



Quadratics: Solving and Graphing

Four focused lessons, worked examples, a mistake clinic, and 60 practice problems with full explanations.

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14.1 Solving Quadratics by Factoring and Square Roots

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Every section teaches the idea, walks through the most common problem types, flags the traps students fall into, and ends with fifteen practice problems. Full answers and explanations are at the back.

14.1 Solving Quadratics by Factoring and Square Roots

Solve quadratic equations by writing them in standard form and using the zero-product property, and by taking square roots when the variable appears only inside a square.

Why This Matters

A quadratic equation is an equation that can be written as $ax^2 + bx + c = 0$ with $a \neq 0$. Quadratics describe the path of a thrown ball, the area of a rectangle, the profit of a business, and the shape of a satellite dish. Whenever a quantity depends on a square, a quadratic equation appears.

Students most often struggle in three places. They forget to move everything to one side so that the other side is **zero** before factoring. They solve $x^2 = 25$ and give only one answer. And they divide both sides of an equation by a variable, which silently throws away a solution. This section gives you a safe routine that avoids all three.

Core Ideas

A quadratic equation is in **standard form** when it is written $ax^2 + bx + c = 0$. Many equations arrive in disguise, such as $x^2 + 3x = 28$ or $x(x + 3) = 40$. The first step is always to expand if needed and move every term to one side so that the other side is 0.

Zero-Product Property

If $A \cdot B = 0$, then $A = 0$ or $B = 0$ (or both).

This is why factoring works: once a quadratic equals zero and is written as a product of factors, you set each factor equal to zero and solve each small equation. It does **not** work for any other number: $(x - 3)(x - 4) = 12$ does not mean $x - 3 = 12$ or $x - 4 = 12$.

- 1 Write in standard form.** Move all terms to one side so the other side is 0.
- 2 Factor.** Take out the GCF, then use the product-and-sum method, the AC method, or a special pattern.
- 3 Set each factor equal to 0.** Apply the zero-product property.
- 4 Solve each equation and check.** Substitute your answers into the original equation.

For equations where the variable appears only in a squared term, no factoring is needed. Isolate the square and take the square root of both sides.

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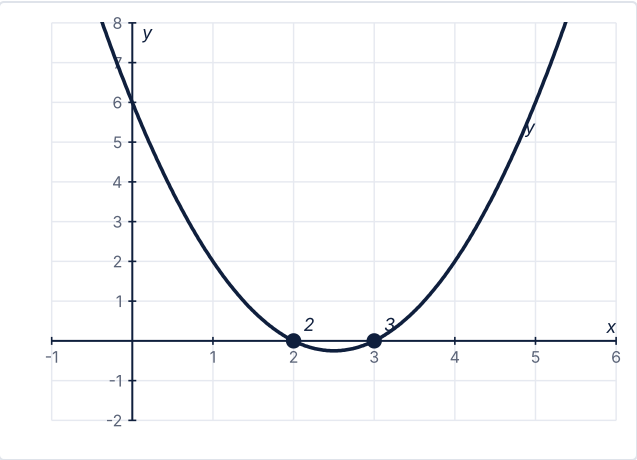
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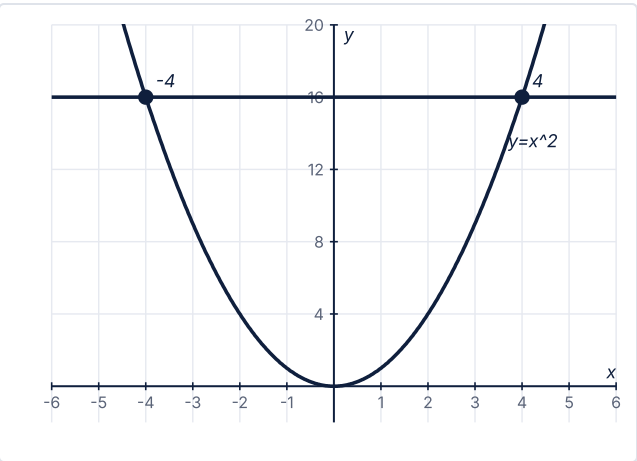
Square Root Property

If $x^2 = k$ with $k > 0$, then $x = \sqrt{k}$ or $x = -\sqrt{k}$, written $x = \pm\sqrt{k}$.
If $k = 0$ there is one solution, $x = 0$. If $k < 0$ there is no real solution, because a real number squared is never negative. The same idea applies to $(x - h)^2 = k$, which gives $x = h \pm \sqrt{k}$.

The graph of $y = x^2 - 5x + 6$ explains why solving $x^2 - 5x + 6 = 0$ gives two answers: the parabola meets the x-axis, where $y = 0$, at two points. The x-coordinates of those points are the solutions, also called the **roots** or **zeros**. The equation $x^2 = 16$ can be pictured as the parabola $y = x^2$ meeting the horizontal line $y = 16$ at two points.



The parabola $y = x^2 - 5x + 6$ crosses the x-axis at $x = 2$ and $x = 3$, the solutions of $x^2 - 5x + 6 = 0$.



The line $y = 16$ meets $y = x^2$ at $x = -4$ and $x = 4$, so $x^2 = 16$ has two solutions.

Equation	Method	Solutions
$(x - 4)(x + 9) = 0$	Zero product	4, -9
$x^2 = 49$	Square root	7, -7
$x^2 - 8x = 0$	Factor out GCF	0, 8
$x^2 + 9 = 0$	Square root	No real solution

Choosing the method takes only a few seconds of looking. If the equation has no first-degree term, as in $3x^2 - 48 = 0$, isolate the square and take roots. If it has no constant term, as in $x^2 - 8x = 0$, factor out x . If it is a full trinomial, factor it. If it is a squared binomial equal to a number, such as $(x + 3)^2 = 49$, take roots without expanding, because expanding only makes the problem harder. Later in this chapter you will meet equations that do not factor at all, and you will use other tools for them. For now, learn to read the shape of each equation before you start.

Worked Examples

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EXAMPLE 1

Solve $x^2 - 5x + 6 = 0$.

SOLUTION

Factor: need product 6 and sum -5 , so -2 and -3 . $(x - 2)(x - 3) = 0$.

Set each factor to zero: $x - 2 = 0$ or $x - 3 = 0$.

Solutions: $x = 2$ or $x = 3$.

EXAMPLE 2

Solve $x^2 + 3x = 28$.

SOLUTION

Move everything to one side: $x^2 + 3x - 28 = 0$.

Factor: product -28 , sum 3, so 7 and -4 . $(x + 7)(x - 4) = 0$.

Solutions: $x = -7$ or $x = 4$. Check $x = 4$: $16 + 12 = 28$.

EXAMPLE 3

Solve $2x^2 - 7x - 15 = 0$.

SOLUTION

$ac = -30$; the pair with sum -7 is -10 and 3.

Split: $2x^2 - 10x + 3x - 15 = 2x(x - 5) + 3(x - 5) = (2x + 3)(x - 5)$.

Set each factor to zero: $2x + 3 = 0$ gives $x = -\frac{3}{2}$, and $x - 5 = 0$ gives $x = 5$.

EXAMPLE 4

Solve $3x^2 - 48 = 0$.

SOLUTION

Isolate the square: $3x^2 = 48$, so $x^2 = 16$.

Take square roots: $x = \pm 4$.

Check: $3(16) - 48 = 0$ for both values.

EXAMPLE 5

Solve $(x - 3)^2 - 5 = 0$.

SOLUTION

Isolate the square: $(x - 3)^2 = 5$.

Take the square root of both sides: $x - 3 = \pm\sqrt{5}$.

Add 3: $x = 3 \pm \sqrt{5}$. As decimals, about 0.76 and 5.24.

EXAMPLE 6

The length of a rectangle is 3 feet more than its width, and the area is 40 square feet. Find the dimensions.

SOLUTION

Let the width be w . Then the length is $w + 3$ and $w(w + 3) = 40$.

Standard form: $w^2 + 3w - 40 = 0$. Factor: $(w + 8)(w - 5) = 0$.

So $w = -8$ or $w = 5$. A width cannot be negative, so $w = 5$ feet.

The length is $5 + 3 = 8$ feet. Check: $5 \cdot 8 = 40$.

Coach's Playbook

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- ◆ Zero on one side first. Always. If a problem is not equal to zero, the zero-product property cannot be used.
- ◆ Never divide by a variable expression. Factor it out instead, and keep both solutions.
- ◆ Check your answers in the original equation. For word problems, also ask whether each answer makes sense (negative lengths and times usually do not).
- ◆ If $b = 0$, as in $ax^2 + c = 0$, skip factoring and use square roots.
- ◆ If $c = 0$, as in $ax^2 + bx = 0$, factor out x : one solution is always $x = 0$.
- ◆ Look at the form of the problem to choose a method: squared binomial equals a number means square roots; zero on one side with factorable trinomial means factoring.

Here is a mental picture to remember the zero-product property. Think of two numbers whose product is exactly zero: at least one of them must be zero. That is true only for zero. If the product is 12, there are infinitely many pairs, such as $2 \cdot 6$, $3 \cdot 4$, or $1.5 \cdot 8$. That is why you must always get 0 on one side first.

When a quadratic has a repeated factor, such as $x^2 - 12x + 36 = (x - 6)^2 = 0$, there is only one solution, $x = 6$. Graphically, the parabola just touches the x -axis at one point. This is called a **double root**. It still counts as an answer, so do not write both 6 and 6 as two different solutions.

Common traps

1. Solving $(x - 3)(x - 4) = 12$ by setting each factor equal to 12. 2. Writing only the positive root for $x^2 = 25$. 3. Dividing both sides by x and losing the solution $x = 0$. 4. Forgetting to reject a negative length or time in a word problem.

Mistake Clinic

Spot the error

A student solves $(x - 3)(x - 4) = 12$ by writing $x - 3 = 12$ or $x - 4 = 12$, so $x = 15$ or $x = 16$. **The error:** the zero-product property needs the product to equal 0. **Right solution:** expand to $x^2 - 7x + 12 = 12$, so $x^2 - 7x = 0$ and $x(x - 7) = 0$. Then $x = 0$ or $x = 7$. Check $x = 7$: $(4)(3) = 12$.

Spot the error

A student solves $x^2 = 25$ and writes $x = 5$. **The error:** both 5 and -5 square to 25. **Right solution:** $x = \pm 5$. The graph of $y = x^2$ meets the line $y = 25$ at two points.

Spot the error

A student solves $x^2 - 5x = 0$ by dividing both sides by x and gets $x - 5 = 0$, so $x = 5$. **The error:** dividing by x removes the solution $x = 0$. **Right solution:** factor, $x(x - 5) = 0$, so $x = 0$ or $x = 5$.

Real-World Connection

A tool is dropped from a platform 64 feet above the ground. Its height after t seconds is $h = 64 - 16t^2$ feet. It hits the ground when $h = 0$: $64 - 16t^2 = 0$, so $t^2 = 4$ and $t = \pm 2$. Time cannot be negative, so the tool lands after 2 seconds.

Challenge Corner

Solve $x^3 - 9x = 0$. **Solution:** Factor out x : $x(x^2 - 9) = 0$. Then factor the difference of squares: $x(x - 3)(x + 3) = 0$. The zero-product property works for any number of factors, so $x = 0$, $x = 3$, or $x = -3$.

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Practice

PRACTICE Solving Quadratics by Factoring and Square Roots

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What are the solutions of $(x - 4)(x + 9) = 0$?
 (A) $x = -4$ and $x = 9$ (B) $x = 4$ and $x = -9$ (C) $x = 4$ and $x = 9$
 (D) $x = 36$
- Solve $x^2 - 7x + 10 = 0$.
 (A) $x = 2$ and $x = 5$ (B) $x = -2$ and $x = -5$ (C) $x = 5$ and $x = -2$
 (D) $x = 1$ and $x = 10$
- Solve $x^2 = 49$.
 (A) $x = 7$ only (B) $x = 24.5$ (C) $x = 49$
 (D) $x = 7$ or $x = -7$
- Solve $x^2 + 4x = 12$.
 (A) $x = 12$ or $x = 8$ (B) $x = 6$ or $x = -2$ (C) $x = -6$ or $x = 2$
 (D) $x = 3$ or $x = 4$
- Solve $5x^2 - 80 = 0$.
 (A) $x = \pm 16$ (B) $x = 4$ or $x = -4$ (C) $x = \pm 8$
 (D) $x = 4$ only
- Solve $(x + 3)^2 = 49$.
 (A) $x = 4$ or $x = -10$ (B) $x = 7$ or $x = -7$ (C) $x = -4$ or $x = 10$
 (D) $x = 4$ only
- Solve $3x^2 + 11x - 4 = 0$.
 (A) $x = -\frac{1}{3}$ or $x = 4$ (B) $x = \frac{1}{3}$ or $x = 4$ (C) $x = 3$ or $x = -4$
 (D) $x = \frac{1}{3}$ or $x = -4$
- Solve $x^2 - 8x = 0$.
 (A) $x = 8$ only (B) $x = 0$ or $x = 8$ (C) $x = -8$ or $x = 0$
 (D) $x = 4$ only
- The product of two consecutive positive integers is 56. What is the larger integer?
 (A) 8 (B) 7 (C) 9 (D) 28
- Solve $2x^2 - 8x = 0$.
- Solve $x^2 - 12x + 36 = 0$.
- Solve $4x^2 - 25 = 0$.
- Solve $3(x - 2)^2 = 27$.
- Solve $x^2 + 6x = 3x + 10$.
- The length of a rectangle is 4 feet more than twice its width, and its area is 48 square feet. Find its perimeter in feet.

14.2 Completing the Square and the Quadratic Formula

Solve any quadratic equation by completing the square or with the quadratic formula, use the discriminant to predict the number of solutions, and choose the best method.

Why This Matters

Many quadratic equations cannot be solved by factoring with whole numbers. The equation $x^2 - 4x + 1 = 0$ has no integer factors at all, yet it has two real solutions. Completing the square and the quadratic formula are tools that solve **every** quadratic equation. They are also the source of the formula for the vertex of a parabola, which you will use when graphing.

Students struggle here because the formula has many parts and signs. The most common errors are using the wrong sign for $-b$, forgetting that the fraction bar covers the whole numerator, and dropping the $\pm$. You will also learn the **discriminant**, a small number that tells you how many real solutions an equation has before you solve it.

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Core Ideas

Completing the square turns an expression like $x^2 + bx$ into a perfect square trinomial by adding the right constant. Since $(x + m)^2 = x^2 + 2mx + m^2$, the constant to add is the square of half the middle coefficient. For $x^2 + 6x$, half of 6 is 3, and $3^2 = 9$, so $x^2 + 6x + 9 = (x + 3)^2$.

Completing the Square

To solve $x^2 + bx + c = 0$: (1) move c to the right side; (2) add $(\frac{b}{2})^2$ to **both sides**; (3) write the left side as $(x + \frac{b}{2})^2$; (4) take square roots of both sides; (5) solve for x .

If the leading coefficient is not 1, divide every term by it first.

Now apply the same steps to the general equation $ax^2 + bx + c = 0$ with letters instead of numbers. Divide by a , complete the square, and take roots, and the result is a formula that works for every quadratic equation.

Here is the idea of the derivation. Start with $ax^2 + bx + c = 0$. Divide by a to get $x^2 + \frac{b}{a}x = -\frac{c}{a}$. Half of $\frac{b}{a}$ is $\frac{b}{2a}$, so add $\frac{b^2}{4a^2}$ to both sides. The left side becomes $(x + \frac{b}{2a})^2$ and the right side becomes $\frac{b^2 - 4ac}{4a^2}$. Taking roots and solving for x gives the formula. So the formula is not magic. It is the method of completing the square, done once and for all, with letters in place of numbers.

Quadratic Formula

If $ax^2 + bx + c = 0$ with $a \neq 0$, then

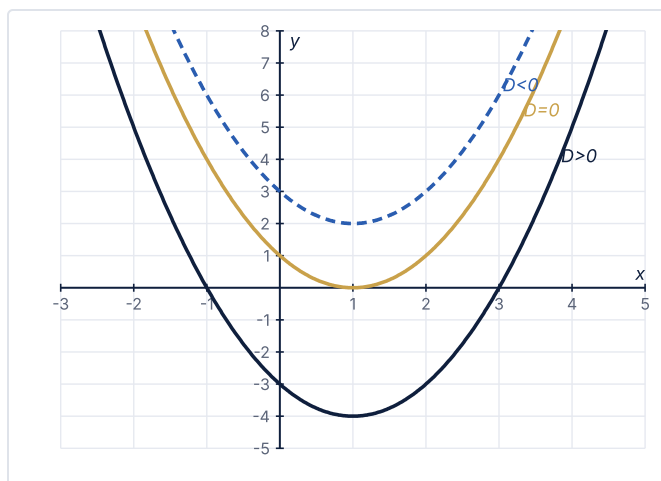
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Identify a , b , and c **with their signs** from the standard form before you substitute.

The Discriminant

The expression $D = b^2 - 4ac$ under the square root decides the type of solutions. If $D > 0$, there are **two** real solutions. If $D = 0$, there is **one** real solution (a double root). If $D < 0$, there are **no** real solutions.

Graphically: the parabola crosses the x-axis twice, touches it once, or misses it.



Three parabolas: $x^2 - 2x - 3$ has $D = 16$ (two x-intercepts), $x^2 - 2x + 1$ has $D = 0$ (one), and $x^2 - 2x + 3$ has $D = -8$ (none).

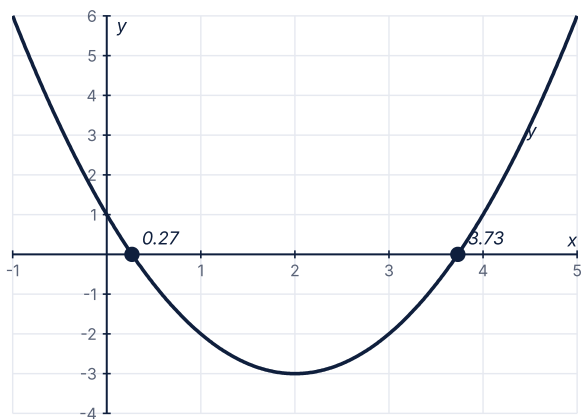
Situation	Best method
No x term, like $x^2 = k$	Square roots
Trinomial that factors easily	Factoring
Leading coefficient 1, even middle term	Completing the square
Anything else, or when unsure	Quadratic formula

The graph of $y = x^2 - 4x + 1$ shows a typical case. It crosses the x-axis at $2 - \sqrt{3} \approx 0.27$ and $2 + \sqrt{3} \approx 3.73$, which are not integers, so factoring over the integers fails but the formula works.

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The roots of $x^2 - 4x + 1 = 0$ are $2 \pm \sqrt{3}$, about 0.27 and 3.73.

Worked Examples

EXAMPLE 1

Solve $x^2 + 6x = 7$ by completing the square.

SOLUTION

Half of 6 is 3, and $3^2 = 9$. Add 9 to both sides: $x^2 + 6x + 9 = 16$.

Write the left side as a square: $(x + 3)^2 = 16$.

Take roots: $x + 3 = \pm 4$, so $x = 1$ or $x = -7$.

EXAMPLE 2

Solve $x^2 - 10x + 3 = 0$ by completing the square.

SOLUTION

Move the constant: $x^2 - 10x = -3$.

Half of -10 is -5 , and $(-5)^2 = 25$. Add 25 to both sides: $(x - 5)^2 = 22$.

Take roots: $x - 5 = \pm\sqrt{22}$, so $x = 5 \pm \sqrt{22}$.

EXAMPLE 3

Solve $2x^2 + 8x - 3 = 0$ by completing the square.

SOLUTION

Divide by 2: $x^2 + 4x - \frac{3}{2} = 0$, so $x^2 + 4x = \frac{3}{2}$.

Add $2^2 = 4$: $(x + 2)^2 = \frac{3}{2} + 4 = \frac{11}{2}$.

Take roots: $x + 2 = \pm\sqrt{\frac{11}{2}} = \pm\frac{\sqrt{22}}{2}$.

So $x = -2 \pm \frac{\sqrt{22}}{2} = \frac{-4 \pm \sqrt{22}}{2}$.

EXAMPLE 4

Solve $3x^2 - 5x - 2 = 0$ with the quadratic formula.

SOLUTION

Here $a = 3$, $b = -5$, $c = -2$. The discriminant is $(-5)^2 - 4(3)(-2) = 25 + 24 = 49$.

$x = \frac{5 \pm 7}{6}$. So $x = \frac{12}{6} = 2$ or $x = \frac{-2}{6} = -\frac{1}{3}$.

Since the discriminant is a perfect square, the equation could also be factored: $(3x + 1)(x - 2) = 0$.

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EXAMPLE 5

Solve $x^2 - 4x + 1 = 0$ and give exact answers.

SOLUTION

$a = 1$, $b = -4$, $c = 1$. $D = 16 - 4 = 12$.

$x = \frac{4 \pm \sqrt{12}}{2}$. Since $\sqrt{12} = 2\sqrt{3}$, this is $\frac{4 \pm 2\sqrt{3}}{2}$.

Divide **both** terms of the numerator by 2: $x = 2 \pm \sqrt{3}$.

EXAMPLE 6

Use the discriminant to count the real solutions of (a) $2x^2 - 4x + 5 = 0$, (b) $x^2 - 6x + 9 = 0$, and (c) $x^2 - 2x - 3 = 0$.

SOLUTION

(a) $D = (-4)^2 - 4(2)(5) = 16 - 40 = -24 < 0$: no real solutions.

(b) $D = 36 - 36 = 0$: one real solution, $x = 3$.

(c) $D = 4 + 12 = 16 > 0$: two real solutions, $x = 3$ and $x = -1$.

Coach's Playbook

- 1 Standard form first.** Rewrite as $ax^2 + bx + c = 0$ and list a , b , c with signs.
- 2 Compute D before anything else.** A negative D saves you from a long calculation. A perfect square D means the equation factors.
- 3 Substitute with parentheses.** Write $(-5)^2$, not -5^2 . Parentheses prevent sign errors.
- 4 Simplify the radical and the fraction.** Factor out perfect squares, and divide every term of the numerator by the common factor.
- 5 Check with a decimal.** Substitute a decimal approximation of your answer into the original equation. It should be close to 0.

Which method should you pick on a test? If you can see the factors within about ten seconds, factor, because it is the fastest. If the problem asks for an exact answer with radicals and the middle coefficient is even, completing the square keeps the numbers small. When in doubt, the quadratic formula never fails. A good habit is to compute the discriminant first: if it is a perfect square such as 25 or 49, the equation factors, and if it is negative, you can stop and write that there are no real solutions.

Also practice simplifying radicals, because exact answers are often required. To simplify $\sqrt{72}$, find the largest perfect square factor: $72 = 36 \cdot 2$, so $\sqrt{72} = 6\sqrt{2}$. In a formula answer such as $\frac{6 \pm \sqrt{72}}{6}$, rewrite it as $\frac{6 \pm 6\sqrt{2}}{6} = 1 \pm \sqrt{2}$. Cancelling is only allowed when the common factor divides **every** term of the numerator.

- ◆ To complete the square, always add the same number to **both** sides.
- ◆ When $a \neq 1$, divide every term by a before halving the middle coefficient.
- ◆ The two solutions from the formula are called conjugates: they differ only in the sign before the radical.
- ◆ If D is a perfect square, the solutions are rational numbers. If not, they are irrational.

Common traps

1. Using the wrong sign for $-b$, such as writing $\frac{-3 \pm 5}{2}$ when $b = -3$. 2. Writing the whole numerator over $2a$ but dividing only the radical. 3. Forgetting to add the new constant to the right side when completing the square. 4. Confusing the discriminant with the whole answer: D is only the number under the radical.

Mistake Clinic**Spot the error**

A student solves $x^2 + 6x = 7$ by writing $x^2 + 6x + 9 = 7$, so $(x + 3)^2 = 7$ and $x = -3 \pm \sqrt{7}$. **The error:** 9 was added to the left side only. **Right solution:** add 9 to both sides: $(x + 3)^2 = 16$, so $x + 3 = \pm 4$ and $x = 1$ or $x = -7$.

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Spot the error

A student solves $x^2 - 3x - 4 = 0$ with $a = 1$, $b = -3$, $c = -4$ and writes $x = \frac{-3 \pm 5}{2}$, getting 1 and -4 . **The error:** the formula starts with $-b$, and $-(-3) = +3$. **Right solution:** $x = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2}$, so $x = 4$ or $x = -1$.

Spot the error

A student simplifies $\frac{4 \pm 2\sqrt{3}}{2}$ as $4 \pm \sqrt{3}$. **The error:** the 2 was divided into only one term. **Right solution:** divide both terms by 2: $\frac{4}{2} \pm \frac{2\sqrt{3}}{2} = 2 \pm \sqrt{3}$. Or factor first: $\frac{2(2 \pm \sqrt{3})}{2}$.

Real-World Connection

A photo that is 6 inches by 8 inches is placed on a mat of equal width x inches on all four sides, so the total area is 120 square inches. Then $(6 + 2x)(8 + 2x) = 120$, which simplifies to $x^2 + 7x - 18 = 0$. The discriminant is $49 + 72 = 121$, so $x = \frac{-7 \pm 11}{2}$. The positive solution is $x = 2$, so the mat is 2 inches wide.

Challenge Corner

The sum of the solutions of $ax^2 + bx + c = 0$ is $-\frac{b}{a}$ and their product is $\frac{c}{a}$. Use this to find the solutions of $2x^2 - 11x + 12 = 0$ without the formula. **Solution:** The sum is $\frac{11}{2}$ and the product is 6. Try $\frac{3}{2}$ and 4: $\frac{3}{2} + 4 = \frac{11}{2}$ and $\frac{3}{2} \cdot 4 = 6$. These work, so the solutions are $x = \frac{3}{2}$ and $x = 4$. (Adding the two formula roots gives $\frac{-2b}{2a} = -\frac{b}{a}$.)

Practice**PRACTICE** Completing the Square and the Quadratic Formula

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What number must be added to $x^2 + 12x$ to make a perfect square trinomial?
(A) 6 (B) 12 (C) 36 (D) 144
- Completing the square on $x^2 - 8x + 5 = 0$ gives which equation?
(A) $(x - 4)^2 = 11$ (B) $(x - 4)^2 = -5$ (C) $(x - 4)^2 = 21$
(D) $(x + 4)^2 = 11$
- What is the discriminant of $2x^2 - 3x - 5 = 0$?
(A) -31 (B) 31 (C) 7 (D) 49
- How many real solutions does $x^2 + 4x + 7 = 0$ have?
(A) One (B) None (C) Two
(D) Cannot be determined
- Which gives the solutions of $x^2 + 5x + 3 = 0$?
(A) $\frac{-5 \pm \sqrt{13}}{2}$ (B) $\frac{5 \pm \sqrt{13}}{2}$ (C) $\frac{-5 \pm \sqrt{37}}{2}$ (D) $\frac{-5 \pm \sqrt{13}}{4}$
- Solve $2x^2 + 3x - 9 = 0$.
(A) $x = -\frac{3}{2}$ or $x = 3$ (B) $x = \frac{3}{2}$ or $x = 3$ (C) $x = \frac{3}{2}$ or $x = -3$
(D) $x = 6$ or $x = -12$
- For $k > 0$, what value of k makes $x^2 + kx + 16 = 0$ have exactly one real solution?
(A) 4 (B) 16 (C) 64 (D) 8
- Solve $x^2 + 10x + 2 = 0$.
(A) $x = 5 \pm \sqrt{23}$ (B) $x = -5 \pm \sqrt{23}$ (C) $x = -5 \pm \sqrt{27}$ (D) $x = -10 \pm \sqrt{23}$
- A ball's height in feet is $h = -16t^2 + 32t + 48$, where t is time in seconds. When does the ball hit the ground?
(A) 3 seconds (B) 1 second (C) 2 seconds (D) 6 seconds
- Solve $x^2 + 8x + 7 = 0$ by completing the square.

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11. Solve $x^2 - 6x - 2 = 0$ and give exact answers.
12. Use the quadratic formula to solve $2x^2 - 7x + 3 = 0$.
13. Use the quadratic formula to solve $x^2 + 2x - 5 = 0$. Give exact answers.
14. How many real solutions does $3x^2 - 2x + 4 = 0$ have?
15. Find the value of m so that $mx^2 + 12x + 9 = 0$ has exactly one real solution.

14.3 Graphing Parabolas

Graph quadratic functions by finding the vertex, axis of symmetry, intercepts, and direction, and move easily between standard, vertex, and factored forms.

Why This Matters

The graph of a quadratic function is a **parabola**. Parabolas appear in the arc of a basketball, the shape of a bridge cable or satellite dish, and the curve of a flashlight reflector. Reading and sketching them is one of the most useful graphing skills in algebra, and it prepares you for functions in Algebra 2 and calculus.

Students most often struggle with three points: finding the vertex when the equation is in standard form, remembering that the vertex form $y = a(x - h)^2 + k$ has a **minus** inside the parentheses, and connecting the numbers in an equation to features of the picture. This section gives you a toolkit with three forms of the equation and a checklist for graphing.

Core Ideas

A quadratic function can be written in three forms. Each form shows a different feature of the graph immediately.

Form	Equation	What it shows
Standard	$y = ax^2 + bx + c$	y-intercept $(0, c)$; axis $x = -\frac{b}{2a}$
Vertex	$y = a(x - h)^2 + k$	Vertex (h, k) ; axis $x = h$
Factored	$y = a(x - p)(x - q)$	x-intercepts p and q

Features of a Parabola

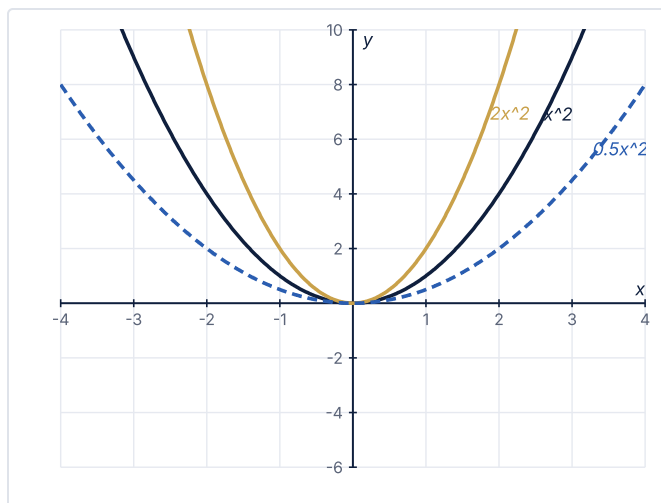
The **vertex** is the turning point. The **axis of symmetry** is the vertical line through the vertex. For $y = ax^2 + bx + c$,

$$x_{\text{vertex}} = -\frac{b}{2a},$$

and the y-coordinate is found by substituting this x-value into the equation.

If $a > 0$ the parabola opens **upward** and the vertex is the minimum. If $a < 0$ it opens **downward** and the vertex is the maximum.

The number a also controls the width. The larger the value of $|a|$, the narrower the parabola. If $|a| < 1$ the parabola is wider than $y = x^2$. The graph below compares four parabolas.



Compare $y = x^2$ (navy), $y = 2x^2$ (narrower), $y = 0.5x^2$ (wider), and $y = -x^2$ (opens downward).

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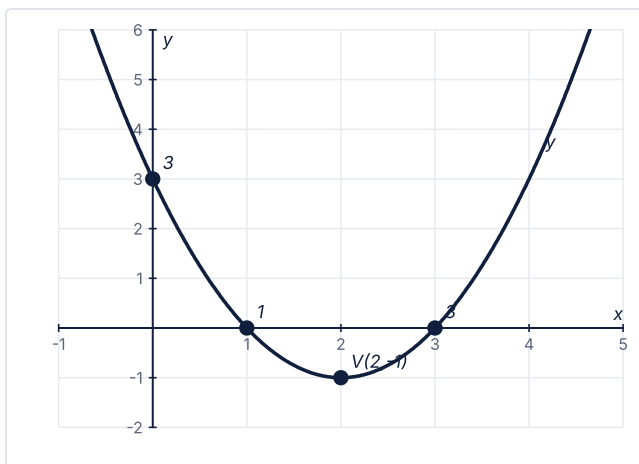
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Vertex Form and Shifts

The graph of $y = a(x - h)^2 + k$ is the graph of $y = ax^2$ moved h units right and k units up. So $y = 2(x + 1)^2 - 8$ has vertex $(-1, -8)$.

The sign inside the parentheses is opposite to the direction of the shift: $x - 3$ moves right 3, and $x + 3$ moves left 3.

Graphing checklist. (1) Find the direction from the sign of a . (2) Find the vertex. (3) Find the y-intercept by setting $x = 0$. (4) Find the x-intercepts by setting $y = 0$ (factor, or use the quadratic formula). (5) Plot the points and use symmetry: every point has a mirror image across the axis. (6) Draw a smooth curve.



The graph of $y = x^2 - 4x + 3$ with its vertex $(2, -1)$, x-intercepts 1 and 3, and y-intercept 3.

Worked Examples

EXAMPLE 1

Find the vertex, axis of symmetry, and intercepts of $y = x^2 - 4x + 3$, and describe the graph.

SOLUTION

Here $a = 1$, $b = -4$, $c = 3$. Opens upward since $a > 0$.

Axis: $x = -\frac{-4}{2(1)} = 2$. Vertex: $y = 4 - 8 + 3 = -1$, so the vertex is $(2, -1)$.

y-intercept: $y = 3$. x-intercepts: $x^2 - 4x + 3 = (x - 1)(x - 3) = 0$, so $x = 1$ and $x = 3$.

EXAMPLE 2

Graph $y = -x^2 + 6x - 5$ by finding its key points.

SOLUTION

$a = -1 < 0$, so it opens downward and the vertex is a maximum.

Axis: $x = -\frac{6}{2(-1)} = 3$. Then $y = -9 + 18 - 5 = 4$, so the vertex is $(3, 4)$.

y-intercept: -5 . x-intercepts: $-(x^2 - 6x + 5) = -(x - 1)(x - 5) = 0$, so $x = 1$ and $x = 5$.

Plot $(1, 0)$, $(3, 4)$, $(5, 0)$, $(0, -5)$ and its mirror image $(6, -5)$.

EXAMPLE 3

For $y = 2(x + 1)^2 - 8$, find the vertex, direction, y-intercept, and x-intercepts.

SOLUTION

Vertex form $a(x - h)^2 + k$ with $h = -1$ and $k = -8$: vertex $(-1, -8)$. Since $a = 2 > 0$ it opens upward.

y-intercept: $y = 2(1)^2 - 8 = -6$.

x-intercepts: $2(x + 1)^2 = 8$, so $(x + 1)^2 = 4$ and $x + 1 = \pm 2$. Then $x = 1$ or $x = -3$.

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EXAMPLE 4

Find the x-intercepts, axis of symmetry, and vertex of $y = (x + 2)(x - 4)$.

SOLUTION

x-intercepts: set each factor to 0, so $x = -2$ and $x = 4$.

The axis of symmetry is halfway between: $x = \frac{-2+4}{2} = 1$.

Vertex: $y = (1 + 2)(1 - 4) = 3 \cdot (-3) = -9$, so the vertex is $(1, -9)$.

EXAMPLE 5

Rewrite $y = x^2 + 6x + 5$ in vertex form.

SOLUTION

Complete the square: half of 6 is 3, and $3^2 = 9$.

$$y = (x^2 + 6x + 9) - 9 + 5 = (x + 3)^2 - 4.$$

The vertex is $(-3, -4)$. Check with $x = -\frac{6}{2} = -3$ and $y = 9 - 18 + 5 = -4$.

EXAMPLE 6

A parabola has vertex $(2, 3)$ and passes through $(4, 11)$. Write its equation.

SOLUTION

Start with vertex form: $y = a(x - 2)^2 + 3$.

Substitute the point: $11 = a(4 - 2)^2 + 3$, so $8 = 4a$ and $a = 2$.

Answer: $y = 2(x - 2)^2 + 3$.

Coach's Playbook

- 1 Identify the form.** Standard, vertex, or factored? The form tells you which feature is free.
 - 2 Find $x = -\frac{b}{2a}$ first.** This gives the axis and the x-coordinate of the vertex. Then substitute to get the y-coordinate.
 - 3 Find intercepts.** Set $x = 0$ for the y-intercept. Set $y = 0$ and solve for the x-intercepts.
 - 4 Use symmetry.** A point d units to the right of the axis has a twin d units to the left at the same height.
 - 5 Sanity check the sketch.** The curve must open the way the sign of a says, and must pass through your points.
- ◆ The vertex is exactly halfway between the x-intercepts, when they exist.
 - ◆ The minimum or maximum value of the function is the y-coordinate of the vertex, not the x-coordinate.
 - ◆ If a parabola has no x-intercepts, the vertex lies above the axis (opening up) or below it (opening down).
 - ◆ To compare widths, ignore the sign of a and compare absolute values.

A quick way to sketch fast: from the vertex, move 1 unit right and go up a units; move 2 units right and go up $4a$ units. This works because the shape of the parabola is $y = ax^2$ measured from the vertex. For $y = 2(x - 2)^2 + 3$ the points are $(3, 5)$ and $(4, 11)$, plus their mirror images $(1, 5)$ and $(0, 11)$.

Finally, remember how the forms connect. The factored form tells you the roots, and the average of the roots gives the x-coordinate of the vertex. The standard form tells you the y-intercept immediately. The vertex form tells you the vertex immediately. A strong student can switch between all three forms, choosing the one that makes the question easy.

Common traps

1. Reading the vertex of $y = (x - 3)^2 + 2$ as $(-3, 2)$.
2. Using $x = \frac{b}{2a}$ and forgetting the negative sign.
3. Mixing up the x- and y-coordinates of the vertex when asked for a maximum value.
4. Thinking a smaller $|a|$ makes a narrower graph.

Mistake Clinic

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Spot the error

A student states that the vertex of $y = (x - 3)^2 + 2$ is $(-3, 2)$. **The error:** the form is $a(x - h)^2 + k$, so $x - 3$ means $h = 3$.

Right solution: the vertex is $(3, 2)$. Test: at $x = 3$ the squared term is 0, which gives the lowest value $y = 2$.

Spot the error

A student finds the axis of symmetry of $y = x^2 + 6x + 5$ using $x = \frac{b}{2a} = 3$. **The error:** the formula has a negative sign.

Right solution: $x = -\frac{6}{2} = -3$. The graph shows the vertex at $(-3, -4)$.

Spot the error

A student claims that $y = \frac{1}{2}x^2$ is narrower than $y = x^2$ because $\frac{1}{2}$ is smaller. **The error:** a smaller $|a|$ gives a wider graph. For the same x -value, the output is smaller, so the curve rises more slowly. **Right solution:** $y = \frac{1}{2}x^2$ is wider; for example at $x = 2$ it reaches 2 while $y = x^2$ reaches 4.

Real-World Connection

Water from a fountain follows the path $y = -x^2 + 4x$, where x is the horizontal distance in feet and y is the height in feet. The vertex is at $x = 2$, $y = -4 + 8 = 4$, so the highest point is 4 feet. The water returns to the ground when $-x(x - 4) = 0$, so at $x = 4$ feet.

Challenge Corner

A parabola passes through $(0, 3)$, $(1, 0)$, and $(3, 0)$. Find its vertex. **Solution:** The x -intercepts are 1 and 3, so $y = a(x - 1)(x - 3)$. At $x = 0$, $3 = a(-1)(-3) = 3a$, so $a = 1$. The axis is $x = 2$, and $y = (1)(-1) = -1$. The vertex is $(2, -1)$.

Practice**PRACTICE Graphing Parabolas**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What is the axis of symmetry of $y = x^2 - 6x + 1$?
 (A) $x = -3$ (B) $x = 6$ (C) $x = 1$ (D) $x = 3$
- What is the vertex of $y = (x - 2)^2 + 5$?
 (A) $(2, 5)$ (B) $(-2, 5)$ (C) $(2, -5)$ (D) $(5, 2)$
- Which equation describes a parabola that opens downward?
 (A) $y = 3x^2 - 2$ (B) $y = (x + 1)^2$ (C) $y = -0.5x^2 + 4$
 (D) $y = x^2 - x - 6$
- Which parabola is the narrowest?
 (A) $y = 0.3x^2$ (B) $y = -4x^2$ (C) $y = -x^2$ (D) $y = 2x^2$
- What is the y -intercept of $y = 2x^2 - 5x + 7$?
 (A) 7 (B) -5 (C) 2 (D) 0
- What are the x -intercepts of $y = (x - 3)(x + 5)$?
 (A) $x = -3$ and $x = 5$ (B) $x = 3$ and $x = 5$ (C) $x = -15$
 (D) $x = 3$ and $x = -5$
- What is the vertex of $y = x^2 - 8x + 10$?
 (A) $(-4, -6)$ (B) $(4, -6)$ (C) $(4, 6)$ (D) $(8, -6)$
- What is the maximum value of $y = -x^2 + 4x + 1$?
 (A) 2 (B) 1 (C) 5 (D) 9

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9. A parabola has vertex $(1, -4)$ and passes through $(3, 0)$. Which is its equation?
 (A) $y = (x - 1)^2 - 4$ (B) $y = (x + 1)^2 - 4$ (C) $y = (x - 1)^2 + 4$
 (D) $y = 2(x - 1)^2 - 4$
10. Find the vertex of $y = x^2 + 10x + 21$.
11. Find the x-intercepts of $y = x^2 - 2x - 15$.
12. Rewrite $y = x^2 - 8x + 19$ in vertex form.
13. Write the equation of the parabola with vertex $(-3, 2)$ that passes through $(-1, -6)$.
14. For $b > 0$, find the value of b so that the vertex of $y = x^2 + bx + 9$ lies on the x-axis.
15. What is the distance between the x-intercepts of $y = -2x^2 + 8x + 10$?

14.4 Quadratic Models and Word Problems

Model real situations with quadratic functions, find maximum and minimum values at the vertex, interpret roots in context, and solve projectile, area, and revenue problems.

Why This Matters

Quadratic functions are the mathematics of things that rise and fall, and of quantities that have a best possible value. A rocket, a thrown ball, the area of a pen for animals, and the profit of a school fundraiser can all be modeled by a quadratic. Knowing how to build the model and how to read the vertex and the roots lets you answer practical questions: How high does it go? When does it land? What price earns the most?

Students find word problems hard because there are two jobs at once: translating the story into an equation, and then interpreting the answer in context. The most common mistakes are answering with the wrong coordinate of the vertex, forgetting units, and keeping an answer that makes no sense, such as a negative time. This section shows a clear routine for every type.

Core Ideas

A **quadratic model** is a function $y = ax^2 + bx + c$ that describes a real situation. The graph is a parabola, and three features of the graph have a meaning in the story.

Feature of the graph	Meaning in the story
Vertex (h, k)	The best value: h is when or how many; k is the maximum or minimum amount
x-intercepts (roots)	The inputs where the output is zero: landing time, break-even points
y-intercept $(0, c)$	The starting value when the input is zero
Axis of symmetry	The midpoint of the roots; the situation is symmetric around it

Projectile Motion

The height of an object thrown or launched upward is modeled by

$$h(t) = -16t^2 + v_0t + h_0,$$

where t is time in seconds, v_0 is the starting upward speed in feet per second, and h_0 is the starting height in feet.

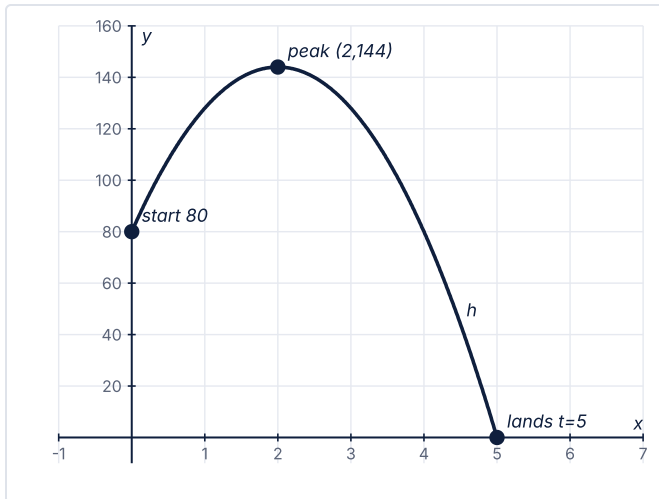
The coefficient -16 comes from gravity (in feet). In metric units the model is $h(t) = -5t^2 + v_0t + h_0$ with height in meters.

Because the coefficient of t^2 is negative, the parabola opens downward, and the vertex is the **highest point**. The time of the highest point is $t = -\frac{b}{2a}$, and the maximum height is h evaluated at that time. The object lands when $h(t) = 0$, and only the positive root makes sense.

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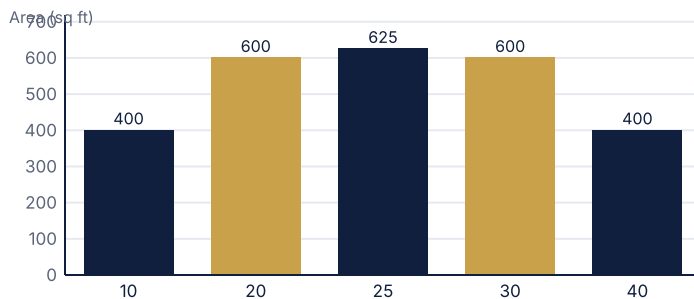
The model $h(t) = -16t^2 + 64t + 80$: starting height 80 feet, peak 144 feet at 2 seconds, landing at 5 seconds.

Optimization with a Quadratic

To find the maximum or minimum value, write the quantity as a function of one variable, put it in the form $ax^2 + bx + c$, and find the vertex. If $a < 0$ the vertex is a maximum. If $a > 0$ it is a minimum.

Always answer the question asked: the **input** at the vertex (how many, what price, what time) or the **output** at the vertex (the maximum area, revenue, or height).

A classic example is a rectangular pen made with 100 feet of fence. If the width is w , the length is $50 - w$, so the area is $A = w(50 - w)$. The bar chart shows that the area rises, peaks when the pen is a square (width 25), and then falls again, symmetric about the peak.



Area of the pen for different widths with 100 feet of fence. The maximum is 625 square feet at width 25 feet.

Worked Examples

EXAMPLE 1

A ball is thrown upward so that its height is $h = -16t^2 + 48t + 4$ feet after t seconds. Find the maximum height and when it occurs.

SOLUTION

Time of the vertex: $t = -\frac{48}{2(-16)} = \frac{48}{32} = 1.5$ seconds.

Height: $h(1.5) = -16(1.5)^2 + 48(1.5) + 4 = -36 + 72 + 4 = 40$.

The ball reaches a maximum height of 40 feet after 1.5 seconds.

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EXAMPLE 2

A rocket's height is $h = -16t^2 + 64t + 80$ feet. When does it hit the ground?

SOLUTION

Set $h = 0$ and divide by -16 : $t^2 - 4t - 5 = 0$.

Factor: $(t - 5)(t + 1) = 0$, so $t = 5$ or $t = -1$.

Time cannot be negative, so the rocket lands after 5 seconds. (The roots are 5 and -1 ; the peak at $t = 2$ is halfway between -1 and 5.)

EXAMPLE 3

A farmer has 80 feet of fence and wants a rectangular pen against a long barn wall, so only three sides need fence. Find the dimensions that give the greatest area.

SOLUTION

Let each of the two sides touching the barn be x . The side parallel to the barn is $80 - 2x$.

Area: $A = x(80 - 2x) = -2x^2 + 80x$.

Vertex: $x = -\frac{80}{2(-2)} = 20$. Then the parallel side is $80 - 40 = 40$.

The pen should be 20 feet by 40 feet, with maximum area $20 \cdot 40 = 800$ square feet.

EXAMPLE 4

A club sells shirts. At a price of p dollars, it sells $120 - 6p$ shirts. What price gives the greatest revenue, and what is that revenue?

SOLUTION

Revenue = price times number sold: $R = p(120 - 6p) = -6p^2 + 120p$.

Vertex: $p = -\frac{120}{2(-6)} = 10$.

$R(10) = 10(120 - 60) = 600$.

A price of 10 dollars gives the greatest revenue, 600 dollars.

EXAMPLE 5

A company's profit is $P(x) = -2x^2 + 40x - 150$ dollars when it makes x hundred items. Find the break-even points and the maximum profit.

SOLUTION

Break-even means $P = 0$: divide by -2 to get $x^2 - 20x + 75 = 0$, so $(x - 5)(x - 15) = 0$ and $x = 5$ or $x = 15$.

The company breaks even at 500 items and at 1,500 items, and it earns a profit in between.

The maximum is at the midpoint $x = 10$: $P(10) = -200 + 400 - 150 = 50$.

Maximum profit is 50 dollars at 1,000 items.

EXAMPLE 6

A ball's height is $h = -16t^2 + 32t + 6$ feet. (a) When is the height 22 feet? (b) When is the height 14 feet?

SOLUTION

(a) Set $-16t^2 + 32t + 6 = 22$, so $-16t^2 + 32t - 16 = 0$, which gives $t^2 - 2t + 1 = 0$, so $(t - 1)^2 = 0$ and $t = 1$.

The only time is $t = 1$ because 22 feet is the maximum height. The ball touches that height once.

(b) Set $-16t^2 + 32t + 6 = 14$: $16t^2 - 32t + 8 = 0$, so $2t^2 - 4t + 1 = 0$.

Quadratic formula: $t = \frac{4 \pm \sqrt{16 - 8}}{4} = 1 \pm \frac{\sqrt{2}}{2}$, about 0.29 and 1.71 seconds: once going up and once coming down.

Coach's Playbook

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- 1 **Define the variable.** Write what x means, with units, before you write any equation.
- 2 **Build the function.** For optimization, write the quantity (area, revenue, profit) in terms of one variable and multiply out.
- 3 **Choose the right feature.** Maximum or minimum: vertex. When it lands or breaks even: roots. Starting value: y-intercept.
- 4 **Answer in context.** Include units and a full sentence. Reject answers that are impossible.
- 5 **Check reasonableness.** Does the answer have the right size? A thrown ball does not reach 4,000 feet.

- ◆ The vertex x -value is the average of the two roots, when they exist. This is a fast way to find it from the factored form.
- ◆ For revenue, use the pattern (price)(quantity). For area of a rectangle with fixed perimeter, the greatest area is a square.
- ◆ Equations like $h(t) = 22$ mean setting the function equal to a number and solving. Move everything to one side.
- ◆ If the problem says "exactly one time", expect a double root, where the line touches the vertex.

A helpful habit is to sketch a quick parabola for every word problem. Mark the starting point, the peak, and the landing point. The picture reminds you that the object passes through each height twice (once going up and once going down), except at the peak. It also shows why the roots are symmetrical about the vertex and why a negative root is not part of the story.

When the model is given as a table, look for symmetry in the outputs. If the heights at $t = 1$ and $t = 3$ are equal, the peak is at $t = 2$, because the axis of symmetry is halfway between equal outputs. This is a clever way to find the vertex without any formula.

Common traps

1. Giving the x -coordinate of the vertex when the question asks for the maximum value. 2. Keeping a negative time or negative length as an answer. 3. Forgetting that a fence along a wall covers only three sides. 4. Leaving off units or the context in the final sentence.

Mistake Clinic

Spot the error

A student finds that the maximum of $h = -16t^2 + 64t$ occurs at $t = 2$ and says the maximum height is 2 feet. **The error:** 2 is the time of the maximum, not the height. **Right solution:** $h(2) = -64 + 128 = 64$, so the maximum height is 64 feet, reached at 2 seconds.

Spot the error

A student solves $-16t^2 + 64t + 80 = 0$ and reports that the object lands at $t = -1$ and $t = 5$ seconds. **The error:** a time of -1 is before the launch and has no meaning here. **Right solution:** $t = 5$ seconds only.

Spot the error

A student has 80 feet of fence for a pen against a barn wall and writes the area as $A = x(40 - x)$, getting a maximum of 400 square feet. **The error:** that formula treats the pen as fenced on all four sides. **Right solution:** with the barn as one side, $A = x(80 - 2x)$, which has its maximum at $x = 20$ and equals $20 \cdot 40 = 800$ square feet.

Real-World Connection

In a soccer kick, the height of the ball in meters is $h = -5t^2 + 20t$ after t seconds. The ball lands when $-5t(t - 4) = 0$, at $t = 4$ seconds. The peak is halfway at $t = 2$, where $h = -20 + 40 = 20$ meters. So the kick stays in the air for 4 seconds and reaches 20 meters.

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Challenge Corner

A school play charges p dollars per ticket. Attendance is $200 - 10p$, and fixed costs are 300 dollars. Find the ticket price that maximizes profit, and the maximum profit. **Solution:** Profit = $p(200 - 10p) - 300 = -10p^2 + 200p - 300$. The vertex is at $p = -\frac{200}{2(-10)} = 10$, and $P(10) = -1000 + 2000 - 300 = 700$. The best price is 10 dollars, giving a profit of 700 dollars.

Practice**PRACTICE Quadratic Models and Word Problems**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- A ball's height is $h = -16t^2 + 32t + 5$ feet. At what time does it reach its maximum height?
 (A) 1 second (B) 2 seconds (C) 0.5 seconds
 (D) 5 seconds
- For the same ball, $h = -16t^2 + 32t + 5$, what is the maximum height?
 (A) 1 foot (B) 5 feet (C) 21 feet (D) 37 feet
- A ball launched from the ground follows $h = -16t^2 + 80t$. After how many seconds does it land?
 (A) 2.5 seconds (B) 5 seconds (C) 80 seconds
 (D) 25 seconds
- A rectangle has a perimeter of 60 feet. What is its greatest possible area?
 (A) 225 square feet (B) 900 square feet (C) 450 square feet
 (D) 15 square feet
- A theater's revenue is $R(p) = p(200 - 5p)$, where p is the ticket price in dollars. Which price gives the greatest revenue?
 (A) 40 dollars (B) 10 dollars (C) 100 dollars
 (D) 20 dollars
- A company's profit is $P(x) = -x^2 + 12x - 20$ dollars. For which values of x is the profit positive?
 (A) $x < 2$ or $x > 10$ (B) $2 < x < 10$ (C) $x > 2$
 (D) $x < 10$
- What is the maximum profit for $P(x) = -x^2 + 12x - 20$?
 (A) 6 dollars (B) 20 dollars (C) 16 dollars
 (D) -20 dollars
- A ball's height is $h = -16t^2 + 48t + 64$ feet. When does it land?
 (A) 4 seconds (B) -1 second (C) 3 seconds
 (D) 1.5 seconds
- In the model $h(t) = -16t^2 + v_0t + h_0$, the y-intercept of the graph represents which quantity?
 (A) The time the object lands (B) The maximum height
 (C) The time of the maximum height (D) The starting height of the object
- An object's height is $h = -16t^2 + 96t$ feet. Find its maximum height in feet.
- A rectangular garden has a perimeter of 44 feet. Find the greatest possible area in square feet.
- At a price of p dollars, a shop sells $150 - 5p$ items. Find the greatest possible revenue in dollars.
- An object's height is $h = -16t^2 + 16t + 32$ feet. After how many seconds does it hit the ground?
- A company's profit in dollars is $P(x) = -2x^2 + 36x - 160$ with x in hundreds of items. Find the two break-even values of x .
- A farmer uses 72 feet of fence for a rectangular pen against a long wall, fencing three sides. Find the greatest possible area in square feet.

Answers and Explanations

Complete solutions for every practice problem, numbered to match. Read the explanation even when your answer is right.

14.1 Solving Quadratics by Factoring and Square Roots

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PRACTICE · SOLVING QUADRATICS BY FACTORING AND SQUARE ROOTS

1. B. Each factor equals zero: $x - 4 = 0$ gives 4 and $x + 9 = 0$ gives -9 . The first wrong choice flips both signs.
2. A. $(x - 2)(x - 5) = 0$. The factors $x - 2$ and $x - 5$ give positive solutions. The negative pair would make the middle term positive.
3. D. Both 7 and -7 square to 49. Choice 24.5 comes from dividing by 2 instead of taking a square root.
4. C. Standard form: $x^2 + 4x - 12 = 0$, so $(x + 6)(x - 2) = 0$. The choice $x = 12$ or $x = 8$ sets factors of $x(x + 4)$ equal to 12, which is not allowed.
5. B. $5x^2 = 80$ gives $x^2 = 16$. The value ± 16 forgets to take the square root, and $x = 4$ alone drops the negative root.
6. A. Take square roots: $x + 3 = \pm 7$, so $x = 4$ or $x = -10$. The pair ± 7 forgets to subtract 3.
7. D. $ac = -12$; the pair is 12 and -1 . Then $(3x - 1)(x + 4) = 0$. Setting $3x - 1 = 0$ gives $\frac{1}{3}$, not 3.
8. B. Factor: $x(x - 8) = 0$. Dividing by x would lose the solution $x = 0$.
9. A. Let the integers be n and $n + 1$: $n^2 + n - 56 = 0$, so $(n + 8)(n - 7) = 0$ and $n = 7$. The larger integer is 8, not 7.
10. $x = 0$ or $x = 4$. Factor out $2x$: $2x(x - 4) = 0$.
11. $x = 6$. It factors as $(x - 6)^2 = 0$, a double root.
12. $x = \pm \frac{5}{2}$. Either factor $(2x - 5)(2x + 5) = 0$ or write $x^2 = \frac{25}{4}$.
13. $x = 5$ or $x = -1$. Divide by 3: $(x - 2)^2 = 9$, so $x - 2 = \pm 3$.
14. $x = -5$ or $x = 2$. Move terms: $x^2 + 3x - 10 = 0$, so $(x + 5)(x - 2) = 0$.
15. 32. With width w : $w(2w + 4) = 48$, so $w^2 + 2w - 24 = 0$ and $(w + 6)(w - 4) = 0$. The width is 4 and the length is 12. Perimeter $= 2(4 + 12) = 32$.

14.2 Completing the Square and the Quadratic Formula**PRACTICE · COMPLETING THE SQUARE AND THE QUADRATIC FORMULA**

1. C. Half of 12 is 6, and $6^2 = 36$. Choice 6 forgets to square, and 144 squares the whole coefficient instead of half of it.
2. A. Move 5: $x^2 - 8x = -5$. Add 16 to both sides: $(x - 4)^2 = 11$. The value 21 comes from adding 5 instead of subtracting it.
3. D. $D = (-3)^2 - 4(2)(-5) = 9 + 40 = 49$. The value -31 uses $-4ac$ with the wrong sign, and 7 is the square root of the discriminant.
4. B. $D = 16 - 28 = -12 < 0$, so there are no real solutions. The parabola stays above the x-axis.
5. A. $D = 25 - 12 = 13$ and the denominator is $2a = 2$. The second choice uses $25 + 12$, and the third uses a denominator of 4.
6. C. $D = 9 + 72 = 81$, so $x = \frac{-3 \pm 9}{4}$, giving $\frac{3}{2}$ and -3 . The last choice forgets to divide by $2a = 4$.
7. D. Set $D = k^2 - 64 = 0$, so $k = 8$. The choice 64 is k^2 , not k .
8. B. Move 2, add 25: $(x + 5)^2 = 23$. The choice with 27 adds the 2 instead of subtracting it, and the sign of 5 must be negative.
9. A. Set $h = 0$ and divide by -16 : $t^2 - 2t - 3 = 0$, so $(t - 3)(t + 1) = 0$. Time cannot be -1 , so $t = 3$.
10. $x = -1$ or $x = -7$. Move 7, add 16: $(x + 4)^2 = 9$, so $x + 4 = \pm 3$.
11. $x = 3 \pm \sqrt{11}$. Move 2, add 9: $(x - 3)^2 = 11$.
12. $x = 3$ or $x = \frac{1}{2}$. $D = 49 - 24 = 25$, so $x = \frac{7 \pm 5}{4}$.
13. $x = -1 \pm \sqrt{6}$. $D = 4 + 20 = 24$, $\sqrt{24} = 2\sqrt{6}$, so $x = \frac{-2 \pm 2\sqrt{6}}{2}$.
14. 0. $D = 4 - 48 = -44 < 0$.
15. $m = 4$. Set $D = 144 - 36m = 0$, so $m = 4$. (Check: $4x^2 + 12x + 9 = (2x + 3)^2$.)

14.3 Graphing Parabolas**PRACTICE · GRAPHING PARABOLAS**

1. D. $x = -\frac{b}{2a} = -\frac{-6}{2} = 3$. The choice $x = -3$ drops the negative sign of the formula.
2. A. In $a(x - h)^2 + k$ the vertex is $(h, k) = (2, 5)$. The first wrong choice reads the sign inside the parentheses directly.
3. C. Opening direction depends on the sign of a . Only $y = -0.5x^2 + 4$ has $a < 0$.
4. B. The narrowest has the largest $|a|$, which is $|-4| = 4$. The negative sign only flips the direction.
5. A. Set $x = 0$: $y = 7$. The y-intercept is the constant term c .
6. D. Set each factor to zero: $x = 3$ and $x = -5$. Choice -15 is the y-intercept.
7. B. $x = -\frac{-8}{2} = 4$ and $y = 16 - 32 + 10 = -6$. The choice $(4, 6)$ gets the y-coordinate by a sign error.
8. C. The vertex is at $x = 2$. Then $y = -4 + 8 + 1 = 5$. The value 2 is the x-coordinate where the maximum occurs.
9. A. With vertex $(1, -4)$: $y = a(x - 1)^2 - 4$. At $x = 3$: $0 = 4a - 4$, so $a = 1$. The choice with $a = 2$ gives $y = 4$ at $x = 3$.
10. $(-5, -4)$. $x = -\frac{10}{2} = -5$ and $y = 25 - 50 + 21 = -4$.
11. $x = 5$ and $x = -3$. Factor: $(x - 5)(x + 3) = 0$.
12. $y = (x - 4)^2 + 3$. Half of -8 is -4 : $(x^2 - 8x + 16) - 16 + 19$.
13. $y = -2(x + 3)^2 + 2$. Substitute: $-6 = a(2)^2 + 2$ gives $a = -2$.
14. $b = 6$. The vertex touches the axis when the discriminant is zero: $b^2 - 36 = 0$. (Then $y = (x + 3)^2$.)
15. 6. Factor $-2(x - 5)(x + 1) = 0$, so the intercepts are 5 and -1 . Distance $= 5 - (-1) = 6$.

14.4 Quadratic Models and Word Problems

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PRACTICE · QUADRATIC MODELS AND WORD PROBLEMS

1. A. $t = -\frac{32}{2(-16)} = 1$. The value 5 is the starting height, not a time.
2. C. $h(1) = -16 + 32 + 5 = 21$. The value 1 is the time of the maximum, and 37 forgets the -16 term.
3. B. Set $-16t(t - 5) = 0$, so $t = 5$. The value 2.5 is the time of the peak, halfway through the flight.
4. A. The sides add to 30, so $A = x(30 - x)$ with its maximum at $x = 15$: $15 \cdot 15 = 225$. The value 15 is a dimension, not the area.
5. D. Expand to $-5p^2 + 200p$. The vertex is at $p = -\frac{200}{2(-5)} = 20$. The price 40 makes the revenue zero, because it is a root.
6. B. The roots are $x = 2$ and $x = 10$. The parabola opens downward, so it is above the axis between the roots.
7. C. The vertex is at $x = -\frac{12}{2(-1)} = 6$. Then $P(6) = -36 + 72 - 20 = 16$. The value 6 is the number of items, not the profit.
8. A. Factor: $-16(t - 4)(t + 1) = 0$, so $t = 4$ or $t = -1$. A negative time is rejected. The value 1.5 is the time of the peak.
9. D. At $t = 0$, $h(0) = h_0$, the starting height. The landing time is an x-intercept, and the maximum is at the vertex.
10. 144. The peak is at $t = -\frac{96}{-32} = 3$ seconds, and $h(3) = -144 + 288 = 144$.
11. 121. The sides add to 22, so $A = x(22 - x)$ is greatest at $x = 11$, and $11 \cdot 11 = 121$.
12. 1125. $R = p(150 - 5p) = -5p^2 + 150p$. The vertex is at $p = 15$, and $R(15) = 15 \cdot 75 = 1125$.
13. 2 seconds. Set $-16(t - 2)(t + 1) = 0$, so $t = 2$ (reject $t = -1$).
14. $x = 8$ and $x = 10$. Divide by -2 : $x^2 - 18x + 80 = 0$, so $(x - 8)(x - 10) = 0$.
15. 648. With the width x for the two sides touching the wall, $A = x(72 - 2x)$. The vertex is at $x = 18$, so $A = 18 \cdot 36 = 648$.

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