



Polynomials and Factoring

Four focused lessons, worked examples, a mistake clinic, and 60 practice problems with full explanations.

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13.1 Adding, Subtracting, and Multiplying Polynomials

13.2 Factoring Out the GCF and Factoring Trinomials

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Every section teaches the idea, walks through the most common problem types, flags the traps students fall into, and ends with fifteen practice problems. Full answers and explanations are at the back.

13.1 Adding, Subtracting, and Multiplying Polynomials

Learn to classify polynomials, combine like terms, and multiply with the distributive property, FOIL, and special products, so that every later factoring and equation-solving skill rests on firm ground.

Why This Matters

Polynomials are the language of algebra. Areas of rooms, paths of thrown balls, the cost of making a product, and the growth of savings are all described with them. Every later topic in this course, including factoring, solving quadratics, and graphing, starts by working comfortably with polynomial expressions.

Students most often get stuck in three places: **subtracting a polynomial** (the minus sign must change every term inside the parentheses), **squaring a binomial** (the middle term does not disappear), and **keeping track of exponents** (you add exponents when multiplying, but you do not add them when combining like terms). This section builds each skill slowly, then shows shortcuts that save time and prevent careless errors.

Core Ideas

A **term** is a number, a variable, or a product of numbers and variables with whole-number exponents, such as 7, x , or $-4x^3$. The number in front of the variable part is the **coefficient**. A **polynomial** is one term or a sum of terms. The **degree** of a term is the exponent of its variable. The **degree of a polynomial** is the largest degree among its terms. We usually write a polynomial in **standard form**, with the highest degree first and the exponents going down.

Name by number of terms	Name by degree	Example
Monomial (1 term)	Constant (degree 0)	9
Binomial (2 terms)	Linear (degree 1)	$3x - 5$
Trinomial (3 terms)	Quadratic (degree 2)	$x^2 + 4x - 1$
Polynomial (4 or more terms)	Cubic (degree 3)	$2x^3 - x^2 + x + 6$

Adding and Subtracting Polynomials

Only **like terms** (same variable raised to the same power) can be combined. Add or subtract their coefficients and keep the variable part unchanged.

To subtract, distribute the negative sign to every term of the second polynomial:

$$A - (B + C - D) = A - B - C + D.$$

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Be careful about what counts as a like term. The terms $5x^2$ and $5x$ have the same coefficient but different exponents, so they are **unlike** and cannot be combined. The terms $4x^2$ and $-9x^2$ are like terms because the variable part is identical, and they combine to $-5x^2$. A term with no variable, such as 8, is a constant and only combines with other constants. If you are unsure, ask yourself: could I replace both terms by the same kind of object, such as 4 squares and 9 squares?

Multiplication is different. When you multiply, exponents of the same base are **added**: $x^2 \cdot x^3 = x^5$. Coefficients are multiplied: $(-3x^2)(4x^3) = -12x^5$. To multiply two polynomials, every term of the first must be multiplied by every term of the second, and then like terms are combined. For two binomials this is the **FOIL** pattern: First, Outer, Inner, Last.

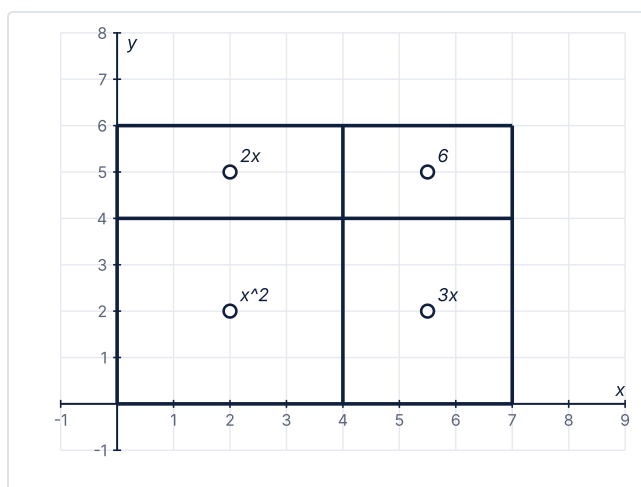
Special Products

$$(a + b)^2 = a^2 + 2ab + b^2 \quad (a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$

The middle term of a squared binomial is **twice the product** of the two terms. The product of a sum and a difference has **no middle term**.

Multiplying polynomials can be pictured with an **area model**. To find $(x + 3)(x + 2)$, build a rectangle whose width is split into pieces of length x and 3, and whose height is split into pieces of length x and 2. The four small rectangles have areas x^2 , $2x$, $3x$, and 6. Adding them gives $x^2 + 5x + 6$. Each box in the model is exactly one of the four FOIL products.



Area model for $(x + 3)(x + 2)$, drawn with $x = 4$: the four boxes are x^2 , $3x$, $2x$, and 6.

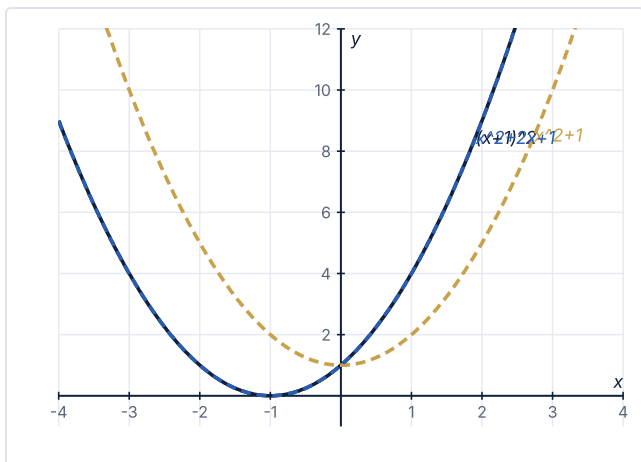
The table and graph below show why $(x + 1)^2$ is not the same as $x^2 + 1$. The expanded form $x^2 + 2x + 1$ matches $(x + 1)^2$ at every input, but $x^2 + 1$ does not.

x	-2	-1	0	1	2
$(x + 1)^2$	1	0	1	4	9
$x^2 + 1$	5	2	1	2	5
$x^2 + 2x + 1$	1	0	1	4	9

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The solid curve and the blue dashed curve coincide, so $(x + 1)^2 = x^2 + 2x + 1$. The gold curve $x^2 + 1$ is a different function.

Worked Examples

EXAMPLE 1

Write $3x - 4x^2 + 9$ in standard form, then name it by its degree and its number of terms.

SOLUTION

Arrange the terms from the highest exponent to the lowest: $-4x^2 + 3x + 9$.

The largest exponent is 2, so it is **quadratic**. It has three terms, so it is a **trinomial**.

EXAMPLE 2

Add: $(3x^2 - 5x + 4) + (-x^2 + 2x - 9)$.

SOLUTION

Group like terms: $(3x^2 - x^2) + (-5x + 2x) + (4 - 9)$.

Combine coefficients: $2x^2 - 3x - 5$.

EXAMPLE 3

Subtract: $(7x^2 + 3x - 8) - (2x^2 - 6x + 5)$.

SOLUTION

Distribute the negative sign: $7x^2 + 3x - 8 - 2x^2 + 6x - 5$.

Combine like terms: $(7 - 2)x^2 + (3 + 6)x + (-8 - 5) = 5x^2 + 9x - 13$.

Check with $x = 1$: the original is $(7 + 3 - 8) - (2 - 6 + 5) = 2 - 1 = 1$, and the answer is $5 + 9 - 13 = 1$. They agree.

EXAMPLE 4

Multiply: $-3x(2x^2 - 4x + 5)$.

SOLUTION

Multiply each term inside by $-3x$.

$(-3x)(2x^2) = -6x^3$, $(-3x)(-4x) = 12x^2$, $(-3x)(5) = -15x$.

Answer: $-6x^3 + 12x^2 - 15x$.

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EXAMPLE 5

Expand (a) $(2x + 5)(3x - 4)$, (b) $(3x - 2)^2$, and (c) $(4x + 5)(4x - 5)$.

SOLUTION

(a) FOIL: First $6x^2$, Outer $-8x$, Inner $15x$, Last -20 . Combine the middle terms: $6x^2 + 7x - 20$.

(b) Use $(a - b)^2 = a^2 - 2ab + b^2$ with $a = 3x$, $b = 2$: $9x^2 - 12x + 4$.

(c) Sum times difference: $(4x)^2 - 5^2 = 16x^2 - 25$.

EXAMPLE 6

Multiply: $(2x - 1)(x^2 + 3x - 5)$.

SOLUTION

Distribute $2x$: $2x^3 + 6x^2 - 10x$.

Distribute -1 : $-x^2 - 3x + 5$.

Add the two rows and combine like terms: $2x^3 + (6 - 1)x^2 + (-10 - 3)x + 5 = 2x^3 + 5x^2 - 13x + 5$.

Coach's Playbook

- 1 Rewrite before you combine.** Put every polynomial in standard form so like terms line up.
- 2 Circle the minus sign.** When subtracting, rewrite the subtraction as adding the opposite of every term.
- 3 Count your products.** A binomial times a trinomial gives $2 \times 3 = 6$ small products before combining. If you wrote fewer, something was skipped.
- 4 Check with a number.** Pick a simple value such as $x = 1$ or $x = 2$. The original and the final answer must give the same number.
- 5 Use the special products.** Spot squares of binomials and sum-difference pairs so you do not waste time with FOIL.

Think of the whole process as three small habits. First, **organize**: write each polynomial in standard form and use parentheses around every polynomial you are about to subtract or multiply. Second, **execute one move at a time**: finish all the distributing before you start combining. Third, **verify**: substitute one easy number. A check takes ten seconds and catches almost every sign error. A good test value is $x = 1$, because all powers of 1 are 1, but if the answer has terms that could cancel by accident, also try $x = 2$ or $x = -1$.

When multiplying a binomial by a trinomial, many students prefer the **vertical format**, the same layout used for multi-digit whole-number multiplication. Write the trinomial on top, the binomial under it, multiply by each term of the bottom row, shift like terms into columns, and add. Whichever layout you use, the result must have degree equal to the sum of the degrees of the factors. For example, a degree 1 times a degree 2 polynomial must be degree 3, and if your answer is not, there is an error to find.

- ◆ Exponent rule for products: add exponents. Rule for combining like terms: keep the exponent.
- ◆ The area model (a rectangle split into boxes) is a great way to organize a multiplication of larger polynomials.
- ◆ Write the sign with each term, including the minus sign, as you copy it.

Common traps

1. Forgetting to change every sign when subtracting a polynomial. 2. Writing $(x + 5)^2 = x^2 + 25$ and dropping the middle term. 3. Multiplying $2x^2 \cdot 3x^3$ and getting $6x^6$ instead of $6x^5$. 4. Combining unlike terms such as $3x^2$ and $5x$.

Mistake Clinic**Spot the error**

A student works $(5x - 3) - (2x - 7)$ and writes $5x - 3 - 2x - 7 = 3x - 10$. **The error:** the minus sign was applied only to the $2x$, not to the -7 . **Right solution:** $5x - 3 - 2x + 7 = 3x + 4$.

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Spot the error

A student expands $(x + 5)^2$ and writes $x^2 + 25$. **The error:** squaring a binomial means multiplying it by itself, so there is a middle term. **Right solution:** $(x + 5)(x + 5) = x^2 + 5x + 5x + 25 = x^2 + 10x + 25$. A check at $x = 1$ shows the difference: $(1 + 5)^2 = 36$, but $1 + 25 = 26$.

Spot the error

A student multiplies $(2x^2)(3x^3)$ and writes $6x^6$. **The error:** the exponents were multiplied. When two powers with the same base are multiplied, the exponents are added. **Right solution:** $(2 \cdot 3)x^{2+3} = 6x^5$.

Real-World Connection

A rectangular garden is 12 feet by 8 feet. You plan a walkway of width x feet all the way around it. The outer rectangle measures $(12 + 2x)$ by $(8 + 2x)$, so the outer area is $(12 + 2x)(8 + 2x) = 4x^2 + 40x + 96$ square feet. Subtracting the garden area, 96, leaves the walkway area $4x^2 + 40x$. For $x = 2$ the walkway is $16 + 80 = 96$ square feet.

Challenge Corner

Simplify $(2x + 3)^2 - (2x - 3)^2$ and use your result to compute $53^2 - 47^2$ without a calculator. **Solution:** $(4x^2 + 12x + 9) - (4x^2 - 12x + 9) = 24x$. Now $53 = 50 + 3$ and $47 = 50 - 3$, so with $2x = 50$, that is $x = 25$, and $53^2 - 47^2 = 24 \cdot 25 = 600$.

Practice**PRACTICE Adding, Subtracting, and Multiplying Polynomials**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What is the degree of $7x^4 - 3x^2 + x - 9$?
 (A) 7 (B) 3 (C) 4 (D) 2
- Simplify $(4x^2 - 3x + 1) + (x^2 + 5x - 6)$.
 (A) $5x^2 + 2x - 5$ (B) $5x^2 + 2x + 5$ (C) $5x^4 + 2x - 5$ (D) $3x^2 - 8x + 7$
- Simplify $(6x - 5) - (2x - 9)$.
 (A) $4x - 14$ (B) $4x - 4$ (C) $8x + 4$ (D) $4x + 4$
- Multiply: $-2x(x^2 - 4x + 3)$.
 (A) $-2x^3 - 8x^2 - 6x$ (B) $-2x^3 + 8x^2 - 6x$ (C) $-2x^2 + 8x - 6$
 (D) $-2x^3 + 8x^2 + 6x$
- Expand $(x + 7)(x - 3)$.
 (A) $x^2 - 21$ (B) $x^2 + 10x - 21$ (C) $x^2 + 4x - 21$
 (D) $x^2 + 4x + 21$
- Expand $(2x - 5)^2$.
 (A) $4x^2 - 20x + 25$ (B) $4x^2 + 25$ (C) $2x^2 - 20x + 25$
 (D) $4x^2 - 10x + 25$
- Expand $(3x + 4)(3x - 4)$.
 (A) $9x^2 + 16$ (B) $6x^2 - 16$ (C) $9x^2 - 24x - 16$
 (D) $9x^2 - 16$
- A rectangle has length $x + 6$ and width $x - 2$. Which expression gives its area?
 (A) $x^2 - 12$ (B) $x^2 + 4x - 12$ (C) $x^2 + 8x - 12$ (D) $2x + 4$

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9. Expand $(x+2)(x^2-x+5)$.

(A) $x^3 - x^2 + 5x + 10$

(B) $x^3 + 3x^2 + 3x + 10$

(C) $x^3 + x^2 + 3x + 10$

(D) $x^3 + x^2 + 7x + 10$

10. Simplify $(5x^2 - 2x + 8) - (3x^2 + 4x - 1)$.

11. Expand and simplify $(4x - 3)(2x^2 + x - 6)$.

12. Expand and simplify $(5x - 2)^2 + (x + 3)(x - 3)$.

13. Use a special product (no calculator) to find 61×59 .

14. A rectangle has sides $2x + 3$ and $x + 5$. Write its area as a polynomial in standard form.

15. The product $(x+3)(x^2+ax+b)$ equals x^3+5x^2+x-15 . Find $a+b$.

13.2 Factoring Out the GCF and Factoring Trinomials

Learn to pull out the greatest common factor, then factor trinomials of the form $x^2 + bx + c$ and $ax^2 + bx + c$ with the AC method, checking every answer by multiplying back.

Why This Matters

Factoring is multiplication run backward. In the last section you turned $(x+3)(x+5)$ into $x^2+8x+15$. Now you start with $x^2+8x+15$ and rebuild the factors. This skill is the key to solving quadratic equations, simplifying fractions that contain variables, and finding where a graph crosses the x-axis.

Students struggle here for three reasons. They forget to pull out the **greatest** common factor first. They guess at number pairs without a system, so they waste time. And they lose track of signs when the last term is negative. This section gives you a reliable system for each case, and a quick way to check every answer by multiplying.

Core Ideas

Before the details, here is the big picture. There are two questions to ask in order. **Question one:** do all terms share a common factor? If yes, pull it out. **Question two:** is what is left a trinomial that factors further? If yes, factor it. Most homework problems in this section are solved by asking just those two questions, in that order.

The **greatest common factor (GCF)** of a set of terms is the largest expression that divides every term evenly. To find it, take the largest number that divides all the coefficients, and then each variable raised to the smallest exponent it has in any term. For $18x^3$ and $24x^2$, the number part is 6 (the largest number dividing both 18 and 24) and the variable part is x^2 (the smaller exponent), so the GCF is $6x^2$.

Factoring Out the GCF

Write the GCF outside parentheses, and inside write what is left after dividing each term by the GCF:

$$ab + ac = a(b + c).$$

The number of terms inside the parentheses must equal the number of terms you started with. Always check by distributing.

Why does the GCF method work? Because of the distributive property read backward. If $6x^2 + 9x$ is written as $3x \cdot 2x + 3x \cdot 3$, the shared piece $3x$ can be moved out front to give $3x(2x + 3)$. Factoring and distributing undo each other, which is why multiplying your answer is a perfect check.

Now look at trinomials of the form $x^2 + bx + c$. A product $(x+m)(x+n)$ expands to $x^2 + (m+n)x + mn$. So to factor, you need two numbers whose **product is** c and whose **sum is** b . The signs tell you a lot, as the table shows.

Sign of c	Sign of b	The two numbers
Positive	Positive	Both positive
Positive	Negative	Both negative
Negative	Positive	One of each; the larger number is positive
Negative	Negative	One of each; the larger number is negative

Factoring $x^2 + bx + c$

Find integers m and n with $mn = c$ and $m + n = b$. Then

$$x^2 + bx + c = (x + m)(x + n).$$

Example: $x^2 + 9x + 20$ needs product 20 and sum 9, so the numbers are 4 and 5.

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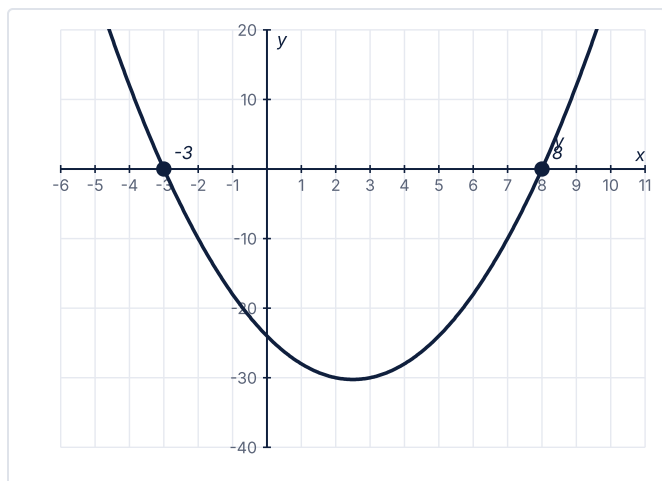
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When the leading coefficient is not 1, use the **AC method**. For $ax^2 + bx + c$: (1) multiply $a \cdot c$; (2) find two numbers whose product is ac and whose sum is b ; (3) split the middle term using those two numbers; (4) factor by grouping. Listing factor pairs in a table keeps you organized. For $x^2 - 5x - 24$ you want a product of -24 and a sum of -5 .

Pair with product -24	Sum
1, -24	-23
2, -12	-10
3, -8	-5 (match)
4, -6	-2

The match is 3 and -8 , so $x^2 - 5x - 24 = (x + 3)(x - 8)$. The graph of $y = x^2 - 5x - 24$ confirms it: the parabola crosses the x -axis exactly at $x = -3$ and $x = 8$, the values that make each factor zero.



The graph of $y = x^2 - 5x - 24 = (x + 3)(x - 8)$ crosses the x -axis at -3 and 8 .

Worked Examples

EXAMPLE 1

Factor $12x^3 - 18x^2 + 6x$.

SOLUTION

The GCF of the coefficients 12, 18, and 6 is 6. The smallest power of x is x^1 . So the GCF is $6x$.

Divide each term: $12x^3 \div 6x = 2x^2$, $18x^2 \div 6x = 3x$, $6x \div 6x = 1$.

Answer: $6x(2x^2 - 3x + 1)$. Note the last term inside is 1, not blank.

EXAMPLE 2

Factor $x^2 + 9x + 20$.

SOLUTION

Need product 20 and sum 9. Both numbers are positive.

Pairs of 20: 1, 20 (21); 2, 10 (12); 4, 5 (9). Use 4 and 5.

Answer: $(x + 4)(x + 5)$.

EXAMPLE 3

Factor $x^2 - 5x - 24$.

SOLUTION

Product -24 and sum -5 : the pair is 3 and -8 (see the table).

Answer: $(x + 3)(x - 8)$. Check: $x^2 - 8x + 3x - 24 = x^2 - 5x - 24$.

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EXAMPLE 4

Factor $2x^2 + 7x + 3$.

SOLUTION

Here $a = 2$, $c = 3$, so $ac = 6$. Need two numbers with product 6 and sum 7: 1 and 6.

Split the middle: $2x^2 + x + 6x + 3$.

Group: $x(2x + 1) + 3(2x + 1) = (2x + 1)(x + 3)$.

EXAMPLE 5

Factor $6x^2 - 11x - 10$.

SOLUTION

$ac = 6 \cdot (-10) = -60$. Need product -60 and sum -11 : the pair is 4 and -15 .

Split the middle: $6x^2 - 15x + 4x - 10$.

Group: $3x(2x - 5) + 2(2x - 5) = (3x + 2)(2x - 5)$.

Check the middle term: outer $-15x$, inner $4x$, total $-11x$.

EXAMPLE 6

Factor completely: $3x^3 - 15x^2 + 18x$.

SOLUTION

First pull out the GCF $3x$: $3x(x^2 - 5x + 6)$.

Factor the trinomial: product 6, sum -5 gives -2 and -3 , so $(x - 2)(x - 3)$.

Answer: $3x(x - 2)(x - 3)$. The GCF must stay in the final answer.

Coach's Playbook

- 1 GCF first, always.** Even if the trinomial looks hard, check whether every term shares a factor. Pulling it out leaves smaller numbers.
- 2 If $a = 1$, play the product-and-sum game.** List factor pairs of c and look for the pair that adds to b .
- 3 If $a \neq 1$, use the AC method.** Multiply ac , find the pair that adds to b , split, and group.
- 4 Check by multiplying.** FOIL your factors. If you do not get the original polynomial, go back.
- 5 Ask: is it finished?** Look at each factor and ask if it can be factored again.

Here is a mental model for the AC method. The middle term of a trinomial is made from two products that were added together (the Outer and the Inner from FOIL). The AC method asks you to find those two hidden pieces. Once you split the middle term into them, the polynomial has four terms and grouping finishes the job. If the two groups do not share a common binomial after grouping, the pair you chose was wrong, so try another pair, or the trinomial may not factor over the integers at all.

Practice speed also matters. Good students list factor pairs systematically from the outside in: 1×24 , 2×12 , 3×8 , 4×6 . They stop as soon as they find the pair with the right sum. Writing the pairs down is faster than guessing, because guessing wastes time and does not prove that a pair does not exist.

- ◆ If c is positive, both numbers share a sign (the sign of b). If c is negative, the numbers have different signs.
- ◆ If the leading coefficient is negative, factor out -1 first. Then $-x^2 + 5x + 6 = -(x^2 - 5x - 6)$.
- ◆ The AC pair can go in either order when you split the middle term. Both give the same factorization.

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Common traps

1. Not taking out the greatest common factor,, such as leaving $2(3x^2 + 6x)$ instead of $6x(x + 2)$. 2. Writing nothing for a term that equals 1, such as $3(3x^2 - 2x)$ when the last term was 3. 3. Choosing the right numbers with the wrong signs, so the middle term has the wrong sign. 4. Forgetting that the GCF stays in the answer when you factor a trinomial further.

Mistake Clinic**Spot the error**

A student factors $6x^2 + 9x$ and writes $3(2x^2 + 3x)$. **The error:** the GCF is $3x$, not 3, so the expression is not factored completely. **Right solution:** $6x^2 + 9x = 3x(2x + 3)$.

Spot the error

A student factors $x^2 - x - 12$ as $(x + 4)(x - 3)$. **The error:** the numbers 4 and -3 do multiply to -12 , but they add to $+1$, not -1 . The larger number must be negative, because b is negative. **Right solution:** use -4 and 3 : $x^2 - x - 12 = (x - 4)(x + 3)$.

Spot the error

A student factors $9x^2 - 6x + 3$ and writes $3(3x^2 - 2x)$. **The error:** the constant term disappeared. Dividing 3 by 3 gives 1, not nothing. **Right solution:** $3(3x^2 - 2x + 1)$. Distributing back shows the difference.

Real-World Connection

A rectangular patio has area $x^2 + 8x + 15$ square feet. Because area equals length times width, factoring gives the dimensions: $(x + 3)(x + 5)$. So if $x = 4$, the patio measures 7 feet by 9 feet, with area 63 square feet. Check: $16 + 32 + 15 = 63$.

Challenge Corner

Factor completely: $4x^3 + 2x^2 - 30x$. **Solution:** The GCF is $2x$, giving $2x(2x^2 + x - 15)$. For the trinomial, $ac = -30$, and the pair with sum 1 is 6 and -5 . Split: $2x^2 + 6x - 5x - 15$. Group: $2x(x + 3) - 5(x + 3) = (2x - 5)(x + 3)$. The complete answer is $2x(2x - 5)(x + 3)$.

Practice**PRACTICE Factoring Out the GCF and Factoring Trinomials**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What is the greatest common factor of $18x^3$ and $24x^2$?
 (A) $6x$ (B) $6x^2$ (C) $3x^2$ (D) $432x^3$
- Factor $15x^2 - 20x$.
 (A) $5x(3x - 4)$ (B) $5(3x^2 - 4x)$ (C) $5x(3x - 20)$
 (D) $x(15x - 20)$
- Factor $x^2 + 11x + 18$.
 (A) $(x + 3)(x + 6)$ (B) $(x - 2)(x - 9)$ (C) $(x + 2)(x + 9)$
 (D) $(x + 1)(x + 18)$

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4. Factor $x^2 - 3x - 28$.
 (A) $(x + 7)(x - 4)$ (B) $(x - 7)(x - 4)$ (C) $(x - 14)(x + 2)$
 (D) $(x - 7)(x + 4)$
5. Which expression is a factor of $x^2 - 2x - 35$?
 (A) $x + 7$ (B) $x - 7$ (C) $x - 5$ (D) $x - 35$
6. Factor $3x^2 + 10x + 8$.
 (A) $(3x + 2)(x + 4)$ (B) $(3x + 8)(x + 1)$ (C) $(3x + 4)(x + 2)$
 (D) $(3x - 4)(x - 2)$
7. Factor $2x^2 - 7x + 6$.
 (A) $(2x + 3)(x + 2)$ (B) $(2x - 2)(x - 3)$ (C) $(2x - 1)(x - 6)$
 (D) $(2x - 3)(x - 2)$
8. A rectangle has area $6x^2 + x - 2$ and length $3x + 2$. What is the width?
 (A) $2x - 1$ (B) $2x + 1$ (C) $3x - 1$ (D) $x - 2$
9. Factor completely: $2x^3 - 8x^2 - 42x$.
 (A) $2x(x^2 - 4x - 21)$ (B) $2x(x - 7)(x + 3)$ (C) $2x(x + 7)(x - 3)$
 (D) $2(x - 7)(x + 3)$
10. Factor $28x^3 + 14x^2 - 21x$.
11. Factor $x^2 - 10x + 24$.
12. Factor $4x^2 + 16x - 9$.
13. Factor $12x^2 - 7x - 10$.
14. The trinomial $x^2 + mx + 12$ can be factored using integers when m is a positive integer. Find the sum of all such values of m .
15. Factor completely: $-6x^3 + 6x^2 + 72x$.

13.3 Special Factoring Patterns

Recognize and factor a difference of squares, perfect square trinomials, sums and differences of cubes, and four-term polynomials by grouping, and learn how to decide which pattern fits.

Why This Matters

Some polynomials follow patterns so reliable that you can factor them in seconds, without searching for number pairs. Recognizing a pattern turns a ten-line problem into a one-line problem. These patterns appear in solving equations, simplifying algebraic fractions, working with areas of squares, and later in calculus and in computer science, where factoring shows up in algorithms.

Students get stuck because the patterns look similar. A **difference** of squares factors, but a **sum** of squares does not. A **perfect square trinomial** has a very specific middle term, and if the middle term is off, the pattern does not apply. Cubes have a trinomial factor that students often mistake for a perfect square. This section sorts the patterns clearly and gives you tests to decide which one you are looking at.

Core Ideas

Start with a skill that makes everything easier: knowing the first perfect squares and cubes by heart. If you see 49, 64, or 121, you should immediately think of 7^2 , 8^2 , and 11^2 . If you see x^6 , remember it is $(x^3)^2$ as well as $(x^2)^3$.

n	1	2	3	4	5	6	7	8	9	10
n^2	1	4	9	16	25	36	49	64	81	100
n^3	1	8	27	64	125	216	343	512	729	1000

Difference of Squares

$$a^2 - b^2 = (a - b)(a + b)$$

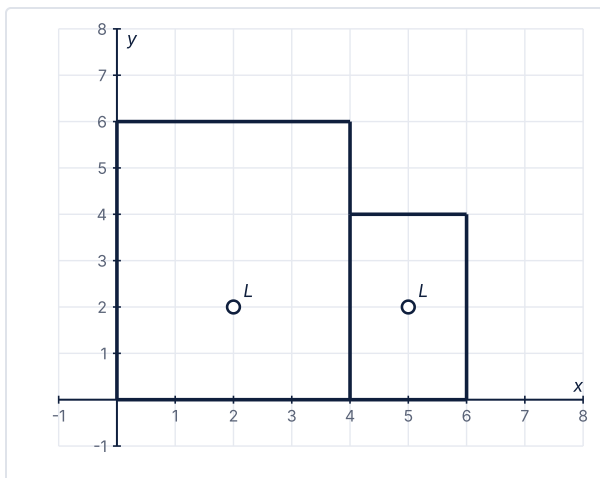
Both terms must be perfect squares and they must be **subtracted**. A sum of squares, such as $x^2 + 9$, cannot be factored over the real numbers.

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Why does the pattern work? Multiply $(a - b)(a + b)$ and the two middle terms ab and $-ab$ cancel, leaving $a^2 - b^2$. The picture shows it geometrically: take a big square of side 6 and cut a small square of side 2 from one corner. The L-shaped region that remains has area $36 - 4 = 32$, and it can be cut and rearranged into a rectangle that is $(6 + 2) = 8$ long and $(6 - 2) = 4$ wide, with area 32.



A square of side 6 with a 2 by 2 corner removed has area $6^2 - 2^2 = 32$, which equals $(6 + 2)(6 - 2)$.

In practice you do not need to draw the picture each time. You need to recognize the shape of the expression. Ask three quick questions. Is it a binomial with a minus sign between two perfect squares? Then it is a difference of squares. Is it a trinomial whose first and last terms are perfect squares? Then test the middle term. Is it a binomial with two perfect cubes? Then the cube formulas apply.

Perfect Square Trinomials

$$a^2 + 2ab + b^2 = (a + b)^2 \quad a^2 - 2ab + b^2 = (a - b)^2$$

Test: the first and last terms are perfect squares, and the middle term is **twice the product** of their square roots.

Sum and Difference of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2) \quad a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Memory aid: **SOAP**. The signs go Same, Opposite, Always Positive. In $a^3 + b^3$ the binomial has the Same sign (+), the trinomial's middle has the Opposite sign (-), and the last term is Always Positive. The trinomial does not factor further.

The last technique is **factoring by grouping**, used for four-term polynomials. Split the polynomial into two pairs, factor the GCF from each pair, and look for a common binomial. If both pairs produce the same binomial, factor it out. For instance, $x^3 + 4x^2 - 9x - 36$ splits into $x^2(x + 4)$ and $-9(x + 4)$, and the shared factor is $x + 4$.

Here is a useful warning. Many polynomials look like they fit a pattern but do not. For example, $x^2 + 8x + 16$ is a perfect square because $2 \cdot 4 = 8$, but $x^2 + 10x + 16$ is not, since $2 \cdot 4 \neq 10$. The second polynomial still factors, but it needs the ordinary product-and-sum method: $(x + 2)(x + 8)$. Always test the middle term before you decide.

Worked Examples

EXAMPLE 1

Factor $x^2 - 49$.

SOLUTION

Both terms are perfect squares: x^2 and 7^2 , and they are subtracted.

Apply $a^2 - b^2 = (a - b)(a + b)$ with $a = x$ and $b = 7$.

Answer: $(x - 7)(x + 7)$.

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EXAMPLE 2

Factor $25x^2 - 36$.

SOLUTION

$25x^2 = (5x)^2$ and $36 = 6^2$.

Apply the pattern with $a = 5x$, $b = 6$: $(5x - 6)(5x + 6)$.

EXAMPLE 3

Factor (a) $x^2 + 10x + 25$ and (b) $9x^2 - 24x + 16$.

SOLUTION

(a) x^2 and $25 = 5^2$ are squares. Twice the product of x and 5 is $10x$, which matches. So $(x + 5)^2$.

(b) $9x^2 = (3x)^2$ and $16 = 4^2$. Twice the product is $2 \cdot 3x \cdot 4 = 24x$, and the middle term is $-24x$, so it is $(3x - 4)^2$.

EXAMPLE 4

Factor completely: $3x^3 - 48x$.

SOLUTION

Pull out the GCF first: $3x(x^2 - 16)$.

Now $x^2 - 16$ is a difference of squares: $(x - 4)(x + 4)$.

Answer: $3x(x - 4)(x + 4)$.

EXAMPLE 5

Factor (a) $x^3 - 27$ and (b) $8x^3 + 125$.

SOLUTION

(a) $x^3 - 3^3$ with $a = x$, $b = 3$: $(x - 3)(x^2 + 3x + 9)$.

(b) $(2x)^3 + 5^3$ with $a = 2x$, $b = 5$: $(2x + 5)(4x^2 - 10x + 25)$. The middle term is $-(2x)(5) = -10x$.

EXAMPLE 6

Factor completely: $x^3 + 4x^2 - 9x - 36$.

SOLUTION

Group the first two terms and the last two: $(x^3 + 4x^2) + (-9x - 36)$.

Factor each group: $x^2(x + 4) - 9(x + 4)$.

Pull out the common binomial: $(x + 4)(x^2 - 9)$.

The second factor is a difference of squares: $(x + 4)(x - 3)(x + 3)$.

Coach's Playbook

- 1 Count the terms.** Two terms: try difference of squares or sum/difference of cubes. Three terms: try perfect square trinomial, then the usual trinomial methods. Four terms: try grouping.
- 2 Take out the GCF first.** This often reveals a pattern. For example, $2x^2 - 50$ becomes $2(x^2 - 25)$.
- 3 Test the pattern.** Write each term as a square or cube, such as $16x^2 = (4x)^2$, and check the signs and the middle term.
- 4 Keep going.** After one step, ask whether any factor still factors, as with $x^4 - 16$.
- 5 Check by multiplying.** The cube patterns are easy to misremember, so multiply the answer back.

When you meet a longer expression, think of it as a decision tree. First look for a GCF. Then count terms and apply the matching pattern. Then look at each factor and ask the question again. Repeating this loop is how you reach a complete factorization. A polynomial is **factored completely** when none of its factors can be factored further using integers.

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Expressions with two variables follow the same patterns. In $49a^2 - 64b^2$ the squares are $(7a)^2$ and $(8b)^2$, so the factorization is $(7a - 8b)(7a + 8b)$. With higher powers, use exponent rules: $x^6 - 1 = (x^3)^2 - 1^2 = (x^3 - 1)(x^3 + 1)$, and each cube factor can be broken down again with the cube formulas. Handling these without panic is a strong sign of algebra skill.

- ◆ Square roots of powers: halve the exponent, so $x^6 = (x^3)^2$. Cube roots: divide the exponent by 3, so $x^6 = (x^2)^3$.
- ◆ A quick mental trick: $a^2 - b^2$ means $(a - b)(a + b)$, so $51^2 - 49^2 = (51 - 49)(51 + 49) = 2 \cdot 100 = 200$.
- ◆ If grouping gives two different binomials, try rearranging the terms, or the polynomial may not factor this way.

Common traps

1. Trying to factor a sum of squares such as $x^2 + 25$. 2. Calling $x^2 + 25$ a perfect square: $(x + 5)^2$ has a middle term of $10x$. 3. Writing $x^3 - 8 = (x - 2)^3$ instead of $(x - 2)(x^2 + 2x + 4)$. 4. Stopping at $4x^2 - 36 = (2x - 6)(2x + 6)$ without noticing that 4 was a common factor.

Mistake Clinic

Spot the error

A student writes $x^2 + 25 = (x + 5)(x + 5)$. **The error:** $(x + 5)(x + 5)$ expands to $x^2 + 10x + 25$, which has a middle term. A sum of squares has no middle term, and it cannot be factored over the real numbers. **Right solution:** $x^2 + 25$ is prime.

Spot the error

A student factors $x^3 - 8$ as $(x - 2)^3$. **The error:** cubing a binomial gives $(x - 2)^3 = x^3 - 6x^2 + 12x - 8$, which has extra terms. **Right solution:** use the difference of cubes pattern with $a = x$, $b = 2$: $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$.

Spot the error

A student factors $4x^2 - 36$ as $(2x - 6)(2x + 6)$ and stops. **The error:** each factor still has a common factor of 2, so the expression is not factored completely. **Right solution:** take out 4 first: $4(x^2 - 9) = 4(x - 3)(x + 3)$.

Real-World Connection

A square room is 15 feet on each side and a square rug measuring 5 feet on each side covers the middle. The bare floor around the rug has area $15^2 - 5^2$ square feet. With the difference of squares, $(15 - 5)(15 + 5) = 10 \cdot 20 = 200$ square feet. This is also the amount of flooring wax needed if one container covers 50 square feet: 4 containers.

Challenge Corner

Factor completely: $x^4 - 5x^2 + 4$. **Solution:** treat x^2 as a single quantity u : $u^2 - 5u + 4 = (u - 1)(u - 4)$. Replace u with x^2 : $(x^2 - 1)(x^2 - 4)$. Both are differences of squares, so the complete answer is $(x - 1)(x + 1)(x - 2)(x + 2)$.

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Practice

PRACTICE Special Factoring Patterns

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- Factor $x^2 - 81$.
 (A) $(x - 9)(x + 9)$ (B) $(x - 9)^2$ (C) $(x + 9)^2$
 (D) $x(x - 81)$
- Factor $16x^2 - 25$.
 (A) $(8x - 5)(8x + 5)$ (B) $(4x - 5)(4x + 5)$ (C) $(4x - 5)^2$
 (D) $(4x + 5)^2$
- Which of these cannot be factored over the integers?
 (A) $x^2 - 16$ (B) $4x^2 - 9$ (C) $x^2 + 16$ (D) $x^2 - 1$
- Factor $x^2 + 14x + 49$.
 (A) $(x + 7)(x - 7)$ (B) $(x + 14)^2$ (C) $(x + 49)(x + 1)$
 (D) $(x + 7)^2$
- Factor $4x^2 - 20x + 25$.
 (A) $(2x - 5)^2$ (B) $(2x + 5)^2$ (C) $(4x - 5)^2$
 (D) $(2x - 5)(2x + 5)$
- Factor completely: $2x^2 - 50$.
 (A) $2(x - 5)^2$ (B) $2(x - 5)(x + 5)$ (C) $(2x - 10)(x + 5)$
 (D) $2(x^2 - 25)$
- Factor $x^3 + 8$.
 (A) $(x + 2)(x^2 + 2x + 4)$ (B) $(x + 2)^3$ (C) $(x + 2)(x^2 - 2x + 4)$
 (D) $(x + 2)(x^2 - 4x + 4)$
- Factor $27x^3 - 1$.
 (A) $(3x - 1)(9x^2 - 3x + 1)$ (B) $(3x - 1)^3$ (C) $(3x + 1)(9x^2 - 3x + 1)$
 (D) $(3x - 1)(9x^2 + 3x + 1)$
- Factor by grouping: $x^3 + 3x^2 + 5x + 15$.
 (A) $(x + 3)(x^2 + 5)$ (B) $(x + 3)(x^2 - 5)$ (C) $(x + 5)(x^2 + 3)$
 (D) $(x + 15)(x^2 + 1)$
- Factor $9x^2 - 100y^2$.
- Factor completely: $12x^3 - 27x$.
- Factor completely: $x^4 - 16$.
- Factor completely: $54x^3 - 2$.
- Factor by grouping: $2x^3 - 6x^2 + 5x - 15$.
- Use a difference of squares to compute $1002^2 - 998^2$ without a calculator.

13.4 Polynomial Division and Factoring Strategies

Divide polynomials with long division and synthetic division, use the Remainder Theorem to find factors, and follow a clear strategy to factor any polynomial completely.

Why This Matters

Dividing polynomials lets you break a complicated expression into simpler pieces. It is how you find the missing factors of a high-degree polynomial, how you simplify algebraic fractions, and how you locate the zeros of a function when no simple pattern is available. Engineers use the idea when they analyze motion and signals, and computer scientists use polynomial division in error-correcting codes.

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Students get stuck on division for two main reasons. First, long division with variables looks very different from long division with numbers, even though it follows the same steps. Second, there are so many factoring methods by now that it is hard to know where to start. This section shows both division methods in detail, then gives you a clear **strategy** to decide which method to use and to know when you have finished.

Core Ideas

Dividing by a monomial is the simplest case. Divide each term of the polynomial by the monomial, and use the rule that exponents are subtracted when you divide powers of the same base: $\frac{x^5}{x^2} = x^3$. For example, $\frac{12x^3-8x^2+4x}{4x} = 3x^2 - 2x + 1$.

To divide by a polynomial with more than one term, use **long division**. The steps repeat in a cycle: **divide** the leading terms, **multiply** the divisor by the result, **subtract**, and **bring down** the next term. Stop when the remainder has a lower degree than the divisor. As with whole numbers, the result is written as a quotient plus a remainder over the divisor.

Division Statement and Remainder Theorem

If you divide a polynomial $p(x)$ by $x - c$, you get a quotient $q(x)$ and a remainder R that is a number:

$$p(x) = (x - c)q(x) + R.$$

The **Remainder Theorem** says that $R = p(c)$. In particular, $x - c$ is a factor of $p(x)$ exactly when $p(c) = 0$.

Synthetic division is a shortcut that works when the divisor has the form $x - c$. You write only the coefficients (use a 0 for any missing power) and the number c , and you do a short routine: bring down the first coefficient, multiply by c , add to the next coefficient, and repeat. The last number is the remainder and the others are the coefficients of the quotient, which has degree one less than the original polynomial.

Step	Action	Example for $(2x^3 - 5x^2 - 4x + 3) \div (x - 3)$
1	Write c and the coefficients	$3 \mid 2, -5, -4, 3$
2	Bring down the first coefficient	2
3	Multiply by c , add to next	$-5 + 6 = 1$
4	Repeat	$-4 + 3 = -1$, then $3 - 3 = 0$
5	Read the result	$2x^2 + x - 1$, remainder 0

Why does the Remainder Theorem work? Start from the division statement $p(x) = (x - c)q(x) + R$ and substitute $x = c$. The first term becomes $(c - c)q(c) = 0$, so $p(c) = R$. This means you can find a remainder without dividing at all, just by evaluating. It also explains the test for factors: if the remainder is zero, then $p(x) = (x - c)q(x)$ exactly, which is what it means for $x - c$ to be a factor.

Factoring Strategy

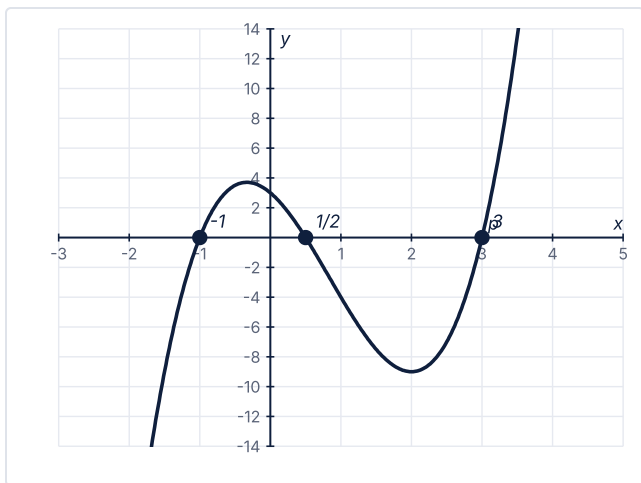
1. Take out the GCF. 2. Count the terms: **two** terms (difference of squares, sums or differences of cubes), **three** terms (perfect square, product-and-sum, or the AC method), **four** terms (grouping). 3. If nothing works, test values with the Remainder Theorem, divide, and factor the quotient. 4. Factor each factor again until none can be factored further.

The graph of $p(x) = 2x^3 - 5x^2 - 4x + 3$ shows why division matters. The curve crosses the x-axis at -1 , $\frac{1}{2}$, and 3, and these x-values match the factorization $p(x) = (x - 3)(2x - 1)(x + 1)$. Once you find one factor by testing, division reveals the others.

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The zeros of $p(x) = 2x^3 - 5x^2 - 4x + 3$ are -1 , $\frac{1}{2}$, and 3 .



The three zeros on a number line. Each zero c gives a factor $x - c$.

Worked Examples

EXAMPLE 1

Divide: $\frac{12x^3 - 8x^2 + 4x}{4x}$.

SOLUTION

Divide each term by $4x$: $\frac{12x^3}{4x} = 3x^2$, $\frac{-8x^2}{4x} = -2x$, $\frac{4x}{4x} = 1$.

Answer: $3x^2 - 2x + 1$.

EXAMPLE 2

Use long division: $(2x^2 + 7x - 4) \div (x + 4)$.

SOLUTION

Divide leading terms: $2x^2 \div x = 2x$. Multiply: $2x(x + 4) = 2x^2 + 8x$. Subtract: $(2x^2 + 7x) - (2x^2 + 8x) = -x$. Bring down -4 to get $-x - 4$.

Divide: $-x \div x = -1$. Multiply: $-1(x + 4) = -x - 4$. Subtract: 0 .

Answer: quotient $2x - 1$, remainder 0 . Check: $(x + 4)(2x - 1) = 2x^2 + 7x - 4$.

EXAMPLE 3

Divide $(x^3 + 2x^2 - 7x + 1)$ by $(x - 2)$ and write the answer with a remainder.

SOLUTION

Use synthetic division with $c = 2$ and coefficients $1, 2, -7, 1$.

Bring down 1 . Then $2 + 2 = 4$. Then $-7 + 8 = 1$. Then $1 + 2 = 3$.

The quotient is $x^2 + 4x + 1$ and the remainder is 3 .

Answer: $x^2 + 4x + 1 + \frac{3}{x-2}$. Check with the Remainder Theorem: $p(2) = 8 + 8 - 14 + 1 = 3$.

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EXAMPLE 4

Divide $(2x^3 - 5x^2 - 4x + 3)$ by $(x - 3)$.

SOLUTION

Synthetic division with $c = 3$: coefficients 2, -5 , -4 , 3.

Bring down 2. Then $-5 + 6 = 1$. Then $-4 + 3 = -1$. Then $3 - 3 = 0$.

The remainder is 0, so $x - 3$ is a factor, and the quotient is $2x^2 + x - 1$.

EXAMPLE 5

Finish factoring $2x^3 - 5x^2 - 4x + 3$ completely.

SOLUTION

From Example 4, $2x^3 - 5x^2 - 4x + 3 = (x - 3)(2x^2 + x - 1)$.

Factor the quadratic: $ac = -2$, the pair with sum 1 is 2 and -1 . Split: $2x^2 + 2x - x - 1 = 2x(x + 1) - (x + 1) = (2x - 1)(x + 1)$.

Answer: $(x - 3)(2x - 1)(x + 1)$.

EXAMPLE 6

Factor completely: $3x^3 + 6x^2 - 12x - 24$.

SOLUTION

Take out the GCF: $3(x^3 + 2x^2 - 4x - 8)$.

Four terms, so group: $x^2(x + 2) - 4(x + 2) = (x + 2)(x^2 - 4)$.

Now $x^2 - 4 = (x - 2)(x + 2)$, so the complete factorization is $3(x + 2)^2(x - 2)$.

Coach's Playbook

- 1 Write in standard form with placeholders.** Before dividing, put terms in descending order and write $0x^2$ or $0x$ for any missing power.
- 2 Use synthetic division only for $x - c$.** For $x + 3$, use $c = -3$. For $2x - 1$, use long division, or divide by $x - \frac{1}{2}$ and adjust.
- 3 Test candidate zeros.** Try small integers such as $\pm 1, \pm 2, \pm 3$. If $p(c) = 0$, then $x - c$ is a factor.
- 4 Check by multiplying.** Divisor times quotient plus remainder must equal the original polynomial.
- 5 Finish the job.** Keep factoring until every factor is linear or prime.

Here is a way to organize your search for a first factor. For a polynomial with integer coefficients, an integer zero must divide the constant term. For $x^3 - 7x + 6$, the constant term is 6, so the only integer candidates are $\pm 1, \pm 2, \pm 3, \pm 6$. Test them in order of size. This turns an open-ended hunt into a short, finite list, and the first success gives you the factor you need to start dividing.

Finally, remember that a factorization is a **product**, while a division answer with a remainder is a **sum**. Mixing them up is a common source of confusion. If the remainder is zero, you can write $p(x) = (x - c) \cdot q(x)$. If the remainder is not zero, you must write $p(x) = (x - c)q(x) + R$ or $\frac{p(x)}{x-c} = q(x) + \frac{R}{x-c}$.

- ◆ If the remainder is not zero, the divisor is not a factor, but the Remainder Theorem still tells you the value of $p(c)$.
- ◆ The degree of the quotient is the degree of the dividend minus the degree of the divisor.
- ◆ For a long division, line up like terms in columns and subtract carefully, because errors almost always come from subtraction signs.

Common traps

1. Using $c = 3$ for $x + 3$ instead of $c = -3$.
2. Skipping a missing power, such as dividing $x^3 - 8$ with coefficients 1, -8 instead of 1, 0, 0, -8 .
3. Forgetting to distribute the minus sign when subtracting in long division.
4. Declaring the problem finished when a factor like $x^2 - 4$ can still be factored.

Mistake Clinic

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Spot the error

A student divides $x^2 + 5x + 6$ by $x + 3$ using synthetic division with $c = 3$ and gets $x + 8$ with remainder 30. **The error:** the divisor $x + 3$ equals $x - (-3)$, so $c = -3$. **Right solution:** $1; 5 - 3 = 2; 6 - 6 = 0$. The quotient is $x + 2$ with remainder 0.

Spot the error

A student divides $x^3 - 8$ by $x - 2$ and writes only the coefficients 1 and -8 . **The error:** the powers x^2 and x are missing, so the columns are wrong. **Right solution:** use 1, 0, 0, -8 with $c = 2$: $1; 0 + 2 = 2; 0 + 4 = 4; -8 + 8 = 0$. The quotient is $x^2 + 2x + 4$.

Spot the error

A student simplifies $\frac{6x^2+8x}{2x}$ and writes $3x + 8$. **The error:** only the first term was divided by $2x$. Every term must be divided. **Right solution:** $\frac{6x^2}{2x} + \frac{8x}{2x} = 3x + 4$.

Real-World Connection

A school fundraiser sells x items per month and earns revenue $R(x) = 6x^2 + 24x$ dollars. The price per item is revenue divided by the number sold: $\frac{6x^2+24x}{x} = 6x + 24$ dollars. If $x = 10$ items are sold, the price per item is 84 dollars, and the revenue is $600 + 240 = 840$ dollars, which is 10 times 84.

Challenge Corner

Factor $x^3 - 7x + 6$ completely. **Solution:** Test small integers. $p(1) = 1 - 7 + 6 = 0$, so $x - 1$ is a factor. Synthetic division with $c = 1$ and coefficients 1, 0, -7 , 6 gives $1; 0 + 1 = 1; -7 + 1 = -6; 6 - 6 = 0$. The quotient is $x^2 + x - 6 = (x + 3)(x - 2)$. So $x^3 - 7x + 6 = (x - 1)(x + 3)(x - 2)$.

Practice**PRACTICE Polynomial Division and Factoring Strategies**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- Divide: $\frac{15x^3-10x^2}{5x}$.
 (A) $3x^2 - 2$ (B) $3x^2 - 10x$ (C) $3x^2 - 2x$ (D) $5x^2 - 5x$
- Divide: $(x^2 - 9x + 14) \div (x - 7)$.
 (A) $x + 2$ (B) $x - 2$ (C) $x - 7$ (D) $x + 7$
- What is the remainder when $x^3 + 2x - 5$ is divided by $x - 2$?
 (A) 7 (B) -17 (C) -5 (D) 3
- Divide: $(x^3 - 4x^2 + x + 6) \div (x - 3)$.
 (A) $x^2 + x - 2$ (B) $x^2 - x + 2$ (C) $x^2 - 7x + 22$ (D) $x^2 - x - 2$
- The polynomial $p(x) = x^3 - 2x^2 - 5x + 6$ has factors $x - 1$, $x + 2$, and $x - 3$. Which of the following is NOT a factor?
 (A) $x - 1$ (B) $x + 2$ (C) $x - 3$ (D) $x + 3$
- Factor completely: $3x^3 - 12x$.
 (A) $3x(x - 2)(x + 2)$ (B) $3x(x^2 - 4)$ (C) $3(x - 2)(x + 2)$
 (D) $3x(x - 2)^2$

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7. Factor completely: $x^3 - 5x^2 - 4x + 20$.
 (A) $(x - 5)(x^2 + 4)$ (B) $(x + 5)(x - 2)(x + 2)$ (C) $(x - 5)(x - 2)(x + 2)$
 (D) $(x - 5)(x - 2)^2$
8. Divide: $(6x^2 + x - 12) \div (2x + 3)$.
 (A) $3x + 4$ (B) $3x - 4$ (C) $2x - 4$ (D) $3x - 12$
9. A box has volume $x^3 + 6x^2 + 11x + 6$ cubic inches, length $x + 3$ and width $x + 1$. What is its height?
 (A) $x + 6$ (B) $x + 4$ (C) $x + 2$ (D) $x - 2$
10. Divide: $\frac{20x^4 - 12x^3 + 8x^2}{4x^2}$.
11. Use synthetic division to divide $3x^3 + 4x^2 - 5x + 2$ by $x + 2$. Write the answer as quotient plus remainder over divisor.
12. Factor completely: $x^3 - 3x^2 - 10x + 24$, given that $x - 4$ is a factor.
13. Find the value of k so that $x - 3$ is a factor of $x^3 - kx^2 + 2x + 3$.
14. Factor completely: $2x^4 - 32$.
15. Factor completely: $x^4 - x^3 - 7x^2 + x + 6$. (Hint: test $x = 1$ and $x = -1$.)

Answers**Answers and Explanations**

Complete solutions for every practice problem, numbered to match. Read the explanation even when your answer is right.

13.1 Adding, Subtracting, and Multiplying Polynomials**PRACTICE · ADDING, SUBTRACTING, AND MULTIPLYING POLYNOMIALS**

- C. The degree is the largest exponent, 4. Choice 7 confuses the degree with the largest coefficient.
- A. Add like terms: $5x^2 - 3x + 5x = 2x$, $1 - 6 = -5$. Adding exponents (choice 3) is wrong; exponents stay the same when you add.
- D. Distribute the negative: $6x - 5 - 2x + 9 = 4x + 4$. Choice 1 keeps -9 unchanged.
- B. $(-2x)(-4x) = +8x^2$. The first choice makes that sign negative.
- C. FOIL gives $x^2 - 3x + 7x - 21 = x^2 + 4x - 21$. Choice 1 forgets the middle terms.
- A. $(2x)^2 - 2(2x)(5) + 25$. Choice 1 drops the middle term, and choice 3 forgets to double the product $2ab$.
- D. Sum times difference gives $(3x)^2 - 4^2 = 9x^2 - 16$. Squaring $3x$ gives $9x^2$, not $6x^2$.
- B. Multiply: $x^2 - 2x + 6x - 12 = x^2 + 4x - 12$. The choice $2x + 4$ is half the perimeter, not the area.
- C. Distribute: $x^3 - x^2 + 5x + 2x^2 - 2x + 10 = x^3 + x^2 + 3x + 10$. The other choices miss a product or mis-sign one.
- $2x^2 - 6x + 9$. Distribute the negative: $5x^2 - 2x + 8 - 3x^2 - 4x + 1$. Combine: $2x^2 - 6x + 9$.
- $8x^3 - 2x^2 - 27x + 18$. Distribute $4x$: $8x^3 + 4x^2 - 24x$. Distribute -3 : $-6x^2 - 3x + 18$. Combine like terms.
- $26x^2 - 20x - 5$. First part: $25x^2 - 20x + 4$. Second part: $x^2 - 9$. Sum: $26x^2 - 20x - 5$.
3599. Write $61 \cdot 59 = (60 + 1)(60 - 1) = 60^2 - 1 = 3600 - 1 = 3599$.
- $2x^2 + 13x + 15$. FOIL: $2x^2 + 10x + 3x + 15$.
- -3 . The x^2 coefficient is $a + 3 = 5$ so $a = 2$. The constant term is $3b = -15$ so $b = -5$. Then $a + b = -3$. (Check the x term: $b + 3a = -5 + 6 = 1$.)

13.2 Factoring Out the GCF and Factoring Trinomials**PRACTICE · FACTORING OUT THE GCF AND FACTORING TRINOMIALS**

- B. The largest common number is 6 and the smaller power of x is x^2 . Choice $6x$ is a common factor but not the greatest one, since x^2 also divides both terms.
- A. The GCF is $5x$. The choice $5x(3x - 20)$ divides the second term wrongly, since $5x \cdot (-20) = -100x$. The other two choices leave part of the common factor behind.
- C. Need product 18 and sum 11: 2 and 9. The pair 3 and 6 multiplies to 18 but adds to 9.
- D. Need product -28 and sum -3 : -7 and 4. The choice $(x + 7)(x - 4)$ has the right numbers with the signs swapped, giving a sum of $+3$.
- B. Product -35 and sum -2 gives -7 and 5, so $x^2 - 2x - 35 = (x - 7)(x + 5)$. The factor $x + 7$ would require the pair 7 and -5 .
- C. $ac = 24$; the pair with sum 10 is 4 and 6. The choice $(3x + 2)(x + 4)$ has the right pieces in the wrong places: its middle term is $12x + 2x = 14x$, not $10x$.
- D. $ac = 12$; both numbers are negative: -3 and -4 . Split: $2x^2 - 3x - 4x + 6 = (2x - 3)(x - 2)$. The choice $(2x - 2)(x - 3)$ expands to $2x^2 - 8x + 6$, so its middle term is wrong.
- A. Factor: $6x^2 + x - 2 = (3x + 2)(2x - 1)$. The remaining factor is $2x - 1$. The choice $2x + 1$ is wrong: $(3x + 2)(2x + 1) = 6x^2 + 7x + 2$.
- B. Pull out $2x$, then factor $x^2 - 4x - 21 = (x - 7)(x + 3)$. The choice $2x(x^2 - 4x - 21)$ is not complete, and $2(x - 7)(x + 3)$ loses the factor x .
- $7x(4x^2 + 2x - 3)$. The GCF of 28, 14, 21 is 7 and the smallest power of x is x .
- $(x - 4)(x - 6)$. Product 24, sum -10 : both numbers are negative, -4 and -6 .
- $(2x - 1)(2x + 9)$. $ac = -36$, pair with sum 16 is 18 and -2 . Split: $4x^2 + 18x - 2x - 9 = 2x(2x + 9) - (2x + 9)$.

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13. $(3x + 2)(4x - 5)$. $ac = -120$, pair with sum -7 is 8 and -15 . Split: $12x^2 - 15x + 8x - 10 = 3x(4x - 5) + 2(4x - 5)$.
14. 28. The factor pairs of 12 with positive sums are (1, 12), (2, 6), (3, 4), giving $m = 13, 8, 7$. Then $13 + 8 + 7 = 28$.
15. $-6x(x - 4)(x + 3)$. Pull out $-6x$: $-6x(x^2 - x - 12)$. Then $x^2 - x - 12 = (x - 4)(x + 3)$.

13.3 Special Factoring Patterns

PRACTICE · SPECIAL FACTORING PATTERNS

- A. Difference of squares: $9^2 = 81$. The square $(x - 9)^2$ would expand with a middle term of $-18x$.
- B. $16x^2 = (4x)^2$ and $25 = 5^2$. The first wrong choice uses $8x$, which would square to $64x^2$.
- C. A sum of squares is prime. The other three are differences of squares.
- D. x^2 and 49 are squares and $2 \cdot 7 = 14$, so it is a perfect square. The product $(x + 7)(x - 7)$ gives $x^2 - 49$.
- A. The middle term is $-2 \cdot 2x \cdot 5 = -20x$, so the sign inside is minus. Squaring $4x$ would give $16x^2$.
- B. Take out 2, then use difference of squares. The product $(2x - 10)(x + 5)$ equals $2x^2 - 50$, but it is not complete because $2x - 10$ has a common factor 2. The choice $2(x^2 - 25)$ stops one step early.
- C. Sum of cubes with $a = x$ and $b = 2$. The trinomial has the middle sign opposite the binomial: $x^2 - 2x + 4$.
- D. Difference of cubes: $a = 3x$, $b = 1$. SOAP gives a plus sign in the middle of the trinomial. The last wrong choice is the factoring of $27x^3 + 1$.
- A. Group: $x^2(x + 3) + 5(x + 3) = (x + 3)(x^2 + 5)$. The factor $x^2 + 5$ is a sum of squares and does not factor further.
- $(3x - 10y)(3x + 10y)$. Both terms are perfect squares: $(3x)^2$ and $(10y)^2$.
- $3x(2x - 3)(2x + 3)$. Pull out $3x$ to get $3x(4x^2 - 9)$, then use difference of squares.
- $(x^2 + 4)(x - 2)(x + 2)$. $x^4 - 16 = (x^2 - 4)(x^2 + 4)$, then factor $x^2 - 4$. The factor $x^2 + 4$ is prime.
- $2(3x - 1)(9x^2 + 3x + 1)$. Take out 2: $2(27x^3 - 1)$, then use the difference of cubes pattern.
- $(x - 3)(2x^2 + 5)$. Group: $2x^2(x - 3) + 5(x - 3)$.
8000. $(1002 - 998)(1002 + 998) = 4 \cdot 2000 = 8000$.

13.4 Polynomial Division and Factoring Strategies

PRACTICE · POLYNOMIAL DIVISION AND FACTORING STRATEGIES

- C. Divide each term: $15x^3 \div 5x = 3x^2$ and $10x^2 \div 5x = 2x$. The wrong choice $3x^2 - 10x$ divides only the first coefficient.
- B. Synthetic division with $c = 7$: $1; -9 + 7 = -2; 14 - 14 = 0$. The quotient is $x - 2$.
- A. By the Remainder Theorem, the remainder is $p(2) = 8 + 4 - 5 = 7$. The value -17 is $p(-2)$, which uses the wrong sign of c .
- D. Synthetic division with $c = 3$: $1; -4 + 3 = -1; 1 - 3 = -2; 6 - 6 = 0$. Quotient $x^2 - x - 2$.
- D. The factors correspond to $p(1) = p(-2) = p(3) = 0$. But $p(-3) = -27 - 18 + 15 + 6 = -24 \neq 0$, so $x + 3$ is not a factor.
- A. GCF $3x$ leaves $x^2 - 4$, which is a difference of squares. The choice $3x(x^2 - 4)$ is not complete.
- C. Group: $x^2(x - 5) - 4(x - 5) = (x - 5)(x^2 - 4)$. Then factor $x^2 - 4$. The factor $x^2 + 4$ would come from a sign error when grouping.
- B. Divide $6x^2$ by $2x$ to get $3x$; multiply $3x(2x + 3) = 6x^2 + 9x$; subtract to get $-8x - 12$; divide to get -4 . Check: $(2x + 3)(3x - 4) = 6x^2 + x - 12$.
- C. Height is volume divided by length times width: $(x + 1)(x + 3) = x^2 + 4x + 3$, and $x^3 + 6x^2 + 11x + 6 = (x^2 + 4x + 3)(x + 2)$.
- $5x^2 - 3x + 2$. Divide each term by $4x^2$.
- $3x^2 - 2x - 1 + \frac{4}{x+2}$. Use $c = -2$: $3; 4 - 6 = -2; -5 + 4 = -1; 2 + 2 = 4$.
- $(x - 4)(x + 3)(x - 2)$. Synthetic division with $c = 4$: $1; -3 + 4 = 1; -10 + 4 = -6; 24 - 24 = 0$. Quotient $x^2 + x - 6 = (x + 3)(x - 2)$.
- $k = 4$. By the Remainder Theorem, $p(3) = 27 - 9k + 6 + 3 = 36 - 9k = 0$, so $k = 4$.
- $2(x - 2)(x + 2)(x^2 + 4)$. Take out 2: $2(x^4 - 16)$. Then $x^4 - 16 = (x^2 - 4)(x^2 + 4)$ and $x^2 - 4 = (x - 2)(x + 2)$.
- $(x - 1)(x + 1)(x - 3)(x + 2)$. Both $p(1) = 1 - 1 - 7 + 1 + 6 = 0$ and $p(-1) = 1 + 1 - 7 - 1 + 6 = 0$. Divide by $x - 1$ to get $x^3 - 7x - 6$, then divide by $x + 1$ to get $x^2 - x - 6 = (x - 3)(x + 2)$.

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