triangle inequality, add the two shortest sides and compare to the longest.
- 3 Count sides carefully.** For polygon angle sums, write n first, then compute $n - 2$.
- 4 Use the exterior-angle shortcut.** For regular polygons, exterior angle $= \frac{360}{n}$, and interior $= 180 - \text{exterior}$. This is faster than the big formula.

Common traps

1. Calling lengths a triangle when the sum of the two shorter sides only equals the longest side. 2. Using $n \cdot 180$ instead of $(n - 2) \cdot 180$. 3. Mixing up interior and exterior angles of a regular polygon. 4. Assuming a triangle is isosceles or right because a sketch looks that way.

Mistake Clinic**Spot the error**

Find the angle sum of a hexagon. A student writes $6 \cdot 180 = 1080$. **Where it went wrong:** the student used $n \cdot 180$. **Right way:** $(n - 2) \cdot 180 = 4 \cdot 180 = 720^\circ$.

Spot the error

A student says sides 5, 6, and 11 make a triangle because $5 + 6 = 11$. **Where it went wrong:** the two short sides must add to **more than** the long side. With equality the sides lie flat in one line. **Right way:** $5 + 6 = 11$ is not greater than 11, so no triangle exists.

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Spot the error

Find each interior angle of a regular hexagon. A student writes $\frac{360}{6} = 60^\circ$. **Where it went wrong:** $\frac{360}{n}$ is the exterior angle. **Right way:** the interior angle is $180 - 60 = 120^\circ$. Check: $6 \cdot 120 = 720$.

Real-World Connection

A stop sign is a regular octagon, so each corner is 135° . Floor tilers use the fact that regular hexagons fit together without gaps because three 120° corners make 360° .

Challenge Corner

A regular polygon has interior angles that are 100° larger than its exterior angles. How many sides does it have?

Solution: let the interior angle be I and the exterior angle be E . They form a straight line, so $I + E = 180$, and $I - E = 100$. Adding gives $2I = 280$, so $I = 140$ and $E = 40$. Then $n = \frac{360}{40} = 9$ sides, a nonagon.

Practice**PRACTICE Triangles and Polygons**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- A triangle has sides 7, 7, and 10. What kind of triangle is it?
(A) isosceles (B) equilateral (C) scalene (D) right
- Which set of lengths cannot form a triangle?
(A) 3, 4, 5 (B) 6, 6, 6 (C) 2, 3, 6 (D) 5, 7, 10
- What is the sum of the interior angles of a decagon (10 sides)?
(A) 1800° (B) 1440° (C) 1620° (D) 1260°
- What is each interior angle of a regular pentagon?
(A) 72° (B) 540° (C) 120° (D) 108°
- Two sides of a triangle are 6 and 11. Which could be the length of the third side?
(A) 5 (B) 17 (C) 10 (D) 20
- An isosceles triangle has a top angle of 40° . What is each base angle?
(A) 70° (B) 100° (C) 40° (D) 140°
- A right isosceles triangle has one right angle. What is each of the other two angles?
(A) 90° (B) 30° (C) 60° (D) 45°
- What is the sum of the interior angles of a heptagon (7 sides)?
(A) 1260° (B) 900° (C) 1080° (D) 720°
- What is each exterior angle of a regular decagon (10 sides)?
(A) 144° (B) 18° (C) 36° (D) 72°
- Find the sum of the interior angles of a polygon with 12 sides, in degrees.
- A quadrilateral has angles x° , $(x + 20)^\circ$, $2x^\circ$, and $(2x + 40)^\circ$. What is its largest angle in degrees?
- Each exterior angle of a regular polygon is 24° . How many sides does the polygon have?
- Two sides of a triangle are 8 cm and 13 cm. If the third side is a whole number of centimeters, how many different lengths are possible?
- Two sides of an isosceles triangle are both 9 cm and the third side is a whole number of centimeters. What is the largest the third side can be?
- Each interior angle of a regular polygon is 162° . How many sides does it have?

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Answers**Answers and Explanations**

Complete solutions for every practice problem, numbered to match. Read the explanation even when your answer is right.

08.1 Angles and Angle Relationships**PRACTICE · ANGLES AND ANGLE RELATIONSHIPS**

1. B. Supplementary angles add to 180: $180 - 112 = 68$. Choice A, 22, comes from $112 - 90$, which does not give a supplement.
2. D. Vertical angles are equal, so the angle across is also 74° . Choice B is its neighbor, not the angle across.
3. A. The sum of the other two is $52 + 71 = 123$, so the third is $180 - 123 = 57$. Choice B stops after adding.
4. C. Same-side interior angles add to 180, so $180 - 125 = 55$. Choice A treats them as equal.
5. B. Let the angle be x . Then $x = 4(90 - x)$, so $5x = 360$ and $x = 72$. Check: complement is 18 and $4 \cdot 18 = 72$. Choice A is the complement.
6. A. Alternate interior angles are equal: $3x + 5 = 5x - 25$, so $x = 15$. The angle is $3(15) + 5 = 50$. Choice B stops at x .
7. D. Complementary angles add to 90: $90 - 63 = 27$. Choice A is the supplement.
8. C. $180 - 90 - 35 = 55$. Choice A is $180 - 35$, which forgets the right angle.
9. B. The exterior angle is the sum of the remote interior angles: $48 + 67 = 115$. Choice A is the interior angle at that vertex.
10. 85. $2x + 3x + 4x + 9 = 180$ gives $9x = 171$, $x = 19$. The largest is $4(19) + 9 = 85$. (Angles are 38, 57, 85.)
11. 132. Vertical angles are equal: $4x + 8 = 6x - 12$, so $x = 10$ and the angle is 48. An adjacent angle is $180 - 48 = 132$.
12. 31. Same-side interior angles add to 180: $5x + 25 = 180$, so $x = 31$. (The angles are 77 and 103.)
13. 28 and 62 degrees. Let the angles be a and $2a + 6$: $3a + 6 = 90$, so $a = 28$ and the other is $2(28) + 6 = 62$.
14. 30. Linear pairs add to 180: $5x + 30 = 180$, so $x = 30$. (The angles are 110 and 70.)
15. 87. Corresponding angles are equal: $4x - 5 = 3x + 18$, so $x = 23$. The angle is $4(23) - 5 = 87$.

08.2 Perimeter and Area of Polygons**PRACTICE · PERIMETER AND AREA OF POLYGONS**

1. C. $8 \times 13 = 104$. Choice A adds the sides and choice B is the perimeter.
2. A. $\frac{1}{2} \times 14 \times 9 = 63$. Choice B forgets the one half.
3. D. Use the height, not the slanted side: $12 \times 4 = 48$. Choice A uses the side 5 and gives 60.
4. B. $\frac{1}{2}(7 + 11)(6) = 9 \times 6 = 54$. Choice A forgets the one half.
5. C. Half the perimeter is 17, so the width is $17 - 11 = 6$. Choice B subtracts $34 - 22 = 12$ but forgets there are two widths.
6. D. Subtract the cut piece: $96 - 12 = 84$. Choice A adds the pieces instead.
7. A. $40 = \frac{1}{2} \cdot 10 \cdot h = 5h$, so $h = 8$. Choice B comes from $40 \div 10$, which forgets the one half.
8. C. $2(9 + 6) = 30$. Choice A is the area and choice B is only half the perimeter.
9. B. $60 = \frac{1}{2}(8 + 12)h = 10h$, so $h = 6$. Choice A forgets the one half.
10. 99. $\frac{1}{2} \times 18 \times 11 = 99$.
11. 7 m. $105 = \frac{1}{2}(12 + 18)h = 15h$, so $h = 7$.
12. Perimeter 48 ft; area 120 square ft. A corner notch does not change the perimeter, so $P = 2(14 + 10) = 48$. Area is $140 - 20 = 120$.
13. 75 square feet. Wall: $12 \times 9 = 108$. Door: 21. Window: 12. $108 - 21 - 12 = 75$.
14. 120 square cm. $A = bh = 15 \times 8$.
15. 516 square feet. Lawn: 600. Patio: 60. Flower bed: $\frac{1}{2} \cdot 8 \cdot 6 = 24$. $600 - 60 - 24 = 516$.

08.3 Circles: Circumference and Area**PRACTICE · CIRCLES: CIRCUMFERENCE AND AREA**

1. D. $C = \pi d = 10\pi$. Choice A uses the radius; choice B is the area of a circle with radius 5.
2. B. $A = \pi r^2 = \pi(36) = 36\pi$. Choice A, 12π , is the circumference.
3. A. $2\pi r = 16\pi$, so $r = 8$. Choice B is the diameter.
4. C. $\pi r^2 = 49\pi$ gives $r = 7$, so $d = 14$. Choice A stops at the radius.
5. B. $\frac{60}{360} = \frac{1}{6}$ and the circumference is 18π , so $\frac{1}{6} \cdot 18\pi = 3\pi$. Choice A is the whole circumference.
6. D. $\pi(8^2 - 5^2) = \pi(64 - 25) = 39\pi$. Choice B, 9π , comes from $(8 - 5)^2$.
7. C. $C = 2\pi r = 2\pi(3.5) = 7\pi$. Choice B is the area.
8. A. The radius is 6, so $A = \pi \cdot 36 = 36\pi$. Choice B uses the diameter as the radius.
9. D. The radius is 5, the full circle has area 25π , and half of that is $\frac{25}{2}\pi$. Choice A forgets to halve.
10. 314 square inches. $r = 10$, so $A = 3.14 \times 100 = 314$.
11. 12π . $\frac{120}{360} = \frac{1}{3}$ and the full area is 36π , so $\frac{1}{3}(36\pi) = 12\pi$.
12. $40 + 10\pi$ meters. The two half circles make one full circle with circumference $\pi(10) = 10\pi$. Add the two straight sides, $2(20) = 40$.

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13. The two 10 inch pizzas, by π square inches. One 14 inch pizza has $r = 7$ and area 49π . Two 10 inch pizzas have $2(25\pi) = 50\pi$. The difference is π (about 3.14 square inches).
14. 7 inches. $44 = 2 \cdot \frac{22}{7} \cdot r = \frac{44}{7}r$, so $r = 7$.
15. $10\pi \cdot \frac{150}{360} \cdot 2\pi(12) = \frac{5}{12} \cdot 24\pi = 10\pi$.

08.4 Triangles and Polygons

PRACTICE · TRIANGLES AND POLYGONS

1. A. Exactly two sides are equal (7 and 7), so it is isosceles. Not equilateral because 10 is different.
2. C. $2 + 3 = 5$, which is less than 6, so the two short sides cannot meet. The other sets pass: $3 + 4 > 5$, $6 + 6 > 6$, $5 + 7 > 10$.
3. B. $(10 - 2) \cdot 180 = 8 \cdot 180 = 1440$. Choice A uses $10 \cdot 180$ and forgets to subtract 2.
4. D. Exterior angle $= \frac{360}{5} = 72$, so interior $= 180 - 72 = 108$. Choice A is the exterior angle and choice B is the sum of all five.
5. C. The third side must satisfy $5 < c < 17$. Only 10 fits. Choices A and B are the boundary values, which only make a flat line, and 20 is too long.
6. A. The two base angles share $180 - 40 = 140$, so each is 70. Choice D forgets to split.
7. D. The other two angles are equal and add to 90° , so each is 45° . Choice C would apply to an equilateral triangle.
8. B. $(7 - 2) \cdot 180 = 5 \cdot 180 = 900$. Choice A is $7 \cdot 180$.
9. C. Exterior angles add to 360, so each is $\frac{360}{10} = 36$. Choice A is the interior angle.
10. 1800 degrees. $(12 - 2) \cdot 180 = 10 \cdot 180 = 1800$.
11. 140 degrees. The sum is 360: $x + (x + 20) + 2x + (2x + 40) = 6x + 60 = 360$, so $x = 50$. The angles are 50, 70, 100, 140.
12. 15 sides. $n = \frac{360}{24} = 15$.
13. 15. The third side c satisfies $13 - 8 < c < 13 + 8$, so $5 < c < 21$. The whole numbers 6 to 20 give $20 - 6 + 1 = 15$ lengths.
14. 17 cm. The third side must be less than $9 + 9 = 18$, and the largest whole number below 18 is 17.
15. 20 sides. The exterior angle is $180 - 162 = 18$, and $n = \frac{360}{18} = 20$.

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