



Similarity and Transformations

Four focused lessons, worked examples, a mistake clinic, and 60 practice problems with full explanations.

In this booster

18.1 Rigid Motions: Translations, Reflections, and Rotations

18.2 Dilations and Similarity

18.3 Using Similar Triangles

18.4 Symmetry, Tessellations, and Coordinate Transformations

Every section teaches the idea, walks through the most common problem types, flags the traps students fall into, and ends with fifteen practice problems. Full answers and explanations are at the back.

18.1 Rigid Motions: Translations, Reflections, and Rotations

Use coordinate rules for translations, reflections, and rotations, compose motions in order, and see why a sequence of rigid motions proves two figures are congruent.

Why This Matters

A **transformation** moves every point of a figure according to a rule. A **rigid motion** is a transformation that keeps the size and shape of the figure the same: it only changes the position. The three rigid motions are translations (slides), reflections (flips), and rotations (turns). Students get stuck here for two main reasons. First, they memorize coordinate rules without a picture, so they mix up $(x, -y)$ with $(-x, y)$, or confuse a 90 degree clockwise turn with a counterclockwise one. Second, they forget that the order of a composition matters.

Rigid motions are the idea behind computer animation, robotics, video game graphics, and machine design. Every time a character slides, spins, or is mirrored on a screen, a program applies exactly these rules to coordinates. In geometry, rigid motions also give a clean definition of congruence: two figures are congruent when a sequence of rigid motions maps one onto the other.

Core Ideas

Translation

A **translation** slides every point the same distance in the same direction. The rule $(x, y) \rightarrow (x + a, y + b)$ moves a point a units right (left if $a < 0$) and b units up (down if $b < 0$).

Reflections

Reflecting over a line flips the figure so that the line is the perpendicular bisector of each segment joining a point to its image.

Over the x -axis: $(x, y) \rightarrow (x, -y)$. Over the y -axis: $(x, y) \rightarrow (-x, y)$. Over $y = x$: $(x, y) \rightarrow (y, x)$. Over $y = -x$: $(x, y) \rightarrow (-y, -x)$. Over the vertical line $x = h$: $(x, y) \rightarrow (2h - x, y)$.

Rotations About the Origin

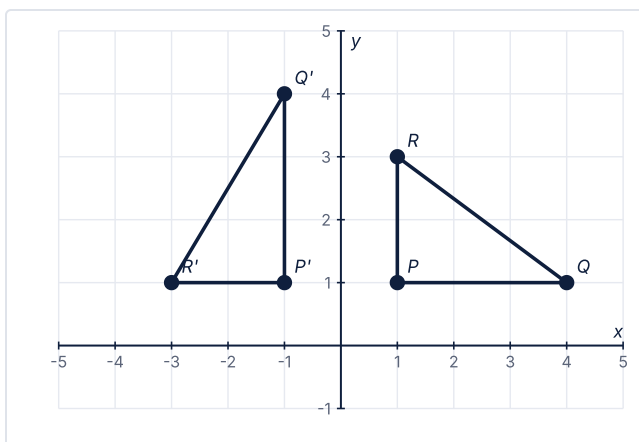
A **rotation** turns every point by the same angle around a fixed center. Counterclockwise is the positive direction.

90° counterclockwise: $(x, y) \rightarrow (-y, x)$. 180°: $(x, y) \rightarrow (-x, -y)$. 270° counterclockwise (same as 90° clockwise): $(x, y) \rightarrow (y, -x)$.

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Triangle PQR rotated 90 degrees counterclockwise about the origin using (x, y) goes to $(-y, x)$. $Q(4, 1)$ moves to $Q'(-1, 4)$.

Motion	Rule	Orientation (order of vertices)
Translation	$(x + a, y + b)$	kept
Reflection	depends on the line	reversed
Rotation	depends on the angle	kept

Every rigid motion preserves **distance** (segment lengths), **angle measure**, **parallelism**, and **area**. Only reflections reverse the orientation, which means a clockwise order of vertices becomes counterclockwise.

Composition and Congruence

A **composition** applies one motion, then another to the image. The order matters: rotating then translating generally gives a different result from translating then rotating.

Two figures are **congruent** exactly when some sequence of rigid motions maps one onto the other. The image and the original have the same side lengths and the same angles.

Worked Examples

EXAMPLE 1

Translate triangle ABC with $A(1, 2)$, $B(4, 2)$, $C(3, 5)$ using the rule $(x, y) \rightarrow (x - 5, y + 3)$.

SOLUTION

Apply the rule to each vertex.

$$A'(1 - 5, 2 + 3) = (-4, 5).$$

$$B'(4 - 5, 2 + 3) = (-1, 5).$$

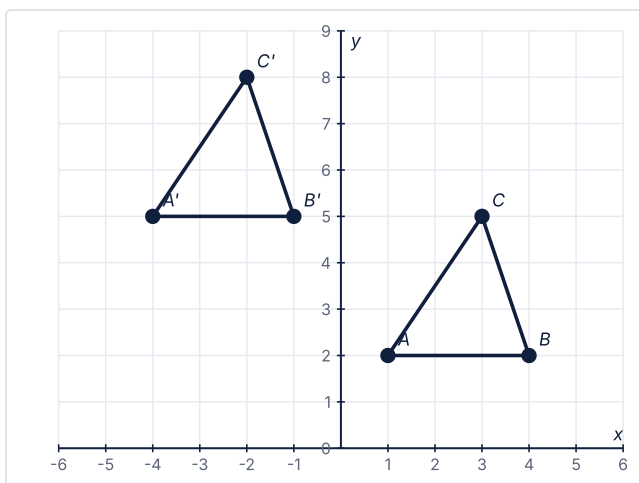
$$C'(3 - 5, 5 + 3) = (-2, 8).$$

Every point moved 5 left and 3 up, so the triangle keeps its size and shape.

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Triangle ABC and its image A'B'C' after a slide of 5 units left and 3 units up.

EXAMPLE 2

Find the image of $P(3, -2)$ under reflections over (a) the x -axis, (b) the y -axis, (c) the line $y = x$, (d) the line $y = -x$.

SOLUTION

(a) $(x, -y)$: $(3, 2)$. (b) $(-x, y)$: $(-3, -2)$.

(c) Swap the coordinates: $(-2, 3)$.

(d) $(-y, -x) = (2, -3)$.

EXAMPLE 3

Rotate $Q(4, 1)$ about the origin by (a) 90° counterclockwise, (b) 180° , (c) 90° clockwise.

SOLUTION

(a) Use $(x, y) \rightarrow (-y, x)$: $(-1, 4)$.

(b) Use $(-x, -y)$: $(-4, -1)$.

(c) A 90° clockwise turn equals 270° counterclockwise, so use $(y, -x)$: $(1, -4)$.

EXAMPLE 4

Triangle ABC has $A(2, 1)$, $B(5, 1)$, $C(5, 3)$. Its image is $A'(-2, 1)$, $B'(-5, 1)$, $C'(-5, 3)$. Describe the motion. Then find the image of A under reflection over the line $x = 4$.

SOLUTION

Each point keeps its y -value and the sign of x flips, so the motion is a reflection over the y -axis, $(x, y) \rightarrow (-x, y)$.

For the line $x = 4$ use $(2h - x, y)$ with $h = 4$: $A(2, 1)$ goes to $(8 - 2, 1) = (6, 1)$.

Check: A is 2 units left of the line, and $(6, 1)$ is 2 units right of it.

EXAMPLE 5

Point $P(2, 1)$ is rotated 90° counterclockwise about the origin and then translated by $(x, y) \rightarrow (x + 3, y)$. Then the two motions are applied in the reverse order. Compare the results.

SOLUTION

Rotate, then translate: $(2, 1) \rightarrow (-1, 2) \rightarrow (2, 2)$.

Translate, then rotate: $(2, 1) \rightarrow (5, 1) \rightarrow (-1, 5)$.

The images $(2, 2)$ and $(-1, 5)$ are different, so the order of a composition matters.

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EXAMPLE 6

Triangle ABC has $A(0, 0)$, $B(3, 0)$, $C(0, 4)$. Rotate it 90° counterclockwise about the origin and then translate by $(x + 5, y + 1)$. Show the final image is congruent to ABC .

SOLUTION

Rotate: $A(0, 0)$, $B'(0, 3)$, $C'(-4, 0)$. Translate: $A''(5, 1)$, $B''(5, 4)$, $C''(1, 1)$.

Original sides: $AB = 3$, $AC = 4$, $BC = \sqrt{9 + 16} = 5$.

Image sides: $A''B'' = 3$, $A''C'' = 4$, $B''C'' = \sqrt{(5 - 1)^2 + (4 - 1)^2} = 5$.

All three sides match (SSS), so the triangles are congruent, as rigid motions guarantee.

Coach's Playbook

- 1 Plot first.** Sketch the figure on a grid, even roughly. A quick picture catches most sign errors.
 - 2 Pick the rule.** Write the rule in coordinate form before you plug in any numbers: for example $(x, y) \rightarrow (-y, x)$.
 - 3 Apply to every vertex.** Do the same rule to each point and label images with primes (A' , B').
 - 4 Check distances.** A rigid motion never changes a length. If a side of the image has a different length, find the arithmetic mistake.
 - 5 Compose in order.** For a composition, write the intermediate image after the first motion before applying the second.
- ◆ A reflection over a line swaps sides of the line: points on the line stay put.
 - ◆ Quick sign test for a rotation: a point in Quadrant I, turned 90° counterclockwise, must land in Quadrant II.
 - ◆ Reflection across $y = x$ just swaps the coordinates; across $y = -x$ it swaps and negates both.

Common traps

1) Using $(y, -x)$ for a 90° counterclockwise rotation (that is clockwise). 2) Changing the wrong coordinate in a reflection: reflecting over the y -axis changes x . 3) Applying a composition in the wrong order. 4) Subtracting when the rule says add (a translation of $(x - 3, y)$ moves LEFT).

Mistake Clinic**Spot the error**

A student rotates $(4, 1)$ by 90° counterclockwise and gets $(1, -4)$. **Where is the error?** The student used $(y, -x)$, which is the clockwise rule. The counterclockwise rule is $(x, y) \rightarrow (-y, x)$, giving $(-1, 4)$, which lands in Quadrant II as it should.

Spot the error

A student reflects $(3, -2)$ over the y -axis and writes $(3, 2)$. **Where is the error?** The student negated y (reflection over the x -axis). Reflecting over the y -axis flips the sign of x : $(-3, -2)$. A picture shows the point should stay at the same height.

Spot the error

A student says: "Reflecting over the x -axis and then over the y -axis is the same as a reflection, so the orientation is reversed." **Where is the error?** Two reflections reverse orientation twice, which returns it to the original. The composition $(x, y) \rightarrow (x, -y) \rightarrow (-x, -y)$ is a 180° rotation, which keeps orientation.

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Real-World Connection

In a video game, a character facing right is turned into one facing left by reflecting the sprite over a vertical line. A character that walks across the screen is translated, and a spinning coin is rotated. The program stores each position as coordinates and updates them using exactly the rules in this section.

Challenge Corner

A point P is reflected over the line $y = x$ and then over the y -axis. Its final image is $(-6, 4)$. Find P . **Solution:** Work backward. Let $P = (a, b)$. Reflection over $y = x$ gives (b, a) . Reflection over the y -axis gives $(-b, a)$. So $-b = -6$ and $a = 4$, giving $b = 6$. $P = (4, 6)$. Check: $(4, 6) \rightarrow (6, 4) \rightarrow (-6, 4)$.

Practice

PRACTICE Rigid Motions: Translations, Reflections, and Rotations

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What is the image of $(-3, 4)$ under the translation $(x, y) \rightarrow (x + 6, y - 7)$?
 (A) $(3, -3)$ (B) $(-9, 11)$ (C) $(3, 11)$ (D) $(-9, -3)$
- What is the image of $(5, -2)$ after a reflection over the y -axis?
 (A) $(5, 2)$ (B) $(-5, -2)$ (C) $(-2, 5)$ (D) $(-5, 2)$
- A point $(2, 6)$ is rotated 90° counterclockwise about the origin. What is the image?
 (A) $(6, -2)$ (B) $(-6, -2)$ (C) $(-2, -6)$ (D) $(-6, 2)$
- What is the image of $(3, 8)$ under reflection over the line $y = x$?
 (A) $(8, 3)$ (B) $(-3, 8)$ (C) $(-8, -3)$ (D) $(3, -8)$
- What is the image of $(-4, 7)$ after a 180° rotation about the origin?
 (A) $(4, 7)$ (B) $(4, -7)$ (C) $(7, -4)$ (D) $(-4, -7)$
- Which property is NOT necessarily preserved by a reflection?
 (A) Side lengths (B) Angle measures
 (C) The clockwise order of the vertices (D) Area
- Which rule describes a 90° counterclockwise rotation about the origin?
 (A) $(x, y) \rightarrow (y, -x)$ (B) $(x, y) \rightarrow (-x, -y)$ (C) $(x, y) \rightarrow (-y, x)$
 (D) $(x, y) \rightarrow (y, x)$
- The point $(3, 5)$ is reflected over the x -axis and then over the y -axis. What is the final image?
 (A) $(-3, -5)$ (B) $(3, -5)$ (C) $(-3, 5)$ (D) $(5, 3)$
- The point $(1, 2)$ is translated by $(x, y) \rightarrow (x + 3, y - 1)$ and then rotated 90° counterclockwise about the origin. What is the final image?
 (A) $(1, 0)$ (B) $(1, 4)$ (C) $(-1, 4)$ (D) $(-2, 1)$
- Find the image of $(-6, -1)$ under reflection over the line $y = -x$. Give the point.
- Rotate $(-3, 2)$ by 270° counterclockwise about the origin. Give the point.
- A translation maps $A(-2, 3)$ to $A'(4, -1)$. Find the image of $B(1, 1)$ under the same translation.
- Point $P(4, -1)$ is rotated 90° clockwise about the origin and then reflected over the x -axis. Find the final image.
- Reflect $(5, 3)$ over the vertical line $x = 2$. Give the image.
- A point P is reflected over the line $y = x$ and then over the y -axis, and the image is $(-9, 2)$. Find P .

18.2 Dilations and Similarity

Learn how dilations change size but not shape, and use AA, SAS, and SSS similarity to find missing lengths and prove triangles similar.

Why This Matters

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Two figures are **similar** when they have the same shape but not necessarily the same size. A photo and its enlargement, a map and the land it shows, and a small model and the real building are all similar. In geometry, similarity is built from a transformation called a **dilation**, which stretches or shrinks a figure from a fixed point. Students get stuck because they confuse similar with congruent, because they match the wrong sides when the triangles are turned, and because they set up a proportion upside down.

Similarity is the foundation of scale drawings, blueprints, map reading, the zoom feature on a phone, and the trigonometric ratios you will use next. If you can recognize similar triangles and write a correct proportion, you can find lengths that are impossible to measure directly.

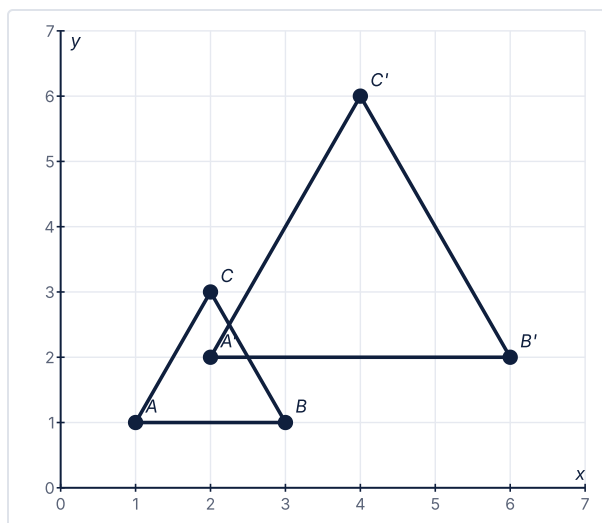
Core Ideas

Dilation

A **dilation** with center O and scale factor $k > 0$ maps each point P to P' on ray OP so that $OP' = k \cdot OP$.

With center at the origin: $(x, y) \rightarrow (kx, ky)$. If $k > 1$ the figure grows (enlargement). If $0 < k < 1$ it shrinks (reduction). If $k = 1$ it does not change.

A dilation multiplies every length by k , keeps angle measures the same, and keeps parallel lines parallel.



Triangle ABC dilated by $k = 2$ from the origin. Every coordinate doubles, every side doubles, and the angles stay the same.

Similar Figures

Two polygons are **similar** ($\sim$) if corresponding angles are congruent and corresponding sides are proportional. The common ratio is the **scale factor**.

Similar means a sequence of rigid motions and dilations maps one figure onto the other. The order of the letters in $\triangle ABC \sim \triangle DEF$ shows which parts correspond.

Triangle Similarity Shortcuts

AA: two pairs of congruent angles.

SAS $\sim$: two pairs of proportional sides and the included angles congruent.

SSS $\sim$: all three pairs of sides proportional.

For triangles, AA is the quickest: the third angles are then automatically equal.

Quantity in similar figures	Ratio of new to old
Side lengths, perimeters, altitudes, medians	k
Angle measures	1 (unchanged)
Areas	k^2

To set up a proportion, write corresponding sides in matching order: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$. Put the same figure on top in every fraction, or keep the same "small over big" pattern, and cross-multiply to solve.

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Worked Examples

EXAMPLE 1

Dilate triangle ABC with $A(1, 1)$, $B(3, 1)$, $C(2, 3)$ by a scale factor of 2 about the origin. Then dilate the point $(6, 8)$ by a scale factor of $\frac{1}{2}$.

SOLUTION

Multiply each coordinate by 2: $A'(2, 2)$, $B'(6, 2)$, $C'(4, 6)$.

Check a length: $AB = 2$ and $A'B' = 4$, so lengths doubled.

For $k = \frac{1}{2}$: $(6, 8) \rightarrow (3, 4)$. This is a reduction.

EXAMPLE 2

A dilation about the origin maps $A(2, 3)$ to $A'(5, 7.5)$. Find the scale factor and the image of $B(4, 2)$.

SOLUTION

Compare coordinates: $k = \frac{5}{2} = 2.5$ and $\frac{7.5}{3} = 2.5$. They agree, so $k = 2.5$.

$B' = (2.5 \cdot 4, 2.5 \cdot 2) = (10, 5)$.

EXAMPLE 3

In $\triangle ABC$, $m\angle A = 50^\circ$ and $m\angle B = 60^\circ$. In $\triangle DEF$, $m\angle D = 70^\circ$ and $m\angle E = 50^\circ$. Are the triangles similar? Write the similarity statement.

SOLUTION

Find the third angles: $m\angle C = 180 - 110 = 70^\circ$ and $m\angle F = 180 - 120 = 60^\circ$.

$\triangle ABC$ has angles $50^\circ, 60^\circ, 70^\circ$ and $\triangle DEF$ has $70^\circ, 50^\circ, 60^\circ$. By AA the triangles are similar.

Match equal angles: $A \leftrightarrow E$ (50°), $B \leftrightarrow F$ (60°), $C \leftrightarrow D$ (70°). So $\triangle ABC \sim \triangle EFD$.

EXAMPLE 4

(a) Is a triangle with sides 6, 8, 10 similar to one with sides 9, 12, 15? (b) Is a triangle with sides 4, 6, 9 similar to one with sides 6, 9, 13?

SOLUTION

(a) Compare sorted sides. For the first pair: $\frac{9}{6} = 1.5$, $\frac{12}{8} = 1.5$, $\frac{15}{10} = 1.5$. All equal, so they are similar by SSS~ with scale factor 1.5.

(b) For the second pair (sides 4, 6, 9 and 6, 9, 13): $\frac{6}{4} = 1.5$, $\frac{9}{6} = 1.5$, but $\frac{13}{9} \approx 1.44$. The ratios are not all equal, so they are NOT similar.

EXAMPLE 5

$\triangle ABC \sim \triangle DEF$ with $AB = 6$, $BC = 10$, $AC = 8$ and $DE = 9$. Find EF and DF .

SOLUTION

Scale factor from ABC to DEF : $\frac{DE}{AB} = \frac{9}{6} = 1.5$.

$EF = 1.5 \cdot 10 = 15$ and $DF = 1.5 \cdot 8 = 12$.

Check by proportion: $\frac{6}{9} = \frac{10}{15} = \frac{8}{12} = \frac{2}{3}$.

EXAMPLE 6

In the figure, $A(0, 0)$, $B(8, 0)$, $C(0, 6)$, $D(6, 0)$, $E(0, 4.5)$. Show $\triangle ADE \sim \triangle ABC$ and find DE .

SOLUTION

Both triangles contain the right angle at A , so the included angles are congruent.

$\frac{AD}{AB} = \frac{6}{8} = \frac{3}{4}$ and $\frac{AE}{AC} = \frac{4.5}{6} = \frac{3}{4}$. The sides around the angle are proportional.

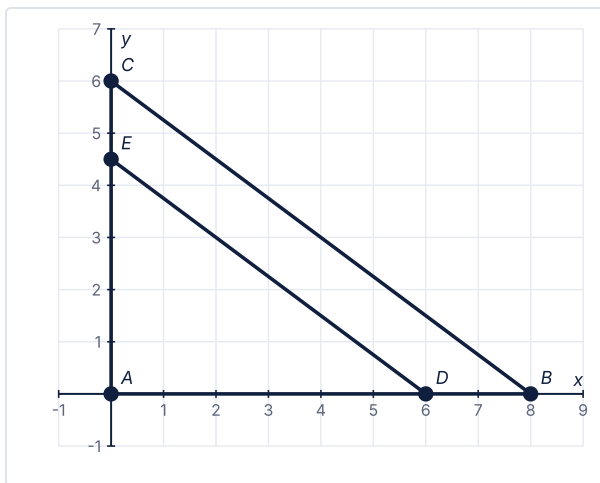
By SAS~, $\triangle ADE \sim \triangle ABC$ with scale factor $\frac{3}{4}$.

$BC = \sqrt{8^2 + 6^2} = 10$, so $DE = \frac{3}{4} \cdot 10 = 7.5$.

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Triangle ADE sits inside triangle ABC. DE is parallel to BC, and the scale factor from ADE to ABC is $4/3$ (or $3/4$ the other way).

Coach's Playbook

- 1 **Name the correspondence.** Write the similarity statement so the vertices line up. Then list the proportional sides from the letters, not from the picture.
 - 2 **Find the scale factor first.** Use one pair of known corresponding sides. Then multiply or divide every other side by the same number.
 - 3 **Decide big or small.** If you go from small to big, the answer must be larger. A quick size check catches flipped fractions.
 - 4 **Prove with AA when you can.** Look for shared angles, vertical angles, or parallel-line angles. Two pairs are enough.
 - 5 **Use cross-products last.** Write the proportion, check that corresponding sides are in the same positions, then cross-multiply.
- ◆ Dilation from the origin: multiply both coordinates by k . The origin itself never moves.
 - ◆ To compare triangles by SSS~, pair the shortest with the shortest, the middle with the middle, and the longest with the longest.
 - ◆ A scale factor between 0 and 1 means the image is smaller.

Common traps

1) Calling triangles similar because two sides look proportional (you need all three, or two plus the included angle). 2) Setting up $\frac{AB}{DE} = \frac{EF}{BC}$ with the sides in different orders. 3) Multiplying by k when you should divide (going from image to original). 4) Thinking similar triangles have the same size.

Mistake Clinic

Spot the error

A student dilates $A(3, 5)$ by $k = 2$ and gets $(5, 7)$. **Where is the error?** The student added 2 to each coordinate. A dilation multiplies: $(2 \cdot 3, 2 \cdot 5) = (6, 10)$. Adding 2 would be a translation.

Spot the error

Given $\triangle ABC \sim \triangle DEF$ with $AB = 6$, $DE = 9$, and $BC = 10$, a student writes $\frac{6}{9} = \frac{EF}{10}$ and gets $EF = 6.67$. **Where is the error?** The proportion pairs AB/DE with EF/BC , which is flipped. The correct proportion is $\frac{6}{9} = \frac{10}{EF}$, giving $EF = 15$. Since DEF is larger, EF must exceed BC , so 6.67 should have raised a flag.

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Spot the error

A student says that two triangles with sides 5, 12, 13 and 10, 24, 27 are similar because "the sides are about double."

Where is the error? Similarity needs exact proportions. $10/5 = 2$ and $24/12 = 2$, but $27/13 \approx 2.08$. The ratios are not equal, so $SSS \sim$ fails. The second triangle would need a side of 26.

Real-World Connection

A map has a scale of 1 inch for every 20 miles, so 3 inches on the map stand for 60 miles. Every distance on the map is the same fraction of the real distance, so the map is a dilation (reduction) of the land. Angles between roads are the same on the map and in real life, which is why directions on a map are accurate.

Challenge Corner

In $\triangle ABC$, point D is on $\overline{AB}$ and E is on $\overline{AC}$ with $\overline{DE} \parallel \overline{BC}$. If $AD = 9$, $DB = 3$, and $BC = 16$, find DE . **Solution:**

Because $DE \parallel BC$, corresponding angles are equal ($\angle ADE \cong \angle ABC$) and $\angle A$ is shared, so $\triangle ADE \sim \triangle ABC$ by AA.

The full side is $AB = 9 + 3 = 12$. So $\frac{DE}{BC} = \frac{AD}{AB} = \frac{9}{12} = \frac{3}{4}$, and $DE = \frac{3}{4} \cdot 16 = 12$.

Practice**PRACTICE Dilations and Similarity**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- A dilation about the origin with scale factor 3 is applied to $(2, -5)$. What is the image?
(A) $(5, -2)$ (B) $(6, -15)$ (C) $(-6, 15)$ (D) $(6, -2)$
- A dilation about the origin with scale factor $\frac{1}{2}$ is applied to $(8, -6)$. What is the image?
(A) $(4, -3)$ (B) $(16, -12)$ (C) $(8, -3)$ (D) $(-4, 3)$
- A dilation about the origin maps $A(2, 4)$ to $A'(5, 10)$. What is the scale factor?
(A) 0.4 (B) 3 (C) 2.5 (D) 6
- Which fact is enough to prove two triangles similar?
(A) One pair of congruent sides (B) Two pairs of congruent angles
(C) One pair of congruent angles (D) Two triangles with the same perimeter
- A triangle has sides 4, 5, 7. A similar triangle has a shortest side of 12. How long is its longest side?
(A) 15 (B) 19 (C) 21 (D) 28
- $\triangle ABC \sim \triangle XYZ$, $AB = 8$, $XY = 12$, and $BC = 10$. What is YZ ?
(A) 6.67 (B) 15 (C) 20 (D) 14
- A dilation with scale factor 2 is applied to a figure. Which is NOT preserved?
(A) The lengths of the sides (B) Angle measures (C) Parallelism of sides
(D) The shape
- Are triangles with sides 5, 12, 13 and 10, 24, 27 similar?
(A) Yes, the scale factor is 2.
(B) Yes, the scale factor is 2.08.
(C) No, because they have different perimeters.
(D) No, the ratios are $2, 2,$ and $\frac{27}{13}$, which are not equal.
- Two similar triangles have a scale factor of 3 : 2 (larger to smaller). The larger has a perimeter of 45. What is the smaller triangle's perimeter?
(A) 67.5 (B) 22.5 (C) 15 (D) 30
- A dilation about the origin with scale factor $\frac{1}{3}$ maps a polygon with vertex $(9, -12)$. Find the image of that vertex.
- A dilation maps a segment of length 7 to a segment of length 17.5. What is the scale factor?
- $\triangle ABC \sim \triangle DEF$, $m\angle A = (3x + 5)^\circ$ and $m\angle D = (5x - 25)^\circ$. Find $m\angle D$ in degrees.

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13. $\triangle ABC$ has sides 9, 15, 12. A similar triangle DEF has $DE = 6$ (matching the side of length 9). Find the perimeter of $\triangle DEF$.
14. Triangle P has sides 8, 12, 16 and triangle Q has sides 6, 9, 12. Are they similar? If so, give the scale factor from P to Q .
15. In $\triangle ABC$, D is on $\overline{AB}$ and E is on $\overline{AC}$ with $\overline{DE} \parallel \overline{BC}$. $AD = 10$, $DB = 5$, and $BC = 21$. Find DE .

18.3 Using Similar Triangles

Apply similar triangles to indirect measurement, the side-splitter theorem, and the angle bisector theorem, and learn how perimeter, area, and volume change with the scale factor.

Why This Matters

Similar triangles turn measuring problems that look impossible into one-line proportions. How tall is a tree you cannot climb? How wide is a river you cannot cross? The answer is to build a small triangle you CAN measure that has the same shape as the big one. Students get stuck for three reasons: they cannot tell which triangles in a busy figure are the similar ones, they write the proportion with sides in the wrong order, and they forget that areas and volumes scale differently from lengths.

Surveyors, architects, photographers, and engineers use these ideas daily. Scale models of airplanes and buildings, enlarged prints, and the mixing of paint or concrete in different batch sizes all rely on ratios of lengths, areas, and volumes. This section gives you the tools to handle all three.

Core Ideas

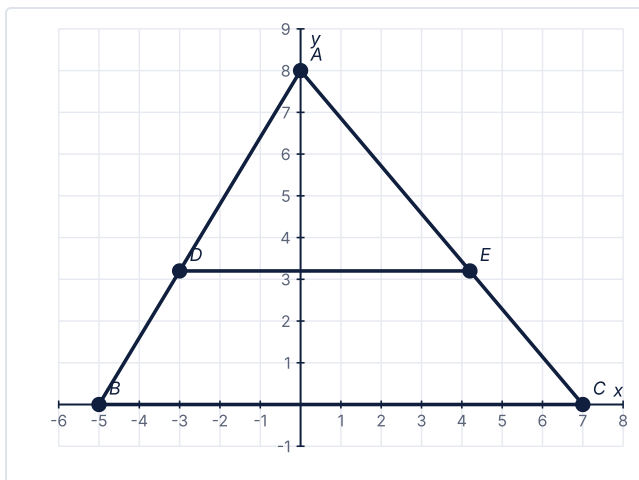
Triangle Proportionality (Side-Splitter) Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.

In $\triangle ABC$ with D on $\overline{AB}$, E on $\overline{AC}$, and $\overline{DE} \parallel \overline{BC}$:

$$\frac{AD}{DB} = \frac{AE}{EC}, \quad \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}.$$

The converse is also true: if a line divides two sides proportionally, it is parallel to the third side.



DE is parallel to BC . Then $AD : DB = AE : EC$. Here $AD = 6$, $DB = 4$, $AE = 9$, $EC = 6$, and both ratios equal $3 : 2$.

Angle Bisector Theorem

If $\overline{AD}$ bisects $\angle A$ of $\triangle ABC$, with D on $\overline{BC}$, then

$$\frac{BD}{DC} = \frac{AB}{AC}.$$

The bisector splits the opposite side in the same ratio as the two sides next to the angle.

Perimeter, Area, and Volume Ratios

If two similar figures have scale factor k (lengths in ratio $k : 1$), then perimeters are in ratio k , areas are in ratio k^2 , and volumes are in ratio k^3 .

If lengths are in ratio $a : b$, then areas are in ratio $a^2 : b^2$ and volumes in ratio $a^3 : b^3$.

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Lengths ratio	Area ratio	Volume ratio
2 : 1	4 : 1	8 : 1
3 : 2	9 : 4	27 : 8
5 : 3	25 : 9	125 : 27

Indirect measurement works because the sun hits a tree and a nearby person at the same angle, so their shadow triangles share angles and are similar by AA. A mirror on the ground works the same way: the angle at which the light comes in equals the angle at which it leaves.

Worked Examples

EXAMPLE 1

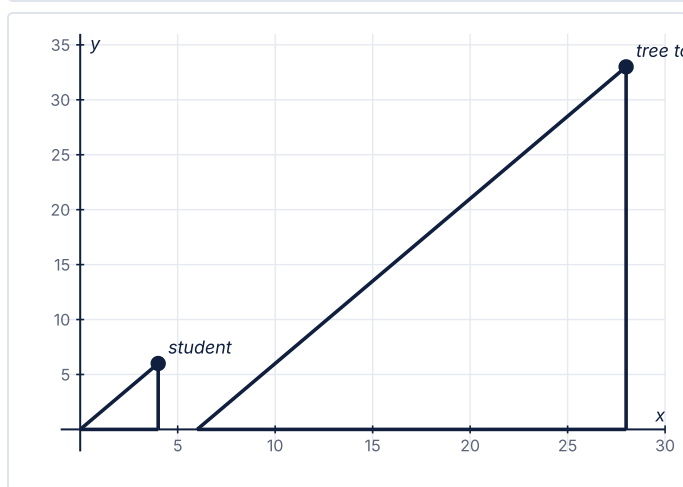
A 6 ft tall student casts a shadow 4 ft long. At the same time a tree casts a shadow 22 ft long. How tall is the tree?

SOLUTION

The sun's rays hit at the same angle, and both objects stand vertically, so the two right triangles are similar by AA.

Proportion (height over shadow): $\frac{6}{4} = \frac{h}{22}$.

$4h = 132$, so $h = 33$ ft.



Shadow triangles for Example 1: the student (6 ft, shadow 4 ft) and the tree (33 ft, shadow 22 ft) have the same shape.

EXAMPLE 2

A student whose eyes are 5 ft above the ground stands 3 ft from a small mirror lying on the ground. The mirror is 21 ft from the base of a building, and the student sees the top of the building in the mirror. How tall is the building?

SOLUTION

The light path makes equal angles at the mirror, so the triangle formed by the student and the triangle formed by the building are similar (AA).

$\frac{5}{3} = \frac{h}{21}$ so $3h = 105$ and $h = 35$ ft.

EXAMPLE 3

In $\triangle ABC$, $\overline{DE} \parallel \overline{BC}$ with D on $\overline{AB}$ and E on $\overline{AC}$. $AD = 6$, $DB = 4$, $AE = 9$. Find EC and AC .

SOLUTION

Side-splitter: $\frac{AD}{DB} = \frac{AE}{EC}$, so $\frac{6}{4} = \frac{9}{EC}$.

$6 \cdot EC = 36$, so $EC = 6$.

$AC = AE + EC = 9 + 6 = 15$.

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EXAMPLE 4

In $\triangle ABC$, $\overline{DE} \parallel \overline{BC}$. $AD = x + 1$, $DB = 6$, $AE = x + 3$, $EC = 10$. Find x and AD .

SOLUTION

$$\frac{x+1}{6} = \frac{x+3}{10}.$$

Cross-multiply: $10(x+1) = 6(x+3)$, so $10x + 10 = 6x + 18$, giving $4x = 8$ and $x = 2$.

$AD = 3$ and $AE = 5$. Check: $\frac{3}{6} = \frac{5}{10} = \frac{1}{2}$.

EXAMPLE 5

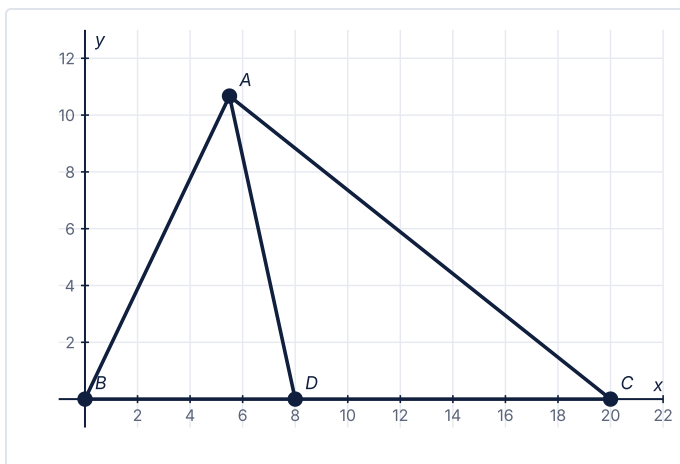
In $\triangle ABC$, $\overline{AD}$ bisects $\angle A$, with D on $\overline{BC}$. $AB = 12$, $AC = 18$, and $BD = 8$. Find DC and BC .

SOLUTION

Angle Bisector Theorem: $\frac{BD}{DC} = \frac{AB}{AC}$, so $\frac{8}{DC} = \frac{12}{18}$.

$12 \cdot DC = 144$, so $DC = 12$.

$BC = 8 + 12 = 20$.



Angle bisector AD in the triangle with $AB = 12$, $AC = 18$, $BC = 20$. D splits BC into $BD = 8$ and $DC = 12$, the same ratio as $12 : 18$.

EXAMPLE 6

Two similar triangles have corresponding sides in the ratio $3 : 5$. The smaller triangle has an area of 27 square units. (a) Find the area of the larger triangle. (b) Two similar solids have lengths in ratio $2 : 3$, and the smaller has volume 40 cubic units and surface area 52 square units. Find the volume and surface area of the larger solid.

SOLUTION

(a) Area ratio $= 3^2 : 5^2 = 9 : 25$. So $\frac{27}{A} = \frac{9}{25}$ and $A = 75$ square units.

(b) Volume ratio $= 2^3 : 3^3 = 8 : 27$, so $V = 40 \cdot \frac{27}{8} = 135$ cubic units.

Surface area ratio $= 4 : 9$, so $S = 52 \cdot \frac{9}{4} = 117$ square units.

Coach's Playbook

- 1 Spot the parallel or the shared angle.** A line parallel to a side, a shared vertex angle, or vertical angles are the clues that create similar triangles.
- 2 Redraw the two triangles.** Draw the small and large triangles separately and label corresponding vertices. This prevents mismatched sides.
- 3 Write the proportion with labels.** Use words like (height over shadow) or (small over big) to keep the order the same in both fractions.

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- 4 Check reasonableness.** A taller object has a longer shadow; a bigger triangle has bigger sides. If your answer breaks this, flip the fraction.
- 5 Match the ratio type.** Ask: is this a length, an area, or a volume? Square the ratio for areas and cube it for volumes.
- ◆ Side-splitter has two versions: part to part ($AD/DB = AE/EC$) and part to whole ($AD/AB = AE/AC$). Both are correct; do not mix them within one fraction.
 - ◆ To go from an area ratio back to a length ratio, take the square root. For volume take the cube root.
 - ◆ In real-world problems, convert to the same units before forming a ratio.

Common traps

1) Using $AD/DB = AE/AC$ (part over part on one side, part over whole on the other). 2) Using the scale factor for area and volume instead of squaring or cubing it. 3) Putting the unknown over the wrong side so the answer is too small. 4) Forgetting to add segments to get a whole side, such as $AB = AD + DB$.

Mistake Clinic

Spot the error

A student solves $\overline{DE} \parallel \overline{BC}$ with $AD = 6$, $DB = 4$, $AE = 9$ and writes $\frac{6}{4} = \frac{9}{AC}$, getting $AC = 6$. **Where is the error?** The proportion mixes part over part with part over whole. The segment EC pairs with DB in $\frac{AD}{DB} = \frac{AE}{EC}$. The correct result is $EC = 6$ and $AC = 15$, which is longer than $AE = 9$ as it should be.

Spot the error

Two similar pentagons have a scale factor of $2 : 3$. The smaller one has area 40. A student says the larger has area $40 \cdot \frac{3}{2} = 60$. **Where is the error?** Area scales by the SQUARE of the scale factor. The ratio is $4 : 9$, so the larger area is $40 \cdot \frac{9}{4} = 90$.

Spot the error

In a triangle with $AB = 12$ and $AC = 18$, a student says the bisector from A cuts $BC = 20$ into two equal pieces of 10 "because it is a bisector." **Where is the error?** The angle bisector divides the OPPOSITE side in the ratio of the adjacent sides, not in half. $BD : DC = 12 : 18 = 2 : 3$, so $BD = 8$ and $DC = 12$. A median would cut the side in half.

Real-World Connection

A cook has a recipe for a cake in an 8-inch round pan and wants to use a 12-inch pan of the same shape and depth ratio. The lengths scale by 1.5, but the area of the pan scales by $1.5^2 = 2.25$, so the cook needs about 2.25 times as much batter, not 1.5 times. This is why scaling recipes by length goes wrong.

Challenge Corner

In $\triangle ABC$, $\overline{DE} \parallel \overline{BC}$ with D on $\overline{AB}$ and E on $\overline{AC}$. The area of $\triangle ADE$ is 20 and the area of $\triangle ABC$ is 45. If $AD = 8$, find AB . **Solution:** $\triangle ADE \sim \triangle ABC$ by AA. The area ratio is $\frac{20}{45} = \frac{4}{9}$, so the scale factor is $\sqrt{\frac{4}{9}} = \frac{2}{3}$. Then $\frac{AD}{AB} = \frac{2}{3}$, so $AB = 8 \cdot \frac{3}{2} = 12$.

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Practice

PRACTICE Using Similar Triangles

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- A 3 ft stick casts a 2 ft shadow. At the same time a flagpole casts a 14 ft shadow. How tall is the flagpole?
(A) 9.3 ft (B) 16 ft (C) 21 ft (D) 28 ft
- In $\triangle ABC$, $\overline{DE} \parallel \overline{BC}$, $AD = 4$, $DB = 6$, $AE = 6$. What is EC ?
(A) 9 (B) 4 (C) 10 (D) 15
- Two similar rectangles have side lengths in the ratio 3 : 4. What is the ratio of their areas?
(A) 3 : 4 (B) 6 : 8 (C) 9 : 16 (D) 27 : 64
- Two similar solids have lengths in the ratio 2 : 3. What is the ratio of their volumes?
(A) 2 : 3 (B) 4 : 9 (C) 6 : 9 (D) 8 : 27
- In $\triangle ABC$, $\overline{AD}$ bisects $\angle A$ with D on $\overline{BC}$. $AB = 10$, $AC = 15$, $BC = 20$. What is BD ?
(A) 10 (B) 8 (C) 12 (D) 6
- A student with eyes 1.6 m above the ground stands 2 m from a mirror on the ground and sees the top of a building. The mirror is 30 m from the building. How tall is the building?
(A) 24 m (B) 19.2 m (C) 3.2 m (D) 48 m
- Two similar triangles have areas 18 and 50. What is the ratio of their corresponding sides?
(A) 9 : 25 (B) 3 : 5 (C) 18 : 50 (D) 3 : 25
- In $\triangle ABC$ with D on $\overline{AB}$, E on $\overline{AC}$, and $\overline{DE} \parallel \overline{BC}$, which proportion is true?
(A) $\frac{AD}{DB} = \frac{AE}{EC}$ (B) $\frac{AD}{DB} = \frac{AE}{AC}$ (C) $\frac{AD}{AB} = \frac{AE}{EC}$
(D) $\frac{DB}{AB} = \frac{AE}{EC}$
- $\overline{DE} \parallel \overline{BC}$, $AD = x$, $DB = 8$, $AE = 9$, $EC = 12$. What is x ?
(A) 10.7 (B) 4.5 (C) 12 (D) 6
- A 5 ft post casts a 3 ft shadow. At the same time a tower casts a 42 ft shadow. How tall is the tower in feet?
- Two similar polygons have lengths in ratio 4 : 7. The smaller has area 48 square units. Find the area of the larger.
- Two similar cans have heights of 6 cm and 9 cm. The smaller holds 200 cubic centimeters. How many cubic centimeters does the larger hold?
- In $\triangle ABC$, $\overline{AD}$ bisects $\angle A$ with D on $\overline{BC}$. $AB = 14$, $AC = 21$, $BC = 25$. Find BD .
- In $\triangle ABC$, $\overline{DE} \parallel \overline{BC}$ with $AD = 2x$, $DB = 9$, $AE = x + 4$, $EC = 6$. Find AD .
- In $\triangle ABC$, $\overline{DE} \parallel \overline{BC}$ with D on $\overline{AB}$. The area of $\triangle ADE$ is 18 and the area of $\triangle ABC$ is 50. If $AB = 15$, find AD .

18.4 Symmetry, Tessellations, and Coordinate Transformations

Explore line and rotational symmetry, learn why only some shapes tessellate, and get a first look at using matrices to apply and combine transformations.

Why This Matters

Symmetry is the pattern of balance in a shape: a figure has **line symmetry** if a flip over some line maps it onto itself, and **rotational symmetry** if a turn of less than 360° maps it onto itself. Tessellations are patterns of shapes that cover a flat surface with no gaps or overlaps, like floor tiles or honeycombs. Students get stuck counting symmetries (they miss diagonal lines or count a turn of 360°), and they have trouble explaining why only certain shapes tessellate. In the second half of this section you meet a powerful tool for coordinate transformations: matrices.

Symmetry shows up in logos, architecture, snowflakes, molecules, and bridge design. Tessellations are used in tile floors, quilting, and the repeating patterns in art. Matrices are the language of computer graphics and robotics: a single matrix can move every point of a 3D model, and multiplying matrices combines transformations. This section gives you a first preview of that idea for 2D.

Core Ideas

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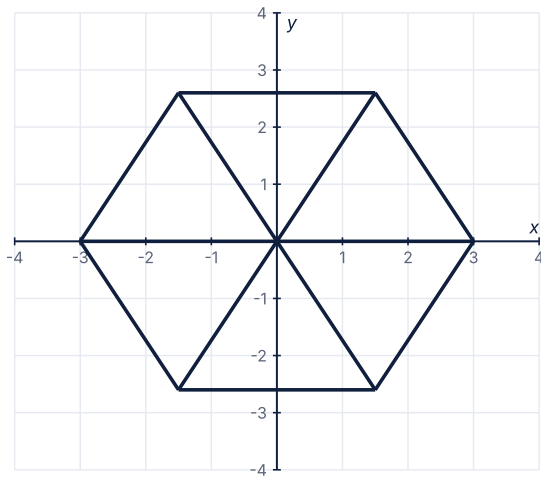
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Line and Rotational Symmetry

A figure has **line symmetry** if there is a line (the line of symmetry) such that reflecting the figure over it gives the same figure.

A figure has **rotational symmetry** if some rotation of less than 360° about its center maps it onto itself. The **order** is the number of times the figure fits onto itself in one full turn, and the **smallest angle** of rotation is $\frac{360^\circ}{\text{order}}$.

A regular polygon with n sides has n lines of symmetry and rotational symmetry of order n (smallest angle $360^\circ/n$).



A regular hexagon with three of its six lines of symmetry (those through opposite vertices). The other three pass through midpoints of opposite sides. Rotational symmetry: order 6, smallest angle 60 degrees.

Shape	Lines of symmetry	Rotational order
Isosceles triangle (not equilateral)	1	1 (none)
Equilateral triangle	3	3
Rectangle (not a square)	2	2
Rhombus (not a square)	2	2
Square	4	4
Parallelogram (not a rectangle or rhombus)	0	2
Regular pentagon	5	5

Tessellation Rule

A **tessellation** covers the plane with no gaps or overlaps. At each vertex, the angles of the shapes that meet must add to exactly 360° .

The interior angle of a regular n -gon is $180^\circ - \frac{360^\circ}{n}$. A regular polygon tessellates alone only if this angle divides 360° evenly: triangles (60), squares (90), and hexagons (120).

Transformation Matrices

Write a point (x, y) as a column $\begin{bmatrix} x \\ y \end{bmatrix}$. A 2×2 matrix times the column gives the image.

Reflection over the x -axis: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Over the y -axis: $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. Over $y = x$: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Rotation 90° counterclockwise: $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Rotation 180° : $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$. Dilation by k : $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$.

To compose, multiply the matrices with the FIRST transformation on the RIGHT: first A then B is BA .

Worked Examples

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EXAMPLE 1

Count the lines of symmetry of (a) a rectangle that is not a square, (b) a rhombus that is not a square, (c) a regular pentagon, (d) a parallelogram with unequal adjacent sides and no right angles.

SOLUTION

- (a) 2: one vertical and one horizontal through the midpoints of opposite sides. The diagonals are NOT lines of symmetry for a non-square rectangle.
 (b) 2: the two diagonals. The lines through midpoints of sides do not work for a rhombus that is not a square.
 (c) 5: each passes through a vertex and the midpoint of the opposite side.
 (d) 0. A flip over either diagonal or any midpoint line does not map it onto itself.

EXAMPLE 2

Find the order of rotational symmetry and smallest angle of rotation for (a) a regular hexagon, (b) a non-square rectangle, (c) an equilateral triangle.

SOLUTION

- (a) The order is 6, and the smallest angle is $360/6 = 60^\circ$.
 (b) A non-square rectangle looks the same after 180° , so order 2 with smallest angle $360/2 = 180^\circ$.
 (c) Order 3, smallest angle 120° .

EXAMPLE 3

Which regular polygons tessellate by themselves: triangle, square, pentagon, hexagon, octagon? Can regular octagons and squares tessellate together?

SOLUTION

Interior angles: triangle 60° , square 90° , pentagon 108° , hexagon 120° , octagon 135° .

Divide 360 by each: 6, 4, 3.33, 3, 2.67. Only the whole-number results work: triangle, square, hexagon.

For octagons and squares: $135 + 135 + 90 = 360$. So two octagons and one square fit around every vertex, and they tessellate together.

EXAMPLE 4

Use a matrix to rotate $(3, 2)$ by 90° counterclockwise, and to reflect it over the y -axis.

SOLUTION

$$\text{Rotation: } \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \cdot 3 + (-1) \cdot 2 \\ 1 \cdot 3 + 0 \cdot 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}.$$

$$\text{Reflection: } \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}.$$

These match the coordinate rules $(-y, x)$ and $(-x, y)$.

EXAMPLE 5

Point $(4, 1)$ is reflected over the x -axis and then rotated 90° counterclockwise. Find the single matrix for this composition and the image.

SOLUTION

First is the reflection $F = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, second is the rotation $R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. The composite is RF (first on the right).

$$RF = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

This is the matrix for reflection over $y = x$, and $(4, 1) \rightarrow (1, 4)$.

Check by steps: $(4, 1) \rightarrow (4, -1) \rightarrow (1, 4)$.

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EXAMPLE 6

Triangle PQR has $P(1, 1)$, $Q(4, 1)$, $R(1, 3)$. It is reflected over the y -axis and then over the line $y = x$. Describe the single transformation that does the same job. Then, what happens when you reflect over the two parallel lines $x = 1$ and $x = 4$ in that order?

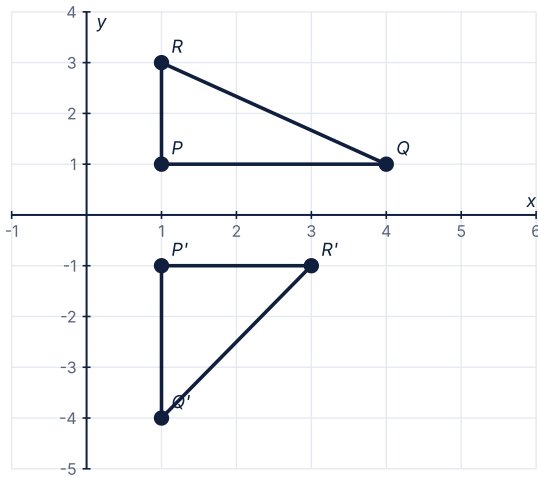
SOLUTION

First: $(x, y) \rightarrow (-x, y)$. Second: swap the coordinates, giving $(y, -x)$ overall.

$(x, y) \rightarrow (y, -x)$ is a 90° clockwise rotation about the origin. The mirror lines meet at 45° and the rotation angle is double that, 90° .

So $P \rightarrow (1, -1)$, $Q \rightarrow (1, -4)$, $R \rightarrow (3, -1)$.

For parallel lines: $x \rightarrow 2(1) - x = 2 - x$, then $2(4) - (2 - x) = x + 6$. The composition is a translation 6 units right, which is twice the distance between the lines.



Triangle PQR and its image after reflecting over the y -axis then over $y = x$, which equals a 90 degree clockwise rotation about the origin.

Coach's Playbook

- 1 For symmetry, test, do not guess.** Trace the figure. Try folding along each possible line: vertical, horizontal, and both diagonals. For rotations, turn the figure by $360/n$ for $n = 2, 3, 4, \dots$
 - 2 Use the formula for regular polygons.** A regular n -gon has n lines and order n . The interior angle is $180 - 360/n$.
 - 3 For tessellations, add angles at a vertex.** The angles meeting at one vertex must total 360° . If a combination does not sum to exactly 360, there will be a gap or overlap.
 - 4 For matrices, write rows as dot products.** Row 1 times the column gives the new x ; row 2 times the column gives the new y .
 - 5 Order of composition.** The transformation applied first goes on the right: first A then B is the product BA .
- ◆ Use the point $(1, 0)$ to test a rotation matrix: the first column of the matrix is where $(1, 0)$ lands, and the second column is where $(0, 1)$ lands.
 - ◆ Two reflections over intersecting lines make a rotation by twice the angle between the lines. Two reflections over parallel lines make a translation by twice the distance between them.
 - ◆ A figure with no line symmetry can still have rotational symmetry (a parallelogram).

Common traps

1) Counting the identity (a 360° turn) as a rotation, or counting the diagonals of a non-square rectangle as lines of symmetry. 2) Assuming every regular polygon tessellates (pentagons and octagons do not by themselves). 3) Multiplying matrices in the wrong order for a composition. 4) Treating a parallelogram as having line symmetry.

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Mistake Clinic

Spot the error

A student says a rectangle that is not a square has 4 lines of symmetry because "the diagonals fold it in half." **Where is the error?** Folding along a diagonal of a non-square rectangle does not make the two halves land on each other (a long side would fall on a short side). Only the vertical and horizontal midlines work, so the answer is 2.

Spot the error

A student states that regular pentagons tessellate because "five-sided shapes are easy to fit together." **Where is the error?** The interior angle of a regular pentagon is 108° , and $360/108$ is not a whole number, so three pentagons leave a gap of 36° at a vertex and four overlap. Pentagons cannot tile the plane by themselves.

Spot the error

To find the image of $(4, 1)$ under a reflection over the x -axis followed by a 90° rotation, a student computes FR instead of RF and gets the matrix $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$. **Where is the error?** That matrix does the rotation first. The first transformation must be on the right: $RF = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, giving $(1, 4)$.

Real-World Connection

A honeycomb is a tessellation of regular hexagons. Three hexagons meet at each vertex ($3 \cdot 120^\circ = 360^\circ$), and hexagons use the least wall material for a given area compared to squares or triangles. Bees do not do the math, but the result is the same.

Challenge Corner

A figure is dilated by a factor of 3 about the origin and then rotated 90° counterclockwise. Find the single matrix for the combined transformation and use it to find the image of $(2, 1)$. **Solution:** The dilation matrix is $3I = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ and the rotation is $R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. The composite is $R \cdot 3I = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$. Apply it: $\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \end{bmatrix}$. Check: dilate to $(6, 3)$, then rotate to $(-3, 6)$.

Practice

PRACTICE Symmetry, Tessellations, and Coordinate Transformations

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- How many lines of symmetry does a regular octagon have?
 (A) 4 (B) 16 (C) 8 (D) 2
- What is the order of rotational symmetry of a rectangle that is not a square?
 (A) 1 (B) 2 (C) 4 (D) 0
- What is the smallest angle of rotation that maps a regular pentagon onto itself?
 (A) 108° (B) 36° (C) 60° (D) 72°

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4. Which figure has NO line of symmetry?
 - (A) A parallelogram with unequal sides and no right angles
 - (B) A rhombus
 - (C) An isosceles triangle
 - (D) A rectangle
5. Which regular polygon cannot tessellate the plane by itself?
 - (A) Regular pentagon
 - (B) Equilateral triangle
 - (C) Square
 - (D) Regular hexagon
6. How many regular hexagons meet at each vertex of a hexagonal tessellation?
 - (A) 4
 - (B) 6
 - (C) 2
 - (D) 3
7. What is the image of $(5, 2)$ under the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$?
 - (A) $(2, 5)$
 - (B) $(-2, 5)$
 - (C) $(2, -5)$
 - (D) $(-5, -2)$
8. What is the image of $(-3, 4)$ under the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$?
 - (A) $(3, 4)$
 - (B) $(3, -4)$
 - (C) $(-3, -4)$
 - (D) $(4, -3)$
9. Which transformation does the matrix $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ represent?
 - (A) Reflection over the y -axis
 - (B) Rotation of 90° counterclockwise
 - (C) Reflection over the x -axis
 - (D) Rotation of 180° about the origin
10. A regular 12-sided polygon is rotated about its center. State its order of rotational symmetry and its smallest angle of rotation in degrees.
11. The smallest angle of rotation that maps a regular polygon onto itself is 24° . How many sides does the polygon have?
12. Two regular octagons meet at a vertex of a tessellation with one more regular polygon. How many sides does that polygon have?
13. Reflection over the x -axis is followed by a 90° counterclockwise rotation. Use the composite matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ to find the image of $(6, -2)$. Give the point.
14. A point is reflected over the line $x = 1$ and then over the line $x = 4$. Find the image of $(-2, 5)$. Give the point.
15. A figure is dilated by a factor of 4 and then reflected over the y -axis. Write the single matrix for the combination and use it to find the image of $(3, -2)$. Give the matrix and the point.

Answers**Answers and Explanations**

Complete solutions for every practice problem, numbered to match. Read the explanation even when your answer is right.

18.1 Rigid Motions: Translations, Reflections, and Rotations**PRACTICE · RIGID MOTIONS: TRANSLATIONS, REFLECTIONS, AND ROTATIONS**

1. A. Add 6 to x : $-3 + 6 = 3$. Subtract 7 from y : $4 - 7 = -3$. Choice B subtracts and adds in the wrong places.
2. B. Reflecting over the y -axis changes only the sign of x . Choice A is the reflection over the x -axis.
3. D. Use $(x, y) \rightarrow (-y, x)$ to get $(-6, 2)$. Choice A is the clockwise rotation.
4. A. Reflecting over $y = x$ swaps the coordinates. Choice C is the reflection over $y = -x$.
5. B. A 180° turn negates both coordinates. Choices that negate only one coordinate are reflections, not rotations.
6. C. Reflections reverse orientation, so a clockwise order of vertices becomes counterclockwise. Lengths, angles, and area are preserved by every rigid motion.
7. C. Test with $(1, 0)$, which should move to $(0, 1)$. Only $(-y, x)$ does that. Choice A is clockwise.
8. A. First $(3, -5)$, then $(-3, -5)$. The composition equals a 180° rotation.
9. C. Translate to $(4, 1)$, then rotate with $(-y, x)$ to get $(-1, 4)$. Choice A applies the motions in the reverse order.
10. $(1, 6)$. Use $(x, y) \rightarrow (-y, -x)$: $(-(-1), -(-6)) = (1, 6)$.
11. $(2, 3)$. Use $(x, y) \rightarrow (y, -x)$: $(2, -(-3)) = (2, 3)$.
12. $(7, -3)$. The rule is $(x + 6, y - 4)$ because $-2 + 6 = 4$ and $3 - 4 = -1$. Then $B' = (1 + 6, 1 - 4) = (7, -3)$.
13. $(-1, 4)$. Clockwise 90° uses $(y, -x)$: $(-1, -4)$. Reflection over the x -axis: $(-1, 4)$.
14. $(-1, 3)$. Use $(2h - x, y)$ with $h = 2$: $(4 - 5, 3) = (-1, 3)$. The point was 3 units right of the line, so the image is 3 units left.
15. $(2, 9)$. Let $P = (a, b)$. The two reflections give $(-b, a) = (-9, 2)$, so $b = 9$ and $a = 2$.

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18.2 Dilations and Similarity

PRACTICE · DILATIONS AND SIMILARITY

1. B. Multiply each coordinate by 3: $(6, -15)$. Choice D multiplies only x .
2. A. Multiply by $\frac{1}{2}$: $(4, -3)$. Choice B is the enlargement by a factor of 2.
3. C. $5/2 = 2.5$ and $10/4 = 2.5$. Choice A is the reciprocal, which would go from the image back to the original.
4. B. AA is a similarity shortcut. A pair of congruent sides alone, or equal perimeters, tells nothing about shape.
5. C. The scale factor is $12/4 = 3$, so the longest side is $3 \cdot 7 = 21$. Choice B adds 7 to 12, which is not proportional.
6. B. Scale factor $12/8 = 1.5$, so $YZ = 1.5 \cdot 10 = 15$. Choice A divides instead of multiplying.
7. A. Lengths are multiplied by 2. Angles, parallelism, and shape are preserved by dilations.
8. D. For SSS~ all three ratios must match; $27/13$ is not 2. Different perimeters alone do not rule out similarity.
9. D. Perimeters scale by the same factor: $45 \cdot \frac{2}{3} = 30$. Choice A multiplies by $\frac{3}{2}$ and gets the larger direction wrong.
10. $(3, -4)$. Multiply each coordinate by $\frac{1}{3}$.
11. 2.5. Scale factor $= \frac{17.5}{7} = 2.5$.
12. 50. Corresponding angles are equal: $3x + 5 = 5x - 25$ gives $x = 15$, so $m\angle D = 5(15) - 25 = 50$.
13. 24. Scale factor $\frac{6}{9} = \frac{2}{3}$. The original perimeter is $9 + 15 + 12 = 36$, so the new perimeter is $\frac{2}{3} \cdot 36 = 24$ (sides 6, 10, 8).
14. Yes, by SSS~. Ratios $\frac{8}{12} = \frac{9}{16} = \frac{12}{16} = \frac{3}{4}$. The scale factor from P to Q is $\frac{3}{4}$.
15. 14. $\triangle ADE \sim \triangle ABC$ by AA, with $AB = 15$. $\frac{DE}{21} = \frac{10}{15} = \frac{2}{3}$, so $DE = 14$.

18.3 Using Similar Triangles

PRACTICE · USING SIMILAR TRIANGLES

1. C. $\frac{3}{2} = \frac{h}{14}$ gives $h = 21$. Choice A flips the ratio: $14 \cdot \frac{2}{3}$.
2. A. $\frac{AD}{DB} = \frac{AE}{EC}$ gives $\frac{4}{6} = \frac{6}{EC}$, so $EC = 9$. Choice B (4) just repeats AD .
3. C. Area ratio is the square of the length ratio: $9 : 16$. Choice D is the volume ratio.
4. D. Volume ratio is the cube of the length ratio: $8 : 27$. Choice B is the area ratio.
5. B. $\frac{BD}{DC} = \frac{10}{15} = \frac{2}{3}$, so $BD = \frac{2}{5} \cdot 20 = 8$. Choice A assumes the bisector cuts the side in half.
6. A. $\frac{16}{2} = \frac{h}{30}$ gives $h = 24$. Choice C shifts the decimal and ignores the 30.
7. B. The side ratio is the square root of the area ratio: $\sqrt{18/50} = \sqrt{9/25} = 3/5$. Choice A is the area ratio.
8. A. Part-to-part on both sides is correct. The other options mix part-to-part with part-to-whole.
9. D. $\frac{x}{8} = \frac{9}{12}$ gives $x = 6$. Choice A inverts the fraction.
10. 70. $\frac{5}{8} = \frac{h}{12}$ gives $h = 70$.
11. 147. Area ratio $16 : 49$, so $48 \cdot \frac{49}{16} = 147$.
12. 675. Volume ratio $= (9/6)^3 = (3/2)^3 = 27/8$, so $200 \cdot \frac{27}{8} = 675$.
13. 10. $\frac{BD}{DC} = \frac{14}{21} = \frac{2}{3}$, so $BD = \frac{2}{5} \cdot 25 = 10$ (and $DC = 15$).
14. 24. $\frac{2x}{9} = \frac{x+4}{6}$ gives $12x = 9x + 36$, so $x = 12$ and $AD = 24$. Check: $AE = 16$ and $\frac{24}{9} = \frac{16}{6} = \frac{8}{3}$.
15. 9. Area ratio $\frac{18}{50} = \frac{9}{25}$, so the scale factor is $\frac{3}{5}$, and $AD = \frac{3}{5} \cdot 15 = 9$.

18.4 Symmetry, Tessellations, and Coordinate Transformations

PRACTICE · SYMMETRY, TESSELLATIONS, AND COORDINATE TRANSFORMATIONS

1. C. A regular n -gon has n lines of symmetry. Choice B doubles the count.
2. B. It maps onto itself after 180° and 360° , so order 2. Choice C would hold only for a square.
3. D. $360/5 = 72^\circ$. Choice A is the interior angle, not the rotation angle.
4. A. A generic parallelogram has rotational symmetry of order 2 but no line symmetry. The other three each have at least one line.
5. A. Its interior angle 108° does not divide 360. The others have angles of 60° , 90° , and 120° .
6. D. Each interior angle is 120° and $360/120 = 3$. Choice B confuses the number of sides with the number at a vertex.
7. B. Row 1: $0 \cdot 5 + (-1) \cdot 2 = -2$. Row 2: $1 \cdot 5 + 0 \cdot 2 = 5$. So $(-2, 5)$. Choice A is a reflection over $y = x$.
8. C. This is a reflection over the x -axis, which negates y only. Choice A is the reflection over the y -axis.
9. D. It sends (x, y) to $(-x, -y)$, which negates both coordinates, a 180° rotation. A reflection would negate only one coordinate.
10. Order 12, smallest angle 30° . The smallest angle is $360/12 = 30$.
11. 15. The number of sides equals $360/24 = 15$.
12. 4 (a square). Each octagon angle is 135° , so the third angle is $360 - 270 = 90^\circ$, the angle of a square.
13. $(-2, 6)$. Row 1: $0 \cdot 6 + 1 \cdot (-2) = -2$. Row 2: $1 \cdot 6 + 0 \cdot (-2) = 6$. Check by steps: $(6, -2) \rightarrow (6, 2) \rightarrow (-2, 6)$.
14. $(4, 5)$. After the first reflection: $(2(1) - (-2), 5) = (4, 5)$. After the second: $(2(4) - 4, 5) = (4, 5)$. Overall this is a translation 6 units to the right.
15. Matrix $\begin{bmatrix} -4 & 0 \\ 0 & 4 \end{bmatrix}$, image $(-12, -8)$. The product is $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$. Check: dilate to $(12, -8)$, then reflect to $(-12, -8)$.

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