



Volume, Surface Area, and the Pythagorean Theorem

Four focused lessons, worked examples, a mistake clinic, and 60 practice problems with full explanations.

In this booster

09.1 Volume of Prisms and Cylinders

09.2 Pyramids, Cones, and Spheres

09.3 Surface Area and Nets

09.4 The Pythagorean Theorem

Every section teaches the idea, walks through the most common problem types, flags the traps students fall into, and ends with fifteen practice problems. Full answers and explanations are at the back.

09.1 Volume of Prisms and Cylinders

Learn the one big idea behind volume: the area of the base times the height. Practice with boxes, triangular prisms, cylinders, tanks, and pouring problems, and keep your units straight.

Why This Matters

Volume tells you how much space a solid takes up. It answers questions like how much water fits in a fish tank, how much concrete a builder orders for a step, or how much soup a can holds. Students usually get stuck for three reasons: they mix up area (flat, square units) with volume (solid, cubic units), they forget which measurement is the height of the solid, and they leave out units or change units in the wrong way.

Volume shows up again in science (density), cooking, shipping, and later courses in geometry, chemistry, and physics. The good news is that almost every volume in this section comes from one simple idea: **area of the base times height**.

Core Ideas

Picture a rectangular box filled with unit cubes, each 1 cm on every side. The bottom layer has some number of cubes. If the box is 4 layers tall, you just repeat that layer 4 times. So the volume is the number of cubes in one layer (the base area) times the number of layers (the height).

Volume of a Prism or Cylinder

For any prism or cylinder:

$$V = B \cdot h$$

where B is the area of the base and h is the height (the distance between the two bases).

Rectangular prism: $V = l \cdot w \cdot h$. Cube with edge s : $V = s^3$.

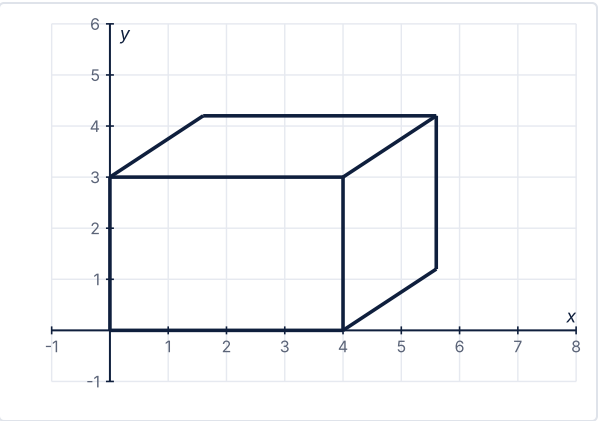
Cylinder: the base is a circle, so $B = \pi r^2$ and

$$V = \pi r^2 h.$$

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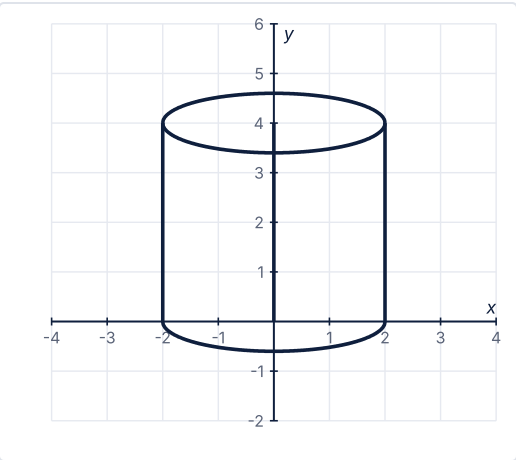
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A rectangular prism drawn in 3D. Picture one layer of cubes on the bottom, then stack 3 layers to reach the top.

The word **base** does not mean the bottom. The base is the face that is repeated all the way through the solid. In a triangular prism lying on its side, the base is the triangle on the end, and the height is the long distance between the two triangles. Always ask: what shape gets stacked, and how far does it stack?



A cylinder with radius 2 and height 4. The circular base has area πr^2 , and the height is the stack.

Units of Volume

Volume is measured in **cubic units**: cm^3 , m^3 , in^3 , ft^3 .
 $1 \text{ mL} = 1 \text{ cm}^3$, $1 \text{ L} = 1000 \text{ cm}^3$, and $1 \text{ m}^3 = 1000 \text{ L}$.

Solid	Base area B	Volume
Rectangular prism	$l \cdot w$	lwh
Triangular prism	$\frac{1}{2}bh_{\text{tri}}$	$B \cdot h_{\text{prism}}$
Cylinder	πr^2	$\pi r^2 h$
Cube	s^2	s^3

Capacity is how much a container holds, and it is found with volume. A tank that is 100 cm by 50 cm by 40 cm has volume $200,000 \text{ cm}^3$, which is 200 L. To fill a container only part way, use the water height instead of the full height. For a cylinder, remember that the radius is half the diameter. In this booster we use $\pi \approx 3.14$ when a decimal answer is needed, and we leave answers "in terms of π " when asked.

Worked Examples

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EXAMPLE 1: RECTANGULAR PRISM

A shoe box is 8 cm long, 5 cm wide, and 3 cm tall. Find its volume.

SOLUTION

Use $V = lwh$.

$$V = 8 \cdot 5 \cdot 3 = 120.$$

The volume is 120 cm^3 .

EXAMPLE 2: TRIANGULAR PRISM

The base of a prism is a triangle with base 6 cm and height 4 cm. The prism is 10 cm long. Find its volume.

SOLUTION

The base shape is the triangle: $B = \frac{1}{2}(6)(4) = 12 \text{ cm}^2$.

The height of the prism is the length, 10 cm.

$$V = Bh = 12 \cdot 10 = 120 \text{ cm}^3.$$

EXAMPLE 3: CYLINDER

A can has radius 3 cm and height 10 cm. Find its volume. Use $\pi \approx 3.14$.

SOLUTION

$$B = \pi r^2 = 3.14 \cdot 3^2 = 28.26 \text{ cm}^2.$$

$$V = 28.26 \cdot 10 = 282.6 \text{ cm}^3.$$

Since $1 \text{ cm}^3 = 1 \text{ mL}$, the can holds about 282.6 mL.

EXAMPLE 4: LIQUID AND CAPACITY

A fish tank is 60 cm long, 30 cm wide, and 40 cm tall. It is filled with water to $\frac{3}{4}$ of its height. How many liters of water are in the tank?

SOLUTION

Water height: $\frac{3}{4}(40) = 30 \text{ cm}$.

$$V = 60 \cdot 30 \cdot 30 = 54,000 \text{ cm}^3.$$

Convert: $54,000 \div 1000 = 54$ liters.

EXAMPLE 5: POURING BETWEEN CONTAINERS

A cylinder with radius 5 cm holds water 8 cm deep. All the water is poured into an empty box whose base is 10 cm by 10 cm. How deep is the water in the box? Use $\pi \approx 3.14$.

SOLUTION

Water volume: $3.14 \cdot 5^2 \cdot 8 = 628 \text{ cm}^3$.

The volume stays the same after pouring. In the box, $V = (10 \cdot 10) \cdot h = 100h$.

$$100h = 628, \text{ so } h = 6.28 \text{ cm}.$$

EXAMPLE 6: DISPLACEMENT

A rectangular tank has a base 40 cm by 25 cm and holds water. A small rock is dropped in and sinks, and the water level rises by 2 cm. What is the volume of the rock?

SOLUTION

The rock pushes aside its own volume of water, so the rock's volume equals the volume of the extra water layer.

The extra layer has the tank's base and a height of 2 cm: $V = 40 \cdot 25 \cdot 2 = 2000$.

The rock has volume 2000 cm^3 , which is 2 liters.

Coach's Playbook

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- 1 **Name the base.** Write down the shape that is stacked and find its area first.
- 2 **Find the height of the solid.** This is the distance between the two bases, not a slant or a side of the base.
- 3 **Check units.** Make every length the same unit before multiplying, then write cubic units.
- 4 **Estimate.** Round the numbers and see if your answer is in the right range. A soda can is a few hundred mL, not a few thousand.
- 5 **Use the volume for pouring problems.** The amount of liquid stays the same. Find it in one container, then use it in the other.

Common traps

(1) Using the diameter where the radius belongs: πr^2 needs the radius. (2) Forgetting the $\frac{1}{2}$ when the base is a triangle. (3) Writing square units for volume, or mixing cm with m. (4) Using the full height of a tank when it is only partly filled.

Mistake Clinic

Spot the error

Problem: A can has a diameter of 10 cm and a height of 12 cm. Find its volume using $\pi \approx 3.14$. **Wrong work:** $V = 3.14 \cdot 10^2 \cdot 12 = 3768 \text{ cm}^3$. **What went wrong:** The 10 is the diameter, but the formula needs the radius. **Right solution:** $r = 5$, so $V = 3.14 \cdot 25 \cdot 12 = 942 \text{ cm}^3$.

Spot the error

Problem: A triangular prism has a triangle base with base 6 cm and height 5 cm. The prism is 9 cm long. **Wrong work:** $V = 6 \cdot 5 \cdot 9 = 270 \text{ cm}^3$. **What went wrong:** The base is a triangle, so its area is half of base times height. The student found the area of a rectangle. **Right solution:** $B = \frac{1}{2}(6)(5) = 15$, and $V = 15 \cdot 9 = 135 \text{ cm}^3$.

Spot the error

Problem: A tank is 2 m long, 50 cm wide, and 40 cm tall. How many liters does it hold? **Wrong work:** $V = 2 \cdot 50 \cdot 40 = 4000$, so 4000 liters. **What went wrong:** The lengths are in different units, and the student also skipped the conversion to liters. **Right solution:** Change 2 m to 200 cm. $V = 200 \cdot 50 \cdot 40 = 400,000 \text{ cm}^3$, and $400,000 \div 1000 = 400$ liters.

Real-World Connection

A builder pouring a concrete slab 6 m long, 3 m wide, and 0.1 m thick needs $6 \cdot 3 \cdot 0.1 = 1.8 \text{ m}^3$ of concrete. Ordering too little is costly, so people calculate volume first.

Challenge Corner

A can has radius 4 cm and height 10 cm. It fits snugly inside a box with a square base, so the box is 8 cm by 8 cm by 10 cm. What fraction of the box is empty space? **Solution:** Box volume: $8 \cdot 8 \cdot 10 = 640$. Can volume: $\pi \cdot 4^2 \cdot 10 = 160\pi$. Empty space: $640 - 160\pi$. Fraction empty: $\frac{640 - 160\pi}{640} = 1 - \frac{\pi}{4}$. Using $\pi \approx 3.14$, this is $1 - 0.785 = 0.215$, so about 21.5 percent of the box is empty.

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Practice

PRACTICE Volume of Prisms and Cylinders

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- A box is 7 cm long, 4 cm wide, and 5 cm tall. What is its volume?
(A) 16 cubic centimeters (B) 140 cubic centimeters (C) 166 cubic centimeters
(D) 35 cubic centimeters
- A cube has edges of 6 inches. What is its volume?
(A) 36 in^3 (B) 18 in^3 (C) 216 in^3 (D) 108 in^3
- A cylinder has radius 4 cm and height 9 cm. What is its volume in terms of π ?
(A) $72\pi \text{ cm}^3$ (B) $576\pi \text{ cm}^3$ (C) $36\pi \text{ cm}^3$ (D) $144\pi \text{ cm}^3$
- A rectangular tank is 2 m long, 1.5 m wide, and 1 m tall. When completely full, how many liters of water does it hold?
(A) 3000 liters (B) 300 liters (C) 30,000 liters
(D) 4.5 liters
- A triangular prism has a right-triangle base with legs 5 cm and 12 cm. The prism is 8 cm long. What is its volume?
(A) 480 cm^3 (B) 240 cm^3 (C) 25 cm^3 (D) 30 cm^3
- A juice can has a diameter of 6 cm and a height of 10 cm. About how many milliliters does it hold? Use $\pi \approx 3.14$.
(A) 94.2 mL (B) 188.4 mL (C) 282.6 mL (D) 1130.4 mL
- A cylinder has a diameter of 10 cm and a height of 6 cm. What is its volume in terms of π ?
(A) $600\pi \text{ cm}^3$ (B) $150\pi \text{ cm}^3$ (C) $60\pi \text{ cm}^3$ (D) $300\pi \text{ cm}^3$
- A rectangular container has a base 50 cm by 20 cm. It holds water 15 cm deep. How many liters of water are in it?
(A) 150 liters (B) 1.5 liters (C) 15,000 liters
(D) 15 liters
- A triangular prism has a triangular base with base 8 cm and height 5 cm. Its volume is 200 cm^3 . How long is the prism?
(A) 5 cm (B) 10 cm (C) 25 cm (D) 40 cm
- A rectangular prism has volume 504 cm^3 , length 9 cm, and width 7 cm. What is its height?
- A cylinder has radius 2 cm and volume 150.72 cm^3 . Use $\pi \approx 3.14$. What is its height?
- A swimming pool is 12 m long and 5 m wide. The water is 1.5 m deep. (a) How many liters of water are in the pool? (b) A pump adds 450 liters per minute to fill an empty pool to this level. How many minutes does it take?
- A cylinder with radius 6 cm is filled with water to a depth of 5 cm. The water is poured into an empty rectangular box with a base of 12 cm by 5 cm. How deep is the water in the box? Use $\pi \approx 3.14$.
- A soup can has radius 3.5 cm and height 11 cm. Find its volume to the nearest tenth of a cubic centimeter. Use $\pi \approx 3.14$.
- A water trough is a triangular prism. Each end is a triangle with base 40 cm and height 30 cm, and the trough is 2 m long. How many liters does it hold when full?

09.2 Pyramids, Cones, and Spheres

Discover why pyramids and cones hold one third of the matching prism or cylinder, learn the sphere formula, and practice with composite solids and missing dimensions.

Why This Matters

Prisms and cylinders have the same shape from bottom to top. Pyramids, cones, and spheres do not: they get narrower, or they curve. That is why their volume formulas feel new and strange. Students commonly forget the fractions in the formulas, cube the wrong number, or use the slant length instead of the true height.

These solids are everywhere: party hats, ice cream cones, funnels, roofs, tents, basketballs, and water towers. Engineers, architects, and scientists use these formulas to size containers and estimate materials. Many real objects are combinations of these solids, so learning to split a shape into pieces is a powerful skill.

Core Ideas

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Here is the big idea. A pyramid has exactly $\frac{1}{3}$ of the volume of a prism with the same base and the same height. A cone has exactly $\frac{1}{3}$ of the volume of a cylinder with the same base and the same height. If you fill a cone with water and pour it into the matching cylinder, you need to do it 3 times to fill the cylinder.



Same radius and same height: the cone is one third of the cylinder.

Pyramid and Cone

Pyramid:

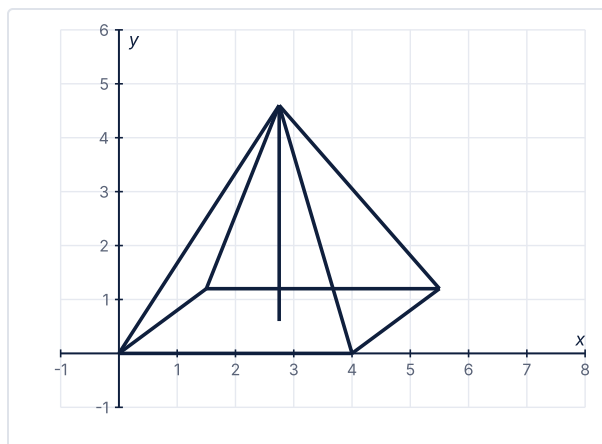
$$V = \frac{1}{3}Bh$$

where B is the area of the base and h is the perpendicular height.

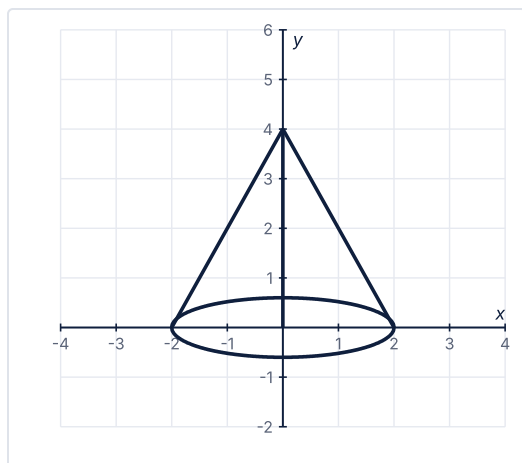
Cone:

$$V = \frac{1}{3}\pi r^2 h$$

The height is measured straight up, not along the slanted side.



A square pyramid. The height is the straight-up distance from the apex to the middle of the base, not the slanted edge.



A cone with radius 2 and height 4. The vertical segment is the height h , not the slanted side.

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Sphere and Hemisphere

Sphere:

$$V = \frac{4}{3}\pi r^3$$

The radius is cubed (multiply it by itself three times), not squared.

Hemisphere (half a sphere):

$$V = \frac{2}{3}\pi r^3.$$

The table below compares solids that share a radius of 3 cm. The cylinder and cone also have height 6 cm. We use $\pi \approx 3.14$.

Solid	Formula	Volume
Cylinder, $r = 3, h = 6$	$\pi r^2 h$	169.56 cm ³
Cone, $r = 3, h = 6$	$\frac{1}{3}\pi r^2 h$	56.52 cm ³
Sphere, $r = 3$	$\frac{4}{3}\pi r^3$	113.04 cm ³
Hemisphere, $r = 3$	$\frac{2}{3}\pi r^3$	56.52 cm ³

For **composite solids**, split the shape into pieces you know, find each volume, and add (or subtract if a piece is cut out). For **missing dimensions**, write the formula, substitute everything you know, and solve the resulting equation.

Worked Examples

EXAMPLE 1: SQUARE PYRAMID

A square pyramid has a base 6 cm by 6 cm and a height of 8 cm. Find its volume.

SOLUTION

Base area: $B = 6 \cdot 6 = 36 \text{ cm}^2$.

$$V = \frac{1}{3}(36)(8) = \frac{288}{3} = 96.$$

The volume is 96 cm³.

EXAMPLE 2: CONE

A cone has radius 3 cm and height 7 cm. Find its volume using $\pi \approx 3.14$.

SOLUTION

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}(3.14)(3^2)(7).$$

$$3^2 \cdot 7 = 63, \text{ and } \frac{63}{3} = 21.$$

$$V = 3.14 \cdot 21 = 65.94 \text{ cm}^3.$$

EXAMPLE 3: SPHERE

A sphere has radius 6 inches. Find its volume in terms of π .

SOLUTION

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3.$$

$$6^3 = 216, \text{ and } \frac{4}{3}(216) = 288.$$

$$V = 288\pi \text{ in}^3.$$

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EXAMPLE 4: COMPOSITE SOLID

An ice cream treat is a cone with radius 3 cm and height 9 cm, topped by a hemisphere of radius 3 cm that sits on the open end. Find the total volume in terms of π , then use $\pi \approx 3.14$.

SOLUTION

Cone: $\frac{1}{3}\pi(3^2)(9) = 27\pi$.

Hemisphere: $\frac{2}{3}\pi(3^3) = 18\pi$.

Total: $27\pi + 18\pi = 45\pi \text{ cm}^3$.

With $\pi \approx 3.14$: $45(3.14) = 141.3 \text{ cm}^3$.

EXAMPLE 5: MISSING DIMENSION

A cone has radius 4 cm and volume 100.48 cm^3 . Find its height. Use $\pi \approx 3.14$.

SOLUTION

Write the formula: $100.48 = \frac{1}{3}(3.14)(4^2)h$.

Multiply both sides by 3 to clear the fraction: $301.44 = 3.14 \cdot 16 \cdot h = 50.24h$.

$h = 301.44 \div 50.24 = 6 \text{ cm}$.

EXAMPLE 6: BALL IN A CAN

A ball of radius 3 cm fits snugly in a can with radius 3 cm and height 6 cm. What fraction of the can's volume does the ball fill?

SOLUTION

Ball: $V = \frac{4}{3}\pi(3^3) = \frac{4}{3}\pi(27) = 36\pi$.

Can: $V = \pi(3^2)(6) = 54\pi$.

Fraction: $\frac{36\pi}{54\pi} = \frac{2}{3}$. The ball fills two thirds of the can.

Coach's Playbook

- 1 Match the solid to its formula.** Pointed solids (pyramid, cone) get the $\frac{1}{3}$. Spheres get $\frac{4}{3}\pi r^3$.
- 2 Find the true height.** If you are given a slant length, use the Pythagorean Theorem later, or look for the vertical height in the problem.
- 3 Cube the radius for spheres.** Compute r^3 first, then multiply by $\frac{4}{3}\pi$. Cancel fractions before using a calculator.
- 4 Split composites.** Draw each piece separately and label its radius and height.
- 5 Check with the 3-to-1 rule.** A cone should be a third of its cylinder. If your cone is bigger than the cylinder, something is wrong.

Common traps

(1) Forgetting the $\frac{1}{3}$ in pyramids and cones. (2) Squaring the radius in a sphere instead of cubing it. (3) Using the slant side as the height. (4) Using the diameter as the radius, especially when a sphere or ball is described by its width.

Mistake Clinic

Spot the error

Problem: A cone has radius 3 cm and height 5 cm. Find its volume using $\pi \approx 3.14$. **Wrong work:** $V = 3.14 \cdot 3^2 \cdot 5 = 141.3 \text{ cm}^3$. **What went wrong:** That is the volume of a cylinder. A cone holds only one third as much. **Right solution:** $V = \frac{1}{3}(3.14)(9)(5) = 47.1 \text{ cm}^3$.

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Spot the error

Problem: Find the volume of a sphere with radius 3 cm in terms of π . **Wrong work:** $V = \frac{4}{3}\pi(3^2) = 12\pi$. **What went wrong:** The radius must be cubed, not squared. **Right solution:** $V = \frac{4}{3}\pi(3^3) = \frac{4}{3}\pi(27) = 36\pi \text{ cm}^3$.

Spot the error

Problem: A cone has radius 6 cm, slant height 10 cm, and height 8 cm. Find its volume. **Wrong work:** $V = \frac{1}{3}\pi(6^2)(10) = 120\pi$. **What went wrong:** The formula needs the height straight up from the center of the base, 8 cm, not the slant height along the side. **Right solution:** $V = \frac{1}{3}\pi(36)(8) = 96\pi \text{ cm}^3$.

Real-World Connection

A funnel cake cone, a grain pile, and a roof are all cone or pyramid shapes. A pile of sand shaped like a cone with radius 2 m and height 1.5 m holds about $\frac{1}{3}(3.14)(4)(1.5) = 6.28 \text{ m}^3$.

Challenge Corner

A solid metal sphere with radius 6 cm is melted down and recast into small cones, each with radius 3 cm and height 12 cm. How many cones can be made, assuming no metal is lost? **Solution:** Sphere: $V = \frac{4}{3}\pi(6^3) = \frac{4}{3}\pi(216) = 288\pi$. One cone: $V = \frac{1}{3}\pi(3^2)(12) = 36\pi$. Number of cones: $288\pi \div 36\pi = 8$. Exactly 8 cones can be made.

Practice**PRACTICE** **Pyramids, Cones, and Spheres**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- A pyramid has a rectangular base 5 cm by 4 cm and a height of 9 cm. What is its volume?
(A) 180 cm^3 (B) 20 cm^3 (C) 60 cm^3 (D) 30 cm^3
- A cone has radius 6 in and height 5 in. What is its volume in terms of π ?
(A) $60\pi \text{ in}^3$ (B) $180\pi \text{ in}^3$ (C) $20\pi \text{ in}^3$ (D) $300\pi \text{ in}^3$
- A ball has a diameter of 12 cm. What is its volume in terms of π ?
(A) $144\pi \text{ cm}^3$ (B) $2304\pi \text{ cm}^3$ (C) $864\pi \text{ cm}^3$ (D) $288\pi \text{ cm}^3$
- A cone and a cylinder have the same radius and the same height. The cylinder has volume 90 cm^3 . What is the volume of the cone?
(A) 270 cm^3 (B) 30 cm^3 (C) 45 cm^3 (D) 90 cm^3
- A bowl is a hemisphere with radius 3 cm. About how much does it hold? Use $\pi \approx 3.14$.
(A) 113.04 cm^3 (B) 28.26 cm^3 (C) 56.52 cm^3
(D) 84.78 cm^3
- A solid is a cylinder of radius 2 and height 5 with a cone of radius 2 and height 3 on top. What is its volume in terms of π ?
(A) 24π (B) 20π (C) 32π (D) 12π
- A cone has a diameter of 8 cm and a height of 9 cm. What is its volume in terms of π ?
(A) $144\pi \text{ cm}^3$ (B) $48\pi \text{ cm}^3$ (C) $192\pi \text{ cm}^3$ (D) $16\pi \text{ cm}^3$
- A sphere has a radius of 5 inches. What is its volume in terms of π ?
(A) $\frac{4000\pi}{3} \text{ in}^3$ (B) $100\pi \text{ in}^3$ (C) $\frac{125\pi}{3} \text{ in}^3$
(D) $\frac{500\pi}{3} \text{ in}^3$
- A square pyramid has a base side of 9 cm and a volume of 189 cm^3 . What is its height?
(A) 21 cm (B) 63 cm (C) 7 cm (D) 14 cm
- A pyramid has volume 120 cm^3 and a rectangular base 6 cm by 5 cm. What is its height?
- A sphere has volume $36\pi \text{ cm}^3$. What is its radius?

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12. A paper cup is a cone with radius 3 cm and height 10 cm. (a) How many milliliters does it hold? Use $\pi \approx 3.14$. (b) How many completely full cups can you pour from a 1-liter bottle?
13. A grain silo is a cylinder with radius 3 m and height 8 m, topped with a hemisphere of radius 3 m. Find the total volume in terms of π . Then give the volume using $\pi \approx 3.14$.
14. A cone-shaped cup and a cylinder both have radius 4 cm and height 9 cm. How many more cubic centimeters does the cylinder hold than the cone? Use $\pi \approx 3.14$.
15. A hemisphere-shaped bowl with radius 6 cm is full of water. The water is poured into a cylinder with radius 3 cm. How deep is the water in the cylinder? Give the exact answer.

09.3 Surface Area and Nets

Unfold solids into nets and add up their faces. Find the surface area of prisms and cylinders, and solve painting and wrapping problems with the right units.

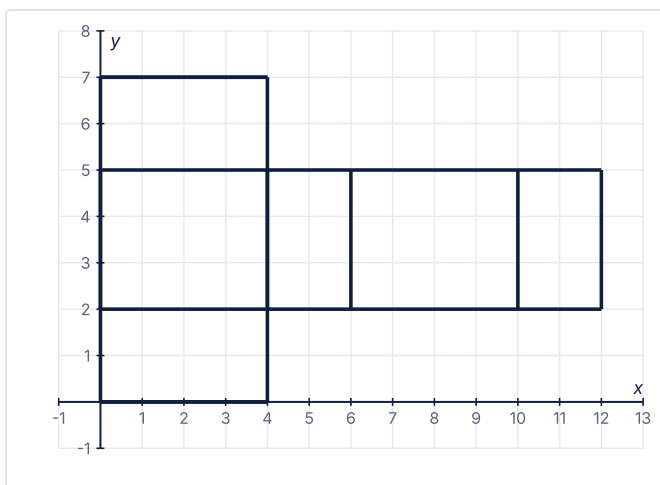
Why This Matters

Surface area measures the outside skin of a solid: how much paint covers a room, how much paper wraps a gift, or how much metal makes a can. Volume tells you what fits inside. Surface area tells you what covers the outside. Students get stuck because they forget a face, count the same face twice, or use a volume formula by mistake.

Packaging designers, painters, builders, and engineers use surface area every day. A **net** is a flat pattern that folds up into a solid. If you can sketch the net, you can see every face, and then finding the surface area is just adding up areas.

Core Ideas

Imagine cutting a box along its edges and flattening it on the table. The flat shape is a net. Surface area is the total area of all the pieces of the net. Because area is always measured in square units, surface area is always in **square units** such as cm^2 or ft^2 .



A net of a box 4 units long, 2 wide, and 3 tall: front, right, back, and left sides in a strip, plus the bottom and top. Six rectangles in all.

Surface Area of a Prism

Surface area = the sum of the areas of all faces.

Rectangular prism:

$$SA = 2(lw + lh + wh)$$

Cube: $SA = 6s^2$.

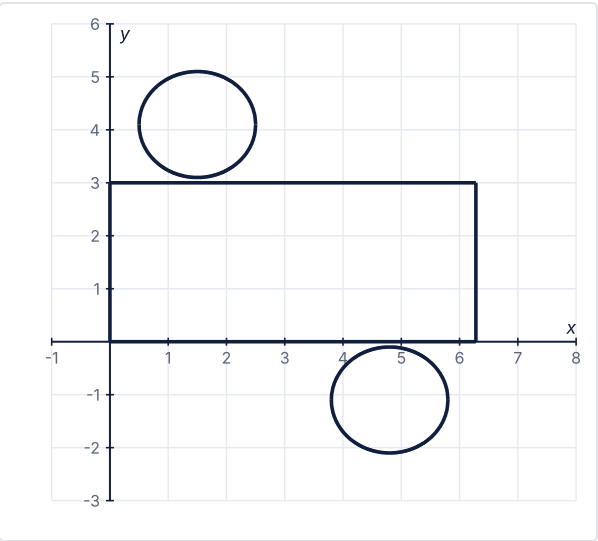
Any prism: $SA = 2B + P \cdot h$, where B is the area of one base, P is the perimeter of the base, and h is the prism's length.

A box has three pairs of matching faces: top and bottom, front and back, left and right. That is why the formula has a 2 outside: it counts each pair. For a triangular prism, there are two triangle ends and three rectangles. Each rectangle has one side of the triangle as its width and the prism length as its other side.

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Net of a cylinder (radius 1, height 3): one rectangle and two circles. The rectangle is as wide as the circumference, $2\pi r \approx 6.28$.

Surface Area of a Cylinder
Lateral (side) area = $2\pi rh$. Two circular bases = $2\pi r^2$.
$$SA = 2\pi r^2 + 2\pi rh$$

The side of a cylinder unrolls into a rectangle with width $2\pi r$ (the circumference) and height h .

Solid	Faces	Surface area
Cube	6 squares	$6s^2$
Rectangular prism	3 pairs of rectangles	$2(lw + lh + wh)$
Triangular prism	2 triangles, 3 rectangles	$2B + Ph$
Cylinder	2 circles, 1 rolled rectangle	$2\pi r^2 + 2\pi rh$

For painting and wrapping problems, first decide which faces get covered. A room has four walls and a ceiling but you may not paint the floor. A box with no lid is missing a face. Then subtract areas like doors and windows, and finally divide by how much one can or sheet covers. Because you cannot buy part of a can, round up.

Worked Examples

EXAMPLE 1: CUBE
Find the surface area of a cube with edge 6 cm.
SOLUTION
A cube has 6 identical square faces.
One face: $6 \cdot 6 = 36 \text{ cm}^2$.
 $SA = 6 \cdot 36 = 216 \text{ cm}^2$.

EXAMPLE 2: RECTANGULAR PRISM
A box is 8 cm long, 3 cm wide, and 4 cm tall. Find its surface area.
SOLUTION
Find the three different faces: $lw = 24$, $lh = 32$, $wh = 12$.
Add them: $24 + 32 + 12 = 68$.
Double it for the matching faces: $SA = 2(68) = 136 \text{ cm}^2$.

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EXAMPLE 3: TRIANGULAR PRISM

A prism has right-triangle ends with legs 3 cm and 4 cm and hypotenuse 5 cm. The prism is 10 cm long. Find the surface area.

SOLUTION

Two triangle ends: $2 \cdot \frac{1}{2}(3)(4) = 12 \text{ cm}^2$.

Three rectangles, each 10 cm long: $3(10) + 4(10) + 5(10) = 30 + 40 + 50 = 120 \text{ cm}^2$.

$$SA = 12 + 120 = 132 \text{ cm}^2.$$

EXAMPLE 4: CYLINDER

Find the surface area of a cylinder with radius 3 cm and height 10 cm. Use $\pi \approx 3.14$.

SOLUTION

Bases: $2\pi r^2 = 2(3.14)(9) = 56.52$.

Side: $2\pi rh = 2(3.14)(3)(10) = 188.4$.

$$SA = 56.52 + 188.4 = 244.92 \text{ cm}^2.$$

EXAMPLE 5: PAINTING A ROOM

A room is 5 m long, 4 m wide, and 3 m high. You will paint the four walls and the ceiling, but not the floor. The door and window take up 6 m^2 that will not be painted. One 1-liter can of paint covers 10 m^2 . How many cans do you need?

SOLUTION

Walls: $2(5 \cdot 3) + 2(4 \cdot 3) = 30 + 24 = 54 \text{ m}^2$.

Ceiling: $5 \cdot 4 = 20 \text{ m}^2$.

Subtract door and window: $54 + 20 - 6 = 68 \text{ m}^2$.

Cans: $68 \div 10 = 6.8$. You cannot buy 0.8 of a can, so round up to **7 cans**.

EXAMPLE 6: OPEN-TOP PAIL

A pail is a cylinder with no lid. Its radius is 10 cm and its height is 25 cm. How much metal is needed to make it? Use $\pi \approx 3.14$.

SOLUTION

An open pail has one circle (the bottom) and the curved side. No top circle.

Bottom: $\pi r^2 = 3.14 \cdot 100 = 314 \text{ cm}^2$.

Side: $2\pi rh = 2 \cdot 3.14 \cdot 10 \cdot 25 = 1570 \text{ cm}^2$.

Total: $314 + 1570 = 1884 \text{ cm}^2$.

Coach's Playbook

- 1 Sketch the net.** Draw each face and write its dimensions. Count the faces: a box has 6, a triangular prism has 5.
- 2 Find the area of each different face.** Group matching faces so you can double them.
- 3 Add everything.** Check that your total uses every face you listed once.
- 4 Decide what gets covered.** Open top, no floor, or a label that only wraps the side all change which faces count.
- 5 Round the right way.** Rolls, cans, and sheets are whole items. Round up when you buy.

Common traps

(1) Giving volume units (cubic) for surface area. (2) Forgetting that every face of a box has a matching face. (3) Using $\pi r^2 h$ (volume) instead of the surface area parts. (4) Including a face that is not covered, such as the floor or the open top.

Mistake Clinic

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Spot the error

Problem: Find the total surface area of a closed cylinder with radius 2 cm and height 7 cm. **Wrong work:** $2\pi(2)(7) + \pi(2^2) = 28\pi + 4\pi = 32\pi$. **What went wrong:** A closed cylinder has two circles, a top and a bottom, but the student used only one. **Right solution:** $28\pi + 2\pi(2^2) = 28\pi + 8\pi = 36\pi \text{ cm}^2$.

Spot the error

Problem: Find the surface area of a box that is 5 cm long, 4 cm wide, and 3 cm tall. **Wrong work:** $5 \cdot 4 + 5 \cdot 3 + 4 \cdot 3 = 20 + 15 + 12 = 47 \text{ cm}^2$. **What went wrong:** The student counted only three faces. Every face has a matching partner on the opposite side. **Right solution:** $2(20 + 15 + 12) = 94 \text{ cm}^2$.

Spot the error

Problem: A prism has right-triangle ends with legs 6 and 8 and hypotenuse 10. The prism is 4 long. **Wrong work:** Triangle area $= \frac{1}{2}(6)(10) = 30$, so $SA = 2(30) + (6 + 8 + 10)(4) = 156$. **What went wrong:** The triangle's area uses the two legs, which are perpendicular, not the hypotenuse. **Right solution:** Triangle area $= \frac{1}{2}(6)(8) = 24$, so $SA = 2(24) + 24 \cdot 4 = 48 + 96 = 144$.

Real-World Connection

A cereal box maker uses the net to cut one flat sheet of cardboard. Making the net waste less cardboard saves money on thousands of boxes.

Challenge Corner

Two cubes, each with edge 3 cm, are glued together face to face to make a 3 cm by 3 cm by 6 cm block. How much less surface area does the block have than the two separate cubes? **Solution:** One cube has $6 \cdot 9 = 54 \text{ cm}^2$, so two cubes have 108. The block has $2(3 \cdot 3 + 3 \cdot 6 + 3 \cdot 6) = 2(9 + 18 + 18) = 90 \text{ cm}^2$. The difference is $108 - 90 = 18$. This matches the two glued faces, $9 + 9 = 18$, that are now hidden inside.

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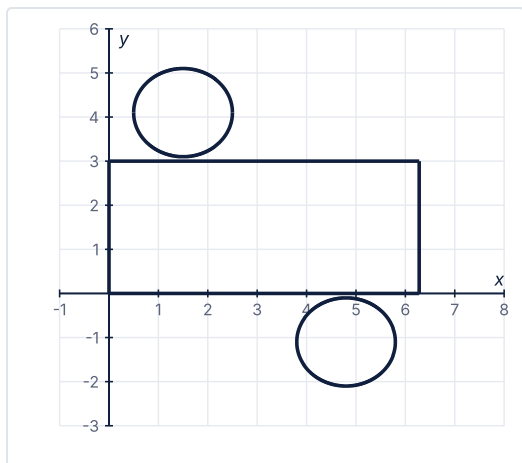
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Practice

PRACTICE Surface Area and Nets

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What is the surface area of a cube with edge 4 cm?
(A) 64 cm^2 (B) 96 cm^2 (C) 16 cm^2 (D) 24 cm^2
- A box is 6 in long, 2 in wide, and 5 in tall. What is its surface area?
(A) 60 in^2 (B) 52 in^2 (C) 104 in^2 (D) 84 in^2
- A net has two circles and one rectangle. Which solid does it fold into?



- (A) Cone (B) Sphere (C) Triangular prism
(D) Cylinder
- A can has radius 3 cm and height 10 cm. A paper label wraps once around the side only. What is the area of the label? Use $\pi \approx 3.14$.
(A) 188.4 cm^2 (B) 282.6 cm^2 (C) 244.92 cm^2
(D) 94.2 cm^2
 - A triangular prism has right-triangle ends with legs 6 and 8 and hypotenuse 10. The prism is 5 long. What is its surface area?
(A) 144 square units (B) 168 square units (C) 120 square units
(D) 118 square units
 - An open box (no lid) is 10 cm long, 6 cm wide, and 4 cm tall. How much cardboard is needed to make it?
(A) 248 cm^2 (B) 128 cm^2 (C) 188 cm^2 (D) 94 cm^2
 - A closed cylinder has radius 3 cm and height 4 cm. What is its total surface area in terms of π ?
(A) $24\pi \text{ cm}^2$ (B) $33\pi \text{ cm}^2$ (C) $84\pi \text{ cm}^2$ (D) $42\pi \text{ cm}^2$
 - A closed box is 10 cm long, 4 cm wide, and 3 cm tall. What is its surface area?
(A) 164 square centimeters (B) 120 square centimeters (C) 82 square centimeters
(D) 52 square centimeters
 - A cube has a surface area of 150 cm^2 . What is the length of one edge?
(A) 25 cm (B) 12.5 cm (C) 30 cm (D) 5 cm
 - A cube has volume 125 cm^3 . What is its surface area?
 - Find the surface area of a cylinder with radius 5 cm and height 8 cm. Leave your answer in terms of π .
 - A gift box is 15 cm long, 10 cm wide, and 6 cm tall. (a) What area of wrapping paper covers the whole box, ignoring overlap? (b) Each sheet of paper has an area of 250 cm^2 . How many sheets are needed?
 - A closed cylindrical tank has radius 2 m and height 5 m. The outside is painted. Each liter of paint covers 4 m^2 . Use $\pi \approx 3.14$. How many whole liters are needed?
 - An open fish tank (no top) is 50 cm long, 30 cm wide, and 40 cm tall. How many square centimeters of glass are needed?
 - A closed box is 6 cm by 5 cm by h cm. Its surface area is 214 cm^2 . Find h .

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09.4 The Pythagorean Theorem

Learn how the three sides of a right triangle are linked. Find missing sides, test whether a triangle is right, measure distance on a grid, and solve ladder, screen, and box-diagonal problems.

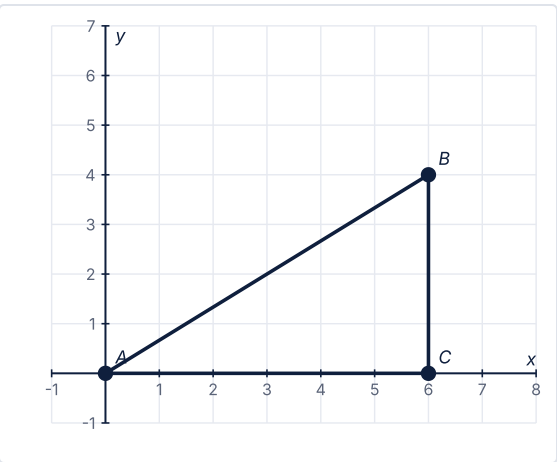
Why This Matters

The Pythagorean Theorem is one of the most useful ideas in all of math. It links the three sides of every right triangle, so if you know two sides you can always find the third. Builders use it to make sure corners are square, painters use it to place ladders safely, and game designers use it to find distances on a screen.

Students get stuck in a few predictable places: they do not know which side is the hypotenuse, they add the squares when they should subtract, or they forget to take the square root at the end. This section fixes all three.

Core Ideas

A **right triangle** has one 90 degree angle. The two sides that form the right angle are the **legs**. The side across from the right angle is the **hypotenuse**, and it is always the longest side.



A right triangle with the right angle at C. The legs are AC (6 units) and BC (4 units). The hypotenuse AB is across from the right angle and is always the longest side.

The Pythagorean Theorem

In a right triangle with legs a and b and hypotenuse c :

$$a^2 + b^2 = c^2.$$

To find the hypotenuse: $c = \sqrt{a^2 + b^2}$. To find a leg: $a = \sqrt{c^2 - b^2}$.

Why does it work? Build a square on each side of the triangle. The area of the square on the hypotenuse equals the sum of the areas of the squares on the two legs. For legs 3 and 4, the squares have areas 9 and 16, and $9 + 16 = 25$, which is the square on the hypotenuse of length 5. Numbers like 3-4-5, 5-12-13, and 8-15-17 are called **Pythagorean triples** because they fit exactly.

Triple	$a^2 + b^2$	c^2
3, 4, 5	$9 + 16 = 25$	25
5, 12, 13	$25 + 144 = 169$	169
8, 15, 17	$64 + 225 = 289$	289
7, 24, 25	$49 + 576 = 625$	625

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The Converse and the Distance Formula

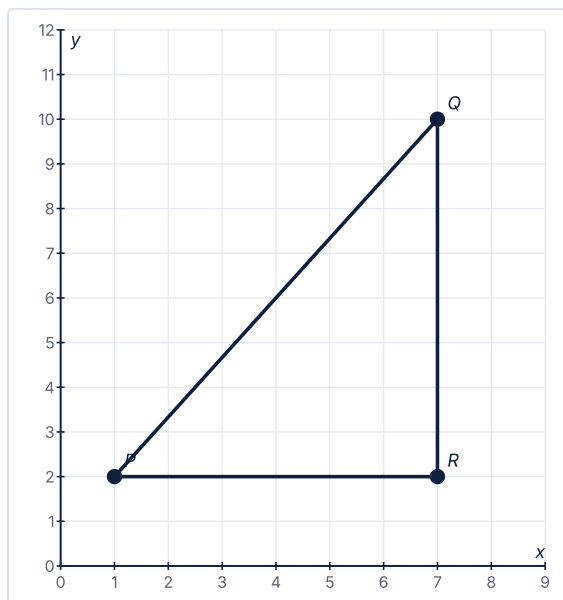
Converse: if the three sides satisfy $a^2 + b^2 = c^2$ (with c the longest), the triangle is a right triangle. If $a^2 + b^2 < c^2$ the angle is wider than 90 degrees, and if $a^2 + b^2 > c^2$ all angles are smaller than 90 degrees.

Distance between (x_1, y_1) and (x_2, y_2) :

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Box diagonal: $d = \sqrt{l^2 + w^2 + h^2}$.

The distance formula is just the theorem in disguise: draw a horizontal leg and a vertical leg between two points, and the distance is the hypotenuse. A box diagonal uses the theorem twice, once across the floor, then once more going up.



Distance from $P(1, 2)$ to $Q(7, 10)$. The horizontal leg is 6 and the vertical leg is 8, so PQ is the hypotenuse.

Worked Examples

EXAMPLE 1: FIND THE HYPOTENUSE

A right triangle has legs of 6 cm and 8 cm. Find the hypotenuse.

SOLUTION

$$c^2 = 6^2 + 8^2 = 36 + 64 = 100.$$

$$c = \sqrt{100} = 10 \text{ cm.}$$

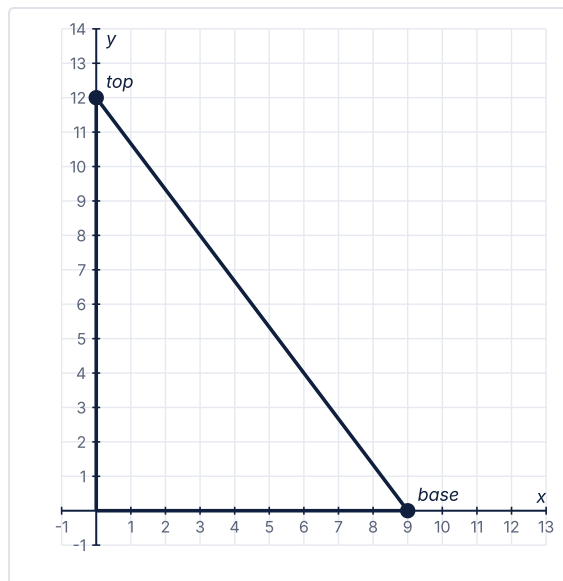
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EXAMPLE 2: A LADDER

A 15-foot ladder leans against a wall. Its base is 9 feet from the wall. How high up the wall does it reach?



A 15-foot ladder leaning on a wall. The wall, the ground, and the ladder form a right triangle.

SOLUTION

The ladder is the hypotenuse, so $c = 15$ and one leg is 9.

$$a^2 = 15^2 - 9^2 = 225 - 81 = 144.$$

$$a = \sqrt{144} = 12. \text{ The ladder reaches 12 feet up.}$$

EXAMPLE 3: IS IT A RIGHT TRIANGLE?

Are the side lengths 7, 24, 25 a right triangle? What about 6, 8, 11?

SOLUTION

Put the longest side as c . For 7, 24, 25: $7^2 + 24^2 = 49 + 576 = 625$ and $25^2 = 625$. Equal, so **yes**, it is a right triangle.

For 6, 8, 11: $6^2 + 8^2 = 100$ but $11^2 = 121$. They are not equal, so it is **not** a right triangle (the big angle is wider than 90 degrees).

EXAMPLE 4: DISTANCE ON A GRID

Find the distance between $P(1, 2)$ and $Q(7, 10)$.

SOLUTION

Horizontal change: $7 - 1 = 6$. Vertical change: $10 - 2 = 8$.

$$d = \sqrt{6^2 + 8^2} = \sqrt{100} = 10 \text{ units.}$$

EXAMPLE 5: A DIAGONAL IN 3D

A box is 3 inches wide, 4 inches long, and 12 inches tall. Find the longest straight line from a bottom corner to the opposite top corner.

SOLUTION

First, the diagonal across the bottom: $\sqrt{3^2 + 4^2} = 5$.

Now use that 5 and the height 12 as legs: $\sqrt{5^2 + 12^2} = \sqrt{169} = 13$.

The long diagonal is 13 inches. (Shortcut: $\sqrt{3^2 + 4^2 + 12^2} = \sqrt{169} = 13$.)

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EXAMPLE 6: CUTTING ACROSS A SQUARE

A baseball infield is a square with 90 feet between bases. How far is it in a straight line from home plate to second base? Give an exact answer and a decimal rounded to the nearest tenth.

SOLUTION

Home plate, first base, and second base form a right triangle. The legs are 90 ft each, and the straight line from home to second is the hypotenuse.

$$c^2 = 90^2 + 90^2 = 8100 + 8100 = 16,200.$$

$$c = \sqrt{16,200} = 90\sqrt{2}.$$

$$90\sqrt{2} \approx 127.3 \text{ feet.}$$

Coach's Playbook

- 1 Draw and label the triangle.** Mark the right angle. The side opposite it is c , no matter how the triangle is turned.
- 2 Decide: add or subtract?** Finding the hypotenuse: add the squares. Finding a leg: subtract the smaller square from the bigger one.
- 3 Take the square root.** The squares are not the final answer. If the result is not a perfect square, estimate: $\sqrt{65}$ is a little over 8.
- 4 Check for reasonableness.** The hypotenuse must be longer than either leg and shorter than their sum.
- 5 Memorize common triples.** 3-4-5, 5-12-13, 8-15-17, and their multiples (such as 6-8-10) save time.

Common traps

(1) Adding the legs instead of adding their squares (for legs 6 and 8, the sum 14 is not the hypotenuse; the answer is 10). (2) Forgetting the square root and answering 100 instead of 10. (3) Treating a leg as the hypotenuse. (4) Testing the converse without putting the longest side in the c position.

Mistake Clinic**Spot the error**

Problem: A right triangle has hypotenuse 13 and one leg 5. Find the other leg. **Wrong work:** $5^2 + 13^2 = 194$, so the leg is $\sqrt{194}$. **What went wrong:** The squares of the legs add up to the square of the hypotenuse. When the hypotenuse is known, subtract. **Right solution:** $b^2 = 13^2 - 5^2 = 169 - 25 = 144$, so $b = 12$.

Spot the error

Problem: A right triangle has legs 6 and 8. Find the hypotenuse. **Wrong work:** $6^2 + 8^2 = 100$, so the hypotenuse is 100. **What went wrong:** The student stopped at c^2 . The value 100 is the square of the side, not the side. **Right solution:** $c = \sqrt{100} = 10$.

Spot the error

Problem: Is a triangle with sides 5, 12, and 13 a right triangle? **Wrong work:** $5^2 + 13^2 = 194$, which is not $12^2 = 144$, so it is not a right triangle. **What went wrong:** The test must treat the longest side as the hypotenuse. **Right solution:** $5^2 + 12^2 = 25 + 144 = 169 = 13^2$, so yes, it is a right triangle.

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Real-World Connection

TV and phone screens are sold by their diagonal. A screen 32 inches wide and 24 inches tall has a diagonal of $\sqrt{32^2 + 24^2} = 40$ inches.

Challenge Corner

A 17-foot ladder leans against a wall with its base 8 feet from the wall. The top of the ladder slides down the wall by 3 feet. How far does the base slide out? Give a decimal rounded to the nearest hundredth. **Solution:** At first the top is at $\sqrt{17^2 - 8^2} = \sqrt{225} = 15$ ft. After sliding down 3 ft, the top is at 12 ft. The new base distance is $\sqrt{17^2 - 12^2} = \sqrt{145} \approx 12.04$ ft. The base moves out $12.04 - 8 = 4.04$ feet. The top dropped 3 feet but the base moved more than 4 feet.

Practice**PRACTICE The Pythagorean Theorem**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- A right triangle has legs of 9 and 12. What is the hypotenuse?
(A) 15 (B) 21 (C) 225 (D) 108
- The hypotenuse of a right triangle is 17 and one leg is 8. What is the other leg?
(A) 9 (B) 15 (C) $\sqrt{353}$ (D) 225
- Which set of numbers can be the side lengths of a right triangle?
(A) 8, 15, 16 (B) 10, 24, 27 (C) 9, 40, 41
(D) 12, 16, 22
- What is the distance between $(-2, 3)$ and $(4, -5)$?
(A) 14 (B) 100 (C) 2 (D) 10
- A television screen is 32 inches wide and 24 inches tall. How long is its diagonal?
(A) 40 inches (B) 56 inches (C) 8 inches
(D) 1600 inches
- A 26-foot ladder reaches a window 24 feet above the ground. How far is the base of the ladder from the wall?
(A) 2 feet (B) 10 feet (C) 50 feet (D) 35.4 feet
- The hypotenuse of a right triangle is 20 and one leg is 12. What is the other leg?
(A) 23.3 (B) 32 (C) 8 (D) 16
- What is the distance between $(3, -1)$ and $(11, 5)$?
(A) 14 (B) 100 (C) 10 (D) 2
- A rectangular garden is 20 m long and 21 m wide. How long is a path along its diagonal?
(A) 41 m (B) 29 m (C) 841 m (D) 420 m
- A right triangle has legs 7 cm and 4 cm. Find the hypotenuse. Give an exact answer and a decimal rounded to the nearest tenth.
- A triangle has sides 11, 60, and 61. Is it a right triangle? Show a check.
- Find the distance between $(1, 1)$ and $(6, 13)$.
- A shipping box is 6 inches wide, 8 inches long, and 24 inches tall. What is the length of the longest straight rod that fits inside it?
- An isosceles triangle has two equal sides of length 10 and a base of 12. What is its height from the top vertex to the base?
- One hiker walks 7 km north while another walks 10 km east from the same camp. How far apart are they in a straight line? Round to the nearest tenth.

Answers **Answers and Explanations**

Complete solutions for every practice problem, numbered to match. Read the explanation even when your answer is right.

09.1 Volume of Prisms and Cylinders

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PRACTICE · VOLUME OF PRISMS AND CYLINDERS

1. B. $7 \cdot 4 \cdot 5 = 140$. Adding the sides gives 16, and 166 is the surface area of this box, not its volume.
2. C. $6^3 = 216$. The 36 is only one face (6^2), and 18 comes from multiplying by 3 instead of cubing.
3. D. $V = \pi(4^2)(9) = 144\pi$. The 576 comes from using the diameter 8 as the radius, and 72 is a side-area idea, not the volume.
4. A. $V = 2 \cdot 1.5 \cdot 1 = 3 \text{ m}^3$, and $1 \text{ m}^3 = 1000 \text{ L}$, so 3000 L. The 4.5 comes from adding the sides.
5. B. Base area is $\frac{1}{2}(5)(12) = 30$, and $V = 30 \cdot 8 = 240$. Choice 480 forgets the $\frac{1}{2}$, and 30 is only the base area.
6. C. The radius is 3 cm. $V = 3.14 \cdot 9 \cdot 10 = 282.6 \text{ cm}^3 = 282.6 \text{ mL}$. Using 6 as the radius gives the tempting 1130.4.
7. B. The radius is 5, so $V = \pi(5^2)(6) = 150\pi$. Choice 600 comes from using the diameter 10 as the radius.
8. D. $V = 50 \cdot 20 \cdot 15 = 15,000 \text{ cm}^3$, and $15,000 \div 1000 = 15$ liters. Leaving the answer as 15,000 skips the conversion.
9. B. $B = \frac{1}{2}(8)(5) = 20$, so $20h = 200$ and $h = 10$. The tempting 5 comes from forgetting the $\frac{1}{2}$ and using $B = 40$.
10. 8 cm. $504 = 9 \cdot 7 \cdot h = 63h$, so $h = 504 \div 63 = 8$.
11. 12 cm. The base area is $3.14 \cdot 4 = 12.56$. Then $12.56h = 150.72$, so $h = 12$.
12. (a) 90,000 liters. (b) 200 minutes. $V = 12 \cdot 5 \cdot 1.5 = 90 \text{ m}^3 = 90,000 \text{ L}$. Then $90,000 \div 450 = 200$.
13. 9.42 cm. Water volume is $3.14 \cdot 36 \cdot 5 = 565.2 \text{ cm}^3$. The box base area is 60, so $h = 565.2 \div 60 = 9.42$.
14. 423.1 cubic centimeters. $B = 3.14 \cdot 3.5^2 = 38.465$, and $V = 38.465 \cdot 11 = 423.115 \text{ cm}^3$.
15. 120 liters. Change 2 m to 200 cm. $B = \frac{1}{2}(40)(30) = 600 \text{ cm}^2$, $V = 600 \cdot 200 = 120,000 \text{ cm}^3$, and $120,000 \div 1000 = 120 \text{ L}$.

09.2 Pyramids, Cones, and Spheres**PRACTICE · PYRAMIDS, CONES, AND SPHERES**

1. C. $V = \frac{1}{3}(20)(9) = 60$. The 180 is the volume of a prism, which forgets the $\frac{1}{3}$.
2. A. $V = \frac{1}{3}\pi(36)(5) = 60\pi$. The 180 forgets the $\frac{1}{3}$.
3. D. The radius is 6. $V = \frac{4}{3}\pi(216) = 288\pi$. The 2304 uses the diameter as the radius, and 144 is $4\pi r^2$, a surface idea.
4. B. A cone is one third of the matching cylinder: $90 \div 3 = 30$. The 270 multiplies by 3 instead.
5. C. $\frac{2}{3}(3.14)(27) = 56.52$. The 113.04 is a whole sphere, not half.
6. A. Cylinder: $\pi(4)(5) = 20\pi$. Cone: $\frac{1}{3}\pi(4)(3) = 4\pi$. Total 24π . Choice 32π forgets the $\frac{1}{3}$ on the cone, and 20π ignores the cone.
7. B. The radius is 4. $V = \frac{1}{3}\pi(16)(9) = 48\pi$. Using 8 as the radius gives the tempting 192.
8. D. $V = \frac{4}{3}\pi(5^3) = \frac{4}{3}\pi(125) = \frac{500\pi}{3}$. The 100π is the surface area idea $4\pi r^2$, and 125 over 3 forgets the 4.
9. C. $189 = \frac{1}{3}(81)h = 27h$, so $h = 7$. Dividing by 9 gives the tempting 21, which ignores the base area and the $\frac{1}{3}$.
10. 12 cm. $120 = \frac{1}{3}(30)h = 10h$, so $h = 12$.
11. 3 cm. $\frac{4}{3}\pi r^3 = 36\pi$ gives $r^3 = 27$, so $r = 3$.
12. (a) 94.2 mL. (b) 10 cups. $V = \frac{1}{3}(3.14)(9)(10) = 94.2$. Then $1000 \div 94.2 \approx 10.6$, so 10 full cups.
13. $90\pi \text{ m}^3$, which is about 282.6 m^3 . Cylinder: $\pi(9)(8) = 72\pi$. Hemisphere: $\frac{2}{3}\pi(27) = 18\pi$. Total 90π , and $90(3.14) = 282.6$.
14. 301.44 cubic centimeters. Cylinder: $3.14 \cdot 16 \cdot 9 = 452.16$. Cone: $452.16 \div 3 = 150.72$. Difference: $452.16 - 150.72 = 301.44$.
15. 16 cm. Hemisphere: $\frac{2}{3}\pi(6^3) = 144\pi$. Cylinder: $\pi(3^2)h = 9\pi h$. So $9\pi h = 144\pi$ and $h = 16$.

09.3 Surface Area and Nets**PRACTICE · SURFACE AREA AND NETS**

1. B. One face is 16, and $6 \cdot 16 = 96$. The 64 is the volume, and 16 is a single face.
2. C. $2(12 + 30 + 10) = 104$. The 60 is the volume, 52 forgot to double, and 84 leaves out one pair of faces.
3. D. The two circles are the bases and the rectangle wraps around as the side. A cone has only one circle, and a triangular prism has triangles.
4. A. The label is the lateral area: $2\pi rh = 2(3.14)(3)(10) = 188.4$. The 282.6 is the volume and 244.92 includes both circles.
5. B. Ends: $2 \cdot 24 = 48$. Rectangles: $(6 + 8 + 10) \cdot 5 = 120$. Total 168. The 144 counts only one triangle end, and 118 leaves out the hypotenuse rectangle.
6. C. Bottom: 60. Four sides: $2(40) + 2(24) = 128$. Total 188. The 248 includes the lid that is not there.
7. D. Side: $2\pi(3)(4) = 24\pi$. Two circles: $2\pi(9) = 18\pi$. Total 42π . Choice 24π forgets the circles, and 33π uses only one.
8. A. $2(10 \cdot 4 + 10 \cdot 3 + 4 \cdot 3) = 2(40 + 30 + 12) = 164$. Choice 120 is the volume, and 82 forgets to double.
9. D. A cube has 6 equal faces, so one face is $150 \div 6 = 25$. Then the edge is $\sqrt{25} = 5$. Choice 25 stops at the face area.
10. 150 cm^2 . The edge is 5 cm because $5^3 = 125$. Then $SA = 6(25) = 150$.
11. $130\pi \text{ cm}^2$. Bases: $2\pi(25) = 50\pi$. Side: $2\pi(5)(8) = 80\pi$. Total 130π .
12. (a) 600 cm^2 . (b) 3 sheets. $2(150 + 90 + 60) = 600$. Then $600 \div 250 = 2.4$, which rounds up to 3.
13. 22 liters. $SA = 2\pi(4) + 2\pi(2)(5) = 28\pi \approx 87.92 \text{ m}^2$. Then $87.92 \div 4 = 21.98$, so round up to 22.
14. 7900 square centimeters. Bottom: $50 \cdot 30 = 1500$. Two long sides: $2(50 \cdot 40) = 4000$. Two short sides: $2(30 \cdot 40) = 2400$. Total $1500 + 4000 + 2400 = 7900$.
15. 7 cm. $2(6 \cdot 5 + 6h + 5h) = 214$, so $30 + 11h = 107$, which gives $11h = 77$ and $h = 7$.

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09.4 The Pythagorean Theorem**PRACTICE · THE PYTHAGOREAN THEOREM**

1. A. $81 + 144 = 225$, and $\sqrt{225} = 15$. Choice 21 adds the legs, and 225 forgets the square root.
2. B. $17^2 - 8^2 = 289 - 64 = 225$, and $\sqrt{225} = 15$. Subtracting $17 - 8$ gives 9, and $\sqrt{353}$ comes from adding the squares by mistake.
3. C. Only $9^2 + 40^2 = 81 + 1600 = 1681 = 41^2$ works. Check 8, 15, 16: $64 + 225 = 289$, but $16^2 = 256$.
4. D. Changes: $4 - (-2) = 6$ and $-5 - 3 = -8$. $\sqrt{36 + 64} = 10$. Choice 14 adds 6 and 8, and 100 forgets the square root.
5. A. $\sqrt{32^2 + 24^2} = \sqrt{1024 + 576} = \sqrt{1600} = 40$. Adding $32 + 24$ gives 56, which is too long for a diagonal.
6. B. $26^2 - 24^2 = 676 - 576 = 100$, so the base is 10 feet away. Choice 35.4 adds the squares, which would make a leg longer than the ladder.
7. D. $b^2 = 20^2 - 12^2 = 400 - 144 = 256$, so $b = 16$. Choice 23.3 adds the squares, and 8 subtracts the lengths.
8. C. The horizontal change is 8 and the vertical change is 6. $d = \sqrt{64 + 36} = \sqrt{100} = 10$. Choice 14 adds 8 and 6, and 100 forgets the square root.
9. B. $d^2 = 20^2 + 21^2 = 400 + 441 = 841$, so $d = 29$. Choice 841 forgets the square root, and 41 adds the sides.
10. $\sqrt{65}$ cm, about 8.1 cm. $7^2 + 4^2 = 49 + 16 = 65$.
11. Yes. $11^2 + 60^2 = 121 + 3600 = 3721$, and $61^2 = 3721$.
12. 13 units. The changes are 5 and 12, and $\sqrt{25 + 144} = \sqrt{169} = 13$.
13. 26 inches. $\sqrt{6^2 + 8^2 + 24^2} = \sqrt{36 + 64 + 576} = \sqrt{676} = 26$.
14. 8. The height splits the base into two parts of 6. Then $h^2 + 6^2 = 10^2$, so $h^2 = 64$ and $h = 8$.
15. About 12.2 km. The paths are perpendicular. $d^2 = 7^2 + 10^2 = 149$, and $d = \sqrt{149} \approx 12.2$.

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