



Functions: Notation, Domain, and Range

Four focused lessons, worked examples, a mistake clinic, and 60 practice problems with full explanations.

In this booster

12.1 What Is a Function?

12.2 Evaluating and Interpreting Functions

12.3 Domain and Range

12.4 Transformations, Composition, and Inverses

Every section teaches the idea, walks through the most common problem types, flags the traps students fall into, and ends with fifteen practice problems. Full answers and explanations are at the back.

12.1 What Is a Function?

Understand relations and functions, use the vertical line test, and read and evaluate function notation $f(x)$ with confidence.

Why This Matters

A **function** is a rule that gives exactly one output for each input. A vending machine is a function: press B4 and you always get the same snack. Functions are the language of algebra, science and computing. A spreadsheet formula, a phone app's price calculator and the path of a thrown ball are all functions.

Students get stuck on three ideas. First, the strict meaning of "exactly one output." Second, the notation $f(x)$, which looks like multiplication but is not. Third, the difference between an input and an output when the problem uses unfamiliar letters. This section builds those ideas from the ground up.

Core Ideas

Relation and Function

A **relation** is any set of ordered pairs (x, y) , where x is the **input** and y is the **output**.

A **function** is a relation in which **each input has exactly one output**. The same output may be used by many inputs.

You can test for this in four ways. **Pairs and tables**: look for an input that appears twice with different outputs. **Mapping**: every input has exactly one arrow leaving it. **Equation**: solve for y and see if one x can give two y -values. **Graph**: use the vertical line test.

Input x	1	2	3	4
Output (function)	6	6	9	12
Output (NOT a function, input 2 repeats)	6	6, 8	9	12

Vertical Line Test

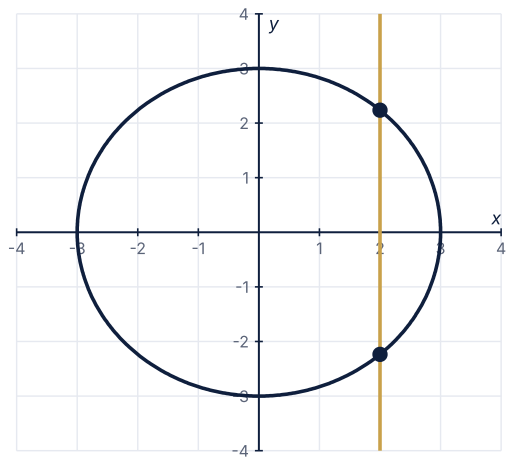
A graph is the graph of a function if **no vertical line crosses it more than once**.

If some vertical line crosses the graph twice, one input has two outputs, so it is not a function.

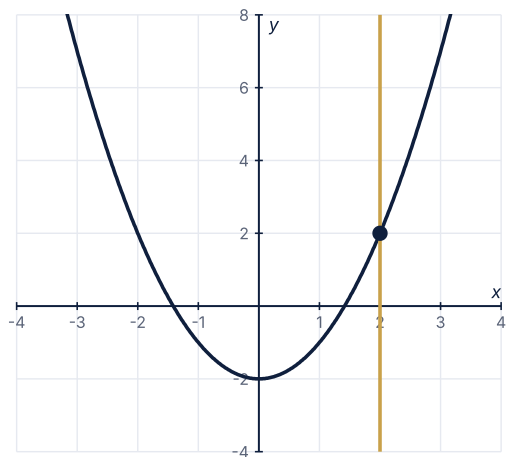
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A circle fails the vertical line test: the line $x = 2$ meets it twice.



$y = x^2 - 2$ passes the test: every vertical line meets it once.

Function Notation

$f(x)$ is read "**f of x**." It means the output of the function f when the input is x . It is **not** f times x .

To evaluate $f(4)$, replace every x in the rule with 4 (use parentheses) and simplify.

Worked Examples

EXAMPLE 1: ORDERED PAIRS

Which relations are functions? (a) $\{(1, 3), (2, 5), (3, 7), (1, 9)\}$ (b) $\{(1, 4), (2, 4), (3, 4)\}$

SOLUTION

(a) The input 1 is paired with both 3 and 9. One input, two outputs, so it is **not** a function.

(b) Each input appears once. The outputs are all 4, which is allowed. It **is** a function (a constant function).

EXAMPLE 2: VERTICAL LINE TEST

Does $x^2 + y^2 = 9$ define y as a function of x ? Does $y = x^2 - 2$?

SOLUTION

For $x^2 + y^2 = 9$, take $x = 2$: $y^2 = 5$, so $y = \pm\sqrt{5}$. One input gives two outputs. **Not** a function (see the circle above).

For $y = x^2 - 2$, each x gives only one y . It **is** a function, and the vertical line test agrees.

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EXAMPLE 3: EVALUATING WITH NOTATION

Let $f(x) = 3x - 5$. Find $f(4)$, $f(-2)$ and $f(a + 1)$.

SOLUTION

$$f(4) = 3(4) - 5 = 12 - 5 = 7.$$

$$f(-2) = 3(-2) - 5 = -6 - 5 = -11.$$

$$f(a + 1) = 3(a + 1) - 5 = 3a + 3 - 5 = 3a - 2. \text{ The input can be an expression.}$$

EXAMPLE 4: WHICH EQUATIONS ARE FUNCTIONS

Decide if y is a function of x : (a) $y = 2x + 1$ (b) $x = y^2$ (c) $y = |x|$ (d) $x = 5$.

SOLUTION

(a) Each x gives one y . Function.

(b) For $x = 4$, $y = 2$ or $y = -2$. Not a function.

(c) The absolute value gives one nonnegative output for each x . Function.

(d) The graph is a vertical line, and $x = 5$ pairs with every y . Not a function.

EXAMPLE 5: SOLVING FOR AN INPUT

Let $f(x) = x^2 - 3x$. Find $f(-2)$, find all x with $f(x) = 10$, and simplify $f(a + 2)$.

SOLUTION

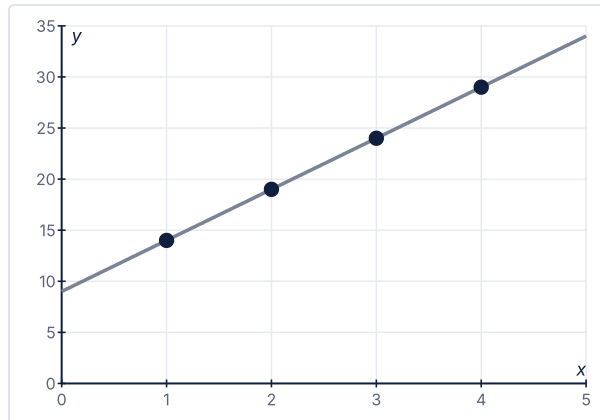
$$f(-2) = (-2)^2 - 3(-2) = 4 + 6 = 10.$$

Set $x^2 - 3x = 10$, so $x^2 - 3x - 10 = 0$ and $(x - 5)(x + 2) = 0$. Then $x = 5$ or $x = -2$. Two different inputs may share one output.

$$f(a + 2) = (a + 2)^2 - 3(a + 2) = a^2 + 4a + 4 - 3a - 6 = a^2 + a - 2.$$

EXAMPLE 6: RULE FROM A TABLE

A tutoring center's price depends on the number of hours t . The table shows the cost $c(t)$. Find a rule for $c(t)$ and find $c(10)$. The values are: $t = 1, 2, 3, 4$ and $c = 14, 19, 24, 29$.



The table points $(1, 14)$, $(2, 19)$, $(3, 24)$, $(4, 29)$ lie on the line $c = 5t + 9$.

SOLUTION

Look at the differences: the cost rises by 5 each hour (14 to 19 to 24 to 29), so the rule looks like $c(t) = 5t + \text{something}$.

Use $t = 1$: $5(1) + b = 14$, so $b = 9$. The rule is $c(t) = 5t + 9$.

Check with $t = 4$: $5(4) + 9 = 29$. It matches the table.

$c(10) = 5(10) + 9 = 59$. The center charges 5 dollars per hour plus a 9-dollar fee, so 10 hours cost 59 dollars.

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- 1 **Say it in words.** Read $f(5)$ as "the output when the input is 5." It keeps you from treating it like multiplication.
- 2 **Use parentheses.** Replace x with the input inside parentheses, especially with negatives: $(-2)^2$ is 4, but -2^2 is -4 .
- 3 **Check repeats.** With a list of pairs, scan the first numbers. A repeated first number with different second numbers means no function.
- 4 **Slide a vertical pencil.** On a graph, sweep an imaginary vertical line from left to right. If it ever touches two points at once, stop: not a function.
- 5 **Mind the letters.** The input may be called t , n or h . The rule is what matters, not the letter.

Common traps

Four slips to watch: (1) calling a relation "not a function" because two inputs share the same output, which is allowed; (2) reading $f(x)$ as $f \cdot x$; (3) dropping the parentheses and getting -2^2 instead of $(-2)^2$; (4) using a horizontal line instead of a vertical line for the test.

Mistake Clinic

Spot the error

A student evaluates $f(x) = x^2 - 3x$ at $x = -2$ and writes $f(-2) = -2^2 - 3(-2) = -4 + 6 = 2$. The error is that -2^2 means the square of 2 with a minus in front. The input -2 must be in parentheses: $(-2)^2 = 4$. So $f(-2) = 4 + 6 = 10$.

Spot the error

A student looks at $\{(1, 4), (2, 4), (3, 4)\}$ and says, "It is not a function because 4 appears three times." The error is that repeated **outputs** are allowed. A function only forbids one **input** with two different outputs. Here each input appears once, so it is a function.

Spot the error

A student has $f(x) = 2x + 3$ and finds $f(a + 1)$ as $2a + 3 + 1 = 2a + 4$. The error is that the whole input $a + 1$ replaces x . Use parentheses: $f(a + 1) = 2(a + 1) + 3 = 2a + 2 + 3 = 2a + 5$.

Real-World Connection

A parking meter is a function: the amount of money you insert (input) gives a single parking time (output). Two different amounts could give the same time, but one amount never gives two different times.

Challenge Corner

A linear function $f(x) = ax + b$ has $f(1) = 7$ and $f(3) = 15$. Find a , b and $f(10)$. Solution: $a + b = 7$ and $3a + b = 15$. Subtract the first from the second: $2a = 8$, so $a = 4$ and $b = 3$. The rule is $f(x) = 4x + 3$, and $f(10) = 43$. Check: $f(3) = 12 + 3 = 15$.

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Practice

PRACTICE What Is a Function?

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

1. Which relation is NOT a function?

(A) $\{(0, 1), (1, 2), (2, 3)\}$ (B) $\{(0, 1), (1, 1), (2, 1)\}$ (C) $\{(1, 0), (1, 1), (2, 2)\}$
 (D) $\{(5, 2), (6, 3)\}$

2. If $f(x) = -2x^2 + 3x$, what is $f(-1)$?

(A) -5 (B) -1 (C) 1 (D) 5

3. The table shows a function with a constant rate of change. What is $f(10)$?

x	0	1	2	3
$f(x)$	5	8	11	14

(A) 30 (B) 35 (C) 38 (D) 50

4. Which equation does NOT define y as a function of x ?

(A) $y = 3x - 1$ (B) $y = x^2 + 4$ (C) $y = |x - 2|$
 (D) $x^2 + y^2 = 25$

5. The function $f(h) = 12h + 8$ gives the cost, in dollars, of renting a bike for h hours. What does $f(3) = 44$ mean?

(A) Renting for 44 hours costs 3 dollars
 (B) Each hour costs 44 dollars
 (C) Renting for 3 hours costs 44 dollars
 (D) The bike is rented 3 times for 44 dollars each

6. If $f(x) = x^2 - 4x$, which expression equals $f(a + 1)$?

(A) $a^2 - 2a - 3$ (B) $a^2 - 4a - 3$ (C) $a^2 - 4a + 1$ (D) $a^2 - 2a + 1$

7. Which set of ordered pairs is a function?

(A) $\{(2, 1), (2, 3), (4, 5)\}$ (B) $\{(1, 2), (3, 2), (5, 2)\}$ (C) $\{(0, 1), (0, 2), (0, 3)\}$
 (D) $\{(4, 1), (5, 2), (4, 3)\}$

8. If $f(x) = \sqrt{2x + 1}$, what is $f(12)$?

(A) 3 (B) 6 (C) 25 (D) 5

9. If $f(x) = 2x - 1$ and $f(k) = 9$, what is k ?

(A) 17 (B) 4 (C) 5 (D) 19

10. If $f(x) = 4x - 7$, find $f(6) - f(2)$.

11. If $g(x) = x^2 + 2x - 8$, find all values of x for which $g(x) = 0$.

12. The function $f(x) = 3x + k$ satisfies $f(2) = 11$. Find $f(10)$.

13. The relation $\{(1, 4), (3, 7), (1, 2a - 6), (5, 9)\}$ is a function. Find a .

14. If $g(x) = |x - 5| + 2$, find $g(1) + g(8)$.

15. If $f(x) = x^2 - 6x + 9$, find all values of x for which $f(x) = 16$.

12.2 Evaluating and Interpreting Functions

Evaluate functions from formulas, tables and graphs, solve for inputs, and explain what function values and rates of change mean in real situations.

Why This Matters

Knowing what a function is gets you started. The real power comes from using it: plugging in numbers, reading values from a graph or a table, and explaining what an answer means in a story. When a weather app gives temperature as a function of time, or a store gives cost as a function of items, every question you ask is one of two kinds: "What is the output for this input?" or "What input gives this output?"

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Students most often mix up these two directions, make sign mistakes when substituting negative numbers, and read the wrong axis on a graph. Many also feel lost when a function appears in words, such as $C(25) = 1150$. This section shows how to evaluate, read and interpret with confidence.

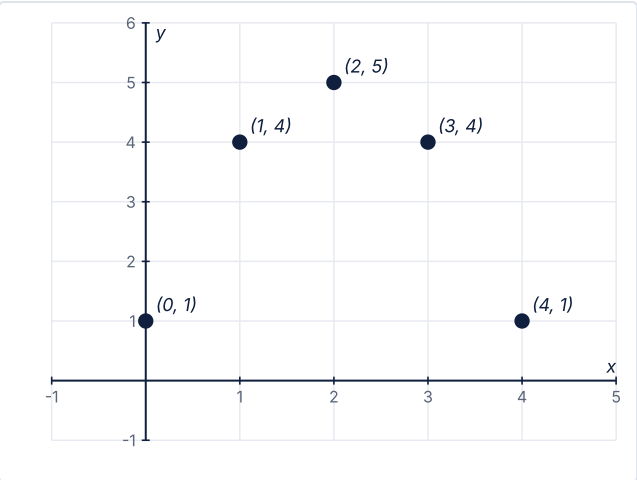
Core Ideas

Evaluating a Function

To find $f(a)$, replace every x in the rule with a (in parentheses) and simplify using the order of operations.
The input can be a number, a negative number, a fraction or an expression like $a + 1$.

There are three ways functions are shown, and each has its own way of evaluating.

Representation	To find $f(a)$ (input to output)	To solve $f(x) = k$ (output to input)
Formula	Substitute a for x and simplify	Set the formula equal to k and solve the equation
Table	Find a in the input row and read the output	Find k in the output row and read the inputs (there can be several)
Graph	Go to $x = a$ on the horizontal axis, then up or down to the curve, then across to the y -axis	Go to $y = k$ on the vertical axis, then across to the curve, then down to the x -axis



The graph of $f(x) = -(x - 2)^2 + 5$ with five labeled points.

Reading Notation in Context

$f(a) = b$ means: **when the input is a , the output is b** . It is the point (a, b) on the graph.
In a story, say it with units: "When the time is 3 seconds, the height is 45 meters."

A function in context often has a **starting value** (the output when the input is 0) and a **rate of change** (how much the output changes for each 1 unit of input). For $C(n) = 40n + 150$, the 150 is the starting value, a fixed fee, and the 40 is the rate, dollars per student.

Worked Examples

EXAMPLE 1: EVALUATE A FORMULA

Let $f(x) = 2x^2 - 5x + 1$. Find $f(3)$, $f(-2)$ and $f(\frac{1}{2})$.

SOLUTION

$f(3) = 2(9) - 5(3) + 1 = 18 - 15 + 1 = 4$.
 $f(-2) = 2(4) - 5(-2) + 1 = 8 + 10 + 1 = 19$. Square before multiplying by 2.
 $f(\frac{1}{2}) = 2 \cdot \frac{1}{4} - 5 \cdot \frac{1}{2} + 1 = \frac{1}{2} - \frac{5}{2} + 1 = -1$.

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EXAMPLE 2: READ A TABLE

The table gives a function g . Find $g(2)$, find all x with $g(x) = 4$, and find $g(g(1))$. Values: $x = 0, 1, 2, 3, 4$ and $g(x) = 6, 4, 3, 3, 4$.

SOLUTION

$g(2) = 3$: look under $x = 2$.

$g(x) = 4$ happens in two columns, $x = 1$ and $x = 4$.

For $g(g(1))$ work from the inside: $g(1) = 4$, then $g(4) = 4$. So $g(g(1)) = 4$.

x	0	1	2	3	4
$g(x)$	6	4	3	3	4

EXAMPLE 3: READ A GRAPH

Use the graph of f above. Find $f(0)$, $f(2)$, and all x with $f(x) = 4$.

SOLUTION

$f(0) = 1$: at $x = 0$ the curve is at height 1.

$f(2) = 5$: the top of the curve.

For $f(x) = 4$, go across at height 4. The line meets the curve at $x = 1$ and $x = 3$.

Check with the formula: $f(1) = -(1-2)^2 + 5 = 4$ and $f(3) = -(3-2)^2 + 5 = 4$.

EXAMPLE 4: A BALL IN THE AIR

A ball's height in meters after t seconds is $h(t) = -5t^2 + 20t$. Find $h(1)$, $h(2)$, $h(4)$ and the times when the height is 15 meters.

SOLUTION

$h(1) = -5 + 20 = 15$. $h(2) = -20 + 40 = 20$. $h(4) = -80 + 80 = 0$.

$h(4) = 0$ means the ball is back on the ground after 4 seconds.

Solve $-5t^2 + 20t = 15$: divide by -5 to get $t^2 - 4t + 3 = 0$, so $(t-1)(t-3) = 0$. The height is 15 meters at $t = 1$ and $t = 3$ seconds.

EXAMPLE 5: COST FUNCTION

A class trip costs $C(n) = 40n + 150$ dollars for n students. Find $C(25)$, find n if $C(n) = 1350$, and explain what 40 and 150 mean.

SOLUTION

$C(25) = 40(25) + 150 = 1000 + 150 = 1150$. Twenty-five students cost 1,150 dollars in total.

Solve $40n + 150 = 1350$: $40n = 1200$ and $n = 30$ students.

The 40 is the cost per student. The 150 is a fixed fee that applies even with zero students.

Per-student cost for 25 students: $1150 \div 25 = 46$ dollars, which is more than 40 because of the fixed fee.

EXAMPLE 6: AVERAGE RATE OF CHANGE

Using $h(t) = -5t^2 + 20t$, find the average rate of change of the height from $t = 0$ to $t = 2$ and from $t = 2$ to $t = 4$. Interpret each.

SOLUTION

Average rate of change is $\frac{h(b)-h(a)}{b-a}$.

From 0 to 2: $\frac{20-0}{2} = 10$ meters per second. The ball rises 10 meters each second on average.

From 2 to 4: $\frac{0-20}{2} = -10$ meters per second. The ball falls 10 meters each second on average.

The negative sign means the height is decreasing.

Coach's Playbook

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- 1 **Substitute in parentheses.** Write $f(-2) = 2(-2)^2 - 5(-2) + 1$. Parentheses protect every sign.
- 2 **Follow the order of operations.** Exponents, then multiplication, then addition and subtraction. Do not combine across terms early.
- 3 **Know which axis is which.** Inputs live on the horizontal axis and outputs on the vertical axis. $f(a)$ starts at a on the horizontal axis.
- 4 **Expect more than one.** An output can come from several inputs, so $f(x) = k$ may have two or more answers.
- 5 **Write sentences with units.** "After 3 hours the cost is 44 dollars" proves you understand the notation.

Common traps

Four slips to watch: (1) squaring only the coefficient or forgetting the negative inside the square; (2) confusing $f(a) = b$ with $f(b) = a$; (3) reading an x -value from the y -axis; (4) stopping after finding one input for $f(x) = k$ when the graph shows two.

Mistake Clinic

Spot the error

A student evaluates $f(x) = 2x^2 - 5x + 1$ at $x = 3$ and writes $(2 \cdot 3)^2 - 5 \cdot 3 + 1 = 36 - 15 + 1 = 22$. The error is that the exponent applies only to x , not to the 2. The correct work is $2(3)^2 - 5(3) + 1 = 18 - 15 + 1 = 4$.

Spot the error

The graph of $f(x) = -(x - 2)^2 + 5$ is shown above. A student is asked for which x the value $f(x) = 4$ and answers " $x = 4$." The error is reading 4 as an input. The question gives an **output** of 4, so find height 4 on the vertical axis and read across. The inputs are $x = 1$ and $x = 3$.

Spot the error

A student sees $C(25) = 1150$ for the trip cost $C(n) = 40n + 150$ and says, "Each student pays 1,150 dollars." The error is misreading the output. $C(25)$ is the **total** cost for 25 students. The cost per student is $1150 \div 25 = 46$ dollars.

Real-World Connection

A phone plan, a taxi ride and a streaming service all follow cost functions like $C(n) = 40n + 150$. Reading the starting value and the rate tells you the plan's fixed fee and the price of each additional unit.

Challenge Corner

The height of a ball is $h(t) = -5t^2 + 20t$ meters. For which times is the ball higher than 15 meters? Solution: solve $-5t^2 + 20t > 15$. Divide by -5 and reverse the sign: $t^2 - 4t + 3 < 0$, so $(t - 1)(t - 3) < 0$. This is true between the roots: $1 < t < 3$. The ball is above 15 meters between 1 and 3 seconds (it is exactly 15 at both ends, which matches Example 4).

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Practice

PRACTICE Evaluating and Interpreting Functions

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

1. If $f(x) = 3x^2 - x + 2$, what is $f(-2)$?

(A) 12

(B) -8

(C) 16

(D) 40

2. The table shows a function f . What is $f(f(3))$?

x	1	2	3	4	5
$f(x)$	9	7	5	3	1

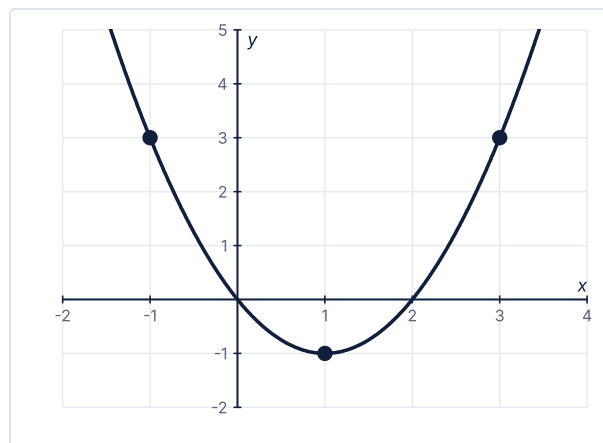
(A) 5

(B) 1

(C) 3

(D) 7

3. The graph shows $f(x) = x^2 - 2x$. For which values of x does $f(x) = 3$?



Graph of $f(x) = x^2 - 2x$.

(A) $x = 3$ only(B) $x = -1$ only(C) $x = 1$ and $x = 3$ (D) $x = -1$ and $x = 3$

4. A phone's battery percentage is $B(h) = 100 - 8h$ after h hours. What does $B(5) = 60$ mean?

(A) After 5 hours, 60 percent of the battery remains

(B) After 60 hours, 5 percent remains

(C) The battery loses 60 percent each hour

(D) The battery loses 5 percent every 60 hours

5. A ball's height is $h(t) = -5t^2 + 30t$ meters after t seconds. At which times is the height 40 meters?

(A) $t = 2$ only(B) $t = 4$ only(C) $t = 2$ and $t = 4$ (D) $t = 40$

6. A taxi fare is $F(m) = 2.5m + 4$ dollars for m miles. What does $\frac{F(10) - F(4)}{6} = 2.5$ represent?

(A) The starting fee in dollars

(B) The cost of the whole 10-mile ride

(C) The number of miles in 6 dollars

(D) The average cost per additional mile, in dollars

7. If $f(x) = x^2 - 4x$, what is $f(5) - f(1)$?

(A) 8

(B) 0

(C) 2

(D) -8

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8. The table shows a function p . For which input or inputs is $p(x) = 7$?

x	0	1	2	3	4
$p(x)$	10	7	7	5	2

- (A) $x = 7$ (B) $x = 1$ and $x = 2$ (C) $x = 2$ only
 (D) $x = 1$ only
9. If $f(x) = 5 - 2x$ and $f(a) = -3$, what is a ?
 (A) -4 (B) 1 (C) 4 (D) 8
10. If $f(x) = 2x^2 - x - 5$, find $f(-2)$.
11. The table gives a linear function r : $x = 0, 2, 4, 6, 8$ and $r(x) = 3, 7, 11, 15, 19$. Find $r(15)$.
12. If $g(x) = 2x^2 - 8$, find all x with $g(x) = 24$.
13. A water tank holds $A(t) = 120 - 15t$ gallons after t minutes. After how many minutes does the tank hold 45 gallons?
14. If $f(x) = 3x^2 - 2x$, simplify $f(a + 1) - f(a)$.
15. The function $f(x) = ax^2 + 1$ satisfies $f(2) = 13$. Find $f(5)$.

12.3 Domain and Range

Find the domain and range of a function from its graph, its equation, and its context, including the restrictions caused by fractions and square roots.

Why This Matters

Every function has two sets attached to it. The **domain** is the set of inputs that are allowed. The **range** is the set of outputs that actually happen. A calculator that shows "error" when you try to divide by zero is telling you that the input is outside the domain. A store that sells 120 seats cannot sell -5 tickets or 20.5 tickets: the story restricts the domain.

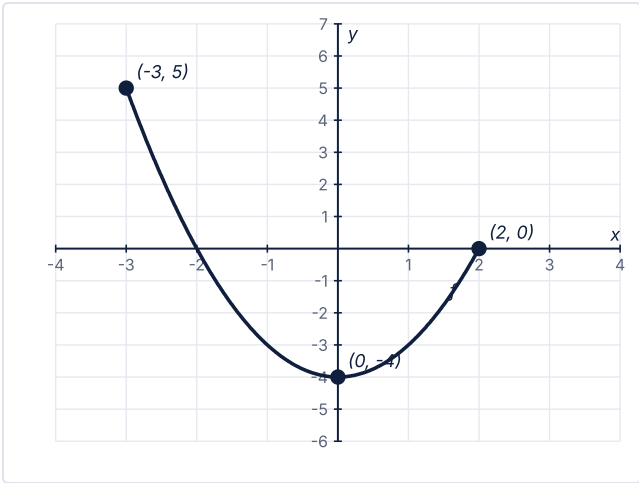
Students struggle for three reasons. They mix up domain and range, they forget that some inputs are forbidden (zero in a denominator, negatives under a square root), and they have trouble writing answers in the right notation. Mastering this section prepares you for every later function topic.

Core Ideas

Domain and Range

The **domain** is the set of all allowed inputs (x -values).
 The **range** is the set of all outputs (y -values) the function produces.
 On a graph, the domain is the horizontal spread of the graph and the range is the vertical spread.

Think of the domain as the left-to-right extent and the range as the bottom-to-top extent. To read a domain, look at how far the graph goes left and right. To read a range, look at how low and high it goes.



$f(x) = x^2 - 4$ on $-3 \leq x \leq 2$. Domain: x from -3 to 2 . Range: y from -4 to 5 .

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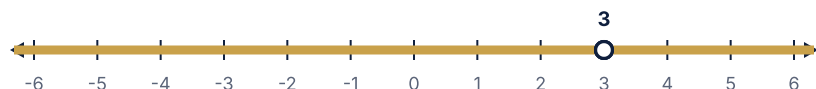
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Restrictions on the Domain

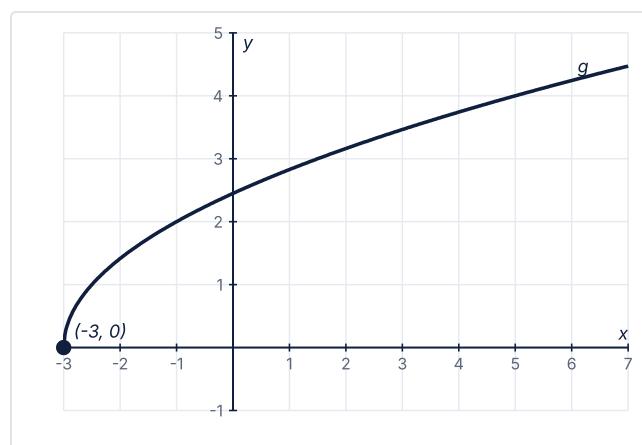
- 1. Denominators cannot be zero.** For $\frac{1}{g(x)}$, exclude any x with $g(x) = 0$.
- 2. Even roots need nonnegative inputs.** For $\sqrt{g(x)}$, require $g(x) \geq 0$.
- 3. Context may restrict inputs.** Counts must be whole numbers, and time and length cannot be negative.

Answers can be written in three ways. Choose the style your teacher prefers, and be able to move between them.

Inequality	Interval notation	Number line
$x \geq -3$	$[-3, \infty)$	Closed circle at -3 , arrow right
$x > 2$	$(2, \infty)$	Open circle at 2 , arrow right
$-2 \leq x < 3$	$[-2, 3)$	Closed at -2 , open at 3
$x \neq 3$	$(-\infty, 3) \cup (3, \infty)$	Open circle at 3 , line on both sides



The domain of $5/(x - 3)$: every number except 3.



$g(x) = \sqrt{2x + 6}$ starts at $x = -3$. Domain: $x \geq -3$. Range: $y \geq 0$.

Worked Examples

EXAMPLE 1: FROM A GRAPH

Use the graph of $f(x) = x^2 - 4$ above (from $x = -3$ to $x = 2$, endpoints included). Find the domain and the range.

SOLUTION

Domain: the graph runs from $x = -3$ to $x = 2$, both included, so $-3 \leq x \leq 2$, or $[-3, 2]$.

Range: the lowest point is $(0, -4)$ and the highest is the left endpoint $(-3, 5)$ since $f(-3) = 9 - 4 = 5$ and $f(2) = 0$.

So the range is $-4 \leq y \leq 5$, or $[-4, 5]$.

The range is not $[-3, 2]$. Those are inputs.

EXAMPLE 2: A FRACTION

Find the domain and range of $f(x) = \frac{5}{x-3}$.

SOLUTION

The denominator is zero when $x - 3 = 0$, so $x = 3$. Exclude it.

Domain: all real numbers except 3, written $x \neq 3$ or $(-\infty, 3) \cup (3, \infty)$.

Range: a fraction with a nonzero numerator can never equal 0, since $5 \div (x - 3) = 0$ has no solution. Every other number is possible.

Range: all real numbers except 0, $y \neq 0$.

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EXAMPLE 3: A SQUARE ROOT

Find the domain and range of $g(x) = \sqrt{2x+6}$.

SOLUTION

Require $2x+6 \geq 0$. Subtract 6: $2x \geq -6$. Divide by 2: $x \geq -3$.

Domain: $x \geq -3$, or $[-3, \infty)$.

A square root gives a nonnegative result, and it grows without bound, so the range is $y \geq 0$, or $[0, \infty)$.

The graph above starts at $(-3, 0)$ and rises to the right.

EXAMPLE 4: A DOMAIN FROM CONTEXT

A theater has 120 seats. The revenue for selling n tickets at 9 dollars each is $R(n) = 9n$. Find the sensible domain and range.

SOLUTION

Tickets are counted, so n is a whole number, and $0 \leq n \leq 120$.

Domain: $\{0, 1, 2, \dots, 120\}$.

Range: the outputs are $9 \cdot 0, 9 \cdot 1, \dots, 9 \cdot 120$, so the multiples of 9 from 0 to 1080.

Notice the formula $9n$ alone allows any real number. The story is what restricts it.

EXAMPLE 5: COMBINING RESTRICTIONS

Find the domain of $h(x) = \frac{\sqrt{x+5}}{x-1}$ and the range of $q(x) = x^2 + 3$.

SOLUTION

For h : the root needs $x+5 \geq 0$, so $x \geq -5$. The denominator needs $x \neq 1$.

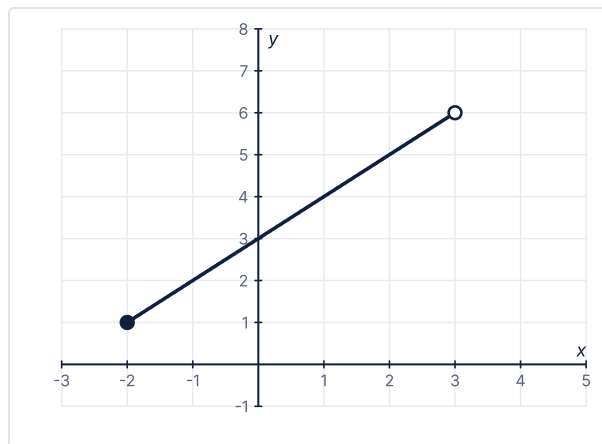
Domain of h : $x \geq -5$ and $x \neq 1$, or $[-5, 1) \cup (1, \infty)$.

For q : $x^2 \geq 0$ for every real x , so $x^2 + 3 \geq 3$.

Range of q : $y \geq 3$, or $[3, \infty)$. The minimum 3 occurs at $x = 0$.

EXAMPLE 6: OPEN AND CLOSED ENDPOINTS

The graph of a function is the segment of $y = x + 3$ that starts at $(-2, 1)$ with a closed dot and ends at $(3, 6)$ with an open dot. Find the domain and range.



A segment with one closed and one open endpoint.

SOLUTION

Domain: from $x = -2$ (included) to $x = 3$ (excluded): $-2 \leq x < 3$, or $[-2, 3)$.

Range: from $y = 1$ (included) to $y = 6$ (excluded): $1 \leq y < 6$, or $[1, 6)$.

Closed dot means included (square bracket). Open dot means excluded (parenthesis).

Coach's Playbook

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- 1 **Domain is left to right.** Scan the graph horizontally for domain and vertically for range. Say it out loud: "x for domain."
- 2 **Hunt the troublemakers.** Check each formula for a denominator or an even root. Those are the only two common sources of restrictions at this level.
- 3 **Set it up.** For a denominator, solve (denominator) = 0 and exclude the answers. For a root, solve (inside) ≥ 0 .
- 4 **Think about shape for range.** A parabola opening up has a lowest point. A square root starts at 0. Ask: what is the smallest and largest output?
- 5 **Check the dots.** Closed dot means include (bracket or $\leq$). Open dot means exclude (parenthesis or $<$).

Common traps

Four slips to watch: (1) giving the x -extent when the question asks for the range; (2) forgetting that a square root allows zero inside, so the sign is $\geq$, not $>$; (3) excluding only one value when the denominator has two zeros, such as $x^2 - 9$; (4) ignoring the story and giving all real numbers when the quantity must be a whole number or nonnegative.

Mistake Clinic

Spot the error

A student finds the domain of $f(x) = \sqrt{x-4}$ and writes $x > 4$. The error is excluding 4. At $x = 4$ the function is $\sqrt{0} = 0$, a fine output. The inside only needs to be nonnegative: $x - 4 \geq 0$ gives $x \geq 4$.

Spot the error

A student finds the domain of $\frac{1}{x^2-9}$ and writes " $x \neq 3$." The error is stopping at one zero. Solve $x^2 - 9 = 0$: $(x-3)(x+3) = 0$ gives $x = 3$ **and** $x = -3$. The domain is all real numbers except 3 and -3 .

Spot the error

For the graph of $y = x^2 - 4$ on $-3 \leq x \leq 2$, a student says the range is $[-3, 2]$. The error is reading the horizontal extent. Those are inputs. The lowest output is -4 and the highest is 5, so the range is $[-4, 5]$.

Real-World Connection

A phone app limits you to between 1 and 10 photos per post. The domain is whole numbers from 1 to 10. The total storage used has a range you can list. Every time a real quantity has a limit, you are using domain and range.

Challenge Corner

Find the domain of $g(x) = \frac{\sqrt{6-2x}}{x+1}$. Solution: the root needs $6 - 2x \geq 0$. Subtract 6 and divide by -2 , reversing the sign: $x \leq 3$. The denominator needs $x \neq -1$. Both conditions apply, and -1 is less than 3, so remove it from the interval. Domain: $x \leq 3$ with $x \neq -1$, or $(-\infty, -1) \cup (-1, 3]$.

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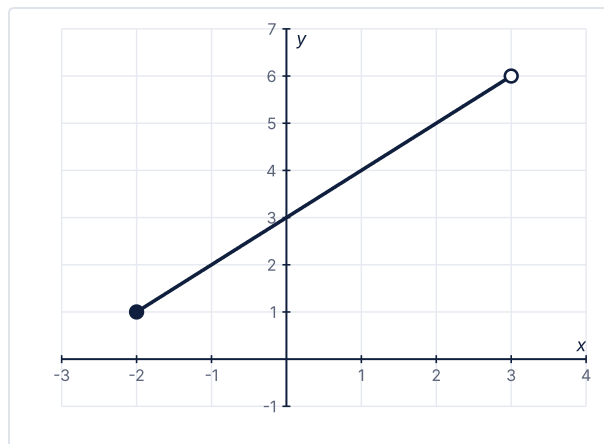
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Practice

PRACTICE Domain and Range

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What is the domain of $f(x) = \frac{7}{x+2}$?
 (A) $x \neq 2$ (B) $x \neq -2$ (C) $x \neq 7$
 (D) All real numbers
- What is the domain of $g(x) = \sqrt{x-4}$?
 (A) $x \leq 4$ (B) $x > 4$ (C) $x \geq 4$ (D) $x \geq -4$
- What is the range of $y = x^2 + 1$?
 (A) $y \geq 1$ (B) $y \geq 0$ (C) $y \leq 1$
 (D) All real numbers
- A function is graphed as a segment from a closed dot at $(-2, 1)$ to an open dot at $(3, 6)$. What is its domain?



A segment with a closed left end and an open right end.

- (A) $-2 < x \leq 3$ (B) $1 \leq x < 6$ (C) $-2 \leq x \leq 3$ (D) $-2 \leq x < 3$
- A bakery packs 6 muffins in each box and has at most 20 boxes. The number of muffins is $N(b) = 6b$. What is the sensible domain?
 (A) All real numbers (B) The whole numbers from 0 to 20
 (C) The multiples of 6 from 0 to 120 (D) $0 \leq b \leq 120$
- What is the domain of $g(x) = \frac{\sqrt{x+1}}{x-3}$?
 (A) $x \geq -1$ (B) $x \neq 3$ (C) $x \geq -1$ and $x \neq 3$
 (D) $x \leq -1$ and $x \neq 3$
- What is the range of $y = |x| + 2$?
 (A) $y \geq 0$ (B) $y \leq 2$ (C) $y \geq -2$ (D) $y \geq 2$
- What values of x are NOT in the domain of $f(x) = \frac{1}{x^2-16}$?
 (A) $x = 4$ and $x = -4$ (B) $x = 4$ only (C) $x = 16$
 (D) $x = 0$
- A ball's height is $h(t) = -16t^2 + 64t$ feet for $0 \leq t \leq 4$ seconds, and its highest point is 64 feet. What is the range?
 (A) $0 \leq h \leq 4$ (B) $0 \leq h \leq 64$ (C) $h \leq 64$ (D) $0 \leq h \leq 128$
- Write the domain of $f(x) = \sqrt{3x-12}$ as an inequality.
- Write the range of $y = -x^2 + 6$ as an inequality.
- List all values excluded from the domain of $f(x) = \frac{x+1}{x^2-9}$.
- A rectangle has perimeter 40 and one side x , so its area is $A(x) = x(20-x)$ for $0 < x < 20$. What is the largest value in the range of A ?
- Write the domain of $f(x) = \frac{\sqrt{x+1}}{x-2}$ in words or inequalities.
- Write the range of $f(x) = \sqrt{x-1} + 3$ as an inequality.

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12.4

Transformations, Composition, and Inverses

Shift, stretch and reflect graphs, compose functions, find and check inverses, and evaluate piecewise functions step by step.

Why This Matters

Once you know what functions are, the next skill is changing them and combining them. A **transformation** moves, stretches or flips a graph, which is how designers adjust a curve without starting over. **Composition** chains two functions so that the output of one becomes the input of the next, like converting units in two steps. An **inverse** undoes a function, like turning a Fahrenheit reading back into Celsius. **Piecewise** functions use different rules on different parts of the domain, such as taxi fares or tax brackets.

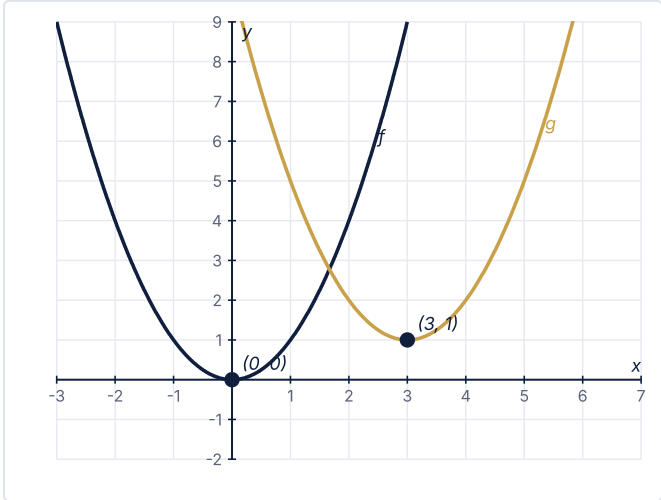
Students struggle with the direction of horizontal shifts (it feels backward), with the order in which functions are applied in a composition, and with the difference between an inverse and a reciprocal. This section gives you clear rules and checks for each.

Core Ideas

Transformations of $y = f(x)$

$y = f(x) + k$ shifts the graph **up** k units (down if $k < 0$).
 $y = f(x - h)$ shifts the graph **right** h units (left if $h < 0$).
 $y = -f(x)$ reflects over the x -axis. $y = f(-x)$ reflects over the y -axis.
 $y = a f(x)$ stretches vertically by a factor of $|a|$ (and reflects if $a < 0$).

Change to the rule	Effect on the graph	Effect on a point (p, q)
$f(x) + 3$	Up 3	$(p, q + 3)$
$f(x - 2)$	Right 2	$(p + 2, q)$
$f(x + 2)$	Left 2	$(p - 2, q)$
$-f(x)$	Reflect over the x -axis	$(p, -q)$
$f(-x)$	Reflect over the y -axis	$(-p, q)$
$2f(x)$	Vertical stretch by 2	$(p, 2q)$



$g(x) = (x - 3)^2 + 1$ is $f(x) = x^2$ moved right 3 and up 1: the vertex goes from $(0,0)$ to $(3,1)$.

Composition and Inverse

Composition: $(f \circ g)(x) = f(g(x))$. Work from the inside out: compute $g(x)$ first, then apply f to the result.

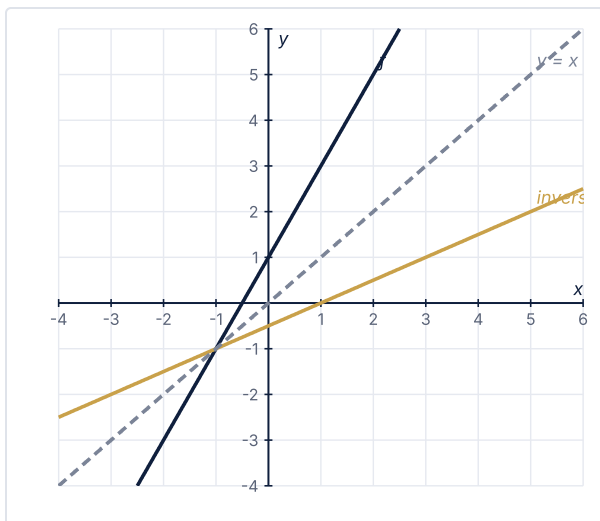
Inverse: f^{-1} undoes f . If $f(a) = b$ then $f^{-1}(b) = a$, and $f(f^{-1}(x)) = x$. To find it, write $y = f(x)$, swap x and y , and solve for y .

The graph of an inverse is the mirror image of the original across the line $y = x$. Inputs and outputs swap, so the domain of f becomes the range of f^{-1} , and the other way around.

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$f(x) = 2x + 1$ and its inverse $(x - 1)/2$ are mirror images across the dashed line $y = x$.

Piecewise Functions

A **piecewise function** uses different rules on different parts of its domain. To evaluate it, first decide which interval the input belongs to, then use only that rule.

At a boundary, check which piece includes the endpoint ($\leq$ or $\geq$ versus $<$ or $>$).

Worked Examples

EXAMPLE 1: DESCRIBE A TRANSFORMATION

Describe how the graph of $g(x) = (x - 3)^2 + 1$ and the graph of $h(x) = -2x^2$ come from $f(x) = x^2$.

SOLUTION

For g : $x - 3$ inside means **right 3**. The $+1$ outside means **up 1**. The vertex moves from $(0, 0)$ to $(3, 1)$ (see the graph above).

For h : the minus sign **reflects** the parabola over the x -axis, and the 2 **stretches** it vertically by 2, so it opens downward and is narrower. The vertex stays at $(0, 0)$.

EXAMPLE 2: MOVE A POINT

The point $(1, 4)$ is on the graph of f . Find the corresponding point on (a) $g(x) = 2f(x + 1) - 1$, (b) $-f(x)$, (c) $f(-x)$.

SOLUTION

(a) Inside: $x + 1 = 1$ gives $x = 0$ (shift left 1). Output: $2(4) - 1 = 7$. The point is $(0, 7)$.

(b) The output changes sign: $(1, -4)$.

(c) The input needs $-x = 1$, so $x = -1$. The point is $(-1, 4)$.

EXAMPLE 3: COMPOSITION

Let $f(x) = 2x + 1$ and $g(x) = x^2 - 3$. Find $f(g(2))$, $g(f(2))$, $f(g(x))$ and $g(f(x))$.

SOLUTION

$g(2) = 4 - 3 = 1$, so $f(g(2)) = f(1) = 3$.

$f(2) = 5$, so $g(f(2)) = g(5) = 25 - 3 = 22$.

$f(g(x)) = 2(x^2 - 3) + 1 = 2x^2 - 5$.

$g(f(x)) = (2x + 1)^2 - 3 = 4x^2 + 4x + 1 - 3 = 4x^2 + 4x - 2$.

Composition is **not** commutative: the order changes the answer.

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EXAMPLE 4: FIND AN INVERSE

Find the inverse of $f(x) = 3x - 5$ and verify it.

SOLUTION

Write $y = 3x - 5$.

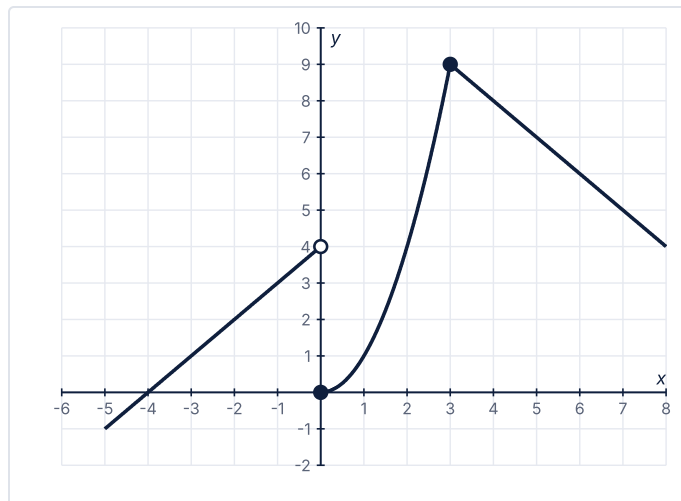
Swap x and y : $x = 3y - 5$.

Solve for y : $x + 5 = 3y$, so $y = \frac{x+5}{3}$. Thus $f^{-1}(x) = \frac{x+5}{3}$.

Check: $f(f^{-1}(x)) = 3 \cdot \frac{x+5}{3} - 5 = x$. It returns the input.

EXAMPLE 5: PIECEWISE FUNCTION

Let $f(x) = x + 4$ if $x < 0$, $f(x) = x^2$ if $0 \leq x \leq 3$, and $f(x) = 12 - x$ if $x > 3$. Find $f(-5)$, $f(0)$, $f(2)$, $f(3)$ and $f(5)$.



A piecewise function with a jump at $x = 0$ and a corner at $x = 3$.

SOLUTION

$f(-5)$: $-5 < 0$, use $x + 4$: -1 .

$f(0)$: $0 \leq 0 \leq 3$, use x^2 : 0 . (The first piece does not include 0.)

$f(2) = 2^2 = 4$. $f(3) = 3^2 = 9$ since 3 belongs to the middle piece.

$f(5)$: $5 > 3$, use $12 - x$: 7 .

EXAMPLE 6: INVERSE IN CONTEXT

Temperature in degrees Fahrenheit is $F = \frac{9}{5}C + 32$. Find the formula for Celsius in terms of F and convert 68 degrees Fahrenheit.

SOLUTION

The inverse undoes the formula. Start with $F = \frac{9}{5}C + 32$.

Subtract 32: $F - 32 = \frac{9}{5}C$. Multiply by $\frac{5}{9}$: $C = \frac{5}{9}(F - 32)$.

For $F = 68$: $C = \frac{5}{9}(36) = 20$.

Check forward: $\frac{9}{5}(20) + 32 = 36 + 32 = 68$. Yes.

Coach's Playbook

- 1 Inside changes are opposite.** What happens to x inside is the reverse of what you expect: $x - 3$ moves right, $x + 3$ moves left. Outside changes act as expected.
- 2 Track one point.** To avoid confusion, follow the vertex or one labeled point through each transformation.
- 3 Work inside out.** For $f(g(x))$, compute the inner function first. Circle it before you substitute.
- 4 Swap, then solve.** For an inverse, swap x and y , then solve for y . Check with $f(f^{-1}(x)) = x$.

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- 5 Pick the piece first.** For a piecewise function, write down which interval the input falls in before computing anything.

Common traps

Four slips to watch: (1) shifting $f(x - 3)$ to the left; (2) computing $g(f(x))$ when the problem says $f(g(x))$; (3) writing $f^{-1}(x) = \frac{1}{f(x)}$, which is a reciprocal, not an inverse; (4) using the wrong piece at a boundary value.

Mistake Clinic

Spot the error

A student says that $g(x) = (x - 3)^2$ is the graph of x^2 shifted **left** 3 because of the minus sign. The error is thinking the sign tells the direction. Set the inside equal to zero: $x - 3 = 0$ gives $x = 3$, which is where the vertex now sits. The graph moves **right** 3.

Spot the error

With $f(x) = 2x + 1$ and $g(x) = x^2$, a student computes $f(g(2))$ by first finding $f(2) = 5$ and then squaring: 25. The error is applying the functions in the wrong order. For $f(g(2))$, do g first: $g(2) = 4$, then $f(4) = 9$.

Spot the error

A student says the inverse of $f(x) = 3x - 5$ is $\frac{1}{3x-5}$. The error is using the reciprocal. An inverse swaps input and output. Solve $x = 3y - 5$ for y : $y = \frac{x+5}{3}$. Check with $f(1) = -2$ and $f^{-1}(-2) = \frac{3}{3} = 1$.

Real-World Connection

Photo filters, game characters and animation software shift, flip and stretch shapes using exactly these rules. Converting currency or temperature and then converting back is an inverse function in action.

Challenge Corner

Let $f(x) = 2x - 3$ and $g(x) = x + 4$. Show that $f(g(x)) \neq g(f(x))$, then solve $f(g(x)) = 11$. Solution: $f(g(x)) = 2(x + 4) - 3 = 2x + 5$ and $g(f(x)) = (2x - 3) + 4 = 2x + 1$. These are different for every x . Now solve $2x + 5 = 11$: $2x = 6$, so $x = 3$. Check: $g(3) = 7$ and $f(7) = 11$.

Practice

PRACTICE Transformations, Composition, and Inverses

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- The graph of $g(x) = f(x + 2) - 3$ is the graph of f shifted in which way?
 (A) Right 2, down 3 (B) Left 2, up 3 (C) Right 2, up 3
 (D) Left 2, down 3
- If $f(x) = x^2 + 1$ and $g(x) = 2x - 3$, what is $f(g(3))$?
 (A) 10 (B) 17 (C) 13 (D) 30
- What is the inverse of $f(x) = 4x + 8$?
 (A) $\frac{x+8}{4}$ (B) $\frac{x}{4} + 8$ (C) $\frac{x-8}{4}$ (D) $\frac{1}{4x+8}$
- Let $f(x) = 2x + 1$ if $x < 2$ and $f(x) = x^2 - 3$ if $x \geq 2$. What is $f(2) + f(1)$?
 (A) 8 (B) 4 (C) 2 (D) 6

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5. The point $(3, 5)$ is on the graph of f . Which point must be on the graph of $g(x) = f(-x)$?
 (A) $(-3, 5)$ (B) $(3, -5)$ (C) $(-3, -5)$ (D) $(5, 3)$
6. If $f(x) = x + 3$ and $g(x) = 2x^2$, what is $g(f(x))$?
 (A) $2x^2 + 3$ (B) $2x^2 + 12x + 18$ (C) $2x^2 + 6$
 (D) $2x^2 + 9$
7. Which function is the graph of $y = x^2$ reflected over the x -axis and then shifted up 4?
 (A) $y = -(x + 4)^2$ (B) $y = x^2 + 4$ (C) $y = -x^2 - 4$
 (D) $y = -x^2 + 4$
8. If $f(x) = 2x + 6$, what is $f^{-1}(10)$?
 (A) 2 (B) 26 (C) 8 (D) 3.5
9. Let $f(x) = x + 2$ if $x < 1$ and $f(x) = x^2 + 4$ if $x \geq 1$. What is $f(1) + f(0)$?
 (A) 5 (B) 3 (C) 7 (D) 9
10. If $f(x) = x^2 + 2x$ and $g(x) = x - 4$, find $f(g(6))$.
11. If $f(x) = 2x - 9$, find $f^{-1}(7)$.
12. Let $f(x) = 3x - 2$ if $x < 1$ and $f(x) = x^2 + 4$ if $x \geq 1$. Find $f(-2) + f(1) + f(3)$.
13. The point $(4, 6)$ is on the graph of f . Find the corresponding point on the graph of $g(x) = 2f(x - 1) + 3$. Write it as an ordered pair.
14. Find the inverse function of $f(x) = \frac{x-1}{4}$.
15. If $f(x) = 2x + 1$ and $g(x) = x^2 - 3$, find all x with $f(g(x)) = 7$.

Answers**Answers and Explanations**

Complete solutions for every practice problem, numbered to match. Read the explanation even when your answer is right.

12.1 What Is a Function?**PRACTICE · WHAT IS A FUNCTION?**

1. C. In choice C the input 1 has two outputs, 0 and 1. In choice B all outputs are 1, but each input appears once, so it is still a function.
2. A. $f(-1) = -2(-1)^2 + 3(-1) = -2 - 3 = -5$. Squaring first gives $(-1)^2 = 1$; the -2 multiplies afterward. Choice 1 results from squaring $-2 \cdot (-1)$.
3. B. The outputs go up by 3 each step, so $f(x) = 3x + 5$. Then $f(10) = 35$. Choice 38 uses $3(11) + 5$.
4. D. The equation $x^2 + y^2 = 25$ gives two y -values for most x (for $x = 3$, $y = \pm 4$). The other three give exactly one y for each x .
5. C. $f(3)$ means the output when the input h is 3. The input is hours and the output is dollars, so 3 hours costs 44 dollars. Choice A swaps input and output.
6. A. $f(a + 1) = (a + 1)^2 - 4(a + 1) = a^2 + 2a + 1 - 4a - 4 = a^2 - 2a - 3$. Choice $a^2 - 4a + 1$ is $f(a) + 1$, which adds after instead of substituting.
7. B. In choice B each input appears once (the outputs all being 2 is fine). Choices A, C and D each have an input that appears with two different outputs.
8. D. $f(12) = \sqrt{2(12) + 1} = \sqrt{25} = 5$. Choice 25 forgets the square root.
9. C. Set $2k - 1 = 9$, so $2k = 10$ and $k = 5$. Choice 17 is $f(9)$, which swaps the roles of input and output.
10. 16. $f(6) = 4(6) - 7 = 17$ and $f(2) = 4(2) - 7 = 1$. Then $17 - 1 = 16$.
11. $x = 2$ or $x = -4$. Factor $x^2 + 2x - 8 = (x - 2)(x + 4)$. Set each factor to zero.
12. 35. From $f(2) = 11$: $3(2) + k = 11$, so $k = 5$. Then $f(10) = 3(10) + 5 = 35$.
13. 5. A function cannot give input 1 two outputs. So $2a - 6$ must equal 4. Then $2a = 10$ and $a = 5$.
14. 11. $g(1) = |-4| + 2 = 6$ and $g(8) = |3| + 2 = 5$. Sum: 11.
15. $x = 7$ or $x = -1$. Notice $f(x) = (x - 3)^2$. Set $(x - 3)^2 = 16$, so $x - 3 = 4$ or $x - 3 = -4$.

12.2 Evaluating and Interpreting Functions**PRACTICE · EVALUATING AND INTERPRETING FUNCTIONS**

1. C. $f(-2) = 3(-2)^2 - (-2) + 2 = 12 + 2 + 2 = 16$. Choice 40 squares $3 \cdot (-2)$ instead of just -2 . Choice -8 treats $(-2)^2$ as negative.
2. B. $f(3) = 5$, then $f(5) = 1$. Work from the inside out. The value 5 is only $f(3)$, and 3 is $f(4)$.
3. D. Go across at height 3. The curve is at that height at $x = -1$ and $x = 3$. Check: $(-1)^2 - 2(-1) = 3$ and $3^2 - 2(3) = 3$. The point $x = 1$ gives 3 only if misread.
4. A. The input is hours and the output is percent. When $h = 5$, the battery is at 60 percent. Choice B swaps input and output.
5. C. Solve $-5t^2 + 30t = 40$, so $t^2 - 6t + 8 = 0$ and $(t - 2)(t - 4) = 0$. The ball is at 40 meters going up (2 s) and going down (4 s).
6. D. Dividing the change in cost by the change in miles gives the cost per extra mile. The starting fee is $F(0) = 4$.
7. A. $f(5) = 25 - 20 = 5$ and $f(1) = 1 - 4 = -3$. The difference is $5 - (-3) = 8$. Choice 0 is $f(4)$, which wrongly subtracts the inputs first.

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8. B. Look for 7 in the output row. It appears under $x = 1$ and $x = 2$. Choice $x = 7$ treats the output as an input.
9. C. Set $5 - 2a = -3$, so $-2a = -8$ and $a = 4$. Choice 1 is $f(2)$, not the input.
10. 5. $f(-2) = 2(4) - (-2) - 5 = 8 + 2 - 5 = 5$.
11. 33. Each step of 2 in x adds 4 to r , so the rate is 2 and $r(x) = 2x + 3$. Then $r(15) = 33$.
12. $x = 4$ or $x = -4$. Set $2x^2 - 8 = 24$, so $x^2 = 16$.
13. 5 minutes. Solve $120 - 15t = 45$: $15t = 75$, so $t = 5$.
14. $6a + 1$. $f(a + 1) = 3(a^2 + 2a + 1) - 2a - 2 = 3a^2 + 4a + 1$. Subtract $f(a) = 3a^2 - 2a$: the result is $6a + 1$.
15. 76. From $f(2) = 13$: $4a + 1 = 13$, so $a = 3$. Then $f(5) = 3(25) + 1 = 76$.

12.3 Domain and Range

PRACTICE · DOMAIN AND RANGE

1. B. The denominator is zero when $x + 2 = 0$, so $x = -2$. Choice $x \neq 2$ drops the sign.
2. C. The inside must be nonnegative: $x - 4 \geq 0$, so $x \geq 4$. At $x = 4$ the value is 0, which is fine. Choice $x > 4$ wrongly excludes 4.
3. A. Since $x^2 \geq 0$, adding 1 gives $y \geq 1$. The minimum is 1 at $x = 0$. Choice $y \geq 0$ forgets the +1.
4. D. The left end is closed, so -2 is included; the right end is open, so 3 is excluded: $-2 \leq x < 3$. Choice $1 \leq x < 6$ is the range.
5. B. Boxes are counted, so b is a whole number from 0 to 20. Choice C describes the range (the muffin counts), not the domain.
6. C. The root needs $x + 1 \geq 0$, so $x \geq -1$. The denominator needs $x \neq 3$. Both conditions must hold. Choice A forgets 3; choice B forgets the root.
7. D. $|x| \geq 0$, so $|x| + 2 \geq 2$. The minimum 2 occurs at $x = 0$. Choice $y \geq 0$ forgets the +2.
8. A. Solve $x^2 - 16 = 0$: $x = 4$ or $x = -4$. Both make the denominator zero. Choice $x = 16$ forgets to take the square root.
9. B. The ball starts on the ground ($h = 0$ at $t = 0$), rises to 64 feet and returns to 0 at $t = 4$. The range is $0 \leq h \leq 64$. Choice $0 \leq h \leq 4$ confuses time with height.
10. $x \geq 4$. Require $3x - 12 \geq 0$, so $3x \geq 12$ and $x \geq 4$.
11. $y \leq 6$. Since $-x^2 \leq 0$, adding 6 gives $y \leq 6$, with maximum 6 at $x = 0$.
12. $x = 3$ and $x = -3$. Solve $x^2 - 9 = 0$: $(x - 3)(x + 3) = 0$.
13. 100. Complete the square: $A(x) = 100 - (x - 10)^2$, which is at most 100, reached at $x = 10$ (a 10 by 10 square).
14. $x \geq -7$ and $x \neq 2$. The root needs $x + 7 \geq 0$. The denominator needs $x \neq 2$. In interval form, $[-7, 2) \cup (2, \infty)$.
15. $y \geq 3$. The square root is at least 0, so $\sqrt{x - 1} + 3 \geq 3$, with minimum 3 at $x = 1$.

12.4 Transformations, Composition, and Inverses

PRACTICE · TRANSFORMATIONS, COMPOSITION, AND INVERSES

1. D. Inside changes act opposite: $x + 2$ shifts left 2. The -3 outside shifts down 3. Choice A reads $x + 2$ as right.
2. A. $g(3) = 2(3) - 3 = 3$, then $f(3) = 9 + 1 = 10$. Choice 17 computes $g(f(3)) = 2(10) - 3$, the wrong order.
3. C. Write $y = 4x + 8$, swap: $x = 4y + 8$, so $y = \frac{x-8}{4}$. Choice $\frac{1}{4x+8}$ is the reciprocal, not the inverse.
4. B. Use the first rule for $x < 2$: $f(1) = 3$. At $x = 2$ use the second rule: $f(2) = 4 - 3 = 1$. The sum is 4. Choice 8 uses the first rule for $f(2)$.
5. A. Replacing x by $-x$ reflects over the y -axis, so $(3, 5)$ becomes $(-3, 5)$. Choice $(3, -5)$ is $-f(x)$.
6. B. $g(f(x)) = 2(x + 3)^2 = 2(x^2 + 6x + 9) = 2x^2 + 12x + 18$. Choice $2x^2 + 3$ is $f(g(x))$, the wrong order.
7. D. Reflecting over the x -axis gives $-x^2$. Shifting up 4 adds 4: $-x^2 + 4$. Choice $x^2 + 4$ forgets the reflection.
8. A. Solve $2x + 6 = 10$: $x = 2$. Choice 26 is $f(10)$, applying f instead of undoing it.
9. C. $f(1)$ uses the second rule: $1 + 4 = 5$. $f(0) = 0 + 2 = 2$. The sum is 7. Choice 5 uses the first rule for $f(1)$.
10. 8. $g(6) = 2$, then $f(2) = 4 + 4 = 8$.
11. 8. Solve $2x - 9 = 7$: $2x = 16$, so $x = 8$. (Equivalent: $f^{-1}(x) = \frac{x+9}{2}$, and $\frac{16}{2} = 8$.)
12. 10. $f(-2) = 3(-2) - 2 = -8$. $f(1) = 1 + 4 = 5$ (since $1 \geq 1$). $f(3) = 9 + 4 = 13$. Sum: $-8 + 5 + 13 = 10$.
13. $(5, 15)$. The input $x - 1 = 4$ gives $x = 5$ (shift right 1). The output is $2(6) + 3 = 15$.
14. $f^{-1}(x) = 4x + 1$. Write $x = \frac{y-1}{4}$, so $y = 4x + 1$.
15. $x = \sqrt{6}$ or $x = -\sqrt{6}$. $f(g(x)) = 2(x^2 - 3) + 1 = 2x^2 - 5$. Set $2x^2 - 5 = 7$: $x^2 = 6$.

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