



Data, Statistics, and Probability Basics

Four focused lessons, worked examples, a mistake clinic, and 60 practice problems with full explanations.

In this booster

10.1 Mean, Median, Mode, and Range

10.2 Dot Plots, Histograms, and Box Plots

10.3 Simple Probability

10.4 Compound Events and Counting

Every section teaches the idea, walks through the most common problem types, flags the traps students fall into, and ends with fifteen practice problems. Full answers and explanations are at the back.

10.1 Mean, Median, Mode, and Range

Find and compare the mean, median, mode, and range. See how an outlier pulls the mean, and use totals to solve missing-value problems.

Why This Matters

Data is everywhere: test scores, game stats, weather, prices, and survey results. A pile of numbers is hard to understand, so statisticians boil it down to a few **summary numbers**. The mean, median, mode, and range each tell a different part of the story. Students get stuck because they mix up the names, forget to put numbers in order before finding the median, or do not know which measure to trust when one strange value shows up.

You will use these ideas in science labs, in sports, in business, and in every later statistics course. Learning when each measure works best is just as important as calculating it.

Core Ideas

The **mean**, **median**, and **mode** are three ways to describe the center (a typical value). The **range** describes the spread (how far apart the data are).

The Four Measures

Mean (average) = $\frac{\text{sum of all values}}{\text{number of values}}$.

Median = the middle value after the data are put in order. With an even number of values, it is the mean of the two middle values.

Mode = the value that appears most often. A set can have one mode, more than one, or none.

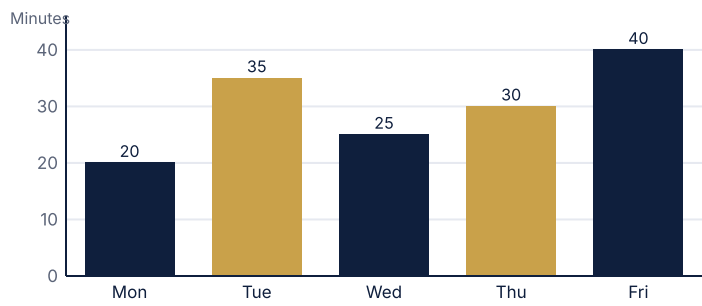
Range = largest value – smallest value.

Think of the mean as a **fair share**. If everyone in a group put their money in a pile and shared it equally, each person would get the mean. Think of the median as the **middle person** in a line sorted from least to most. Think of the mode as the **most popular** answer.

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Minutes of reading each day. The mean is 30, and the middle bar in sorted order (the median) is also 30.

Outliers

An **outlier** is a value far away from the rest of the data.

An outlier pulls the **mean** toward it a lot. The **median** and **mode** barely move. When data have an outlier, the median is usually the better description of a typical value.



For the data 2, 3, 4, 5, 36, the one large value pulls the mean (10) far to the right of the median (4).

Here is a quick comparison to remember how each measure behaves.

Measure	Uses all values?	Affected by outliers?	Best used for
Mean	Yes	Strongly	Fair share, data without outliers
Median	No (only the middle)	Barely	Typical value, data with outliers
Mode	No	No	Most common choice, category data
Range	No (only ends)	Strongly	How spread out the data are

Missing-value problems work backward from the mean. If you want a mean of m with n values, the total must be $m \times n$. Subtract the sum of the values you already have to find the one you need.

Worked Examples

EXAMPLE 1: ALL FOUR MEASURES

Six students scored 4, 8, 6, 9, 8, and 7 on a short quiz. Find the mean, median, mode, and range.

SOLUTION

Mean: $4 + 8 + 6 + 9 + 8 + 7 = 42$, and $42 \div 6 = 7$.

Median: order the data: 4, 6, 7, 8, 8, 9. The middle two values are 7 and 8, so the median is $\frac{7+8}{2} = 7.5$.

Mode: 8 appears twice, more than any other value.

Range: $9 - 4 = 5$.

EXAMPLE 2: EFFECT OF AN OUTLIER

A runner's mile times in minutes over five days are 12, 13, 12, 14, 13. On day six she twists an ankle and takes 40 minutes. Find the mean and median before and after day six.

SOLUTION

Before: mean = $\frac{64}{5} = 12.8$. Ordered: 12, 12, 13, 13, 14, so the median is 13.

After: mean = $\frac{104}{6} \approx 17.3$. Ordered: 12, 12, 13, 13, 14, 40, so the median is $\frac{13+13}{2} = 13$.

The mean jumped by about 4.5 minutes but the median did not change. The median describes her usual time better.

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EXAMPLE 3: MISSING TEST SCORE

Maya's first four test scores are 82, 90, 78, and 85. What score does she need on the fifth test to have a mean of exactly 85?

SOLUTION

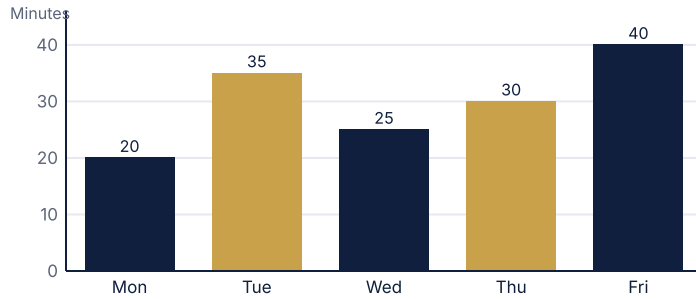
Total needed: $85 \times 5 = 425$.

Points so far: $82 + 90 + 78 + 85 = 335$.

Needed: $425 - 335 = 90$. Maya needs a 90.

EXAMPLE 4: READING A BAR CHART

The bar chart shows minutes of reading Monday to Friday. How many minutes must Saturday have so that the six-day mean is 32?



Reading minutes, Monday to Friday.

SOLUTION

Read the bars: 20, 35, 25, 30, 40. Sum = 150.

For a mean of 32 over 6 days, the total must be $32 \times 6 = 192$.

Saturday: $192 - 150 = 42$ minutes.

EXAMPLE 5: A FREQUENCY TABLE

Twenty students were asked how many pets they have. 0 pets: 5 students; 1 pet: 8 students; 2 pets: 4 students; 3 pets: 2 students; 4 pets: 1 student. Find the mean, median, mode, and range.

Pets	0	1	2	3	4
Students	5	8	4	2	1

SOLUTION

Mean: total pets = $0(5) + 1(8) + 2(4) + 3(2) + 4(1) = 26$. Then $26 \div 20 = 1.3$.

Median: with 20 values, use the 10th and 11th. The first 5 are 0s and the next 8 are 1s (positions 6 to 13), so both are 1. The median is 1.

Mode: 1 pet (8 students).

Range: $4 - 0 = 4$.

EXAMPLE 6: COMBINING TWO GROUPS

A class of 10 students has a mean quiz score of 80. A second class of 20 students has a mean of 74. What is the mean score of all 30 students together?

SOLUTION

You cannot just average 80 and 74, because the classes have different sizes.

Total points in class 1: $10 \cdot 80 = 800$. Total points in class 2: $20 \cdot 74 = 1480$.

All points: $800 + 1480 = 2280$ for 30 students.

Mean: $2280 \div 30 = 76$.

Coach's Playbook

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- 1 **Order the data first.** Always write the numbers from least to greatest before finding the median, mode, or range.
- 2 **Count the values.** Odd count: the median is the middle one. Even count: average the two middle ones.
- 3 **Use total = mean times count.** This one line solves every missing-value problem.
- 4 **Ask about outliers.** If one value is far from the others, expect the mean to move and the median to stay put.
- 5 **Check that your answer makes sense.** The mean and median must be between the smallest and largest values.

Common traps

(1) Finding the median without sorting the list first. (2) Calling the range a single middle number instead of the largest minus the smallest. (3) Dividing the sum by the wrong count, or forgetting to count repeated values. (4) In a frequency table, adding the numbers in the top row instead of multiplying each value by its frequency.

Mistake Clinic

Spot the error

Problem: Find the median of 12, 5, 20, 9, 7. **Wrong work:** The middle number in the list is 20, so the median is 20.

What went wrong: The data must be put in order first. **Right solution:** Order: 5, 7, 9, 12, 20. The middle value is 9, so the median is 9.

Spot the error

Problem: Find the median of 3, 5, 8, 10. **Wrong work:** The two middle numbers are 5 and 8, and $5 + 8 = 13$, so the median is 13.

What went wrong: With an even number of values, the median is the mean of the two middle values, so you must divide by 2. **Right solution:** $(5 + 8) \div 2 = 6.5$.

Spot the error

Problem: The temperatures at dawn were -4 and 6 degrees at the coldest and warmest readings. Find the range.

Wrong work: $6 - 4 = 2$. **What went wrong:** The student ignored the negative sign. Range is the highest minus the lowest, and subtracting a negative adds. **Right solution:** $6 - (-4) = 10$ degrees.

Real-World Connection

When news reports talk about the "typical" house price in a city, they often use the median. A few mansions would pull the mean up and make typical homes look more expensive than they are.

Challenge Corner

Five whole numbers have mean 12, median 11, range 10, and exactly one mode, 9. Find all five numbers. **Solution:** The sum is $5 \cdot 12 = 60$. In order the numbers are $a \leq b \leq c \leq d \leq e$. The median gives $c = 11$. The only mode is 9, so 9 must appear twice (and no other number can), so $a = b = 9$. The range is 10, so $e = 9 + 10 = 19$. Then $9 + 9 + 11 + d + 19 = 60$, which gives $d = 12$. Check: the numbers 9, 9, 11, 12, 19 have mean 12, median 11, range 10, and mode 9.

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Practice

PRACTICE Mean, Median, Mode, and Range

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What is the mean of 4, 9, 11, 6, and 10?
(A) 8 (B) 9 (C) 7 (D) 11
- What is the median of 14, 9, 21, 11, 16, 12?
(A) 16 (B) 12 (C) 14 (D) 13
- What is the mode of 2, 2, 2, 5, 7, 9, 10?
(A) 5 (B) 8 (C) 2 (D) 10
- The temperatures (in degrees) on five winter mornings were -3 , 5, 0, 8, and -6 . What is the range?
(A) 2 (B) 14 (C) 8 (D) 4
- The data set 20, 22, 24, 26, 28 has mean 24 and median 24. The value 100 is added. Which statement is true?
(A) Both the mean and the median stay at 24.
(B) The mean rises to about 36.7 and the median rises to 25.
(C) The median rises to about 36.7 and the mean stays at 24.
(D) The median rises more than the mean.
- The mean of five numbers is 14. Four of the numbers are 10, 12, 15, and 18. What is the fifth number?
(A) 14 (B) 70 (C) 13.75 (D) 15
- The scores 70, 85, 85, 90, 95, and 100 are listed. What is the median?
(A) 85 (B) 90 (C) 87.5 (D) 87
- The mean of 7 numbers is 12. An eighth number, 20, is added. What is the new mean?
(A) 16 (B) 13 (C) 12 (D) 14.5
- A data set of 9 numbers has a median of 20. The largest number is increased by 100. What is the new median?
(A) 120 (B) 70 (C) cannot be determined
(D) 20
- Find the mean, median, mode, and range of 12, 15, 11, 15, 17, 10, 15, 13.
- The mean of six numbers is 9. Five of the numbers are 4, 8, 11, 12, and 7. What is the sixth number?
- Five students saved 5, 8, 8, 10, and 12 dollars. A sixth student saved 50 dollars. By how much does the mean change when the 50 is included, and does the median change by more or less?
- A survey asked 20 students how many siblings they have. 0: 3 students; 1: 7 students; 2: 6 students; 3: 3 students; 4: 1 student. Find the mean and the median.
- A bowler's scores in six games were 120, 135, 150, 105, 135, and 165. Find the mean, median, mode, and range.
- The mean of a student's first four test scores is 78. After the fifth test, the mean is 80. What did the student score on the fifth test?

10.2 Dot Plots, Histograms, and Box Plots

Read and build dot plots, histograms, and box plots. Find quartiles, use the quarters in a box plot, and compare the center and spread of two data sets.

Why This Matters

A list of 30 test scores is hard to read. A good picture makes the pattern jump out: where most of the data sit, how spread out they are, and whether anything is unusual. Dot plots, histograms, and box plots are three of the most common data displays. Students struggle because the pictures look similar but show different things, because histogram bars hide the exact values, and because a box plot's long box does not mean "more data" in that part.

News reports, science fair posters, and sports websites all use these displays. Being able to read them carefully, and build them, helps you spot good arguments and weak ones.

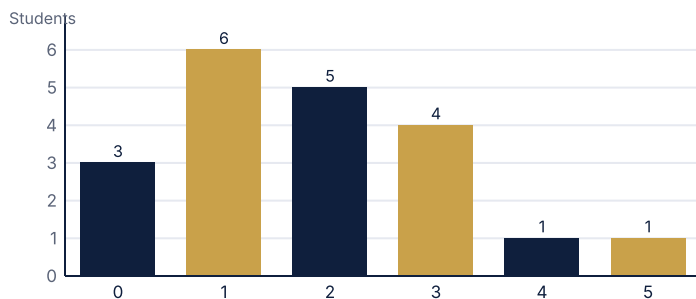
Core Ideas

A **dot plot** puts one dot above a number line for each data value. Stacks of dots show how often each value happens, and you can still see every data value. A dot plot works best for small data sets with fairly few different values.

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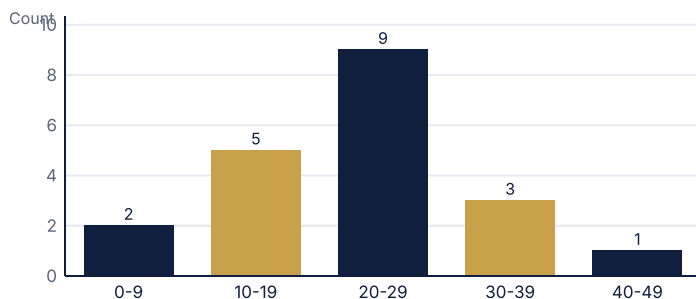
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A dot plot written as counts. The number of siblings is on the bottom, and each bar height equals the number of dots stacked at that value. There are 20 students.

A **histogram** groups the data into equal-width intervals (bins). The height of each bar is how many values fall in that bin. The bars touch because the bins run continuously. A histogram is good for large data sets, but you cannot read exact values from it: a bar for 20 to 29 only says how many values are in that range.



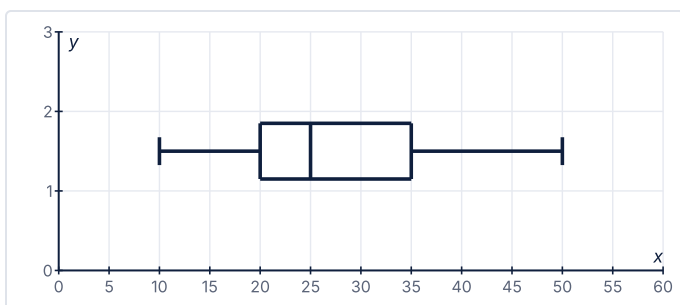
A histogram of 20 values, in bins of width 10. The tallest bin is 20 to 29.

Five-Number Summary and Box Plots

The five-number summary is: **minimum**, **first quartile (Q1)**, **median (Q2)**, **third quartile (Q3)**, **maximum**.

Q1 is the median of the lower half of the data, and Q3 is the median of the upper half. If the number of values is odd, leave the overall median out of both halves.

Interquartile range (IQR) = $Q3 - Q1$. It measures the spread of the middle half of the data.



A box plot of 30 trail lengths in miles. The five-number summary is min 10, Q1 20, median 25, Q3 35, max 50. Each section of the plot holds about one quarter of the data.

Reading a Box Plot

Each of the four sections (min to Q1, Q1 to median, median to Q3, Q3 to max) contains about **25%** of the data.

The box itself (Q1 to Q3) holds about **50%** of the data. A long section means the data are more spread out there, not that there are more values.

To compare two distributions, compare **center** (medians) and **spread** (range or IQR). Say it in a sentence: "Class A has a higher median, but class B is more consistent because its IQR is smaller."

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Worked Examples

EXAMPLE 1: READING A DOT PLOT

The bar chart above shows the number of siblings for 20 students (a dot plot as counts). (a) How many students have 3 or more siblings? (b) What is the mode?

SOLUTION

(a) Add the counts at 3, 4, and 5: $4 + 1 + 1 = 6$ students.

(b) The tallest stack is at 1 sibling, with 6 students. The mode is 1.

EXAMPLE 2: READING A HISTOGRAM

Use the histogram above (bins 0-9, 10-19, 20-29, 30-39, 40-49 with counts 2, 5, 9, 3, 1). (a) How many values are below 30? (b) What percent of the values are in the 20-29 bin? (c) Which bin contains the median?

SOLUTION

(a) $2 + 5 + 9 = 16$ values.

(b) $\frac{9}{20} = 0.45 = 45\%$.

(c) With 20 values, the median is between the 10th and 11th. The first two bins hold $2 + 5 = 7$ values, and the first three hold 16, so the 10th and 11th are in the 20-29 bin.

EXAMPLE 3: FINDING QUANTILES

Find the five-number summary and IQR of 2, 4, 6, 8, 9, 10, 12, 14, 20.

SOLUTION

The data are already in order and there are 9 values.

Median: the 5th value is 9.

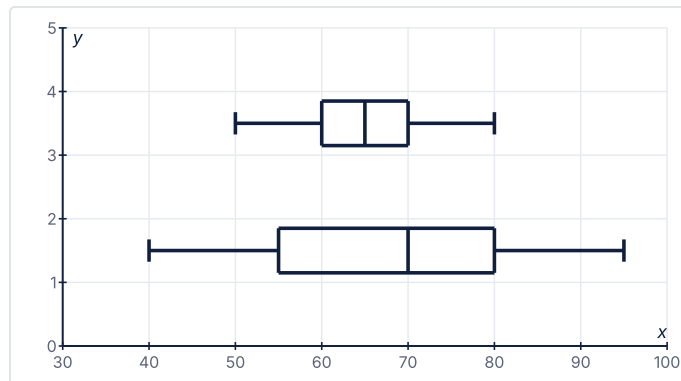
Lower half (leave out the 9): 2, 4, 6, 8. $Q1 = \frac{4+6}{2} = 5$.

Upper half: 10, 12, 14, 20. $Q3 = \frac{12+14}{2} = 13$.

Min is 2, max is 20. $IQR = 13 - 5 = 8$.

EXAMPLE 4: COMPARING TWO BOX PLOTS

Two classes took the same test. Class A: min 40, Q1 55, median 70, Q3 80, max 95. Class B: min 50, Q1 60, median 65, Q3 70, max 80. Compare the classes.



Test scores. Top box: Class B. Bottom box: Class A.

SOLUTION

Center: Class A has a higher median (70 vs 65).

Spread: IQR for A is $80 - 55 = 25$. IQR for B is $70 - 60 = 10$. Range for A is 55 and for B is 30.

Conclusion: Class A typically scored higher, but its scores were much more spread out. Class B was more consistent.

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EXAMPLE 5: USING THE QUARTERS

Class A in Example 4 has 40 students. About how many scored 80 or higher? About how many scored between 55 and 80?

SOLUTION

Q3 is 80, so the top section (80 to 95) holds about 25% of the data: $0.25 \times 40 = 10$ students.

The box (55 to 80) holds about 50%: $0.50 \times 40 = 20$ students.

EXAMPLE 6: BUILDING A HISTOGRAM

Fourteen students recorded how many minutes they practiced piano: 3, 12, 7, 15, 22, 18, 9, 25, 14, 11, 28, 5, 16, 20. Make a frequency table with bins 0-9, 10-19, and 20-29, and describe the shape.

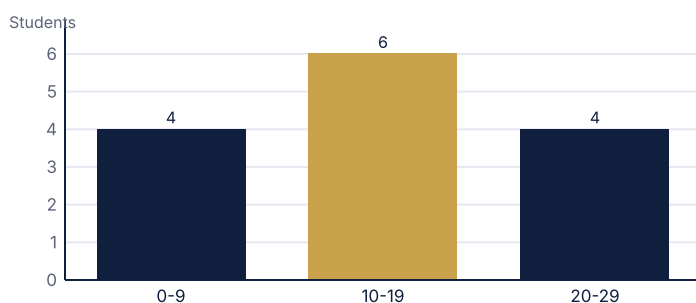
SOLUTION

Go through the list once and tally each value in its bin.

0-9: 3, 7, 9, 5, so 4 values. 10-19: 12, 15, 18, 14, 11, 16, so 6 values. 20-29: 22, 25, 28, 20, so 4 values.

Check: $4 + 6 + 4 = 14$, the number of students.

The middle bin is tallest and the two ends match, so the shape is roughly symmetric with a peak at 10-19 minutes. The chart below shows it.



Piano practice minutes for 14 students: the tallest bar (10-19) holds 6 students.

Coach's Playbook

- 1 Order the data first.** Quartiles only make sense after you sort.
- 2 Find the median, then split.** Split the data into lower and upper halves, and find the median of each half.
- 3 Label the five numbers.** Write min, Q1, median, Q3, max in a row before drawing.
- 4 Think in quarters.** Each section of a box plot is 25% of the data. Turn that into a count by multiplying by the total.
- 5 Compare with two words.** Always name a center (median) and a spread (IQR or range).

Common traps

(1) Thinking a long box means there are more data values there, when each section still has about 25%. (2) Reading exact values from a histogram. (3) Including the median in both halves when the count is odd. (4) Comparing only the ranges and ignoring the medians.

Mistake Clinic**Spot the error**

Problem: A box plot has min 4, Q1 8, median 11, Q3 15, max 30. Find the IQR. **Wrong work:** $30 - 4 = 26$. **What went wrong:** That is the range. The interquartile range uses only the box, from Q1 to Q3. **Right solution:** $IQR = 15 - 8 = 7$.

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Spot the error

Problem: A histogram has a bar over 10-19 with height 6. A student says, "The typical score is 6." **What went wrong:** The height of a bar is a count of how many values fall in the bin, not a data value. **Right reading:** 6 students had values from 10 to 19.

Spot the error

Problem: A box plot of 80 test scores has min 10, Q1 20, median 25, Q3 45, max 50. A student says, "More students scored between 25 and 45 than between 20 and 25 because that part of the box is longer." **What went wrong:** Each section of a box plot holds about the same number of values, one quarter of the data. Length shows how spread out the values are, not how many there are. **Right reading:** About 20 students are in each section.

Real-World Connection

Weather sites use box plots to show a month of daily high temperatures. A tall box means the weather swings a lot; a short box means days are steady.

Challenge Corner

A set of 11 values is 4, 6, 7, 9, 12, 13, 15, 18, 19, 22, 40. Find the five-number summary and the IQR. A value is called far-out if it is more than $1.5 \times \text{IQR}$ above Q3 or below Q1. Is 40 far-out? **Solution:** The median is the 6th value, 13. The lower half (4, 6, 7, 9, 12) has median 7, so $Q1 = 7$. The upper half (15, 18, 19, 22, 40) has median 19, so $Q3 = 19$. The summary is 4, 7, 13, 19, 40, and $\text{IQR} = 19 - 7 = 12$. Then $1.5 \times 12 = 18$, so the upper fence is $19 + 18 = 37$. Since $40 > 37$, the value 40 is far-out.

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Practice

PRACTICE Dot Plots, Histograms, and Box Plots

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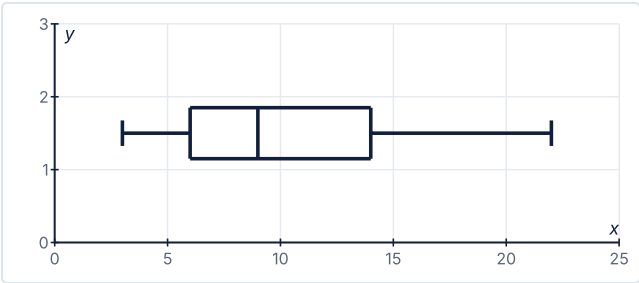
1. The table shows how many pets each of 20 students has. How many students have at least 2 pets?

Pets	0	1	2	3	4
Students	4	7	5	3	1

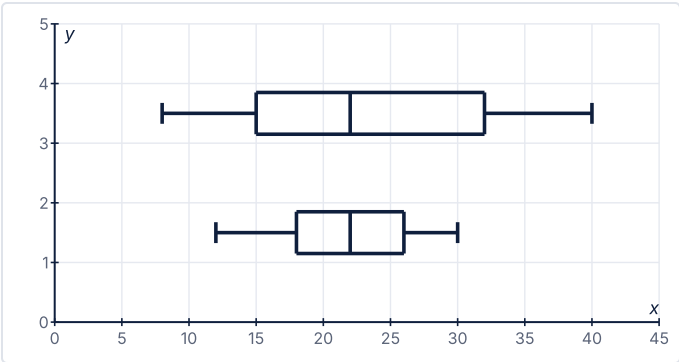
- (A) 4 (B) 5 (C) 9 (D) 11
2. A histogram of 20 values has these bins and counts. Which bin contains the median?

Bin	0-4	5-9	10-14	15-19	20-24
Count	7	5	3	3	2

- (A) 5-9 (B) 0-4 (C) 10-14 (D) 15-19
3. A box plot shows min 3, Q1 6, median 9, Q3 14, max 22. What is the interquartile range?



- (A) 19 (B) 8 (C) 5 (D) 9
4. The box plot of 60 runners' finishing times has a median of 41 minutes and Q3 of 47 minutes. About how many runners finished between the median and Q3?
- (A) 30 (B) 20 (C) 45 (D) 15
5. The box plots show times in minutes for two teams. Which statement is supported?



Bottom: Team X. Top: Team Y.

- (A) Team X has the higher median.
- (B) Both teams have the same median, but team Y is more spread out.
- (C) Team Y has the higher median because its maximum is greater.
- (D) The spread is the same because the medians are equal.
6. The data set 5, 7, 8, 10, 12, 13, 15, 18 has median 11. What is the interquartile range?
- (A) 13 (B) 3 (C) 6.5 (D) 7
7. A box plot has $Q1 = 20$ and $Q3 = 50$ with median 32. About what percent of the values are between 20 and 50?
- (A) 25% (B) 75% (C) 50% (D) 30%

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8. A dot plot has 3 dots at 5, 4 dots at 6, and 3 dots at 7. What is the median?
(A) 5 (B) 6.5 (C) 7 (D) 6
9. A histogram has bins 0-4, 5-9, 10-14, and 15-19 with counts 3, 7, 6, and 4. How many values are 10 or greater?
(A) 13 (B) 10 (C) 6 (D) 20
10. Find the five-number summary for 3, 5, 6, 6, 8, 9, 11, 12, 14, 15.
11. A histogram of 25 quiz scores has counts 2, 5, 9, 6, and 3 for the bins 50-59, 60-69, 70-79, 80-89, and 90-99. What percent of the scores are 80 or higher?
12. A box plot of 40 quiz scores has $Q1 = 62$, median = 70, and $Q3 = 84$. About how many students scored between 62 and 84?
13. Twenty students reported how many hours they slept. 6 hours: 2 students; 7 hours: 5; 8 hours: 8; 9 hours: 4; 10 hours: 1. Find the median and the mean.
14. Find the five-number summary and the IQR of 15, 18, 20, 22, 25, 27, 30, 34.
15. Group P has a box plot with min 20, $Q1$ 30, median 40, $Q3$ 50, max 60. Group Q has min 25, $Q1$ 35, median 40, $Q3$ 45, max 55. Compare the centers and the spreads.

10.3 Simple Probability

Learn to measure chance with fractions. Build sample spaces, use the complement rule, compare theoretical and experimental probability, and make predictions.

Why This Matters

Probability is the math of chance. It tells you how likely something is, from the chance that it rains tomorrow to the chance a game spinner lands on your color. Students get stuck because they write a probability as a part-to-part ratio instead of part-to-whole, forget to count every possible outcome, or confuse what should happen (theoretical) with what did happen (experimental).

Weather forecasts, game design, medical tests, and quality checks in factories all depend on probability. In later math courses it grows into statistics and data science, so a solid start now pays off.

Core Ideas

An **experiment** is an action with uncertain results, such as flipping a coin or drawing a marble. Each possible result is an **outcome**. The list of all possible outcomes is the **sample space**. An **event** is a set of outcomes we care about, such as "rolling an even number."

Theoretical Probability

When all outcomes are equally likely:

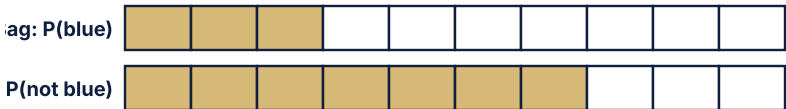
$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

A probability is always between 0 and 1 (or 0% and 100%). $P = 0$ means impossible, $P = 1$ means certain.



The probability scale: 0 means it cannot happen, 1 means it must happen. Most events fall in between.

For one roll of a fair die, the sample space is $\{1, 2, 3, 4, 5, 6\}$. The event "rolling an even number" has 3 favorable outcomes (2, 4, 6), so $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$. Probabilities can be written as fractions, decimals, or percents.



A bag has 10 marbles and 3 are blue. Three of ten parts are blue, so $P(\text{blue}) = \frac{3}{10}$. The other seven parts are the complement.

Complement Rule

The **complement** of an event is "the event does not happen."

$$P(\text{not } A) = 1 - P(A).$$

The two probabilities always add to 1.

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Experimental Probability and Predictions

Experimental probability = $\frac{\text{number of times the event happened}}{\text{total number of trials}}$.

Prediction: expected number of times $\approx P \times \text{number of trials}$.

Experimental results usually differ a little from theory because of chance. The more trials you do, the closer the experimental probability tends to get to the theoretical probability. If you flip a fair coin 10 times you might see 7 heads, but after 1000 flips the fraction of heads will be very close to $\frac{1}{2}$.

Event	Probability	Meaning
Sun rises tomorrow	about 1	certain
Fair coin lands heads	0.5	even chance
Roll a 7 on one die	0	impossible
Roll a 6 on one die	$\frac{1}{6}$	unlikely

Worked Examples

EXAMPLE 1: A BAG OF MARBLES

A bag has 5 red, 3 blue, and 2 green marbles. One marble is picked at random. Find $P(\text{blue})$ and $P(\text{not red})$.

SOLUTION

Total marbles: $5 + 3 + 2 = 10$.

$P(\text{blue}) = \frac{3}{10}$.

Not red means blue or green: $3 + 2 = 5$ marbles, so $P(\text{not red}) = \frac{5}{10} = \frac{1}{2}$.

EXAMPLE 2: A SPINNER

A spinner has 8 equal sections numbered 1 to 8. Find $P(\text{even})$, $P(\text{prime})$, $P(\text{greater than 5})$, and $P(\text{multiple of 3})$.

SOLUTION

Even: 2, 4, 6, 8, so $\frac{4}{8} = \frac{1}{2}$.

Prime: 2, 3, 5, 7, so $\frac{4}{8} = \frac{1}{2}$ (1 is not prime).

Greater than 5: 6, 7, 8, so $\frac{3}{8}$.

Multiple of 3: 3 and 6, so $\frac{2}{8} = \frac{1}{4}$.

EXAMPLE 3: COMPLEMENTS

The chance of rain tomorrow is 35%. What is the chance it does not rain? In the bag from Example 1, what is $P(\text{not blue})$?

SOLUTION

$P(\text{no rain}) = 1 - 0.35 = 0.65 = 65\%$.

$P(\text{not blue}) = 1 - \frac{3}{10} = \frac{7}{10}$.

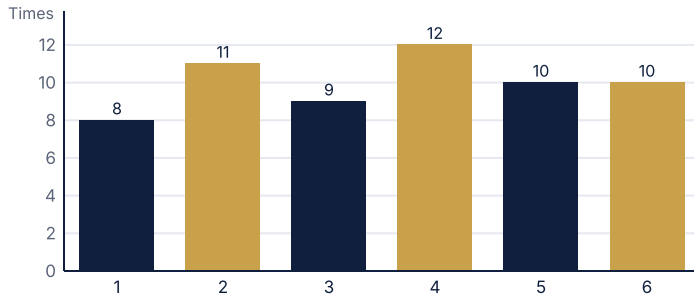
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EXAMPLE 4: EXPERIMENTAL VS. THEORETICAL

A fair die is rolled 60 times with the results shown in the chart. Find the experimental probability of rolling a 6 and of rolling a 4. Compare with the theoretical probability.



Results of 60 rolls.

SOLUTION

Experimental $P(6) = \frac{10}{60} = \frac{1}{6}$. Experimental $P(4) = \frac{12}{60} = \frac{1}{5}$.

Theoretical $P(\text{any one face}) = \frac{1}{6}$.

The 6 matched theory exactly. The 4 came up a bit more often than expected, which is normal random variation.

EXAMPLE 5: WORKING BACKWARD

A bag has 12 marbles. $P(\text{red}) = \frac{1}{4}$ and $P(\text{blue}) = \frac{1}{3}$. The rest are green. How many marbles of each color are there?

SOLUTION

Red: $\frac{1}{4} \times 12 = 3$. Blue: $\frac{1}{3} \times 12 = 4$.

Green probability: $1 - \frac{1}{4} - \frac{1}{3} = \frac{12}{12} - \frac{3}{12} - \frac{4}{12} = \frac{5}{12}$.

So there are 5 green marbles. Check: $3 + 4 + 5 = 12$.

EXAMPLE 6: OVERLAPPING EVENTS

A card is drawn at random from 20 cards numbered 1 to 20. Find the probability that the number is a multiple of 4 or greater than 15.

SOLUTION

Multiples of 4: 4, 8, 12, 16, 20 (5 cards).

Greater than 15: 16, 17, 18, 19, 20 (5 cards).

Cards 16 and 20 are in both lists. Count each card once: 4, 8, 12, 16, 17, 18, 19, 20, which is 8 cards. (Adding $5 + 5 = 10$ would count 16 and 20 twice.)

$P = \frac{8}{20} = \frac{2}{5}$.

Coch's Playbook

- 1 List the sample space.** Count every possible outcome so the bottom of your fraction is the whole.
- 2 Count the favorable outcomes.** Write them out when the list is short.
- 3 Write part over whole.** Favorable on top, total on the bottom. Simplify at the end.
- 4 Use the complement for "at least" or "not" questions.** If counting "not A" is easier, find it and subtract from 1.
- 5 Sanity check.** Your answer must be between 0 and 1. Probabilities of all outcomes should add to 1.

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Common traps

- (1) Writing favorable over unfavorable (like $\frac{3}{7}$ for blue) instead of favorable over total. (2) Counting 1 as a prime number. (3) Thinking that after 5 heads in a row, tails is "due." Each flip is independent. (4) Giving an answer bigger than 1.

Mistake Clinic**Spot the error**

Problem: A bag has 3 red and 5 blue marbles. Find $P(\text{red})$. **Wrong work:** $P(\text{red}) = \frac{3}{5}$. **What went wrong:** The student compared red to blue instead of red to all marbles. Probability is favorable outcomes over total outcomes. **Right solution:** $P(\text{red}) = \frac{3}{3+5} = \frac{3}{8}$.

Spot the error

Problem: The chance of rain is 70 percent, or 0.7. Find the chance of no rain. **Wrong work:** $100 - 0.7 = 99.3$ percent. **What went wrong:** The student mixed a percent with a decimal. Subtract from the same scale, either $1 - 0.7$ or $100 - 70$. **Right solution:** $1 - 0.7 = 0.3$, which is 30 percent.

Spot the error

Problem: A fair coin lands heads 4 times in a row. A student says the next flip is more likely to be tails. **What went wrong:** Each flip is separate, and the coin has no memory. **Right reasoning:** The probability of tails on the next flip is still $\frac{1}{2}$.

Real-World Connection

A factory knows that 2% of its phone chargers are defective. In a shipment of 1500, it expects about $0.02 \times 1500 = 30$ defective ones, and plans to test accordingly.

Challenge Corner

A bag holds red, blue, and green marbles. $P(\text{red}) = \frac{2}{5}$, $P(\text{blue}) = \frac{1}{4}$, and there are 14 green marbles. How many marbles of each color are in the bag? **Solution:** The three probabilities add to 1, so $P(\text{green}) = 1 - \frac{2}{5} - \frac{1}{4} = 1 - \frac{8}{20} - \frac{5}{20} = \frac{7}{20}$. Since 14 marbles are green, $\frac{7}{20}$ of the total is 14, so the total is $14 \div \frac{7}{20} = 40$. Red: $\frac{2}{5}(40) = 16$. Blue: $\frac{1}{4}(40) = 10$. Check: $16 + 10 + 14 = 40$.

Practice**PRACTICE Simple Probability**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- A fair die is rolled once. What is the probability of rolling a number greater than 4?
(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{1}{4}$
- A bag has 4 yellow marbles and 6 purple marbles. One is picked at random. What is $P(\text{yellow})$?
(A) $\frac{2}{3}$ (B) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $\frac{1}{4}$
- Which of these numbers cannot be the probability of an event?
(A) 0 (B) 0.999 (C) $\frac{7}{5}$ (D) 100%
- The chance of snow tomorrow is 0.15. What is the chance of no snow?
(A) 0.15 (B) 0.75 (C) 1.15 (D) 0.85

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5. A spinner was spun 40 times and landed on green 14 times. What is the experimental probability of landing on green?
(A) $\frac{7}{20}$ (B) $\frac{7}{13}$ (C) $\frac{1}{4}$ (D) $\frac{13}{20}$
6. In a carnival game, the chance of winning is $\frac{1}{8}$. If you play 200 times, about how many wins do you expect?
(A) 8 (B) 1600 (C) 25 (D) 175
7. A fair 12-sided die has faces numbered 1 to 12. What is the probability of rolling a number less than 4?
(A) $\frac{1}{3}$ (B) $\frac{3}{4}$ (C) $\frac{1}{12}$ (D) $\frac{1}{4}$
8. A bag has 8 marbles and $P(\text{green}) = \frac{3}{8}$. How many marbles are not green?
(A) 3 (B) 11 (C) 5 (D) 8
9. A coin was flipped 50 times and landed tails 22 times. What is the experimental probability of heads?
(A) $\frac{11}{25}$ (B) $\frac{1}{2}$ (C) $\frac{14}{11}$ (D) $\frac{14}{25}$
10. A bag has 6 red, 9 blue, and 5 white marbles. One is picked at random. What is the probability that it is not white?
11. Twenty cards are numbered 1 to 20. One is drawn at random. What is the probability that it shows a prime number?
12. A bag has red, blue, and green marbles. There are 3 red marbles, and the probability of picking red is $\frac{1}{5}$. (a) How many marbles are in the bag? (b) If 5 of them are blue, how many are green?
13. In practice, a player made 36 of 48 free throws. (a) What is the experimental probability that she makes a shot? (b) About how many of her next 80 shots should she make?
14. In a quality check, 3 of 120 light bulbs were defective. About how many defective bulbs should be expected in a shipment of 2000 bulbs?
15. A letter is picked at random from the word MATHEMATICS. Find (a) $P(M)$, (b) $P(\text{vowel})$, and (c) $P(\text{not a vowel})$.

10.4 Compound Events and Counting

Count outcomes with tree diagrams and the counting principle. Find probabilities of independent events, work with two-dice tables, and count arrangements with permutations.

Why This Matters

Real events often come in steps: flip a coin and then roll a die, pick a shirt and then pick pants, or choose a password one character at a time. To handle them, you need two tools: a way to **count** all the possible outcomes quickly, and a way to find the probability when one event follows another. Students get stuck because they add when they should multiply, forget to list every outcome, or mix up when order matters.

These ideas power passwords and phone numbers, scheduling, game design, and later topics like combinations, statistics, and computer science.

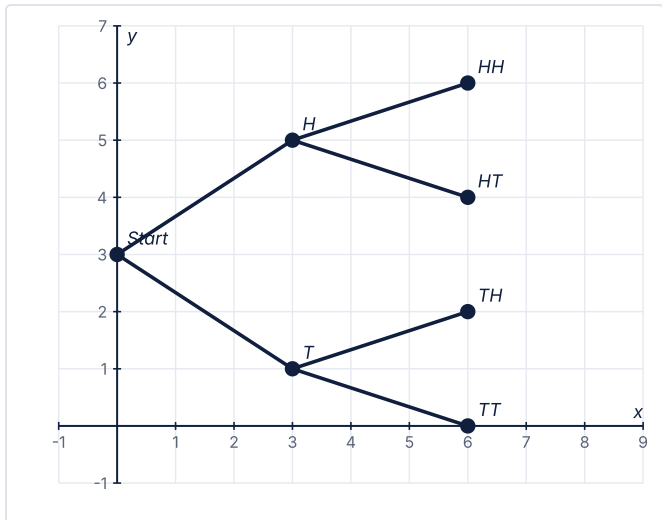
Core Ideas

A **compound event** combines two or more events. A **tree diagram** is a picture that lists every outcome: each branch is a choice, and each complete path is one outcome.

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A tree diagram for two coin flips. Each path from Start to the right is one outcome, so there are 4 outcomes, and each has probability $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

The Counting Principle

If one choice can be made in m ways and a second choice in n ways, the two choices together can be made in $m \times n$ ways. This extends to any number of choices: multiply the number of options at each step.

Suppose you have 4 shirts, 3 pairs of pants, and 2 pairs of shoes. There are $4 \times 3 \times 2 = 24$ outfits. Notice that we **multiply** the options; we do not add them.

Independent Events

Two events are **independent** if one does not change the probability of the other (for example, a coin flip and a die roll). For independent events:

$$P(A \text{ and } B) = P(A) \times P(B).$$

Picking marbles with **replacement** (putting the first one back before the second pick) keeps the events independent. If you do not put it back, the bag changes and the second probability is different. In this section we replace the item or use separate objects like a coin and a die.

Permutations

A **permutation** is an arrangement where order matters.
 The number of ways to arrange n different things in a line is $n! = n \times (n - 1) \times \cdots \times 2 \times 1$.
 The number of ways to choose and arrange r of n different things is $n \times (n - 1) \times \cdots$ (r factors).

For example, 5 runners can finish 1st, 2nd, and 3rd in $5 \times 4 \times 3 = 60$ ways. There are 5 choices for first place, then 4 left for second, then 3 left for third. The same runner cannot win twice, so each step has one fewer option.

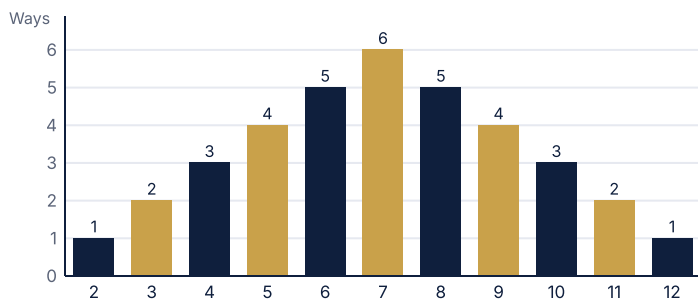
When you roll two dice, there are $6 \times 6 = 36$ equally likely outcomes. This table shows the sum for each pair.

Die 1 \ Die 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

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Number of ways to get each sum with two dice (out of 36). A sum of 7 is the most likely.

Worked Examples

EXAMPLE 1: COUNTING PRINCIPLE

A school lunch line offers 4 sandwiches, 3 fruits, and 2 drinks. A student takes one of each. How many different lunches are possible?

SOLUTION

Multiply the choices: $4 \times 3 \times 2 = 24$.

There are 24 different lunches.

EXAMPLE 2: TREE DIAGRAM

A fair coin is flipped twice. Find $P(\text{two heads})$ and $P(\text{exactly one head})$.

SOLUTION

From the tree, the outcomes are HH, HT, TH, TT. Each has probability $\frac{1}{4}$.

Two heads: only HH, so $\frac{1}{4}$.

Exactly one head: HT and TH, so $\frac{2}{4} = \frac{1}{2}$.

EXAMPLE 3: INDEPENDENT EVENTS

A spinner has 4 equal sections numbered 1 to 4. It is spun and a die is rolled. Find $P(3 \text{ on the spinner and an even number on the die})$. Then, a bag has 3 red and 2 blue marbles. A marble is drawn, replaced, and drawn again. Find $P(\text{red, then red})$.

SOLUTION

Spinner: $P(3) = \frac{1}{4}$. Die: $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$.

Multiply: $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$.

Marbles: $P(\text{red}) = \frac{3}{5}$ each time (because of the replacement), so $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$.

EXAMPLE 4: TWO DICE

Two fair dice are rolled. Find $P(\text{sum of 9 or more})$ using the table above.

SOLUTION

Count sums that are 9 or more: sum 9 appears 4 times, 10 appears 3 times, 11 appears 2 times, 12 appears 1 time.

$4 + 3 + 2 + 1 = 10$ favorable outcomes out of 36.

$P = \frac{10}{36} = \frac{5}{18}$.

EXAMPLE 5: PERMUTATIONS

Eight runners compete for gold, silver, and bronze medals. How many different ways can the medals be awarded? How many ways can 4 different books be lined up on a shelf?

SOLUTION

Medals: 8 choices for gold, 7 for silver, 6 for bronze. $8 \times 7 \times 6 = 336$.

Books: $4! = 4 \times 3 \times 2 \times 1 = 24$.

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EXAMPLE 6: COUNTING WITH A RULE

How many 3-digit numbers can be made from the digits 1, 2, 3, 4, 5 if no digit can be used twice and the number must be even?

SOLUTION

An even number must end in 2 or 4, so the ones digit has 2 choices. Fill that spot first.

The hundreds digit can be any of the 4 remaining digits: 4 choices.

The tens digit can be any of the 3 digits left: 3 choices.

By the counting principle: $2 \cdot 4 \cdot 3 = 24$ numbers.

Coach's Playbook

- 1 Count the steps.** Break the problem into choices made one after another.
- 2 Ask: can repeats happen?** A PIN can repeat digits, so each step has the same number of options. Medals cannot repeat a winner, so each step has one fewer.
- 3 Multiply for "and," never add.** If a choice happens and then another choice happens, multiply the counts or the probabilities.
- 4 Draw a tree or a table for small problems.** Listing all outcomes helps you check a formula.
- 5 Check the size.** Probabilities of independent events get smaller when multiplied. Your answer should be smaller than each of the pieces.

Common traps

(1) Adding the number of choices instead of multiplying (3 shirts and 4 pants is 12 outfits, not 7). (2) Using the same number of options at every step when the items cannot be reused. (3) Treating draws without replacement as independent. (4) Forgetting that the sum 7 can happen in 6 ways but the sum 12 in only 1 way.

Mistake Clinic**Spot the error**

Problem: A student has 3 shirts and 4 pairs of pants. How many outfits are possible? **Wrong work:** $3 + 4 = 7$. **What went wrong:** Each shirt can be worn with every pair of pants, so the choices multiply. **Right solution:** $3 \cdot 4 = 12$ outfits.

Spot the error

Problem: A fair coin is flipped twice. Find $P(\text{two heads})$. **Wrong work:** $\frac{1}{2} + \frac{1}{2} = 1$. **What went wrong:** Probabilities of independent events that must both happen are multiplied. Also, a probability of 1 would mean the event is certain.

Right solution: $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

Spot the error

Problem: Six runners race for first and second place. How many ways can the two places be filled? **Wrong work:** $6 \cdot 6 = 36$. **What went wrong:** One runner cannot win both places, so the second place has only 5 choices. **Right solution:** $6 \cdot 5 = 30$ ways.

Real-World Connection

A phone lock with a 4-digit PIN using digits 0 to 9 has $10 \times 10 \times 10 \times 10 = 10,000$ possible codes. That is why a longer PIN is much harder to guess.

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Challenge Corner

A password has 3 different capital letters followed by 2 digits (digits may repeat). (a) How many passwords are possible? (b) What fraction begin with a vowel (A, E, I, O, or U)? **Solution:** (a) Letters: $26 \cdot 25 \cdot 24 = 15,600$. Digits: $10 \cdot 10 = 100$. Total: $15,600 \cdot 100 = 1,560,000$. (b) Starting with a vowel: $5 \cdot 25 \cdot 24 \cdot 100 = 300,000$. Fraction: $\frac{300,000}{1,560,000} = \frac{5}{26}$, which also equals the chance that the first letter is a vowel.

Practice

PRACTICE Compound Events and Counting

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

1. A restaurant offers 3 appetizers, 5 main dishes, and 4 desserts. A customer picks one of each. How many different meals are possible?
(A) 12 (B) 15 (C) 20 (D) 60
2. A PIN has 4 digits, each from 0 to 9, and digits may repeat. How many PINs are possible?
(A) 10,000 (B) 40 (C) 5040 (D) 1000
3. In how many different orders can 5 different books be placed on a shelf?
(A) 25 (B) 120 (C) 5 (D) 20
4. A fair coin is flipped and a fair die is rolled. What is $P(\text{tails and a } 5)$?
(A) $\frac{2}{3}$ (B) $\frac{1}{6}$ (C) $\frac{1}{12}$ (D) $\frac{1}{8}$
5. A spinner has 3 equal sections: red, blue, green. It is spun twice. What is the probability that both spins land on the same color?
(A) $\frac{1}{9}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{2}{3}$
6. Two fair dice are rolled. What is the probability that the sum is 8?

Die 1 \ Die 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

- (A) $\frac{1}{6}$ (B) $\frac{2}{9}$ (C) $\frac{5}{36}$ (D) $\frac{1}{12}$
7. A quiz has 5 true/false questions. How many different ways can it be answered?
(A) 10 (B) 25 (C) 32 (D) 5
8. A fair die is rolled twice. What is the probability that the first roll is a 2 and the second roll is greater than 4?
(A) $\frac{1}{36}$ (B) $\frac{1}{3}$ (C) $\frac{1}{12}$ (D) $\frac{1}{18}$
9. A club of 8 members chooses a president and then a vice president, with no one holding both jobs. How many different choices are possible?
(A) 56 (B) 64 (C) 28 (D) 16
10. How many different 3-letter codes can be made from the letters A, B, C, D, E if no letter can be used twice?
11. A spinner has 5 equal sections numbered 1 to 5, and a die is rolled. What is the probability of an odd number on the spinner and a number greater than 4 on the die?
12. A license plate has 2 letters followed by 3 digits. Letters and digits may repeat. How many different plates are possible?
13. A bag has 4 red marbles and 6 blue marbles. A marble is drawn and replaced, then a second is drawn. What is the probability of red first and blue second?
14. How many different 4-digit codes can be made from the digits 1 to 6 if no digit is repeated?
15. A spinner has 4 equal sections: red, blue, green, and yellow. It is spun, a coin is flipped, and a die is rolled. (a) How many outcomes are possible? (b) What is the probability of red, heads, and an even number?

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Answers**Answers and Explanations**

Complete solutions for every practice problem, numbered to match. Read the explanation even when your answer is right.

10.1 Mean, Median, Mode, and Range**PRACTICE · MEAN, MEDIAN, MODE, AND RANGE**

1. A. Sum = 40 and $40 \div 5 = 8$. The 9 is the median, 7 is the range, and 11 is the largest value.
2. D. Order: 9, 11, 12, 14, 16, 21. The middle two are 12 and 14, so the median is 13. Choosing 16 comes from taking the middle of the unsorted list.
3. C. The value 2 appears three times, so the mode is 2. The 5 is the median and 8 is the range.
4. B. Range = $8 - (-6) = 14$. Subtracting 8 - 6 gives 2, which forgets the negative sign. Choice 4 is the sum of the values.
5. B. New mean = $\frac{220}{6} \approx 36.7$ and new median = $\frac{24+26}{2} = 25$. An outlier changes the mean much more than the median.
6. D. Total = $14 \cdot 5 = 70$. The four known numbers sum to 55, so the fifth is $70 - 55 = 15$. The 70 is the total, and 13.75 is the mean of just the four.
7. C. Six values are in order, so average the 3rd and 4th: $(85 + 90) \div 2 = 87.5$. Choice 85 or 90 picks only one middle value.
8. B. The old total is $7 \cdot 12 = 84$. The new total is $84 + 20 = 104$ for 8 numbers, and $104 \div 8 = 13$. Choice 16 averages 12 and 20, ignoring the group sizes.
9. D. The median is the 5th value in order. The largest value stays the largest, so the middle value does not move. The median stays 20, while the mean would go up.
10. Mean 13.5, median 14, mode 15, range 7. Sum = 108 and $108 \div 8 = 13.5$. Ordered: 10, 11, 12, 13, 15, 15, 15, 17, so the median is $\frac{13+15}{2} = 14$. Range = $17 - 10 = 7$.
11. 12. Total = $9 \cdot 6 = 54$. The five known numbers sum to 42, so the sixth is $54 - 42 = 12$.
12. The mean rises by 6.9 dollars (from 8.6 to 15.5). The median rises by only 1 dollar (from 8 to 9), which is less. Mean without: $43 \div 5 = 8.6$. Mean with: $93 \div 6 = 15.5$.
13. Mean 1.6, median 1.5. Total siblings = $0 + 7 + 12 + 9 + 4 = 32$ and $32 \div 20 = 1.6$. The 10th value is 1 and the 11th is 2, so the median is $\frac{1+2}{2} = 1.5$.
14. Mean 135, median 135, mode 135, range 60. Total = 810, and $810 \div 6 = 135$. Order: 105, 120, 135, 135, 150, 165, so the median is $(135 + 135) \div 2 = 135$. The most common score is 135. Range = $165 - 105 = 60$.
15. 88. First four total: $4 \cdot 78 = 312$. Five tests total: $5 \cdot 80 = 400$. Fifth score: $400 - 312 = 88$.

10.2 Dot Plots, Histograms, and Box Plots**PRACTICE · DOT PLOTS, HISTOGRAMS, AND BOX PLOTS**

1. C. At least 2 means 2 or more: $5 + 3 + 1 = 9$. The 5 is exactly 2 pets, and 11 is fewer than 2 pets.
2. A. The 10th and 11th values are the median. The first bin holds 7 values and the first two hold 12, so both are in 5-9. The tallest bin (0-4) is not automatically the median bin.
3. B. IQR = $Q3 - Q1 = 14 - 6 = 8$. The 19 is the range, and 5 is $Q3 - \text{median}$.
4. D. The section from the median to Q3 holds about 25% of the data: $0.25 \times 60 = 15$. The 30 would be the whole box ($Q1$ to $Q3$).
5. B. Both medians are 22. The IQR for X is 8 and for Y is 17, so Y is more spread out. A larger maximum does not raise the median.
6. C. Lower half 5, 7, 8, 10 gives $Q1 = 7.5$. Upper half 12, 13, 15, 18 gives $Q3 = 14$. IQR = $14 - 7.5 = 6.5$. The 13 is the range.
7. C. $Q1$ to $Q3$ is the box, and it holds the middle half of the data, 50 percent. The 75 percent comes from counting the median as a boundary.
8. D. There are 10 values. In order, the 5th and 6th values are both 6, so the median is 6. Choosing 6.5 comes from averaging the three labels 5, 6, 7 badly.
9. B. Add the last two bins: $6 + 4 = 10$. Choice 20 is all the values, and 6 forgets the last bin.
10. Min 3, $Q1$ 6, median 8.5, $Q3$ 12, max 15. The median is $\frac{8+9}{2} = 8.5$. The lower half 3, 5, 6, 6, 8 has median 6. The upper half 9, 11, 12, 14, 15 has median 12.
11. 36%. $6 + 3 = 9$ scores, and $\frac{9}{25} = 0.36$.
12. About 20 students. The box ($Q1$ to $Q3$) holds about 50% of the data, and $0.50 \times 40 = 20$.
13. Median 8, mean 7.85. The 10th and 11th values are both 8. Mean = $\frac{12+35+64+36+10}{20} = \frac{157}{20} = 7.85$.
14. Min 15, $Q1$ 19, median 23.5, $Q3$ 28.5, max 34, and IQR 9.5. The median is $(22 + 25) \div 2 = 23.5$. Lower half 15, 18, 20, 22 has median $(18 + 20) \div 2 = 19$. Upper half 25, 27, 30, 34 has median $(27 + 30) \div 2 = 28.5$. IQR = $28.5 - 19 = 9.5$.
15. The centers are the same (both medians are 40). Group P is more spread out: its IQR is $50 - 30 = 20$, while Group Q's is $45 - 35 = 10$. Group P's range is also larger, 40 compared with 30.

10.3 Simple Probability

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PRACTICE · SIMPLE PROBABILITY

1. A. Only 5 and 6 are greater than 4, so $\frac{2}{6} = \frac{1}{3}$. The $\frac{1}{2}$ wrongly includes the 4, and $\frac{2}{3}$ is the complement.
2. B. $\frac{4}{10} = \frac{2}{5}$. The $\frac{2}{3}$ comes from comparing yellow to purple only (4 to 6), and $\frac{3}{5}$ is P(purple).
3. C. $\frac{7}{5} = 1.4$ is greater than 1. Probabilities must be between 0 and 1, so 0, 0.999, and 100% are all possible.
4. D. $1 - 0.15 = 0.85$. The 0.15 is the chance of snow, and 1.15 adds instead of subtracting.
5. A. $\frac{14}{40} = \frac{7}{20}$. The $\frac{7}{13}$ compares green to non-green, and $\frac{13}{20}$ is the complement.
6. C. $\frac{1}{8} \times 200 = 25$. Multiplying by 8 gives 1600, and 175 is the expected number of losses.
7. D. The numbers 1, 2, and 3 are less than 4, so $P = \frac{3}{12} = \frac{1}{4}$. Choice $\frac{1}{3}$ counts 4 as well.
8. C. Green marbles: 3. Not green: $8 - 3 = 5$. Choice 3 is the number that are green.
9. D. Heads came up $50 - 22 = 28$ times, so $\frac{28}{50} = \frac{14}{25}$. Choice $\frac{11}{25}$ is the experimental probability of tails.
10. $\frac{3}{4}$. Total = 20. Not white = $6 + 9 = 15$, so $\frac{15}{20} = \frac{3}{4}$. Or $1 - \frac{5}{20} = \frac{3}{4}$.
11. $\frac{2}{5}$. The primes are 2, 3, 5, 7, 11, 13, 17, 19, which is 8 cards. $\frac{8}{20} = \frac{2}{5}$.
12. (a) 15 marbles. (b) 7 green. Since $\frac{3}{n} = \frac{1}{5}$, $n = 15$. Green = $15 - 3 - 5 = 7$.
13. (a) $\frac{3}{4}$. (b) 60 shots. $\frac{36}{48} = \frac{3}{4}$, and $\frac{3}{4} \times 80 = 60$.
14. About 50 bulbs. The experimental probability is $\frac{3}{120} = \frac{1}{40}$, and $2000 \cdot \frac{1}{40} = 50$.
15. (a) $\frac{2}{11}$, (b) $\frac{4}{11}$, (c) $\frac{7}{11}$. The word has 11 letters. M appears twice. The vowels A, E, A, I make 4 letters. Not a vowel: $1 - \frac{4}{11} = \frac{7}{11}$.

10.4 Compound Events and Counting

PRACTICE · COMPOUND EVENTS AND COUNTING

1. D. $3 \times 5 \times 4 = 60$. Adding gives 12, which treats the choices as separate instead of combined.
2. A. Each of the 4 digits has 10 options: $10^4 = 10,000$. The 5040 is the count if digits could not repeat, and 40 is 4×10 .
3. B. $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$. The 25 is 5^2 , and 20 stops after two factors.
4. C. $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$. The $\frac{2}{3}$ comes from adding the probabilities instead of multiplying.
5. B. There are $3 \times 3 = 9$ outcomes. The matching ones are RR, BB, GG, which is 3. So $\frac{3}{9} = \frac{1}{3}$. The $\frac{1}{6}$ is the chance of one specific matching pair.
6. C. The sum 8 appears 5 times in the table: (2,6), (3,5), (4,4), (5,3), (6,2). So $\frac{5}{36}$. The $\frac{1}{6}$ is the chance of sum 7, which appears 6 times.
7. C. Each question has 2 choices, so $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$. Choice 10 adds instead of multiplying, and 25 uses 5^2 .
8. D. $P(\text{first is } 2) = \frac{1}{6}$ and $P(\text{greater than } 4) = \frac{2}{6}$. Multiply: $\frac{1}{6} \cdot \frac{2}{6} = \frac{2}{36} = \frac{1}{18}$. Choice $\frac{1}{3}$ comes from adding.
9. A. There are 8 choices for president and then 7 for vice president: $8 \cdot 7 = 56$. Choice 64 allows one person to hold both jobs, and 16 adds.
10. 60. There are 5 choices for the first letter, 4 for the second, and 3 for the third: $5 \times 4 \times 3 = 60$.
11. $\frac{1}{5}$. $P(\text{odd}) = \frac{3}{5}$ and $P(> 4) = \frac{2}{6} = \frac{1}{3}$. Multiply: $\frac{3}{5} \times \frac{1}{3} = \frac{1}{5}$.
12. 676,000. Letters: $26 \times 26 = 676$. Digits: $10 \times 10 \times 10 = 1000$. Total $676 \times 1000 = 676,000$.
13. $\frac{6}{25}$. $P(\text{red}) = \frac{4}{10} = \frac{2}{5}$ and $P(\text{blue}) = \frac{6}{10} = \frac{3}{5}$. Multiply: $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$.
14. 360. There are 6 choices for the first digit, 5 for the second, 4 for the third, and 3 for the fourth: $6 \cdot 5 \cdot 4 \cdot 3 = 360$.
15. (a) 48 outcomes. (b) $\frac{1}{16}$. For (a), $4 \cdot 2 \cdot 6 = 48$. For (b), $\frac{1}{4} \cdot \frac{1}{2} \cdot \frac{3}{6} = \frac{3}{48} = \frac{1}{16}$.

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