



Geometry Proofs and Congruence

Four focused lessons, worked examples, a mistake clinic, and 60 practice problems with full explanations.

In this booster

17.1 Reasoning and the Language of Proof

17.2 Angle and Line Proofs

17.3 Triangle Congruence

17.4 Properties of Triangles and Quadrilaterals

Every section teaches the idea, walks through the most common problem types, flags the traps students fall into, and ends with fifteen practice problems. Full answers and explanations are at the back.

17.1 Reasoning and the Language of Proof

Learn the words that every proof is built from: definitions, postulates, theorems, and conditional statements. You will practice writing converses and contrapositives and hunting for counterexamples.

Why This Matters

Geometry is the first course where you are asked to **prove** that something is true, not just compute it. That shift surprises many students. In earlier courses, a single example felt like enough. Here, one example proves nothing, and a clear chain of reasons proves everything. Students most often get stuck because they mix up a statement with its converse, because they trust a pattern that happens to work a few times, or because they do not know which facts they are allowed to use as reasons.

The language of proof is not only for geometry class. Programmers write conditional statements (if this, then that) every day. Lawyers, scientists, and engineers all need to tell the difference between a claim and its reverse. If you master the vocabulary in this section, every proof in the rest of the booster becomes a matter of putting familiar pieces in order.

Core Ideas

Every proof is built from four kinds of statements. **Definitions** explain what a word means exactly. **Postulates** (also called axioms) are basic facts we accept without proof, such as "through any two points there is exactly one line." **Theorems** are statements that have been proved from definitions, postulates, and earlier theorems. **Given information** is what the problem tells you to assume.

Conditional Statement

A **conditional** has the form "If p , then q ," written $p \rightarrow q$. The part after "if" is the **hypothesis** (p). The part after "then" is the **conclusion** (q).

A conditional is **false** only when the hypothesis is true and the conclusion is false. One such case is called a **counterexample**.

From any conditional you can build three related statements by swapping and negating. Negating means saying the opposite, and we write $\sim p$ for "not p ."

Form	Symbols	Meaning	Same truth value as the original?
Conditional	$p \rightarrow q$	If p , then q	Yes (it is the original)
Converse	$q \rightarrow p$	Swap hypothesis and conclusion	No, not always
Inverse	$\sim p \rightarrow \sim q$	Negate both parts	No, not always
Contrapositive	$\sim q \rightarrow \sim p$	Swap and negate both	Yes, always

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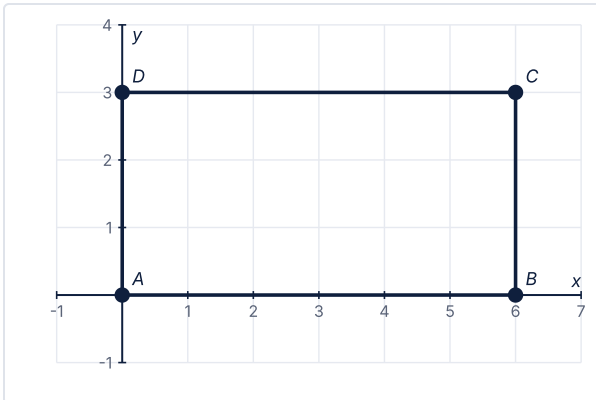
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Contrapositive Rule

A conditional and its **contrapositive** are always either both true or both false. The converse and the inverse are also always the same as each other, but they can differ from the original.

To disprove any conditional, you need only one counterexample. To prove it, you need a reason that works for every case.



Rectangle ABCD is a counterexample to "If a quadrilateral has four right angles, then it has four equal sides." All four angles are right angles, but the sides are 6 and 3.

A **biconditional** joins a conditional and its converse into one statement using the phrase "if and only if." It is true only when both directions are true. Good definitions are always biconditionals: a triangle is equilateral if and only if all three sides are congruent.

Two Ways to Reason

Inductive reasoning looks at several examples, finds a pattern, and makes a guess (a conjecture). It can be wrong.

Deductive reasoning starts from true statements and uses logic to reach a conclusion that must be true. Proofs use deductive reasoning.

Law of Detachment: if $p \rightarrow q$ is true and p is true, then q is true. **Law of Syllogism:** if $p \rightarrow q$ and $q \rightarrow r$ are true, then $p \rightarrow r$ is true.

Statement	Hypothesis	Conclusion
If a figure is a square, then it is a rectangle.	a figure is a square	it is a rectangle
If $x = 3$, then $x^2 = 9$.	$x = 3$	$x^2 = 9$
Two angles that add to 90° are complementary.	two angles add to 90°	they are complementary

Worked Examples

EXAMPLE 1

Identify the hypothesis and conclusion of "If a number is divisible by 6, then it is even." Then write it in the form "If p , then q " for the sentence "A square has four equal sides."

SOLUTION

The hypothesis follows "if": a number is divisible by 6. The conclusion follows "then": the number is even.

For the second sentence, ask what you are given. You are given a square. So: "If a figure is a square, then it has four equal sides." The hypothesis is "a figure is a square."

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EXAMPLE 2

Write the converse, inverse, and contrapositive of "If a quadrilateral is a square, then it has four right angles." Decide whether each is true or false.

SOLUTION

Original: true, because every square has four right angles.

Converse: If a quadrilateral has four right angles, then it is a square. False. A rectangle that is not a square is a counterexample.

Inverse: If a quadrilateral is not a square, then it does not have four right angles. False, for the same rectangle.

Contrapositive: If a quadrilateral does not have four right angles, then it is not a square. True, matching the original.

EXAMPLE 3

Find a counterexample to: "If $x^2 = 49$, then $x = 7$."

SOLUTION

A counterexample makes the hypothesis true and the conclusion false.

Try $x = -7$. Then $x^2 = (-7)^2 = 49$, so the hypothesis is true. But $x = -7 \neq 7$, so the conclusion is false.

The statement is false, and $x = -7$ proves it.

EXAMPLE 4

Is the biconditional "Two angles are supplementary if and only if they form a linear pair" true? Explain.

SOLUTION

Split it into two conditionals. (1) If two angles form a linear pair, then they are supplementary. This is true.

(2) If two angles are supplementary, then they form a linear pair. Try angles of 100° and 80° in different places. They add to 180° but are not adjacent, so they are not a linear pair.

Because direction (2) fails, the biconditional is false.

EXAMPLE 5

These statements are true: "If a student joins the robotics club, then the student attends Friday meetings." "If a student attends Friday meetings, then the student earns a team badge." Ria joins the robotics club. What follows?

SOLUTION

Chain the two conditionals with the Law of Syllogism: if a student joins the club, then the student earns a team badge.

Now Ria joins the club, so the Law of Detachment applies. Conclusion: Ria earns a team badge (and she attends Friday meetings).

EXAMPLE 6

A student notices that $n^2 + n + 41$ is prime for $n = 0, 1, 2$ and 3 . Values: 41, 43, 47, 53. She conjectures it is prime for every whole number n . Is she right?

SOLUTION

This is inductive reasoning, so we look for a counterexample. In fact the expression is prime for every n from 0 to 39, so patience is needed.

Try $n = 40$: $40^2 + 40 + 41 = 1600 + 81 = 1681 = 41 \cdot 41$.

Since $1681 = 41^2$ is not prime, the conjecture is false. A pattern that holds 40 times can still fail. This is why geometry demands proof.

Coach's Playbook

- 1 Underline the hypothesis.** Find the word "if" and mark everything up to "then." Rewrite unusual sentences as "If ..., then ..." first.
- 2 Name the form you need.** Converse: swap. Inverse: negate. Contrapositive: swap and negate. Say it out loud as you do it.
- 3 Test with a quick example.** To check truth, hunt for a counterexample first. If you cannot find one after trying odd cases (zero, negatives, a long thin rectangle), then look for a reason.

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4 Check every direction of a biconditional. Write the two conditionals separately and test each.

5 Cite a reason for each step. Definition, postulate, theorem, or given. If you cannot name the reason, you do not have a step yet.

- ♦ Memory trick: the contrapositive is the converse of the inverse, and it always matches the original.
- ♦ "All A are B" becomes "If A, then B." Never reverse it by accident.
- ♦ Words like "only if" flip direction: " p only if q " means $p \rightarrow q$, not $q \rightarrow p$.

Common traps

1) Assuming the converse is true because the original is. 2) Calling a statement true because it worked for two or three examples. 3) Forgetting to negate BOTH parts when writing the inverse or contrapositive. 4) Using a picture as proof: a diagram that looks like a right angle is not given as a right angle.

Mistake Clinic

Spot the error

A student writes the contrapositive of "If a shape is a square, then it is a rectangle" as "If a shape is a rectangle, then it is a square." **Where is the error?** That is the converse, not the contrapositive. It is also false (a 6×3 rectangle). The contrapositive swaps AND negates: "If a shape is not a rectangle, then it is not a square." This is true.

Spot the error

A student says: "I tested $x = 2, 4, 6$ and each time x^2 was even, so 'If x^2 is even, then x is even' is proved." **Where is the error?** Testing examples is inductive and proves nothing. The statement happens to be true, but the student has not shown it for all numbers. A valid proof writes x as odd, $x = 2k + 1$, shows x^2 is odd, and uses the contrapositive.

Spot the error

A student writes the inverse of "If it is Saturday, then there is no school" as "If there is no school, then it is Saturday." **Where is the error?** The student swapped the parts (converse) but did not negate. The inverse negates both parts and keeps the order: "If it is not Saturday, then there is school."

Real-World Connection

A game app says: "If you collect 10 coins, then you unlock level 2." That does not mean that unlocking level 2 required 10 coins (the converse). A player might unlock level 2 with a key. Reading rules carefully, in games, contracts, and code, depends on keeping a conditional and its converse separate.

Challenge Corner

True statements: (i) If a polygon is a regular hexagon, then all its sides are congruent. (ii) If all of a polygon's sides are congruent, then the polygon is equilateral. Polygon Q is NOT equilateral. What can you conclude about Q ? **Solution:** Take the contrapositive of (ii): if a polygon is not equilateral, then not all of its sides are congruent. So Q has sides that are not all congruent. Now take the contrapositive of (i): if a polygon does not have all sides congruent, then it is not a regular hexagon. Chaining: Q is not a regular hexagon.

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Practice

PRACTICE Reasoning and the Language of Proof

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- In the statement "If it rains, then the game is canceled," what is the hypothesis?
 (A) it rains (B) the game is canceled (C) it does not rain
 (D) the game is not canceled
- What is the converse of "If a figure is a triangle, then it has three sides"?
 (A) If a figure is not a triangle, then it does not have three sides.
 (B) If a figure has three sides, then it is a triangle.
 (C) If a figure does not have three sides, then it is not a triangle.
 (D) If a figure is a triangle, then it has four sides.
- What is the contrapositive of "If $x > 5$, then $x^2 > 25$ "?
 (A) If $x^2 > 25$, then $x > 5$. (B) If $x \leq 5$, then $x^2 \leq 25$. (C) If $x^2 \leq 25$, then $x \leq 5$.
 (D) If $x < 5$, then $x^2 < 25$.
- Which statement is FALSE?
 (A) If $x = 4$, then $x^2 = 16$.
 (B) If $x^2 = 16$, then $x = 4$.
 (C) If a figure is a square, then it is a rectangle.
 (D) If two angles are vertical angles, then they have equal measures.
- A statement that is accepted as true without proof is called a
 (A) postulate (B) theorem (C) conjecture
 (D) counterexample
- True: "If a plant gets light, then it grows." True: "If a plant grows, then it makes seeds." Which must be true?
 (A) If a plant makes seeds, then it gets light.
 (B) If a plant does not grow, then it gets light.
 (C) If a plant gets light, then it makes seeds.
 (D) If a plant does not make seeds, then it grows.
- The statement "If a number ends in 0, then it is divisible by 5" is true. Which number is a counterexample to its CONVERSE?
 (A) 10 (B) 15 (C) 20 (D) 12
- Which is a true biconditional?
 (A) A number is even if and only if it is divisible by 4.
 (B) A polygon is a triangle if and only if it has a right angle.
 (C) A number is prime if and only if it is odd.
 (D) An angle is a right angle if and only if its measure is 90° .
- Using inductive reasoning, what is the next number in the pattern 2, 6, 12, 20, ...?
 (A) 28 (B) 30 (C) 32 (D) 26
- Write the contrapositive of "If an angle is acute, then its measure is less than 90° ."
- Give a counterexample to: "If n is prime, then n is odd."
- Consider "If a figure is a square, then it is a rectangle." Of the statement, its converse, its inverse, and its contrapositive, how many are true?
- Find the smallest positive integer n for which $n^2 - n + 11$ is NOT prime.
- The biconditional "A whole number is divisible by 3 if and only if the sum of its digits is divisible by 3" is true. Use it to decide whether 4,725 is divisible by 3.
- True statements: "If a student is in band, then the student practices daily." "If a student practices daily, then the student improves." Dana does not improve. What can you conclude about Dana?

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17.2 Angle and Line Proofs

Master angle pairs formed by crossing and parallel lines, then learn to write two-column and paragraph proofs that give a reason for every step.

Why This Matters

Angle and line proofs are the training ground for all geometry proofs. The facts are simple (vertical angles are equal, a straight angle is 180°), but the skill of writing them as a clear chain of reasons is new. Students most often get stuck in three places: they do not know which angle pairs the words "corresponding," "alternate interior," and "same-side interior" describe; they use a fact the problem has not allowed (like assuming lines are parallel); and they write steps with no reasons.

These ideas are used by carpenters aligning boards, by designers laying out tile, by surveyors measuring distances, and later in trigonometry and calculus when you reason about slopes and angles. Once you can tell which angles must be equal and which must add to 180° , you can solve for unknown angles quickly and prove why your answer is right.

Core Ideas

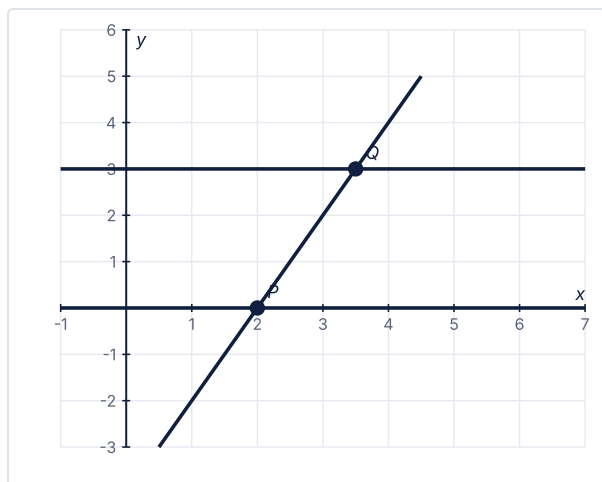
Start with the basic angle facts. A **linear pair** is two adjacent angles whose outer sides form a straight line. **Vertical angles** are the opposite angles formed when two lines cross. When a third line (a **transversal**) crosses two lines, it creates eight angles, and pairs of them have special names.

Angle Pair Theorems

Linear Pair Postulate: the angles of a linear pair are supplementary (sum 180°).

Vertical Angles Theorem: vertical angles are congruent.

Complementary angles sum to 90° . **Supplementary** angles sum to 180° .



Parallel lines (horizontal) cut by a transversal through P and Q. At each crossing there are four angles; the angles at P match the angles at Q when the lines are parallel.

Parallel Lines and a Transversal

If two **parallel** lines are cut by a transversal, then:

corresponding angles are congruent; alternate interior angles are congruent; alternate exterior angles are congruent; same-side interior angles are supplementary.

Converses: if corresponding angles (or alternate interior, or alternate exterior) are congruent, or same-side interior angles are supplementary, then the lines are parallel.

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Angle pair	Where they sit	Parallel lines give
Corresponding	same corner at each crossing	equal
Alternate interior	between the lines, opposite sides	equal
Alternate exterior	outside the lines, opposite sides	equal
Same-side interior	between the lines, same side	add to 180°

Proofs also use properties you already know from algebra. Each step of a proof must be justified by one of these, by a definition, by a theorem, or by given information.

Property	Statement
Reflexive	$a = a$ (or $\angle A \cong \angle A$)
Symmetric	if $a = b$ then $b = a$
Transitive	if $a = b$ and $b = c$ then $a = c$
Substitution	if $a = b$ you may replace a with b
Addition and Subtraction	equals added to (or taken from) equals stay equal
Definition of midpoint	a midpoint splits a segment into two congruent parts
Definition of angle bisector	a bisector splits an angle into two congruent angles

A **two-column proof** lists numbered statements on the left and the reason for each on the right. A **paragraph proof** says the same thing in sentences. Both start with the given information and end with the statement you must prove.

Worked Examples

EXAMPLE 1

Two lines cross. One angle measures $(4x - 7)^\circ$ and its vertical angle measures $(2x + 19)^\circ$. Find x and the angle.

SOLUTION

Vertical angles are congruent: $4x - 7 = 2x + 19$.

$2x = 26$, so $x = 13$.

The angle is $4(13) - 7 = 45^\circ$. Check: $2(13) + 19 = 45$.

EXAMPLE 2

Two parallel lines are cut by a transversal. Two same-side interior angles measure $(3x + 5)^\circ$ and $(2x + 25)^\circ$. Find both angles.

SOLUTION

Same-side interior angles are supplementary: $(3x + 5) + (2x + 25) = 180$.

$5x + 30 = 180$, so $x = 30$.

The angles are $3(30) + 5 = 95^\circ$ and $2(30) + 25 = 85^\circ$. They add to 180° .

EXAMPLE 3

Parallel lines are cut by a transversal. Alternate interior angles measure $(5x - 12)^\circ$ and $(3x + 20)^\circ$. Find x .

SOLUTION

Alternate interior angles are congruent: $5x - 12 = 3x + 20$.

$2x = 32$, so $x = 16$. Each angle is $5(16) - 12 = 68^\circ$, and $3(16) + 20 = 68^\circ$.

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EXAMPLE 4

Write a two-column style proof that vertical angles are congruent. Given: lines AB and CD cross at E . Prove: $\angle AEC \cong \angle BED$.

SOLUTION

1. $\angle AEC$ and $\angle CEB$ form a linear pair, so $m\angle AEC + m\angle CEB = 180^\circ$. Reason: Linear Pair Postulate.
2. $\angle CEB$ and $\angle BED$ form a linear pair, so $m\angle CEB + m\angle BED = 180^\circ$. Reason: Linear Pair Postulate.
3. $m\angle AEC + m\angle CEB = m\angle CEB + m\angle BED$. Reason: Substitution (both equal 180°).
4. $m\angle AEC = m\angle BED$. Reason: Subtraction Property (subtract $m\angle CEB$).
5. $\angle AEC \cong \angle BED$. Reason: Definition of congruent angles.

EXAMPLE 5

Given $a \parallel b$ and a line c perpendicular to a . Prove $c \perp b$ in a paragraph proof.

SOLUTION

Because $c \perp a$, the angle formed where c meets a is a right angle, so it measures 90° by the definition of perpendicular.

Since $a \parallel b$, the corresponding angle where c meets b is congruent to that right angle (Corresponding Angles Theorem). So it also measures 90° .

An intersection that forms a right angle means the lines are perpendicular, so $c \perp b$.

EXAMPLE 6

In the figure, $\overline{AB} \parallel \overline{CD}$, E lies between the lines, $m\angle ABE = 35^\circ$, and $m\angle CDE = 50^\circ$. Find $m\angle BED$.

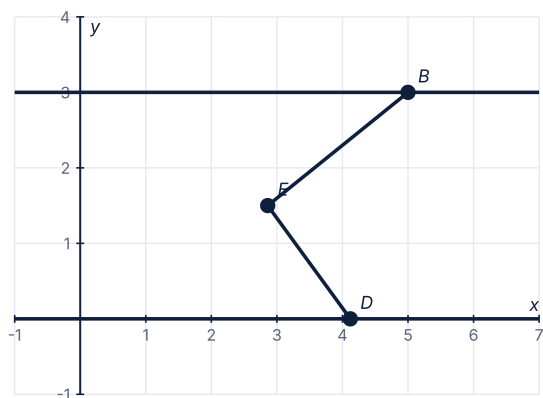
SOLUTION

Draw a helper line through E parallel to $\overline{AB}$ and $\overline{CD}$. It splits $\angle BED$ into two pieces.

The upper piece is an alternate interior angle with $\angle ABE$ (parallel lines), so it measures 35° .

The lower piece is an alternate interior angle with $\angle CDE$, so it measures 50° .

Adding, $m\angle BED = 35^\circ + 50^\circ = 85^\circ$.



Zigzag figure for Example 6: AB is the top line, CD the bottom line. Angle BED is the sum of the two alternate interior angles, $35 + 50 = 85$ degrees.

Coach's Playbook

- 1 **Mark the figure.** Copy the figure and label every given measure or tick mark. Mark right angles only when they are stated.
- 2 **Name the pair.** Say "corresponding," "alternate interior," or "same-side interior" before you write an equation. The name tells you equal or 180° .
- 3 **Equal or supplementary?** Angles that look the same size are equal. Angles that look like one acute and one obtuse add to 180° . Use this as a sanity check on your answer.

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- 4 Solve, then substitute back.** Always find the angle itself, not just x , and check it against the picture (an acute angle should come out less than 90°).
- 5 For a proof, plan backward.** Write what you must prove at the bottom, then ask what single fact would give it. Work up to the givens.
- ◆ Add a helper line through a point parallel to the given parallel lines when the figure has a bend or zigzag.
 - ◆ In algebra problems, two equal angles give an equation with an equals sign; supplementary angles give a total of 180.
 - ◆ A reason column can use only: given, definition, postulate, or theorem. "It looks equal" is never a reason.

Common traps

1) Using alternate interior angles when the lines are not stated to be parallel. 2) Setting two same-side interior angles equal instead of adding them to 180° . 3) Solving for x and forgetting to compute the angle. 4) Assuming a line is perpendicular or parallel because it looks that way.

Mistake Clinic

Spot the error

A student sets same-side interior angles equal: $(3x + 5) = (2x + 25)$ and gets $x = 20$. **Where is the error?** Same-side interior angles are supplementary, not congruent. The correct equation is $(3x + 5) + (2x + 25) = 180$, which gives $x = 30$ and angles of 95° and 85° .

Spot the error

A student writes: " $\angle 1$ and $\angle 2$ are alternate interior angles, so $\angle 1 \cong \angle 2$." The figure shows two lines crossed by a transversal, but nothing says they are parallel. **Where is the error?** Alternate interior angles are congruent only when the lines are parallel. Without that given, the student cannot make this step. A valid proof first states or proves $\ell \parallel m$.

Spot the error

A student finds that vertical angles measure $(6x - 8)^\circ$ and $(4x + 22)^\circ$ and writes $6x - 8 + 4x + 22 = 180$, getting $x = 16.6$. **Where is the error?** Vertical angles are equal, not supplementary. Use $6x - 8 = 4x + 22$, so $2x = 30$ and $x = 15$. Each angle is 82° .

Real-World Connection

The lines painted in a parking lot are parallel, and a car's path crossing them acts as a transversal. Painters check that the angle between the car lane and each stripe is the same. This is the Corresponding Angles Converse in action: equal corresponding angles guarantee the stripes are parallel, so the parking spaces stay the same width.

Challenge Corner

In a zigzag figure, $\overline{AB} \parallel \overline{CD}$, $m\angle ABE = (2x + 5)^\circ$, $m\angle CDE = (3x + 10)^\circ$, and $m\angle BED = 140^\circ$. Find x . **Solution:** Draw the helper line through E parallel to AB . As in Example 6, $m\angle BED = m\angle ABE + m\angle CDE$. So $(2x + 5) + (3x + 10) = 140$, giving $5x + 15 = 140$ and $x = 25$. The angles are 55° and 85° , and $55 + 85 = 140$.

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Practice

PRACTICE Angle and Line Proofs

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- Two lines cross. One angle measures 72° . What is the measure of its vertical angle?
(A) 18° (B) 72° (C) 108° (D) 288°
- Angles $\angle 1$ and $\angle 2$ are a linear pair. If $m\angle 1 = 115^\circ$, what is $m\angle 2$?
(A) 65° (B) 75° (C) 115° (D) 245°
- Parallel lines are cut by a transversal. Alternate interior angles measure $(3x + 10)^\circ$ and $(5x - 30)^\circ$. What is x ?
(A) 10 (B) 25 (C) 20 (D) 5
- Which fact lets you conclude that two lines cut by a transversal are parallel?
(A) Corresponding angles are congruent. (B) Alternate interior angles are supplementary.
(C) Vertical angles are congruent. (D) Same-side interior angles are congruent.
- Parallel lines are cut by a transversal. Corresponding angles measure $(2x + 14)^\circ$ and $(4x - 6)^\circ$. What is the measure of each angle?
(A) 20° (B) 34° (C) 54° (D) 146°
- In a proof, you know $\angle A \cong \angle B$ and $\angle B \cong \angle C$. Which property lets you write $\angle A \cong \angle C$?
(A) Reflexive Property (B) Symmetric Property (C) Transitive Property
(D) Addition Property
- M is the midpoint of $\overline{AB}$. $AM = 3x + 2$ and $MB = 5x - 10$. What is AB ?
(A) 20 (B) 6 (C) 40 (D) 32
- Ray $\overrightarrow{BD}$ bisects $\angle ABC$. $m\angle ABD = (4x + 3)^\circ$ and $m\angle DBC = (6x - 11)^\circ$. What is $m\angle ABC$?
(A) 31° (B) 62° (C) 7° (D) 93°
- Two parallel lines are cut by a transversal. One of the same-side interior angles measures 112° . What is the other?
(A) 68° (B) 112° (C) 78° (D) 248°
- Two complementary angles: the second is 6 degrees more than 3 times the first. Find the larger angle in degrees.
- Parallel lines are cut by a transversal. Same-side interior angles measure $(7x + 3)^\circ$ and $(5x + 9)^\circ$. Find x .
- $\overline{AB} \parallel \overline{CD}$ with a point E between them, $m\angle ABE = 48^\circ$ and $m\angle CDE = 37^\circ$. Find $m\angle BED$ in degrees.
- Two lines cross. Adjacent angles measure $(2x + 10)^\circ$ and $(3x + 20)^\circ$. Find the measure of the smaller angle in degrees.
- Alternate exterior angles measure $(9x - 4)^\circ$ and $(6x + 32)^\circ$. For what value of x are the two lines parallel?
- In a zigzag figure, $AB \parallel CD$, $m\angle ABE = (4x - 3)^\circ$, $m\angle CDE = (2x + 9)^\circ$, and $m\angle BED = 102^\circ$ where E lies between the lines. Find x .
(Hint: use the helper-line method.)

17.3 Triangle Congruence

Learn the five shortcuts that prove triangles congruent (SSS, SAS, ASA, AAS, HL), why SSA and AAA fail, and how to use CPCTC in proofs, including overlapping triangles.

Why This Matters

Two figures are **congruent** when one can be moved on top of the other so that every part matches exactly. For triangles, you do not need to check all six parts (three sides and three angles). A few well-chosen facts are enough, and knowing exactly which few is the heart of this section. Students get stuck because they confuse the shortcuts (is it SAS or SSA?), because they match the wrong parts when the triangles are turned or flipped, and because they forget to prove the triangles congruent before using CPCTC.

Congruent triangles are everywhere in building and design: truss bridges, roof frames, and cranes all depend on the fact that a triangle with fixed side lengths cannot change shape. Triangle congruence is also the engine behind later geometry: almost every proof about parallelograms, circles, and kites ends with "the triangles are congruent, so these parts are equal."

Core Ideas

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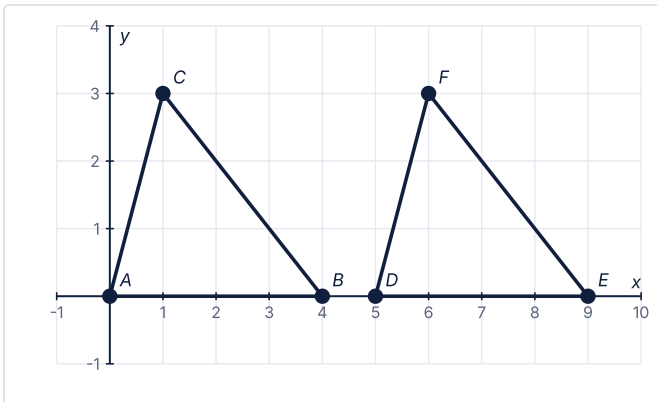
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Congruent Triangles

$\triangle ABC \cong \triangle DEF$ means that $A \leftrightarrow D$, $B \leftrightarrow E$, $C \leftrightarrow F$, all three pairs of corresponding sides are congruent, and all three pairs of corresponding angles are congruent.

The ORDER of the letters tells you which parts match. $\overline{AB}$ corresponds to $\overline{DE}$, and $\angle C$ corresponds to $\angle F$.



Triangle DEF is triangle ABC slid 5 units right, so the triangles are congruent with A-D, B-E, C-F.

Triangle Congruence Shortcuts

SSS: three pairs of congruent sides.

SAS: two pairs of sides and the angle BETWEEN them (the included angle).

ASA: two pairs of angles and the side BETWEEN them.

AAS: two pairs of angles and a side that is NOT between them.

HL (right triangles only): the hypotenuse and one leg.

Shortcut	Parts used	Works?
SSS	side, side, side	Yes
SAS	side, included angle, side	Yes
ASA	angle, included side, angle	Yes
AAS	angle, angle, non-included side	Yes
HL	hypotenuse and leg (right triangles)	Yes
SSA	two sides and a non-included angle	No (can fail)
AAA	three angles	No (same shape, different size)

Why does SSA fail? Imagine two sides and an angle that is not between them. The free side can swing to two different places, giving two different triangles. AAA fails because a small triangle and a large triangle can have the same angles. Neither one is a valid reason in a proof.

CPCTC

Corresponding Parts of Congruent Triangles are Congruent.

Use it only AFTER you have proved the triangles congruent by SSS, SAS, ASA, AAS, or HL. It lets you say that any other pair of corresponding sides or angles are equal.

Overlapping triangles share a side or an angle. Draw them separately, so you can see which sides and angles match. A shared side is congruent to itself by the **Reflexive Property**. Vertical angles, parallel-line angle pairs, and midpoints are the usual sources of the other facts you need.

Worked Examples

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EXAMPLE 1

Which congruence shortcut, if any, is shown? (a) Two sides and the angle between them are congruent to the matching parts of another triangle. (b) Two angles and the side between them are congruent. (c) Two sides and an angle NOT between them are congruent. (d) Two angles and a side not between them are congruent.

SOLUTION

(a) SAS. (b) ASA. (c) None: this is SSA, which does not guarantee congruence. (d) AAS.

Tip: draw the triangle and mark the given parts. The angle is included only if it sits at the meeting point of the two marked sides.

EXAMPLE 2

Triangle ABC has vertices $A(0, 0)$, $B(4, 0)$, $C(1, 3)$. Triangle DEF has $D(5, 0)$, $E(9, 0)$, $F(6, 3)$. Show that $\triangle ABC \cong \triangle DEF$.

SOLUTION

Use the distance formula on each side.

$$AB = 4 \text{ and } DE = 4.$$

$$BC = \sqrt{(1-4)^2 + (3-0)^2} = \sqrt{18} \text{ and } EF = \sqrt{(6-9)^2 + 3^2} = \sqrt{18}.$$

$$AC = \sqrt{1^2 + 3^2} = \sqrt{10} \text{ and } DF = \sqrt{(6-5)^2 + 3^2} = \sqrt{10}.$$

All three pairs of sides are congruent, so $\triangle ABC \cong \triangle DEF$ by SSS.

EXAMPLE 3

Given $\triangle ABC \cong \triangle DEF$, with $AB = 3x + 4$, $DE = 5x - 8$, $m\angle C = (4y + 2)^\circ$, and $m\angle F = (6y - 18)^\circ$. Find x , y , and AB .

SOLUTION

Corresponding sides: $AB = DE$, so $3x + 4 = 5x - 8$, giving $x = 6$ and $AB = 22$.

Corresponding angles: $\angle C \cong \angle F$, so $4y + 2 = 6y - 18$, giving $y = 10$ and $m\angle C = 42^\circ$.

EXAMPLE 4

Given: M is the midpoint of both $\overline{AB}$ and $\overline{CD}$ (the segments cross at M). Prove $\triangle AMC \cong \triangle BMD$, then prove $AC = BD$.

SOLUTION

1. $AM = BM$ and $CM = DM$. Reason: Definition of midpoint.
2. $\angle AMC \cong \angle BMD$. Reason: Vertical Angles Theorem.
3. $\triangle AMC \cong \triangle BMD$. Reason: SAS (the angle is between the two pairs of sides).
4. $AC = BD$. Reason: CPCTC.

EXAMPLE 5

Given: in quadrilateral $ABCD$, $\overline{AB} \cong \overline{DC}$ and $\overline{AC} \cong \overline{DB}$. Prove $\triangle ABC \cong \triangle DCB$ and then $\angle ABC \cong \angle DCB$.

SOLUTION

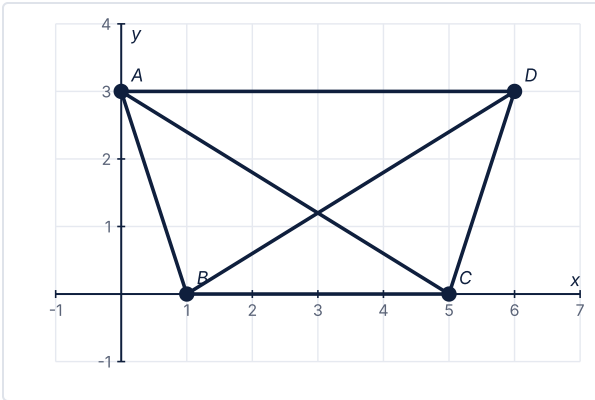
The triangles overlap and share side $\overline{BC}$. Redraw them separately.

1. $AB = DC$ (given). 2. $AC = DB$ (given). 3. $BC = CB$ (Reflexive Property, shared side).
4. $\triangle ABC \cong \triangle DCB$ by SSS. Check the order: $A \rightarrow D$, $B \rightarrow C$, $C \rightarrow B$.
5. $\angle ABC \cong \angle DCB$ by CPCTC.

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Quadrilateral ABCD with diagonals AC and DB. Triangles ABC and DCB overlap and share side BC. Here $AB = DC = \sqrt{10}$ and $AC = DB = \sqrt{34}$.

EXAMPLE 6

Given: $\overline{PQ} \perp \overline{QR}$, $\overline{PS} \perp \overline{SR}$, and $\overline{QR} \cong \overline{SR}$. Prove that $\overline{PR}$ bisects $\angle QPS$.

SOLUTION

1. $\angle Q$ and $\angle S$ are right angles, so $\triangle PQR$ and $\triangle PSR$ are right triangles. Reason: Definition of perpendicular.
2. $PR = PR$ (shared hypotenuse). Reason: Reflexive Property.
3. $QR = SR$. Reason: Given.
4. $\triangle PQR \cong \triangle PSR$. Reason: HL (hypotenuse and a leg).
5. $\angle QPR \cong \angle SPR$. Reason: CPCTC.
6. $\overline{PR}$ bisects $\angle QPS$. Reason: Definition of angle bisector.

Coach's Playbook

1. **Mark the figure.** Use tick marks for equal sides and arcs for equal angles. Mark everything the problem gives, then everything you can add (vertical angles, shared sides, midpoints).
 2. **Count parts.** You need three matched parts in a valid pattern. Write them in order along the triangle (for example S, A, S) to see whether the pattern is allowed.
 3. **Match vertices.** Write the congruence statement with letters in matching order before you start. A wrong order makes CPCTC give a wrong answer.
 4. **Plan with the goal.** Ask: which two triangles contain the parts I must prove equal? Prove those congruent first.
 5. **Finish with CPCTC.** After the triangles are proved congruent, CPCTC is a reason, not a way to prove the triangles congruent.
- ◆ Separate overlapping triangles by drawing them as two figures side by side.
 - ◆ If a shortcut seems one part short, look for a hidden fact: a shared side, vertical angles, a midpoint, or parallel lines.
 - ◆ In coordinate problems, SSS with the distance formula is almost always the quickest route.

Common traps

1) Claiming SSA or AAA as a reason. 2) Using CPCTC before proving the triangles congruent. 3) Writing the congruence statement with mismatched vertex order. 4) Forgetting to state the shared side as a given reason (Reflexive Property), or using a measurement that merely looks equal in the diagram.

Mistake Clinic

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Spot the error

A student writes: " $AB = DE$, $BC = EF$, $\angle A \cong \angle D$, so $\triangle ABC \cong \triangle DEF$ by SAS." **Where is the error?** The angle $\angle A$ is not between sides AB and BC . The included angle for those sides is $\angle B$. This is SSA, which is not a valid shortcut. The student needs $\angle B \cong \angle E$ instead.

Spot the error

A student wants to prove two triangles congruent and writes: " $\angle X \cong \angle Y$ by CPCTC" as step 2, before proving any congruence. **Where is the error?** CPCTC is justified only after the triangles are shown congruent. The correct order is: list the givens, prove the triangles congruent with SSS, SAS, ASA, AAS, or HL, and only then apply CPCTC to the parts you need.

Spot the error

Two triangles have angles of 40° , 60° , 80° . A student says they must be congruent by AAA. **Where is the error?** AAA is not a congruence shortcut. One triangle may have sides of 4, 5, 6 and the other 8, 10, 12. They have the same shape (similar) but different sizes. Congruence needs at least one matching side.

Real-World Connection

A steel roof truss is built from triangles because a triangle is rigid. If two trusses are made with the same three beam lengths (SSS), they are congruent, and a builder can swap one for the other. A four-sided frame has no such guarantee: it can lean, which is why braces are added along a diagonal.

Challenge Corner

Given: segments $\overline{AC}$ and $\overline{BD}$ cross at E , which is the midpoint of both. $AB = 2x + 7$ and $CD = 4x - 5$. Find AB .

Solution: $AE = CE$ and $BE = DE$ (midpoint), and $\angle AEB \cong \angle CED$ (vertical angles). So $\triangle AEB \cong \triangle CED$ by SAS, and $AB = CD$ by CPCTC. Then $2x + 7 = 4x - 5$, so $x = 6$ and $AB = 19$.

Practice**PRACTICE Triangle Congruence**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- Which of these is NOT a valid way to prove triangles congruent?
 (A) SSS (B) SAS (C) ASA (D) SSA
- In $\triangle ABC$ and $\triangle DEF$: $AB = DE$, $\angle B \cong \angle E$, $BC = EF$. Which shortcut proves them congruent?
 (A) SSS (B) SAS (C) ASA (D) AAS
- If $\triangle PQR \cong \triangle XYZ$, which side corresponds to $\overline{QR}$?
 (A) $\overline{YZ}$ (B) $\overline{XY}$ (C) $\overline{XZ}$ (D) $\overline{PR}$
- Two triangles share the side $\overline{BD}$. Which reason justifies $\overline{BD} \cong \overline{BD}$?
 (A) Symmetric Property (B) Transitive Property (C) Reflexive Property
 (D) Substitution Property
- In $\triangle ABC$ and $\triangle DEF$: $\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\overline{BC} \cong \overline{EF}$. Which shortcut applies?
 (A) ASA (B) SAS (C) SSA (D) AAS
- $\triangle ABC \cong \triangle DEF$, $m\angle A = (5x + 2)^\circ$ and $m\angle D = (7x - 22)^\circ$. What is $m\angle A$?
 (A) 12° (B) 62° (C) 82° (D) 106°

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7. You want to prove $\overline{AC} \cong \overline{DF}$. What must you do BEFORE using CPCTC?
- Prove the triangles containing them are congruent.
 - Measure both segments.
 - Prove the triangles are similar.
 - Assume the angles are equal.
8. A right triangle has hypotenuse 13 and one leg 5. Another right triangle has the same hypotenuse and the same leg. What is the length of the other leg in both triangles?
- 8
 - 18
 - 12
 - 144
9. In $\triangle ABC$, $AB = 6$, $BC = 8$, $m\angle A = 40^\circ$. In $\triangle DEF$, $DE = 6$, $EF = 8$, $m\angle D = 40^\circ$. What can you conclude?
- The triangles are congruent by SAS.
 - The triangles are congruent by ASA.
 - The triangles are congruent by SSS.
 - Congruence cannot be concluded from SAS; the angle is not between the sides.
10. $\triangle JKL \cong \triangle MNP$. $JK = 2x + 3$, $MN = 4x - 9$, $KL = 7$, and $JL = 9$. Find the perimeter of $\triangle MNP$.
11. Right triangle ABC has legs 9 and 12. Right triangle DEF has a leg of 9 and a hypotenuse of 15. Are they congruent? If so, name the shortcut.
12. $\triangle ABC \cong \triangle DEF$ with $m\angle A = (2x + 11)^\circ$ and $m\angle D = (4x - 9)^\circ$. Find $m\angle D$.
13. Quadrilateral $ABCD$ has $A(0, 3)$, $B(1, 0)$, $C(5, 0)$, $D(6, 3)$. Name the shortcut that proves $\triangle ABC \cong \triangle DCB$.
14. $\triangle ABC \cong \triangle DEF$, $m\angle B = (3y + 9)^\circ$ and $m\angle E = (5y - 27)^\circ$. Find $m\angle B$ in degrees.
15. Segments $\overline{AC}$ and $\overline{BD}$ cross at their common midpoint E . $AB = 5x - 4$ and $CD = 3x + 8$. Find AB .

17.4 Properties of Triangles and Quadrilaterals

Use the properties of isosceles triangles, medians, perpendicular bisectors, parallelograms, rectangles, rhombi, and trapezoids to find missing measures and justify classifications.

Why This Matters

Once you can prove triangles congruent, you can discover and prove the special properties of triangles and four-sided figures. Many students find this section heavy because there are so many names and facts: isosceles, median, perpendicular bisector, parallelogram, rectangle, rhombus, trapezoid. The trick is to see them as a family tree. Each special shape inherits all the properties of the shape above it and adds one or two of its own. Students most often get stuck when they apply a rhombus fact to a plain parallelogram, or when they forget which angles are the base angles of an isosceles triangle.

These shapes appear in architecture (rectangular frames, rhombus-patterned tiles), engineering (parallelogram linkages in lifts and desk lamps), and art. They also set up later work with coordinates, vectors, and area formulas. Learn the family tree and the key properties of each shape, and a large share of geometry problems turn into quick algebra.

Core Ideas

An **isosceles triangle** has at least two congruent sides, called **legs**. The third side is the **base**. The angles opposite the legs are the **base angles**. The angle between the legs is the **vertex angle**.

Isosceles Triangle Theorems

Base Angles Theorem: if two sides of a triangle are congruent, then the angles opposite them are congruent.

Converse: if two angles of a triangle are congruent, then the sides opposite them are congruent.

The segment from the vertex to the midpoint of the base is also the altitude and the angle bisector. An **equilateral** triangle has three 60° angles.

Perpendicular Bisector Theorem

A point is on the perpendicular bisector of a segment if and only if it is equally far from both endpoints.

A **median** of a triangle joins a vertex to the midpoint of the opposite side. An **altitude** is the perpendicular segment from a vertex to the opposite side.

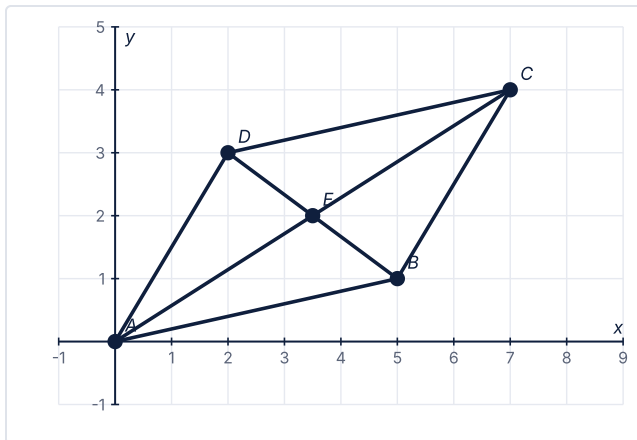
The quadrilateral family tree starts with the **parallelogram** (two pairs of parallel sides). A **rectangle** is a parallelogram with four right angles. A **rhombus** is a parallelogram with four congruent sides. A **square** is both. A **trapezoid** has exactly one pair of parallel sides (the bases), and an **isosceles trapezoid** has congruent legs.

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Shape	Key properties
Parallelogram	opposite sides parallel and congruent; opposite angles congruent; consecutive angles supplementary; diagonals bisect each other
Rectangle	all parallelogram properties, plus four right angles and congruent diagonals
Rhombus	all parallelogram properties, plus four congruent sides; diagonals are perpendicular and bisect the angles
Square	all rectangle and rhombus properties
Isosceles trapezoid	one pair of parallel sides; legs congruent; base angles congruent; diagonals congruent



Parallelogram ABCD with diagonals AC and BD crossing at E. The diagonals bisect each other, so E is the midpoint of both: (3.5, 2).

Two Useful Formulas

Midsegment of a trapezoid: its length is the average of the bases, $m = \frac{b_1 + b_2}{2}$, and it is parallel to the bases.

Rhombus diagonals cut it into four congruent right triangles, so with diagonals d_1 and d_2 each side is $\sqrt{(d_1/2)^2 + (d_2/2)^2}$ and the area is $\frac{d_1 d_2}{2}$.

Worked Examples

EXAMPLE 1

In isosceles triangle ABC with $AB = AC$, the base angles measure $(3x + 7)^\circ$ and $(5x - 13)^\circ$. Find x , the base angles, and the vertex angle.

SOLUTION

The angles opposite the congruent sides are equal: $3x + 7 = 5x - 13$, so $x = 10$.

Each base angle is $3(10) + 7 = 37^\circ$.

The angles of a triangle sum to 180° , so the vertex angle is $180 - 2(37) = 106^\circ$.

EXAMPLE 2

Point P lies on the perpendicular bisector of $\overline{AB}$. If $PA = 4x - 3$ and $PB = 2x + 9$, find PA .

SOLUTION

A point on the perpendicular bisector is equidistant from the endpoints, so $PA = PB$.

$4x - 3 = 2x + 9$ gives $x = 6$.

$PA = 4(6) - 3 = 21$. Check: $PB = 2(6) + 9 = 21$.

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EXAMPLE 3

In parallelogram $ABCD$, $AB = 3x + 2$ and $CD = 5x - 10$. Also $m\angle A = (2y + 10)^\circ$ and $m\angle B = (3y + 20)^\circ$. Find AB and the two angles.

SOLUTION

Opposite sides are congruent: $3x + 2 = 5x - 10$, so $x = 6$ and $AB = 20$.

Angles A and B are consecutive, so they are supplementary: $5y + 30 = 180$, so $y = 30$.

$m\angle A = 70^\circ$ and $m\angle B = 110^\circ$.

EXAMPLE 4

Show that the quadrilateral with vertices $A(0, 0)$, $B(5, 1)$, $C(7, 4)$, $D(2, 3)$ is a parallelogram.

SOLUTION

Use the fact that a quadrilateral whose diagonals bisect each other is a parallelogram.

Midpoint of $\overline{AC}$: $(\frac{0+7}{2}, \frac{0+4}{2}) = (3.5, 2)$.

Midpoint of $\overline{BD}$: $(\frac{5+2}{2}, \frac{1+3}{2}) = (3.5, 2)$.

The diagonals share a midpoint, so they bisect each other and $ABCD$ is a parallelogram. (Check: both $\overline{AB}$ and $\overline{DC}$ run 5 right and 1 up, so they are parallel and equal.)

EXAMPLE 5

A rhombus has diagonals of lengths 10 and 24. Find its side length, perimeter, and area.

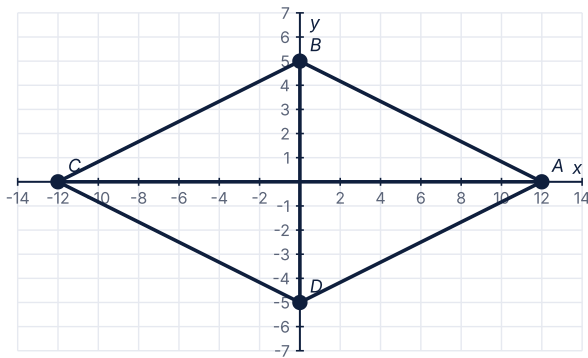
SOLUTION

The diagonals are perpendicular and bisect each other, so each side is the hypotenuse of a right triangle with legs 5 and 12.

Side $= \sqrt{5^2 + 12^2} = \sqrt{169} = 13$.

Perimeter $= 4 \cdot 13 = 52$.

Area $= \frac{10 \cdot 24}{2} = 120$.



Rhombus with diagonals of length 24 (horizontal) and 10 (vertical). Each side has length 13.

EXAMPLE 6

Isosceles trapezoid $ABCD$ has bases $\overline{AB} = 14$ and $\overline{DC} = 22$, legs of length 5, and $m\angle D = 65^\circ$. Find the midsegment length, the perimeter, and the other angles.

SOLUTION

Midsegment $= \frac{14 + 22}{2} = 18$.

Perimeter $= 14 + 22 + 5 + 5 = 46$.

Base angles of an isosceles trapezoid are congruent, so $m\angle C = 65^\circ$.

Each base angle on the top is supplementary to the one below it (same-side interior angles of the parallel bases): $m\angle A = m\angle B = 180 - 65 = 115^\circ$.

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Coach's Playbook

- 1 Draw and label.** Sketch the shape, mark equal sides and angles, and draw diagonals when the problem mentions them.
 - 2 Find the family.** Ask what the shape is and list ALL its properties, including those inherited from parent shapes.
 - 3 Turn properties into equations.** Equal parts give an equation with an equals sign. Consecutive angles in a parallelogram give a total of 180.
 - 4 Use right triangles.** Diagonals of a rhombus or the altitude of an isosceles triangle create right triangles, so the Pythagorean Theorem is ready to use.
 - 5 Check with the picture.** A base angle of an isosceles triangle must be less than 90° . A side that comes out negative signals a mistake.
- ◆ To test for a parallelogram in coordinates, compare the midpoints of the diagonals or check slopes of both pairs of sides.
 - ◆ To classify a quadrilateral, check sides first (all equal?), then diagonals (equal? perpendicular?).
 - ◆ The best name is the most specific one that is true: a square is also a rectangle, but call it a square.

Common traps

1) Using the vertex angle as a base angle. 2) Saying all parallelograms have perpendicular or congruent diagonals (only rhombi and rectangles). 3) Thinking a trapezoid has two pairs of parallel sides. 4) Forgetting to halve the diagonals before using the Pythagorean Theorem in a rhombus.

Mistake Clinic

Spot the error

A student says a parallelogram has diagonals of 10 and 24, so each side is $\sqrt{10^2 + 24^2} = 26$. **Where is the error?** The diagonals of a rhombus meet at their midpoints, so the right triangle has legs of half each diagonal: 5 and 12. The side is $\sqrt{25 + 144} = 13$. Also, a plain parallelogram's diagonals are not necessarily perpendicular, so the shape must be a rhombus.

Spot the error

In isosceles triangle ABC with $AB = AC$, a student writes $m\angle A = m\angle B$ because "the triangle is isosceles." **Where is the error?** The congruent angles are opposite the congruent sides. Sides AB and AC are opposite angles C and B , so $\angle B \cong \angle C$. Angle A is the vertex angle and is generally different.

Spot the error

A student states: "Consecutive angles of a parallelogram are congruent, so with $m\angle A = 68^\circ$, $m\angle B = 68^\circ$." **Where is the error?** Opposite angles are congruent. Consecutive angles are supplementary, so $m\angle B = 180 - 68 = 112^\circ$, and $\angle C = 68^\circ$, $\angle D = 112^\circ$.

Real-World Connection

A folding scissor lift or a desk lamp arm uses parallelograms: the opposite sides stay parallel and equal as the arm moves, so the platform or lamp head stays level while the angles change. The rule that consecutive angles add to 180° shows how raising one angle must lower its neighbor.

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Challenge Corner

An isosceles trapezoid has bases 10 and 26 and legs of length 10. Find its area. **Solution:** Drop perpendiculars from the ends of the shorter base to the longer base. This splits the extra length $26 - 10 = 16$ into two equal pieces of 8.

Each leg is the hypotenuse of a right triangle with a horizontal leg of 8, so the height is $\sqrt{10^2 - 8^2} = 6$. Area = $\frac{10 + 26}{2} \cdot 6 = 108$.

Practice**PRACTICE Properties of Triangles and Quadrilaterals**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- An isosceles triangle has a vertex angle of 50° . What is the measure of each base angle?
(A) 50° (B) 65° (C) 130° (D) 80°
- In isosceles triangle ABC with $AB = AC$, the base angles are $(4x + 6)^\circ$ and $(6x - 14)^\circ$. What is the measure of a base angle?
(A) 10° (B) 34° (C) 46° (D) 134°
- Point P is on the perpendicular bisector of $\overline{AB}$. $PA = 3x + 1$ and $PB = 7x - 11$. What is PA ?
(A) 10 (B) 3 (C) 31 (D) 22
- One angle of a parallelogram measures 68° . What is the measure of an angle next to it?
(A) 68° (B) 22° (C) 292° (D) 112°
- Which property is true for every rhombus but NOT for every parallelogram?
(A) Opposite sides are parallel. (B) The diagonals are perpendicular.
(C) Opposite angles are congruent. (D) The diagonals bisect each other.
- The diagonals of rectangle $ABCD$ are $AC = 3x + 4$ and $BD = 5x - 6$. What is AC ?
(A) 19 (B) 5 (C) 25 (D) 29
- A trapezoid has bases of length 9 and 17. How long is its midsegment?
(A) 8 (B) 26 (C) 13 (D) 6
- The diagonals of parallelogram $ABCD$ meet at E . $AE = 2x + 3$ and $EC = 4x - 7$. What is AC ?
(A) 13 (B) 23 (C) 5 (D) 26
- Which quadrilateral always has congruent diagonals?
(A) Parallelogram (B) Rhombus (C) Kite
(D) Rectangle
- A rhombus has diagonals of length 16 and 30. Find the length of one side.
- Isosceles triangle ABC has legs $(3x - 4)$ and $(x + 10)$ and a base of 13. Find the perimeter.
- Three vertices of parallelogram $ABCD$ are $A(1, 1)$, $B(6, 2)$, $C(8, 6)$. Find the coordinates of D .
- A rectangle has a diagonal of length 26 and one side of length 10. Find its area.
- Quadrilateral $ABCD$ has $A(0, 0)$, $B(4, 3)$, $C(8, 0)$, $D(4, -3)$. Give the most specific name for the shape.
- An isosceles trapezoid has bases 14 and 32 and legs of length 15. Find its area.

Answers and Explanations

Complete solutions for every practice problem, numbered to match. Read the explanation even when your answer is right.

17.1 Reasoning and the Language of Proof**PRACTICE · REASONING AND THE LANGUAGE OF PROOF**

- A. The hypothesis is the part after "if": it rains. Choice B is the conclusion.
- B. B. The converse swaps hypothesis and conclusion. Choice C is the inverse and D is the contrapositive.
- C. C. Swap and negate both parts. Choice A is the converse and B is the inverse, which each forgot one of the two steps.
- B. The value $x = -4$ is a counterexample: $(-4)^2 = 16$ but $x \neq 4$. The other three statements are true.
- A. A. Postulates are basic facts we accept. A theorem is proved, a conjecture is only a guess, and a counterexample disproves a statement.
- C. C. This is the Law of Syllogism, linking the first hypothesis to the last conclusion. Choice A reverses it (converse), which need not be true.

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7. B. B. The converse says that if a number is divisible by 5, it ends in 0. The number 15 is divisible by 5 but ends in 5. Choices 10 and 20 satisfy both parts, and 12 fails the hypothesis.
8. D. Both directions hold for the right angle. The others fail one direction: 6 is even but not divisible by 4; an equilateral triangle has no right angle; 9 is odd but not prime.
9. B. B. The differences are 4, 6, 8, so the next difference is 10 and the next term is $20 + 10 = 30$. Choice A keeps adding 8, which ignores the growing differences.
10. If an angle's measure is not less than 90° , then the angle is not acute. Swap the parts and negate both.
11. $n = 2$. It is prime and even. It satisfies the hypothesis but not the conclusion.
12. 2. The original and the contrapositive are true. The converse and inverse are false (a 6×3 rectangle is a counterexample).
13. $n = 11$. For $n = 1$ to 10 the values are 11, 13, 17, 23, 31, 41, 53, 67, 83, 101, all prime. For $n = 11$: $121 - 11 + 11 = 121 = 11^2$.
14. Yes. The digit sum is $4 + 7 + 2 + 5 = 18$, which is divisible by 3, so by the biconditional 4,725 is divisible by 3 (indeed $4725 = 3 \cdot 1575$).
15. Dana is not in band (and does not practice daily). Chain the statements: band implies improves. The contrapositive says that not improving implies not in band.

17.2 Angle and Line Proofs

PRACTICE · ANGLE AND LINE PROOFS

1. B. Vertical angles are congruent. Choice C is the measure of an adjacent angle (linear pair).
2. A. A linear pair sums to 180° , so $180 - 115 = 65$. Choice C wrongly treats the pair as congruent.
3. C. Alternate interior angles are equal: $3x + 10 = 5x - 30$, so $x = 20$. Choice B comes from wrongly adding them to 180.
4. A. Congruent corresponding angles prove lines parallel. Vertical angles are congruent for any crossing lines, so they say nothing about parallelism.
5. B. Corresponding angles are equal: $2x + 14 = 4x - 6$, so $x = 10$ and the angle is 34° . Choice A is only x doubled, not the angle.
6. C. Transitive: two things equal to a third are equal to each other. Symmetric would only flip $\angle A \cong \angle B$ to $\angle B \cong \angle A$.
7. C. A midpoint gives $AM = MB$: $3x + 2 = 5x - 10$, so $x = 6$, and $AM = MB = 20$. Then $AB = 40$. Choice A is just one half.
8. B. A bisector makes the halves equal: $4x + 3 = 6x - 11$, so $x = 7$. Each half is 31° , so the whole angle is 62° . Choice A is only one half.
9. A. Same-side interior angles are supplementary: $180 - 112 = 68$. Choice B would be right for alternate interior angles.
10. 69. Let the first be x . Then $x + (3x + 6) = 90$, so $x = 21$. The larger is $3(21) + 6 = 69$.
11. 14. They sum to 180: $12x + 12 = 180$, so $x = 14$ (angles 101° and 79°).
12. 85. Draw a helper line through E parallel to the given lines. By alternate interior angles the two pieces are 48° and 37° , which add to 85° .
13. 70. Adjacent angles from crossing lines form a linear pair: $5x + 30 = 180$, so $x = 30$. The angles are 70° and 110° . The smaller is 70.
14. 12. Parallel lines require equal alternate exterior angles: $9x - 4 = 6x + 32$, so $3x = 36$ and $x = 12$ (each angle 104°).
15. 16. The helper line gives $m\angle BED = m\angle ABE + m\angle CDE$. So $6x + 6 = 102$ and $x = 16$ (angles 61° and 41°).

17.3 Triangle Congruence

PRACTICE · TRIANGLE CONGRUENCE

1. D. SSA does not guarantee congruence, since the free side can swing to two positions. The other three are valid.
2. B. The angle $\angle B$ is between AB and BC , and $\angle E$ is between DE and EF , so SAS applies.
3. A. The vertices correspond in order: $Q \rightarrow Y$, $R \rightarrow Z$. Choice B is $\overline{PQ}$'s partner.
4. C. Anything is congruent to itself by the Reflexive Property.
5. D. The side $\overline{BC}$ is not between angles A and B (it is opposite $\angle A$), so the pattern is AAS, not ASA.
6. B. Corresponding angles are equal: $5x + 2 = 7x - 22$ gives $x = 12$. Then $m\angle A = 5(12) + 2 = 62^\circ$. Choice A stops at x .
7. A. CPCTC only applies after the triangles are proved congruent by a valid shortcut.
8. C. By HL the triangles are congruent. By the Pythagorean Theorem the other leg is $\sqrt{13^2 - 5^2} = \sqrt{144} = 12$. Choice D forgets the square root.
9. D. Angle A lies between AB and AC , not AB and BC . This is SSA, which is not enough.
10. 31. $2x + 3 = 4x - 9$ gives $x = 6$, so $JK = MN = 15$. Perimeter = $15 + 7 + 9 = 31$ (congruent triangles have equal perimeters).
11. Yes, by HL. The hypotenuse of ABC is $\sqrt{81 + 144} = 15$, which matches, and the legs of 9 match.
12. 31. $2x + 11 = 4x - 9$ gives $x = 10$, so $m\angle D = 4(10) - 9 = 31$ degrees.
13. SSS. $AB = DC = \sqrt{10}$, $AC = DB = \sqrt{34}$, and $BC = CB = 4$ (shared side).
14. 63. $3y + 9 = 5y - 27$ gives $y = 18$, so $m\angle B = 3(18) + 9 = 63$.
15. 26. SAS (midpoints and vertical angles) gives $\triangle AEB \cong \triangle CED$, so $AB = CD$ by CPCTC. $5x - 4 = 3x + 8$ gives $x = 6$, so $AB = 26$.

17.4 Properties of Triangles and Quadrilaterals

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PRACTICE · PROPERTIES OF TRIANGLES AND QUADRILATERALS

1. B. The base angles share $180 - 50 = 130$ equally: 65° each. Choice C forgets to split the 130 in two.
2. C. Base angles are equal: $4x + 6 = 6x - 14$ gives $x = 10$, and the angle is $4(10) + 6 = 46^\circ$. Choice A is just x .
3. A. Equal distances: $3x + 1 = 7x - 11$ gives $x = 3$, so $PA = 10$. Choice B is x alone.
4. D. Consecutive angles of a parallelogram are supplementary: $180 - 68 = 112$. Choice A is the opposite angle, not the neighbor.
5. B. Only rhombi (and squares) have perpendicular diagonals. The other three hold for every parallelogram.
6. A. Rectangle diagonals are congruent: $3x + 4 = 5x - 6$ gives $x = 5$, so $AC = 19$. Choice A is x .
7. C. The midsegment is the average of the bases: $(9 + 17)/2 = 13$. Choice B adds without dividing.
8. D. Diagonals bisect each other: $2x + 3 = 4x - 7$ gives $x = 5$, so $AE = 13$ and $AC = 26$. Choice A stops at half the diagonal.
9. D. A rectangle's diagonals are congruent. A rhombus or a plain parallelogram can have diagonals of different lengths.
10. 17. Half-diagonals are 8 and 15, so the side is $\sqrt{64 + 225} = \sqrt{289} = 17$.
11. 47. The legs are equal: $3x - 4 = x + 10$ gives $x = 7$, so each leg is 17. Perimeter $= 17 + 17 + 13 = 47$.
12. (3, 5). The diagonals bisect each other, so the midpoint of $\overline{AC}$, (4.5, 3.5), is also the midpoint of $\overline{BD}$. So $D = (9 - 6, 7 - 2) = (3, 5)$.
13. 240. The other side is $\sqrt{26^2 - 10^2} = \sqrt{576} = 24$. Area $= 10 \cdot 24 = 240$.
14. Rhombus. All four sides have length 5, but the diagonals are 8 and 6, which are not equal, so it is not a square.
15. 276. The overhang on each side is $(32 - 14)/2 = 9$, so the height is $\sqrt{15^2 - 9^2} = 12$. Area $= \frac{14+32}{2} \cdot 12 = 23 \cdot 12 = 276$.

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