



# Systems of Equations and Inequalities

Four focused lessons, worked examples, a mistake clinic, and 60 practice problems with full explanations.

## In this booster

**11.1** Solving Systems by Graphing and Substitution

**11.2** Solving Systems by Elimination

**11.3** Systems in Word Problems

**11.4** Linear Inequalities and Systems of Inequalities

Every section teaches the idea, walks through the most common problem types, flags the traps students fall into, and ends with fifteen practice problems. Full answers and explanations are at the back.

## 11.1 Solving Systems by Graphing and Substitution

Learn what the solution of a system means, find it by graphing and by substitution, and recognize the special cases of no solution and infinitely many solutions.

### Why This Matters

A single equation with two variables, like  $y = 2x + 1$ , has endless solutions. A **system** puts two such equations together and asks for the one pair  $(x, y)$  that makes both true at the same time. Companies use systems to find the price where cost equals income, scientists use them to find where two measurements agree, and every later algebra course builds on them.

Students usually get stuck in three places: reading a graph's crossing point as a pair in the right order, substituting an expression without parentheses and making a sign error, and panicking when the algebra ends in a strange statement like  $6 = 6$  or  $-6 = 10$  instead of a number. This section removes all three problems.

### Core Ideas

Every point on the graph of an equation is a solution of that equation. If two lines are drawn on the same grid, a point that sits on **both** lines is a solution of **both** equations. That shared point is the solution of the system.

### Solution of a System

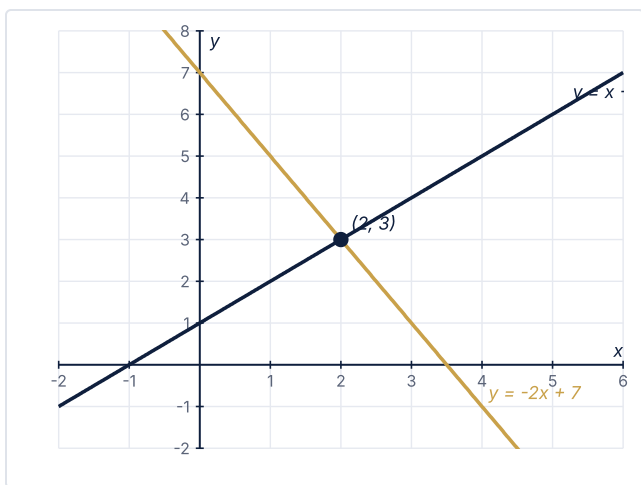
A **solution** of a system of two equations is an ordered pair  $(x, y)$  that makes **both** equations true.

Graphically, it is the point where the two graphs **intersect**.

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The lines cross at  $(2, 3)$ , so that pair solves both equations.

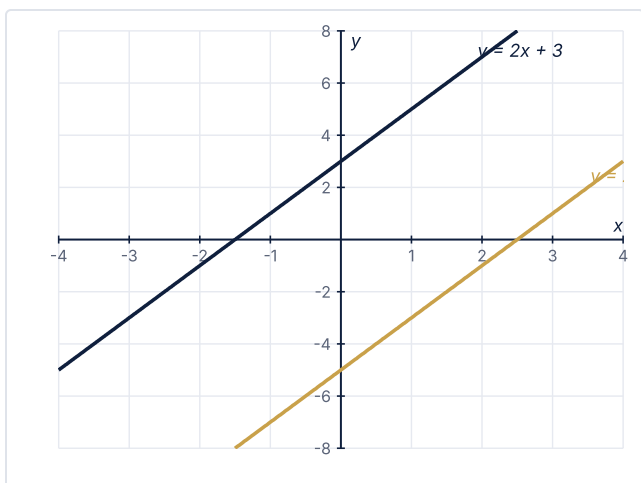
Graphing is good for seeing what is going on, but it can be slow and inexact when the crossing point is not on a grid corner. **Substitution** is the exact, algebraic way. You solve one equation for a variable, then replace that variable in the other equation.

### The Substitution Method

1. Solve one equation for one variable (pick the one with a coefficient of 1 or  $-1$  if you can).
2. Substitute that expression, in parentheses, into the other equation.
3. Solve the one-variable equation.
4. Put the value back into the first expression to find the other variable.
5. Check the pair in **both** original equations.

Two lines in a plane can meet in three ways. The pattern shows up in the slopes and intercepts, so you can often predict the answer before solving.

| What the lines do           | Slopes                     | Algebra ends with                | Number of solutions |
|-----------------------------|----------------------------|----------------------------------|---------------------|
| <b>Cross once</b>           | Different                  | A value like $x = 3$             | Exactly one         |
| <b>Parallel, never meet</b> | Same, different intercepts | A false statement like $-6 = 10$ | None                |
| <b>Same line</b>            | Same, same intercept       | A true statement like $6 = 6$    | Infinitely many     |



Parallel lines: same slope 2, different intercepts, so no point is shared.

### Worked Examples

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**EXAMPLE 1: SOLVE BY GRAPHING**

Solve the system  $y = x + 1$  and  $y = -2x + 7$  by graphing.

**SOLUTION**

Graph  $y = x + 1$ : start at  $(0, 1)$  and go up 1, right 1 each step.

Graph  $y = -2x + 7$ : start at  $(0, 7)$  and go down 2, right 1 each step.

The graphs cross at  $(2, 3)$  (see the graph above).

Check:  $3 = 2 + 1$  is true, and  $-2(2) + 7 = 3$  is true. The solution is  $(2, 3)$ .

**EXAMPLE 2: SUBSTITUTION, ONE VARIABLE ALREADY ISOLATED**

Solve  $y = 3x - 4$  and  $2x + y = 11$ .

**SOLUTION**

The first equation already says  $y = 3x - 4$ . Substitute into the second:  $2x + (3x - 4) = 11$ .

Combine:  $5x - 4 = 11$ , so  $5x = 15$  and  $x = 3$ .

Back-substitute:  $y = 3(3) - 4 = 5$ .

Check in the second equation:  $2(3) + 5 = 11$ . The solution is  $(3, 5)$ .

**EXAMPLE 3: SUBSTITUTION AFTER ISOLATING A VARIABLE**

Solve  $x + 2y = 9$  and  $3x - y = 6$ .

**SOLUTION**

The first equation is easy to solve for  $x$ :  $x = 9 - 2y$ .

Substitute with parentheses:  $3(9 - 2y) - y = 6$ .

Distribute:  $27 - 6y - y = 6$ , so  $27 - 7y = 6$  and  $-7y = -21$ , giving  $y = 3$ .

Then  $x = 9 - 2(3) = 3$ . Check in the second:  $3(3) - 3 = 6$ . The solution is  $(3, 3)$ .

**EXAMPLE 4: NO SOLUTION**

Solve  $y = 2x + 3$  and  $4x - 2y = 10$ . Describe the graphs.

**SOLUTION**

Substitute:  $4x - 2(2x + 3) = 10$ .

Distribute:  $4x - 4x - 6 = 10$ , which becomes  $-6 = 10$ .

This is false for every  $x$ , so no pair works. There is **no solution**.

Rewrite the second equation as  $y = 2x - 5$ . Both lines have slope 2 and different intercepts, so they are parallel.

**EXAMPLE 5: INFINITELY MANY SOLUTIONS**

Solve  $6x - 3y = 9$  and  $y = 2x - 3$ .

**SOLUTION**

Substitute:  $6x - 3(2x - 3) = 9$ .

Distribute:  $6x - 6x + 9 = 9$ , which becomes  $9 = 9$ .

This is true for every  $x$ . Every point of the line  $y = 2x - 3$  also satisfies  $6x - 3y = 9$  (divide it by 3 to see  $2x - y = 3$ ).

The two equations describe the **same line**, so there are **infinitely many solutions**.

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**EXAMPLE 6: COUNT THE SOLUTIONS WITHOUT SOLVING**

Without solving, decide how many solutions each system has: (a)  $y = 4x - 1$  and  $y = -4x - 1$ ; (b)  $y = \frac{1}{2}x + 3$  and  $x - 2y = -6$ ; (c)  $2y = 6x + 4$  and  $y = 3x - 5$ .

**SOLUTION**

(a) Slopes are 4 and  $-4$ , which differ, so the lines cross exactly once. Both have intercept  $-1$ , so the crossing point is  $(0, -1)$ .

(b) Rewrite  $x - 2y = -6$  as  $y = \frac{1}{2}x + 3$ . It is the same line, so there are **infinitely many** solutions.

(c) Rewrite  $2y = 6x + 4$  as  $y = 3x + 2$ . Compare with  $y = 3x - 5$ : same slope 3, different intercepts. Parallel, so **no solution**.

**Coach's Playbook**

- 1 Look first.** Scan for a variable with coefficient 1 or  $-1$ . Solve for that one to avoid fractions.
- 2 Parentheses always.** When you substitute an expression, wrap it in parentheses, especially after a minus sign.
- 3 Read the ending.** A number means one solution. A false statement means none. A true statement means infinitely many.
- 4 Check in both.** Plug the pair into both original equations. It takes ten seconds and catches most mistakes.
- 5 Write ordered pairs.** The answer is  $(x, y)$ , with  $x$  first. Say which is which.

**Common traps**

Four slips to watch: (1) writing the pair backwards, like  $(3, 2)$  when the lines cross at  $x = 2, y = 3$ ; (2) substituting without parentheses, so  $3(9 - 2y)$  turns into  $3 \cdot 9 - 2y$ ; (3) stopping after finding  $x$  and forgetting  $y$ ; (4) calling  $0 = 0$  or  $9 = 9$  "no solution" when a true statement means infinitely many.

**Mistake Clinic****Spot the error**

A student solves  $y = 2x - 1$  and  $3x + 2y = 12$  and writes: " $3x + 2(2x) - 1 = 12$ , so  $7x - 1 = 12$  and  $x = \frac{13}{7}$ ." The mistake is that the substitution replaced  $y$  with  $2x$  and left the  $-1$  outside, so only part of  $y$  was substituted. The whole expression goes inside parentheses:  $3x + 2(2x - 1) = 12$ . Then  $7x - 2 = 12$ ,  $x = 2$ , and  $y = 2(2) - 1 = 3$ . The solution is  $(2, 3)$ .

**Spot the error**

A student solves  $y = 3x - 4$  and  $2x + y = 11$  by substituting  $y = 3x - 4$  into the **first** equation and gets  $3x - 4 = 3x - 4$ , then  $0 = 0$ , and says there are infinitely many solutions. The error is substituting an expression back into the equation it came from, which always gives a true statement. Substitute into the **other** equation:  $2x + (3x - 4) = 11$ , so  $x = 3$  and  $y = 5$ . The solution is  $(3, 5)$ .

**Spot the error**

A student solves  $2x - 4y = 6$  and  $x = 2y + 3$ . After substituting, the line reads  $2(2y + 3) - 4y = 6$ , then  $6 = 6$ . The student writes "no solution." The mistake is the meaning of the ending: a **true** statement means the equations describe the same line, so there are **infinitely many** solutions. A false statement such as  $6 = 7$  is what signals no solution.

**Real-World Connection**

A gym charges 20 dollars a month plus 5 dollars per class. Another charges 35 dollars a month with free classes. Setting the two cost equations equal and solving tells you how many classes make the plans cost the same.

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### Challenge Corner

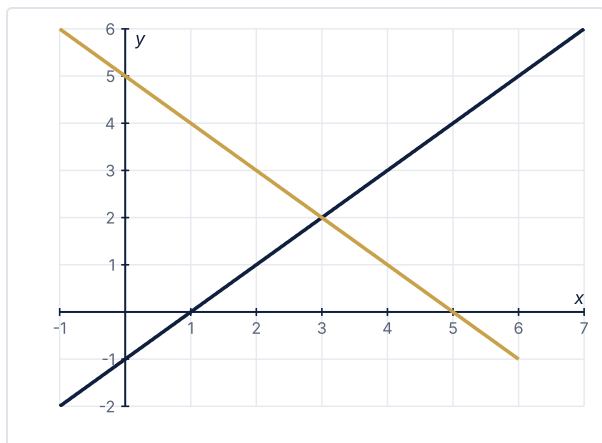
For the system  $y = 2x + k$  and  $4x - 2y = 6$ , find the value of  $k$  that gives infinitely many solutions, and describe what happens for every other  $k$ . Solution: substitute to get  $4x - 2(2x + k) = 6$ , so  $4x - 4x - 2k = 6$  and  $-2k = 6$ . If  $k = -3$  the statement is  $6 = 6$ , true for all  $x$ : infinitely many solutions. For any other  $k$  the statement  $-2k = 6$  is false, so there is no solution. The lines have the same slope 2, so they can never cross exactly once.

### Practice

#### PRACTICE Solving Systems by Graphing and Substitution

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- Which system has  $(4, 1)$  as its solution?  
 (A)  $y = x - 3$  and  $x + 2y = 8$       (B)  $y = x - 3$  and  $x + 2y = 6$       (C)  $y = x + 3$  and  $x + y = 5$   
 (D)  $2x + y = 8$  and  $x - y = 4$
- What is the solution of  $y = 2x + 1$  and  $y = -x + 10$ ?  
 (A)  $(7, 3)$       (B)  $(3, 10)$       (C)  $(3, 7)$       (D)  $(1, 3)$
- How many solutions does the system  $y = -3x + 2$  and  $6x + 2y = -4$  have?  
 (A) Exactly one      (B) Exactly two      (C) Infinitely many  
 (D) None
- If  $2x + y = 12$  and  $x = y + 3$ , what is the value of  $x$ ?  
 (A) 5      (B) 2      (C) 3      (D) 7
- The graph shows two lines. What is the solution of the system?



Two lines on the same grid.

- (A)  $(2, 3)$       (B)  $(5, 0)$       (C)  $(3, 2)$       (D)  $(0, -1)$
- For what value of  $k$  does the system  $y = 4x - 5$  and  $8x - 2y = k$  have infinitely many solutions?  
 (A) 5      (B) 10      (C)  $-10$       (D)  $-5$
- Which ordered pair is the solution of  $x + 3y = 10$  and  $2x - y = 6$ ?  
 (A)  $(4, 2)$       (B)  $(2, 4)$       (C)  $(3, 0)$       (D)  $(10, 0)$
- The lines  $y = -x + 6$  and  $y = 2x$  intersect at a point  $(x, y)$ . What is  $x + y$ ?  
 (A) 2      (B) 4      (C) 8      (D) 6
- The lines  $y = ax + 3$  and  $y = -2x + 7$  intersect where  $x = 1$ . What is the value of  $a$ ?  
 (A)  $-2$       (B) 5      (C) 2      (D) 8
- Solve by substitution:  $y = 2x - 7$  and  $3x + 4y = -6$ . Write your answer as an ordered pair.
- Solve by substitution:  $x = 3y + 2$  and  $2x - 5y = 7$ . Write your answer as an ordered pair.
- The system  $kx + 3y = 9$  and  $4x + 6y = 18$  has infinitely many solutions. Find  $k$ .

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13. Solve:  $y = \frac{2}{3}x + 1$  and  $2x + 3y = 15$ . Write your answer as an ordered pair.
14. Solve:  $y = -x + 9$  and  $4x - 3y = 1$ . Write your answer as an ordered pair.
15. Solve  $x = 4y - 3$  and  $2x + y = 12$ . What is the value of  $x + y$ ?

## 11.2 Solving Systems by Elimination

Learn to add and multiply equations so a variable cancels, handle the special cases, and choose the fastest method for each system.

### Why This Matters

Substitution works well when a variable is already alone. Many real systems are not that tidy. Equations like  $3x + 4y = 1$  and  $5x + 6y = 3$  have no variable by itself, and solving for one creates messy fractions. **Elimination** is the tool for that situation. It lines the equations up and adds them so that one variable disappears.

Students get stuck deciding what to multiply by, forgetting to multiply every term (including the number on the right side), and making sign errors when they subtract. A second struggle is knowing which method to choose. This section gives you a clear routine and a quick way to pick the best method.

### Core Ideas

The key fact is that you may add the same quantity to both sides of an equation. If  $A = B$  and  $C = D$ , then  $A + C = B + D$ . So adding two equations gives a new true equation. If the  $y$ -terms are opposites, such as  $+4y$  and  $-4y$ , they cancel and only  $x$  remains.

#### The Elimination Method

1. Write both equations in standard form  $Ax + By = C$ , with like terms lined up.
2. Multiply one or both equations by numbers so that one variable has **opposite coefficients** (like  $6y$  and  $-6y$ ).
3. **Add** the equations. One variable disappears.
4. Solve for the remaining variable, then substitute back to find the other.
5. Check in both original equations.

When the coefficients are already opposites, no multiplying is needed. When they are the same, you can subtract instead of adding, or multiply one equation by  $-1$  and add. When neither match, multiply both equations to build a common coefficient: use the least common multiple of the two coefficients.

| Situation                             | Coefficients of $y$ | Best move                                     |
|---------------------------------------|---------------------|-----------------------------------------------|
| <b>Opposites already</b>              | $+3y$ and $-3y$     | Add the equations                             |
| <b>Same</b>                           | $+2y$ and $+2y$     | Subtract (or multiply one by $-1$ , then add) |
| <b>One is a multiple of the other</b> | $y$ and $4y$        | Multiply the small one by 4 (or $-4$ )        |
| <b>No relation</b>                    | $3y$ and $5y$       | Multiply by 5 and by 3 (or $-3$ )             |

#### Special Cases in Elimination

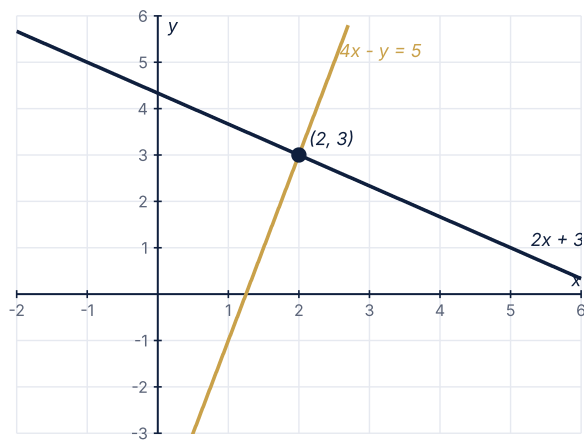
If both variables disappear and you get a **false** statement (like  $0 = -3$ ), the system has **no solution** (parallel lines).

If both variables disappear and you get a **true** statement (like  $0 = 0$ ), the system has **infinitely many solutions** (the same line).

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Example 2: the elimination answer  $(2, 3)$  is the point where the two lines cross.

## Worked Examples

### EXAMPLE 1: ADD THE EQUATIONS DIRECTLY

Solve  $x + y = 11$  and  $x - y = 3$ .

#### SOLUTION

The  $y$ -coefficients are  $+1$  and  $-1$ , which are opposites. Add:  $2x = 14$ , so  $x = 7$ .

Substitute into  $x + y = 11$ :  $7 + y = 11$ , so  $y = 4$ .

Check:  $7 - 4 = 3$ . The solution is  $(7, 4)$ .

### EXAMPLE 2: MULTIPLY ONE EQUATION

Solve  $2x + 3y = 13$  and  $4x - y = 5$ .

#### SOLUTION

To cancel  $y$ , multiply the second equation by 3:  $12x - 3y = 15$ .

Add to the first:  $2x + 3y + 12x - 3y = 13 + 15$ , so  $14x = 28$  and  $x = 2$ .

Substitute into  $4x - y = 5$ :  $8 - y = 5$ , so  $y = 3$ .

Check in the first:  $2(2) + 3(3) = 13$ . The solution is  $(2, 3)$ .

### EXAMPLE 3: MULTIPLY BOTH EQUATIONS

Solve  $3x + 4y = 1$  and  $5x + 6y = 3$ .

#### SOLUTION

Eliminate  $y$ . The coefficients 4 and 6 have least common multiple 12. Multiply the first by 3 and the second by  $-2$ .

First:  $9x + 12y = 3$ . Second:  $-10x - 12y = -6$ .

Add:  $-x = -3$ , so  $x = 3$ .

Substitute into  $3x + 4y = 1$ :  $9 + 4y = 1$ , so  $4y = -8$  and  $y = -2$ .

Check in the second:  $5(3) + 6(-2) = 3$ . The solution is  $(3, -2)$ .

### EXAMPLE 4: THE SPECIAL CASES

Use elimination on (a)  $2x + 3y = 5$  and  $4x + 6y = 7$ , and (b)  $2x - 6y = 10$  and  $-x + 3y = -5$ .

#### SOLUTION

(a) Multiply the first by  $-2$ :  $-4x - 6y = -10$ . Add to  $4x + 6y = 7$ :  $0 = -3$ . False, so **no solution**.

(b) Multiply the second by 2:  $-2x + 6y = -10$ . Add to  $2x - 6y = 10$ :  $0 = 0$ . True, so **infinitely many solutions**.

Check (b): the first equation divided by 2 is  $x - 3y = 5$ , and the second times  $-1$  is also  $x - 3y = 5$ . Same line.

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**EXAMPLE 5: CLEAR FRACTIONS FIRST**

Solve  $\frac{x}{2} + \frac{y}{3} = 4$  and  $\frac{x}{4} - \frac{y}{6} = 1$ .

**SOLUTION**

Multiply the first equation by 6:  $3x + 2y = 24$ .

Multiply the second equation by 12:  $3x - 2y = 12$ .

The  $y$ -terms are opposites. Add:  $6x = 36$ , so  $x = 6$ .

Substitute into  $3x + 2y = 24$ :  $18 + 2y = 24$ , so  $y = 3$ .

Check in the original second equation:  $\frac{6}{4} - \frac{3}{6} = 1.5 - 0.5 = 1$ . The solution is  $(6, 3)$ .

**EXAMPLE 6: CHOOSE THE BEST METHOD**

Pick the most efficient method and solve: (a)  $y = 5x - 3$  and  $2x + y = 11$ ; (b)  $4x + 3y = 17$  and  $4x - 3y = -1$ .

**SOLUTION**

(a) The variable  $y$  is already alone, so use **substitution**:  $2x + (5x - 3) = 11$ , so  $7x = 14$  and  $x = 2$ . Then  $y = 5(2) - 3 = 7$ . The solution is  $(2, 7)$ .

(b) Both equations are in standard form and the  $y$ -terms are opposites, so use **elimination**. Add:  $8x = 16$ , so  $x = 2$ . Then  $4(2) + 3y = 17$ , so  $3y = 9$  and  $y = 3$ . The solution is  $(2, 3)$ .

Trying substitution on (b) would force fractions, and elimination on (a) would need rearranging first. The right choice saves time.

**Coach's Playbook**

- 1 Line up first.** Rewrite both equations as  $Ax + By = C$  so that the  $x$ -terms,  $y$ -terms and constants sit in columns.
- 2 Pick the cheaper variable.** Choose the variable that needs the least multiplying. Opposite or matching coefficients mean zero work.
- 3 Multiply every term.** When you multiply an equation, multiply the right side too.
- 4 Add, do not subtract, when you can.** Making one coefficient negative and then adding causes fewer sign errors than subtracting a whole equation.
- 5 Check in the original.** Use the original equations, not your multiplied versions.

**Choosing a method.** Use substitution when a variable is already isolated or has coefficient  $\pm 1$  in a neat equation (like  $y = 3x - 4$ ). Use elimination when both equations are in standard form with no lone variable. Use graphing when you need a picture, an estimate, or when the problem gives you a graph.

**Common traps**

Four slips to watch: (1) multiplying only the left side of an equation by a number; (2) subtracting equations and forgetting that subtracting a negative term adds it, so  $5y - (-5y)$  is  $10y$ , not 0; (3) finding one variable and stopping; (4) treating  $0 = 0$  as "the answer is 0" instead of recognizing infinitely many solutions.

**Mistake Clinic****Spot the error**

A student solves  $2x + y = 7$  and  $x - 3y = 7$ . To cancel  $y$ , the student multiplies the first equation by 3 and writes  $6x + 3y = 7$ . The error is that the right side was not multiplied. Every term must be multiplied:  $6x + 3y = 21$ . Adding to  $x - 3y = 7$  gives  $7x = 28$ , so  $x = 4$ , and  $y = 7 - 2(4) = -1$ . The solution is  $(4, -1)$ .

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**Spot the error**

A student solves  $4x + 3y = 10$  and  $4x - 5y = -6$  by subtracting the equations and writes  $3y - 5y = -2y$  and  $10 - 6 = 4$ , so  $y = -2$ . The error is the sign: subtracting  $-5y$  gives  $3y - (-5y) = 8y$ , and  $10 - (-6) = 16$ . So  $8y = 16$  and  $y = 2$ . Then  $4x + 6 = 10$  gives  $x = 1$ . The solution is  $(1, 2)$ .

**Spot the error**

A student solves  $5x + 2y = 19$  and  $2x - 2y = 2$ . Adding gives  $7x = 21$ , so the student writes the answer " $x = 3$ ." The error is stopping halfway: a system has two unknowns, so the solution is an ordered pair. Substitute:  $2(3) - 2y = 2$  gives  $y = 2$ . The solution is  $(3, 2)$ .

**Real-World Connection**

A cafe sells 3 muffins and 2 coffees for 13 dollars, and 5 muffins and 3 coffees for 21 dollars. Elimination finds each price without guessing: subtract multiples of the two equations and the muffin or coffee price drops out.

**Challenge Corner**

The system  $ax + 3y = 7$  and  $2x + by = 9$  has the solution  $(2, 1)$ . Find  $a$  and  $b$ , then write the system. Solution: substitute  $x = 2$  and  $y = 1$  into each equation. The first gives  $2a + 3 = 7$ , so  $a = 2$ . The second gives  $4 + b = 9$ , so  $b = 5$ . The system is  $2x + 3y = 7$  and  $2x + 5y = 9$ . Check by subtracting:  $2y = 2$ , so  $y = 1$ , and then  $x = 2$ .

**Practice****PRACTICE Solving Systems by Elimination**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- Adding the equations  $2x + 5y = 9$  and  $3x - 5y = 1$  gives which equation?  
 (A)  $5x + 10y = 10$  (B)  $-x + 10y = 8$  (C)  $5x = 8$  (D)  $5x = 10$
- For the system  $4x + 3y = 18$  and  $2x - 3y = 0$ , what is the value of  $y$ ?  
 (A) 2 (B) 3 (C) 6 (D) 0
- If  $3x + 4y = 10$  and  $2x - 3y = 1$ , what is the value of  $x + y$ ?  
 (A) 1 (B) 3 (C) 2 (D) 5
- How many solutions does the system  $6x - 9y = 12$  and  $2x - 3y = 5$  have?  
 (A) Exactly one (B) Infinitely many (C) None (D) Exactly two
- The sum of two numbers is 25 and their difference is 7. What is the larger number?  
 (A) 16 (B) 9 (C) 18 (D) 7
- For which value of  $k$  does the system  $3x + ky = 6$  and  $x + 2y = 5$  have no solution?  
 (A) 2 (B) 15 (C) 3 (D) 6
- To eliminate  $x$  by adding the equations  $2x + 3y = 8$  and  $3x - 2y = 7$ , which multipliers should be used?  
 (A) First by 3, second by 2 (B) First by 2, second by 3  
 (C) First by 3, second by  $-2$  (D) No multipliers; add as they are
- For the system  $7x + 2y = 8$  and  $3x - 2y = 12$ , what is the value of  $y$ ?  
 (A) 3 (B)  $-3$  (C) 2 (D)  $-2$
- Three bats and two balls cost 46 dollars. Three bats and five balls cost 70 dollars. What is the price of one ball, in dollars?  
 (A) 8 (B) 10 (C) 12 (D) 24
- Solve by elimination:  $5x + 3y = 21$  and  $2x - 3y = 0$ . Write the answer as an ordered pair.

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11. Solve by elimination:  $4x - 3y = -1$  and  $3x + 2y = 12$ . Write the answer as an ordered pair.
12. Solve:  $\frac{x}{3} + \frac{y}{2} = 5$  and  $\frac{x}{2} - \frac{y}{3} = 1$ . Write the answer as an ordered pair.
13. Three times a number  $a$  plus a number  $b$  is 17. The number  $a$  minus twice  $b$  is  $-6$ . Find the product  $ab$ .
14. Solve by elimination:  $3x - 2y = 4$  and  $5x + 4y = 14$ . Write the answer as an ordered pair.
15. For what value of  $c$  does the system  $2x + 3y = 7$  and  $6x + 9y = c$  have infinitely many solutions?

### 11.3 Systems in Word Problems

Turn stories about tickets, mixtures, plans, and businesses into systems of equations, solve them, and interpret the break-even point in context.

#### Why This Matters

Systems of equations exist because real situations have two unknowns that are tied together by two facts. How many adult and child tickets were sold? How much of each solution went into a mixture? When does one phone plan become cheaper than another? Businesses call that last one the **break-even point**, and it decides whether a product makes money.

The algebra in this section is the same as before. The hard part is the reading. Students get stuck on defining the variables, turning sentences into equations, mixing up which price goes with which quantity, and forgetting to answer the question that was actually asked. We will build a four-step routine that makes word problems predictable.

#### Core Ideas

##### The Four-Step Word Problem Routine

1. **Define** two variables with units (for example,  $a$  = number of adult tickets).
2. **Write** two equations from two facts in the story. Each fact is usually a "how many" (count) equation and a "how much" (value) equation.
3. **Solve** with substitution or elimination.
4. **Answer** the question in a sentence, and check the numbers against the story.

Most word problems come from a few families. Each one has a count equation and a value equation, and learning the shapes makes them easy to recognize.

| Problem type               | Count equation                | Value equation                                                                                            |
|----------------------------|-------------------------------|-----------------------------------------------------------------------------------------------------------|
| Tickets or items           | $a + c = \text{total number}$ | $(\text{price}_a)a + (\text{price}_c)c = \text{total money}$                                              |
| Mixtures                   | $x + y = \text{total amount}$ | $(\text{rate}_1)x + (\text{rate}_2)y = (\text{target rate})(\text{total})$                                |
| Rates with wind or current | (none)                        | $b + c = \text{speed with, } b - c = \text{speed against}$                                                |
| Cost comparison            | (none)                        | $\text{cost}_1 = \text{fixed}_1 + \text{rate}_1 n$ and $\text{cost}_2 = \text{fixed}_2 + \text{rate}_2 n$ |

##### Break-Even Point

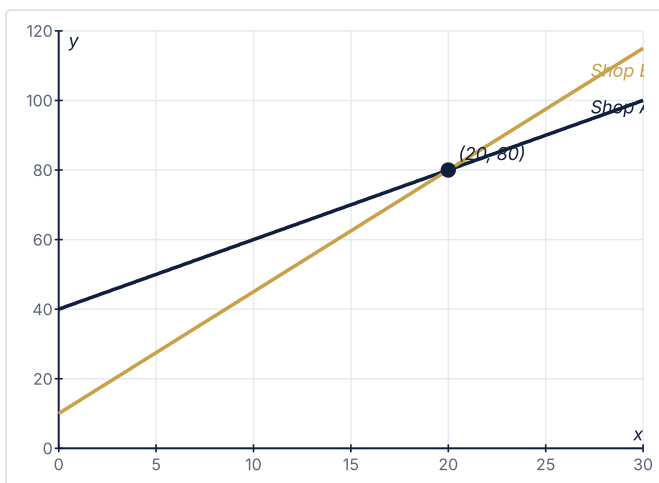
The **break-even point** is where two quantities are equal, such as cost and revenue, or the costs of two plans. Set the two expressions equal and solve for the input  $n$ . The graph shows it as the intersection of the two lines.

A cost line often starts above zero, because of a fixed cost (the setup fee, the equipment). A revenue line usually starts at zero. Before the lines cross, cost is larger and you lose money. After the crossing, revenue is larger and you make a profit.

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Cost of printing  $n$  shirts at two shops. They cost the same at 20 shirts.

## Worked Examples

### EXAMPLE 1: TICKETS

A school play sold 120 tickets. Adult tickets cost 8 dollars and child tickets cost 5 dollars. Total sales were 840 dollars. How many of each were sold?

#### SOLUTION

Let  $a$  = adult tickets and  $c$  = child tickets.

Count:  $a + c = 120$ . Value:  $8a + 5c = 840$ .

Substitute  $c = 120 - a$ :  $8a + 5(120 - a) = 840$ , so  $3a + 600 = 840$  and  $a = 80$ .

Then  $c = 120 - 80 = 40$ .

Check:  $8(80) + 5(40) = 640 + 200 = 840$ . There were 80 adult and 40 child tickets.

### EXAMPLE 2: MIXTURE

A lab technician mixes a 10% acid solution with a 30% acid solution to make 20 liters of a 22% solution. How many liters of each are needed?

#### SOLUTION

Let  $x$  = liters of 10% solution and  $y$  = liters of 30% solution.

Count:  $x + y = 20$ . Acid amount:  $0.10x + 0.30y = 0.22(20) = 4.4$ .

Substitute  $x = 20 - y$ :  $0.10(20 - y) + 0.30y = 4.4$ , so  $2 + 0.20y = 4.4$  and  $y = 12$ .

Then  $x = 8$ .

Check:  $0.10(8) + 0.30(12) = 0.8 + 3.6 = 4.4$ . Use 8 liters of the 10% and 12 liters of the 30% solution.

### EXAMPLE 3: COST COMPARISON

Shop A charges a 40-dollar setup fee plus 2 dollars per shirt. Shop B charges a 10-dollar setup fee plus 3.50 dollars per shirt. For how many shirts do they cost the same? Which shop is cheaper for 50 shirts?

#### SOLUTION

Costs:  $A = 40 + 2n$  and  $B = 10 + 3.5n$ .

Set equal:  $40 + 2n = 10 + 3.5n$ , so  $30 = 1.5n$  and  $n = 20$ .

At 20 shirts both cost  $40 + 2(20) = 80$  dollars (see the graph above).

For 50 shirts:  $A = 40 + 100 = 140$  and  $B = 10 + 175 = 185$ . Shop A is cheaper for larger orders because its per-shirt rate is lower.

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**EXAMPLE 4: BREAK-EVEN FOR A BUSINESS**

A student bakes cookies for a fundraiser. Equipment costs 270 dollars (a fixed cost) and ingredients cost 3 dollars per cookie. Each cookie sells for 9 dollars. How many cookies must be sold to break even?

**SOLUTION**

Cost:  $C = 270 + 3n$ . Revenue:  $R = 9n$ .

Set equal:  $270 + 3n = 9n$ , so  $270 = 6n$  and  $n = 45$ .

Check: cost  $270 + 135 = 405$  and revenue  $9(45) = 405$ .

The student must sell 45 cookies to break even. Every cookie after that earns 6 dollars of profit.

**EXAMPLE 5: WIND AND CURRENT**

A boat travels 36 kilometers downstream in 2 hours. The return trip upstream takes 3 hours. Find the speed of the boat in still water and the speed of the current.

**SOLUTION**

Downstream speed is  $36 \div 2 = 18$  km/h. Upstream speed is  $36 \div 3 = 12$  km/h.

Let  $b$  = boat speed and  $c$  = current speed. Then  $b + c = 18$  and  $b - c = 12$ .

Add:  $2b = 30$ , so  $b = 15$ . Then  $c = 18 - 15 = 3$ .

Check:  $15 + 3 = 18$  and  $15 - 3 = 12$ . The boat goes 15 km/h in still water and the current is 3 km/h.

**EXAMPLE 6: MONEY IN TWO ACCOUNTS**

A family splits 5,000 dollars between two accounts. One earns 3% interest per year and the other earns 5%. After one year the interest is 190 dollars. How much is in each account?

**SOLUTION**

Let  $x$  = dollars at 3% and  $y$  = dollars at 5%.

Total:  $x + y = 5000$ . Interest:  $0.03x + 0.05y = 190$ .

Substitute  $x = 5000 - y$ :  $0.03(5000 - y) + 0.05y = 190$ , so  $150 + 0.02y = 190$  and  $y = 2000$ .

Then  $x = 3000$ .

Check:  $0.03(3000) + 0.05(2000) = 90 + 100 = 190$ . There are 3,000 dollars at 3% and 2,000 dollars at 5%.

## **Coach's Playbook**

- 1 Name the unknowns.** Write "Let  $a$  = number of adult tickets." Include units so you do not mix quantities.
- 2 Hunt for two facts.** Underline each number in the story and decide what it says. Usually one fact is about counts and one is about money, amounts or time.
- 3 Convert percents.** Write 22% as 0.22 before you build a mixture equation.
- 4 Use rate times time.** Distance equals speed times time. Divide first to get speeds if you are given distances and times.
- 5 Reread the question.** The system may give  $a$  and  $c$ , but the question may ask for the total money, the difference, or the cheaper plan.

### **Common traps**

Four slips to watch: (1) matching a price to the wrong quantity, so  $5a + 8c$  instead of  $8a + 5c$ ; (2) forgetting to turn a percent into a decimal; (3) answering with a variable value that does not match the question, such as giving the number of child tickets when asked for adults; (4) accepting an answer that does not fit the story, such as a negative number of tickets. Always check.

## **Mistake Clinic**

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**Spot the error**

A student reads: "120 tickets, adults 8 dollars, children 5 dollars, total 840 dollars." The student writes  $a + c = 120$  and  $5a + 8c = 840$ . The error is that the prices are attached to the wrong ticket types. Adult tickets cost 8 each, so the money equation is  $8a + 5c = 840$ . Solving gives  $a = 80$  and  $c = 40$ . Always say aloud, "adult price times adult count."

**Spot the error**

A student mixes a 10% and a 30% solution to make 20 liters of 22% solution, and writes  $x + y = 20$  and  $0.10x + 0.30y = 22$ . Solving gives  $y = 100$  liters, more than the 20 liters total. The error is that the right side should be the amount of acid in the final mix,  $0.22(20) = 4.4$ , not 22. The correct equation  $0.10x + 0.30y = 4.4$  gives  $y = 12$  and  $x = 8$ . An answer larger than the total is a warning sign.

**Spot the error**

A student finds that cookies break even at  $n = 45$  and writes, "The break-even revenue is 45 dollars." The error is mixing up the input and the output. The solution  $n = 45$  is the **number of cookies**. The revenue at that point is  $9(45) = 405$  dollars, which also equals the cost  $270 + 3(45) = 405$ . Always attach units to the answer.

**Real-World Connection**

Every food truck, app developer and sports team store does a break-even calculation before launching a product. Fixed costs go on one side, per-item profit on the other, and the answer says how many sales are needed.

**Challenge Corner**

A snack bar sells 4 pretzels and 3 juices for 12 dollars, and 2 pretzels and 5 juices for 13 dollars. What do 3 pretzels and 4 juices cost? Solution: let  $p$  be the pretzel price and  $j$  the juice price. Then  $4p + 3j = 12$  and  $2p + 5j = 13$ . Multiply the second equation by 2:  $4p + 10j = 26$ . Subtract the first:  $7j = 14$ , so  $j = 2$ . Then  $4p + 6 = 12$  gives  $p = 1.5$ . Now  $3(1.5) + 4(2) = 4.5 + 8 = 12.5$  dollars.

**Practice****PRACTICE Systems in Word Problems**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- A theater sold 200 tickets for 1,520 dollars. Student tickets cost 6 dollars and adult tickets cost 10 dollars. Which system has  $s$  student tickets and  $a$  adult tickets?
 

|                                         |                                         |                                          |
|-----------------------------------------|-----------------------------------------|------------------------------------------|
| (A) $s + a = 1520$ and $6s + 10a = 200$ | (B) $s + a = 200$ and $10s + 6a = 1520$ | (C) $s + a = 200$ and $16(s + a) = 1520$ |
| (D) $s + a = 200$ and $6s + 10a = 1520$ |                                         |                                          |
- Using the same theater sales (200 tickets, 1,520 dollars, student 6 dollars, adult 10 dollars), how many adult tickets were sold?
 

|         |         |        |        |
|---------|---------|--------|--------|
| (A) 120 | (B) 200 | (C) 80 | (D) 60 |
|---------|---------|--------|--------|
- Plan A costs 20 dollars to join plus 5 dollars per month. Plan B costs 50 dollars to join plus 2 dollars per month. After how many months do the two plans cost the same?
 

|        |        |       |        |
|--------|--------|-------|--------|
| (A) 30 | (B) 10 | (C) 6 | (D) 15 |
|--------|--------|-------|--------|
- Two notebooks and three pens cost 9.50 dollars. Three notebooks and one pen cost 9.00 dollars. What is the price of one pen, in dollars?
 

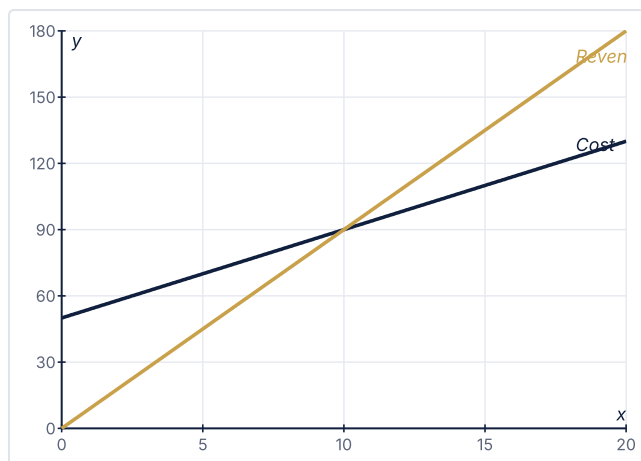
|          |          |          |          |
|----------|----------|----------|----------|
| (A) 1.50 | (B) 1.00 | (C) 2.50 | (D) 1.25 |
|----------|----------|----------|----------|

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5. A cook mixes 20% juice with 50% juice to make 30 liters of 40% juice. How many liters of the 50% juice are used?  
 (A) 10 (B) 20 (C) 15 (D) 25
6. The graph shows the cost and revenue of making and selling  $x$  phone cases. How many cases must be sold to break even?



Cost and revenue for phone cases.

- (A) 10 (B) 9 (C) 90 (D) 50
7. A kayak travels 12 kilometers downstream in 2 hours and returns the same distance in 4 hours. What is the speed of the current, in kilometers per hour?  
 (A) 3 (B) 4.5 (C) 1.5 (D) 6
8. Two numbers add to 40. One number is 4 more than three times the other. What is the smaller number?  
 (A) 31 (B) 10 (C) 13 (D) 9
9. A fund holds 8,000 dollars in two accounts, one paying 2% and one paying 4%. The total yearly interest is 260 dollars. How many dollars are in the 4% account?  
 (A) 3000 (B) 5000 (C) 6000 (D) 2600
10. Movie night sold 150 tickets. Child tickets cost 6 dollars, adult tickets cost 11 dollars, and sales were 1,225 dollars. How many adult tickets were sold?
11. How many milliliters of a 12% salt solution must be mixed with a 4% salt solution to make 40 milliliters of a 6% solution?
12. Ride company A charges 3 dollars plus 1.20 dollars per kilometer. Ride company B charges 1 dollar plus 1.40 dollars per kilometer. For how many kilometers do the two rides cost the same?
13. A plane flies 600 kilometers with a tailwind in 3 hours, then returns the same 600 kilometers against the wind in 4 hours. What is the plane's speed in still air, in kilometers per hour?
14. A theater sold 90 tickets. Adult tickets cost 12 dollars, student tickets cost 8 dollars, and sales were 920 dollars. How many student tickets were sold?
15. How many liters of 20% vinegar solution must be mixed with 5% vinegar solution to make 30 liters of 10% solution?

## 11.4 Linear Inequalities and Systems of Inequalities

Solve and graph linear inequalities, shade half-planes with a test point, find feasible regions of systems, and use real-world constraints to choose the best plan.

### Why This Matters

Not every situation asks for an exact match. A budget is a limit, not a target. A team needs at least 12 players. A recipe can use no more than 2 cups of sugar. These are **inequalities**, and they describe whole ranges of acceptable answers. When two or more limits apply together, you get a **system of inequalities**, and the set of answers that meet every limit is called the **feasible region**.

Students struggle with three things: forgetting to reverse the inequality sign when multiplying or dividing by a negative number, choosing between solid and dashed lines, and deciding which side of a line to shade. A test point removes the guesswork. This section also shows how businesses use feasible regions to plan production.

### Core Ideas

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## Solving a Linear Inequality

Solve it like an equation, with one exception.

When you **multiply or divide both sides by a negative number**, reverse the inequality sign. For example,  $-3x > 9$  becomes  $x < -3$ .

Symbols:  $<$  less than,  $>$  greater than,  $\leq$  at most,  $\geq$  at least.

On a number line, an **open circle** means the endpoint is not included ( $<$  or  $>$ ). A **closed circle** means it is included ( $\leq$  or  $\geq$ ).



Two examples:  $x < -3$  (open circle, arrow left) and  $x \geq 2$  (closed circle, arrow right).

With two variables, an inequality like  $y > 2x - 1$  has a whole region of solutions. Draw the **boundary line** first, then decide which side to shade.

## Graphing a Linear Inequality in Two Variables

1. Graph the boundary line (replace the inequality sign with  $=$ ).
2. Use a **dashed** line for  $<$  or  $>$ , and a **solid** line for  $\leq$  or  $\geq$ .
3. Pick a **test point** not on the line, like  $(0, 0)$ . Substitute it.
4. If the statement is true, shade the side that contains the test point. If false, shade the other side.

## Systems of Inequalities

The solution of a system of inequalities is the set of points that satisfy **every** inequality. On a graph, it is the region where all the shaded parts **overlap**.

The corners of the overlap are found by solving pairs of boundary equations.

| Inequality      | Line style | Shade (if the form is $y$ on the left) |
|-----------------|------------|----------------------------------------|
| $y > mx + b$    | Dashed     | Above the line                         |
| $y \geq mx + b$ | Solid      | Above the line                         |
| $y < mx + b$    | Dashed     | Below the line                         |
| $y \leq mx + b$ | Solid      | Below the line                         |

This table works when  $y$  is alone on the left. If you divide by a negative number while solving for  $y$ , flip the sign first. The test point is the safest check.

## Worked Examples

### EXAMPLE 1: SOLVE AND GRAPH

Solve  $-3x + 5 > 14$  and show the answer on a number line.

#### SOLUTION

Subtract 5:  $-3x > 9$ .

Divide by  $-3$  and reverse the sign:  $x < -3$ .

Check with  $x = -4$ :  $-3(-4) + 5 = 17 > 14$ . True.

On the number line, draw an open circle at  $-3$  and an arrow to the left.

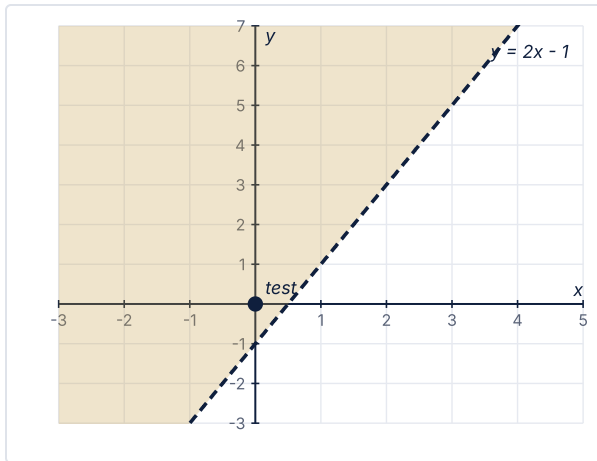
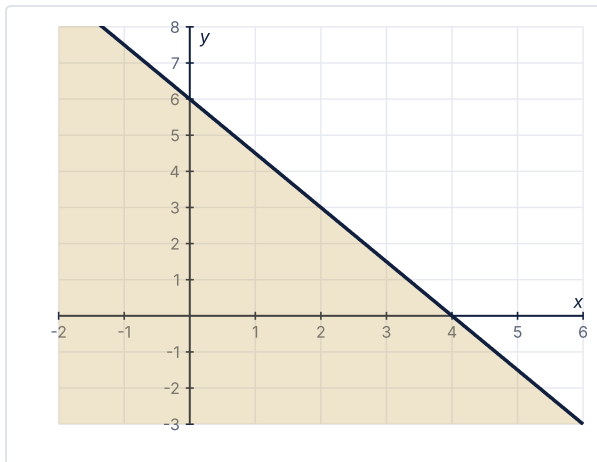


Solution of Example 1:  $x < -3$ .

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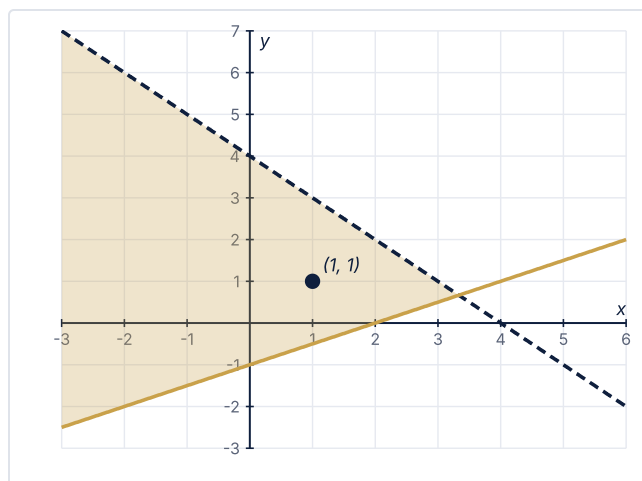
**EXAMPLE 2: GRAPH A TWO-VARIABLE INEQUALITY**Graph  $y > 2x - 1$ . $y > 2x - 1$ : dashed boundary, shade above.**SOLUTION**The boundary is  $y = 2x - 1$ : slope 2,  $y$ -intercept  $-1$ . Because the sign is  $>$ , draw it **dashed**.Test  $(0, 0)$ : is  $0 > 2(0) - 1 = -1$ ? Yes, it is true.Shade the side containing  $(0, 0)$ , which is above the line.Points on the line, such as  $(1, 1)$ , are not solutions, since  $1 > 1$  is false.**EXAMPLE 3: STANDARD FORM AND A SOLID LINE**Graph  $3x + 2y \leq 12$ . $3x + 2y \leq 12$ : solid boundary, shade below.**SOLUTION**Boundary:  $3x + 2y = 12$ . Intercepts:  $(4, 0)$  and  $(0, 6)$ . The sign is  $\leq$ , so the line is **solid**.Test  $(0, 0)$ :  $0 \leq 12$  is true, so shade the side with the origin.Solving for  $y$ :  $y \leq 6 - 1.5x$ , which agrees: shade below the line.Want a plan built around **your** student?

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**EXAMPLE 4: A SYSTEM OF TWO INEQUALITIES**

Graph the system  $y < -x + 4$  and  $y \geq \frac{1}{2}x - 1$ . Is  $(1, 1)$  a solution?



The shaded wedge is the overlap of the two half-planes.

**SOLUTION**

Graph  $y = -x + 4$  as a dashed line and  $y = \frac{1}{2}x - 1$  as a solid line.

First inequality: test  $(0, 0)$ ,  $0 < 4$  true, shade below. Second:  $0 \geq -1$  true, shade above.

The solution region is the overlap: above the solid line and below the dashed line.

The lines meet where  $-x + 4 = \frac{1}{2}x - 1$ , so  $x = \frac{10}{3}$  and  $y = \frac{2}{3}$ .

Test  $(1, 1)$ :  $1 < 3$  true and  $1 \geq -0.5$  true. Yes, it is a solution.

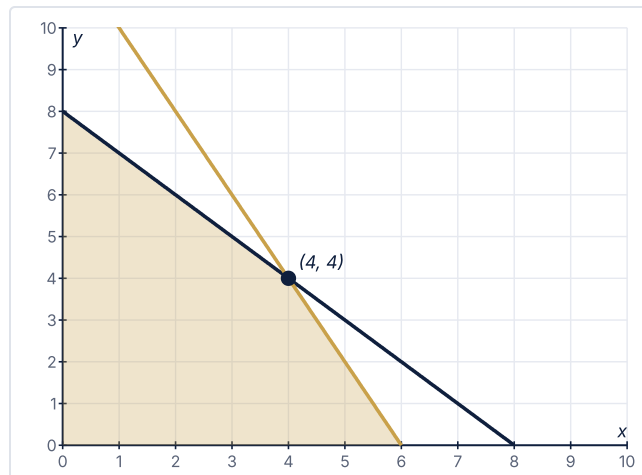
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**EXAMPLE 5: A REAL-WORLD FEASIBLE REGION**

A bakery oven holds at most 8 trays per batch. Cupcake trays use 2 hours of prep, brownie trays use 1 hour, and there are 12 prep hours. Let  $x$  = cupcake trays and  $y$  = brownie trays. Profit is  $3x + 2y$  (in tens of dollars). Find the best plan.



The feasible region for the bakery has corners  $(0,0)$ ,  $(6,0)$ ,  $(4,4)$  and  $(0,8)$ .

**SOLUTION**

Constraints:  $x + y \leq 8$ ,  $2x + y \leq 12$ ,  $x \geq 0$ ,  $y \geq 0$ .

Corners of the feasible region:  $(0,0)$ ,  $(6,0)$ ,  $(0,8)$ , and the crossing of  $x + y = 8$  and  $2x + y = 12$ . Subtract:  $x = 4$ , so  $y = 4$  and the corner is  $(4,4)$ .

Profit at corners:  $(0,0) \rightarrow 0$ ;  $(6,0) \rightarrow 18$ ;  $(0,8) \rightarrow 16$ ;  $(4,4) \rightarrow 12 + 8 = 20$ .

The maximum is 20 (tens of dollars) at 4 cupcake trays and 4 brownie trays.

Is  $(5,4)$  allowed?  $5 + 4 = 9 > 8$ . No, it is outside the region.

**EXAMPLE 6: INEQUALITIES FROM WORDS**

(a) Three less than twice a number is at most 15. Find the numbers. (b) A student scored 78, 90 and 82 on three tests and wants an average of at least 85 after a fourth test. What must the fourth score be?

**SOLUTION**

(a) "Three less than twice a number" is  $2x - 3$ . "At most 15" means  $\leq 15$ . Solve  $2x - 3 \leq 15$ :  $2x \leq 18$ , so  $x \leq 9$ .

(b) Let  $x$  be the fourth score. The average is  $\frac{78+90+82+x}{4} \geq 85$ .

Multiply by 4:  $250 + x \geq 340$ , so  $x \geq 90$ .

The student needs at least 90 on the fourth test. Check:  $\frac{250+90}{4} = 85$ .

**Coach's Playbook**

- 1 **Flip on negatives.** Whenever you multiply or divide by a negative number, reverse the sign. Then test a number to confirm.
- 2 **Line first, shade second.** Draw the boundary with the right style before you think about shading.
- 3 **Always test a point.** Use  $(0,0)$  whenever the line does not pass through it. Substitute and shade toward a true statement.
- 4 **Translate phrases.** "At most" is  $\leq$ , "at least" is  $\geq$ , "fewer than" is  $<$ , "more than" is  $>$ .
- 5 **Respect the story.** Real quantities like hours or items cannot be negative, so add  $x \geq 0$  and  $y \geq 0$  when they apply.

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**Common traps**

Four slips to watch: (1) forgetting to flip the sign when dividing by a negative; (2) using a solid line for a strict inequality, so a boundary point is wrongly counted as a solution; (3) shading "above" without checking after the inequality was rearranged; (4) naming a corner by guessing instead of solving the two boundary equations.

**Mistake Clinic****Spot the error**

A student solves  $-2x + 7 < 1$  and writes: " $-2x < -6$ , so  $x < 3$ ." The error is that the inequality sign was not reversed when dividing by  $-2$ . The correct step is  $x > 3$ . Test it:  $x = 4$  gives  $-8 + 7 = -1 < 1$ , true. The student's  $x = 2$  gives  $3 < 1$ , false.

**Spot the error**

A student graphs  $y > 2x + 1$  and uses the point  $(1, 3)$  to decide where to shade. Since  $3 > 3$  is false, the student shades the side without that point. The error is that  $(1, 3)$  lies **on** the boundary line, so it cannot tell you which side to shade. Use  $(0, 0)$  instead:  $0 > 1$  is false, so shade the side that does **not** contain the origin, which is above the dashed line.

**Spot the error**

A student finds where  $x + y = 8$  and  $2x + y = 12$  cross. Subtracting gives  $x = 4$ , and then the student writes  $y = 12 - 4 = 8$ . The error is leaving out the 2 in  $2x$ . Substitute correctly:  $2(4) + y = 12$ , so  $y = 4$ . The corner is  $(4, 4)$ , which also fits  $4 + 4 = 8$ .

**Real-World Connection**

Factories, farms and delivery companies use feasible regions to decide how much of each product to make. The best plan is almost always at a corner of the region.

**Challenge Corner**

A quiz needs at least 20 points. Multiple-choice items ( $x$  of them) are worth 2 points and short-answer items ( $y$  of them) are worth 3 points. At most 9 items can be answered. Write the system, and find the fewest items that can reach 20 points. Solution:  $2x + 3y \geq 20$ ,  $x + y \leq 9$ ,  $x \geq 0$ ,  $y \geq 0$ . For example  $(4, 4)$  works:  $8 + 12 = 20$  and  $4 + 4 = 8 \leq 9$ . Six items give at most  $6 \cdot 3 = 18$  points, which is not enough. Seven items reach 20 with  $(1, 6)$ :  $2 + 18 = 20$ . So the fewest is 7 items.

**Practice****PRACTICE Linear Inequalities and Systems of Inequalities**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

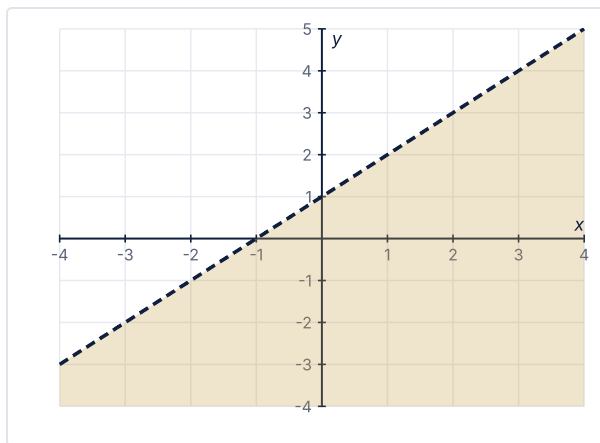
- Which inequality is equivalent to  $4 - 2x \geq 10$ ?  
 (A)  $x \geq -3$                       (B)  $x \geq 3$                       (C)  $x \leq -3$                       (D)  $x \leq 3$
- Which point is a solution of  $y > 3x - 2$ ?  
 (A)  $(1, 2)$                       (B)  $(2, 3)$                       (C)  $(0, -2)$                       (D)  $(-1, -6)$
- Which describes the graph of  $y \leq -2x + 4$ ?  
 (A) A dashed line with shading above                      (B) A solid line with shading above  
 (C) A dashed line with shading below                      (D) A solid line with shading below

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4. Which inequality matches the graph?



Dashed boundary with the lower side shaded.

- (A)  $y > x + 1$       (B)  $y < x + 1$       (C)  $y \leq x + 1$       (D)  $y \geq x + 1$
5. Which point satisfies both  $y \geq x - 1$  and  $y < -x + 5$ ?
- (A) (3, 3)      (B) (1, 3)      (C) (2, 4)      (D) (4, 2)
6. Pencils cost 2 dollars and notebooks cost 5 dollars. A club buys  $x$  pencils and  $y$  notebooks, spends at most 40 dollars, and buys at least 3 notebooks. Which point is a possible purchase?
- (A) (15, 3)      (B) (5, 2)      (C) (10, 4)      (D) (20, 2)
7. Which inequality does the number line show?



A closed circle at -1 with an arrow to the left.

- (A)  $x \leq -1$       (B)  $x < -1$       (C)  $x \geq -1$       (D)  $x > -1$
8. Which inequality is equivalent to  $3(x - 2) \geq 2x + 1$ ?
- (A)  $x \geq -5$       (B)  $x \leq 7$       (C)  $x \geq 5$       (D)  $x \geq 7$
9. A student scored 85 and 92 on two tests and wants an average of at least 88 after a third test. Which inequality gives the possible third score  $x$ ?
- (A)  $x \geq 79$       (B)  $x \geq 88$       (C)  $x \geq 87$       (D)  $x \geq 91$
10. Solve  $-4(x + 1) \leq 2x + 8$ . Write your answer as an inequality.
11. The constraints  $x \geq 0$ ,  $y \geq 0$ ,  $x + y \leq 10$  and  $3x + y \leq 18$  form a region. Find the corner where the lines  $x + y = 10$  and  $3x + y = 18$  cross. Write it as an ordered pair.
12. Using the region in the previous problem, find the maximum value of  $5x + 4y$ .
13. A club buys pens at 3 dollars each and notebooks at 4 dollars each. It must buy at least 10 items in total and can spend at most 36 dollars. What is the greatest number of notebooks it can buy?
14. Solve  $2(3 - x) > 5x - 8$ . Write the answer as an inequality.
15. The boundary lines of  $y \leq 2x - 1$  and  $x + y \leq 11$  cross at one point. Find it as an ordered pair.

## Answers

## Answers and Explanations

Complete solutions for every practice problem, numbered to match. Read the explanation even when your answer is right.

### 11.1 Solving Systems by Graphing and Substitution

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**PRACTICE · SOLVING SYSTEMS BY GRAPHING AND SUBSTITUTION**

1. B. Plug in  $(4, 1)$ . In the second option,  $1 = 4 - 3$  and  $4 + 2(1) = 6$  both hold. The first option fails because  $4 + 2 = 6$ , not 8. The third gives  $1 = 7$ , false. The fourth gives  $2(4) + 1 = 9$ , not 8.
2. C. Set  $2x + 1 = -x + 10$ , so  $3x = 9$  and  $x = 3$ . Then  $y = 2(3) + 1 = 7$ . The pair is  $(3, 7)$ . Choice  $(7, 3)$  swaps the coordinates.
3. D. Rewrite  $6x + 2y = -4$  as  $y = -3x - 2$ . The slope is  $-3$  like the first line, but the intercepts 2 and  $-2$  differ. The lines are parallel, so there is no solution. Substituting gives  $6x + 2(-3x + 2) = -4$ , so  $4 = -4$ , false.
4. A. Substitute:  $2(y + 3) + y = 12$ , so  $3y + 6 = 12$  and  $y = 2$ . Then  $x = 2 + 3 = 5$ . The value 2 is  $y$ , not  $x$ .
5. C. The lines cross where  $x - 1 = -x + 5$ , so  $x = 3$  and  $y = 2$ . The point is  $(3, 2)$ . Choice  $(2, 3)$  has the coordinates reversed.
6. B. Substitute:  $8x - 2(4x - 5) = k$  gives  $8x - 8x + 10 = k$ , so  $k = 10$ . Then the equation reads  $10 = 10$  for every  $x$ . Choosing 5 forgets to multiply by 2;  $-10$  is a sign slip.
7. A. Substitute  $x = 10 - 3y$  into  $2x - y = 6$ :  $2(10 - 3y) - y = 6$ , so  $20 - 7y = 6$  and  $y = 2$ . Then  $x = 4$ . The pair  $(2, 4)$  swaps the coordinates;  $(10, 0)$  fits only the first equation.
8. D. Set  $-x + 6 = 2x$ , so  $x = 2$  and  $y = 4$ . The sum is 6. The values 2 and 4 are single coordinates, not the sum.
9. C. At  $x = 1$  the second line has  $y = -2(1) + 7 = 5$ . The first line must pass through  $(1, 5)$ :  $a(1) + 3 = 5$ , so  $a = 2$ . Choice 5 is the  $y$ -value, not the slope.
10.  $(2, -3)$ . Substitute:  $3x + 4(2x - 7) = -6$ , so  $11x - 28 = -6$ ,  $11x = 22$ ,  $x = 2$ . Then  $y = 2(2) - 7 = -3$ . Check:  $3(2) + 4(-3) = -6$ .
11.  $(11, 3)$ . Substitute:  $2(3y + 2) - 5y = 7$ , so  $6y + 4 - 5y = 7$ ,  $y = 3$ . Then  $x = 3(3) + 2 = 11$ . Check:  $2(11) - 5(3) = 7$ .
12.  $k = 2$ . Divide  $4x + 6y = 18$  by 2 to get  $2x + 3y = 9$ . For infinitely many solutions the first equation must be the same line, so  $kx + 3y = 9$  must equal  $2x + 3y = 9$  and  $k = 2$ .
13.  $(3, 3)$ . Substitute:  $2x + 3(\frac{2}{3}x + 1) = 15$ , so  $2x + 2x + 3 = 15$ ,  $4x = 12$ ,  $x = 3$ . Then  $y = \frac{2}{3}(3) + 1 = 3$ . Check:  $2(3) + 3(3) = 15$ .
14.  $(4, 5)$ . Substitute:  $4x - 3(-x + 9) = 1$ , so  $7x - 27 = 1$  and  $x = 4$ . Then  $y = -4 + 9 = 5$ . Check:  $4(4) - 3(5) = 1$ .
15. 7. Substitute:  $2(4y - 3) + y = 12$ , so  $9y - 6 = 12$  and  $y = 2$ . Then  $x = 4(2) - 3 = 5$ . So  $x + y = 7$ .

**11.2 Solving Systems by Elimination****PRACTICE · SOLVING SYSTEMS BY ELIMINATION**

1. D. Add the left sides and the right sides:  $(2x + 3x) + (5y - 5y) = 9 + 1$ , so  $5x = 10$ . The  $y$ -terms cancel. Choice  $5x = 8$  adds 9 and 1 incorrectly as 8.
2. A. Add the equations:  $6x = 18$ , so  $x = 3$ . Then  $4(3) + 3y = 18$  gives  $3y = 6$  and  $y = 2$ . The value 3 is  $x$ , not  $y$ .
3. B. Multiply the first by 3 and the second by 4:  $9x + 12y = 30$  and  $8x - 12y = 4$ . Add:  $17x = 34$ , so  $x = 2$ . Then  $y = 1$  and  $x + y = 3$ . The values 2 and 1 are single variables.
4. C. Multiply the second equation by 3:  $6x - 9y = 15$ . Subtract from the first:  $0 = -3$ , false. The lines have the same slope and different intercepts, so there is no solution.
5. A. Let the numbers be  $a$  and  $b$ :  $a + b = 25$  and  $a - b = 7$ . Add:  $2a = 32$ , so  $a = 16$  and  $b = 9$ . The larger number is 16; 9 is the smaller.
6. D. Multiply  $x + 2y = 5$  by 3 to get  $3x + 6y = 15$ . The coefficient pair matches when  $k = 6$ , but  $6 \neq 15$  on the right, so the lines are parallel. Choosing 15 mixes up a constant with the coefficient.
7. C. To cancel  $x$  by adding, the  $x$ -coefficients must be opposites. Multiply the first by 3 to get  $6x$  and the second by  $-2$  to get  $-6x$ . Using  $+2$  gives  $6x$  and  $6x$ , which would need subtraction.
8. B. Add the equations:  $10x = 20$ , so  $x = 2$ . Then  $14 + 2y = 8$ , so  $y = -3$ . The value 2 is  $x$ .
9. A. Subtract the equations:  $3y = 24$ , so one ball costs 8 dollars. Then  $3b + 16 = 46$  gives a bat price of 10 dollars. The value 10 is the bat.
10.  $(3, 2)$ . Add the equations:  $7x = 21$ , so  $x = 3$ . Then  $15 + 3y = 21$  gives  $y = 2$ . Check:  $2(3) - 3(2) = 0$ .
11.  $(2, 3)$ . Multiply the first by 2 and the second by 3:  $8x - 6y = -2$  and  $9x + 6y = 36$ . Add:  $17x = 34$ , so  $x = 2$ . Then  $6 + 2y = 12$  gives  $y = 3$ . Check:  $4(2) - 3(3) = -1$ .
12.  $(6, 6)$ . Multiply the first by 6:  $2x + 3y = 30$ . Multiply the second by 6:  $3x - 2y = 6$ . Multiply these by 2 and 3:  $4x + 6y = 60$  and  $9x - 6y = 18$ . Add:  $13x = 78$ , so  $x = 6$ . Then  $y = 6$ .
13. 20. The equations are  $3a + b = 17$  and  $a - 2b = -6$ . Multiply the first by 2:  $6a + 2b = 34$ . Add to the second:  $7a = 28$ , so  $a = 4$ . Then  $b = 17 - 12 = 5$ . The product is  $4 \cdot 5 = 20$ .
14.  $(2, 1)$ . Multiply the first by 2:  $6x - 4y = 8$ . Add to the second:  $11x = 22$ , so  $x = 2$ . Then  $6 - 2y = 4$  gives  $y = 1$ .
15. 21. Multiply the first equation by 3:  $6x + 9y = 21$ . For the second equation to be the same line,  $c = 21$ .

**11.3 Systems in Word Problems****PRACTICE · SYSTEMS IN WORD PROBLEMS**

1. D. Count equation:  $s + a = 200$ . Money equation: student price times students plus adult price times adults,  $6s + 10a = 1520$ . Choice B attaches the prices to the wrong ticket types.
2. C. Substitute  $s = 200 - a$ :  $6(200 - a) + 10a = 1520$ , so  $1200 + 4a = 1520$  and  $a = 80$ . There were 120 student tickets. The value 120 answers a different question.
3. B. Set  $20 + 5m = 50 + 2m$ , so  $3m = 30$  and  $m = 10$ . Both plans cost 70 dollars at 10 months. The value 30 is the difference in joining fees, not the answer.

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4. A. Let  $n$  be the notebook price and  $p$  the pen price:  $2n + 3p = 9.5$  and  $3n + p = 9$ . From the second,  $p = 9 - 3n$ . Then  $2n + 27 - 9n = 9.5$ , so  $n = 2.5$  and  $p = 1.5$ . The value 2.50 is the notebook.
5. B. Let  $x$  be 20% and  $y$  be 50%:  $x + y = 30$  and  $0.2x + 0.5y = 12$ . Substitute  $x = 30 - y$ :  $6 + 0.3y = 12$ , so  $y = 20$ . Choice 10 is the amount of 20% juice.
6. A. Break-even is where cost equals revenue:  $50 + 4x = 9x$ , so  $x = 10$ . At that point both equal 90 dollars. The value 90 is the dollars, not the number of cases.
7. C. Downstream speed is  $12 \div 2 = 6$  and upstream speed is  $12 \div 4 = 3$ . Let  $b + c = 6$  and  $b - c = 3$ . Subtract:  $2c = 3$ , so  $c = 1.5$ . The value 4.5 is the boat speed.
8. D. Let the smaller be  $x$  and the larger  $y = 3x + 4$ . Then  $x + 3x + 4 = 40$ , so  $x = 9$  and  $y = 31$ . The value 31 is the larger number.
9. B. Let  $x$  be the 2% amount and  $y$  the 4% amount:  $x + y = 8000$  and  $0.02x + 0.04y = 260$ . Substitute:  $160 + 0.02y = 260$ , so  $y = 5000$ . The value 3000 is the 2% account.
10. 65 adult tickets. Let  $c + a = 150$  and  $6c + 11a = 1225$ . Substitute  $c = 150 - a$ :  $900 + 5a = 1225$ , so  $a = 65$  (and  $c = 85$ ). Check:  $6(85) + 11(65) = 510 + 715 = 1225$ .
11. 10 milliliters. Let  $x$  be the 12% amount and  $y$  the 4% amount:  $x + y = 40$  and  $0.12x + 0.04y = 2.4$ . Substitute  $y = 40 - x$ :  $1.6 + 0.08x = 2.4$ , so  $x = 10$  (and  $y = 30$ ).
12. 10 kilometers. Set  $3 + 1.2d = 1 + 1.4d$ , so  $2 = 0.2d$  and  $d = 10$ . Both cost 15 dollars at 10 kilometers.
13. 175 kilometers per hour. With the tailwind the speed is  $600 \div 3 = 200$ . Against the wind it is  $600 \div 4 = 150$ . Let  $p + w = 200$  and  $p - w = 150$ . Add:  $2p = 350$ , so  $p = 175$  (and  $w = 25$ ).
14. 40 student tickets. Let  $a + s = 90$  and  $12a + 8s = 920$ . Substitute  $a = 90 - s$ :  $1080 - 4s = 920$ , so  $s = 40$  (and  $a = 50$ ).
15. 10 liters. Let  $x$  be the 5% amount and  $y$  the 20% amount:  $x + y = 30$  and  $0.05x + 0.20y = 3$ . Substitute:  $1.5 + 0.15y = 3$ , so  $y = 10$  (and  $x = 20$ ).

## 11.4 Linear Inequalities and Systems of Inequalities

### PRACTICE · LINEAR INEQUALITIES AND SYSTEMS OF INEQUALITIES

1. C. Subtract 4:  $-2x \geq 6$ . Divide by  $-2$  and reverse the sign:  $x \leq -3$ . Choice  $x \geq -3$  forgets to flip the sign.
2. A. Test each point.  $(1, 2)$ :  $2 > 3 - 2 = 1$  is true.  $(0, -2)$ :  $-2 > -2$  is false, since the inequality is strict and the point lies on the line.  $(2, 3)$ :  $3 > 4$  is false.  $(-1, -6)$ :  $-6 > -5$  is false.
3. D. The sign  $\leq$  includes the line, so it is solid. Test  $(0, 0)$ :  $0 \leq 4$  is true, so shade the side with the origin, which is below. Dashed lines are for strict signs only.
4. B. The line is dashed, so the sign is strict. The shading is below the line, so  $y < x + 1$ . Choice  $y > x + 1$  shades the wrong side.
5. B. Check  $(1, 3)$ :  $3 \geq 0$  and  $3 < 4$ , both true.  $(3, 3)$  fails because  $3 < 2$  is false.  $(2, 4)$  fails because  $4 < 3$  is false.  $(4, 2)$  fails because  $2 \geq 3$  is false.
6. C. Check  $2x + 5y \leq 40$  and  $y \geq 3$ .  $(10, 4)$ :  $20 + 20 = 40 \leq 40$  and  $4 \geq 3$ .  $(15, 3)$  costs  $30 + 15 = 45$ , over budget.  $(5, 2)$  and  $(20, 2)$  have fewer than 3 notebooks.
7. A. The closed circle means the endpoint is included, and the arrow to the left means smaller values. So  $x \leq -1$ . Choice  $x < -1$  would need an open circle.
8. D. Distribute:  $3x - 6 \geq 2x + 1$ . Subtract  $2x$ :  $x - 6 \geq 1$ . Add 6:  $x \geq 7$ . No sign flip occurs because nothing was divided by a negative.
9. C. The average is  $\frac{85+92+x}{3} \geq 88$ . Multiply by 3:  $177 + x \geq 264$ , so  $x \geq 87$ . Choice  $x \geq 88$  forgets that the first two scores already count.
10.  $x \geq -2$ . Distribute:  $-4x - 4 \leq 2x + 8$ . Add  $4x$  and subtract 8:  $-12 \leq 6x$ . Divide by 6:  $-2 \leq x$ , or  $x \geq -2$ . No sign flip is needed since 6 is positive.
11.  $(4, 6)$ . Subtract  $x + y = 10$  from  $3x + y = 18$ :  $2x = 8$ , so  $x = 4$ . Then  $y = 10 - 4 = 6$ . Check:  $3(4) + 6 = 18$ .
12. 44. The corners are  $(0, 0)$ ,  $(6, 0)$ ,  $(4, 6)$  and  $(0, 10)$ . Values of  $5x + 4y$ : 0, 30,  $20 + 24 = 44$ , 40. The largest is 44 at  $(4, 6)$ .
13. 6 notebooks. Let  $x$  be pens and  $y$  notebooks:  $x + y \geq 10$  and  $3x + 4y \leq 36$ . With  $y = 6$ ,  $x \geq 4$  and  $3x \leq 12$ , so  $x = 4$  works (cost  $12 + 24 = 36$ ). With  $y = 7$ ,  $x \geq 3$  costs at least  $9 + 28 = 37 > 36$ . So 6 is the maximum.
14.  $x < 2$ . Distribute:  $6 - 2x > 5x - 8$ . Add  $2x$  and add 8:  $14 > 7x$ . Divide by 7:  $2 > x$ , or  $x < 2$ .
15.  $(4, 7)$ . Substitute  $y = 2x - 1$  into  $x + y = 11$ :  $x + 2x - 1 = 11$ , so  $x = 4$ . Then  $y = 2(4) - 1 = 7$ .

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