



Circles and Coordinate Geometry

Four focused lessons, worked examples, a mistake clinic, and 60 practice problems with full explanations.

In this booster

19.1 Circle Basics and Angle Relationships

19.2 Arc Length, Sectors, and Radians

19.3 Equations of Circles

19.4 Coordinate Geometry Toolkit

Every section teaches the idea, walks through the most common problem types, flags the traps students fall into, and ends with fifteen practice problems. Full answers and explanations are at the back.

19.1 Circle Basics and Angle Relationships

Learn the parts of a circle and the rules that connect central angles, inscribed angles, arcs, chords, tangents and secants, so you can find unknown angles and lengths.

Why This Matters

Circles are everywhere: wheels, clocks, pizza, camera lenses, sports fields, and the orbits studied in science. Geometry gives circles a set of angle and length rules that let you find unknown measures without measuring anything. The same ideas show up in engineering drawings, in map and radar work, and in later courses such as trigonometry and precalculus.

Students usually get stuck in three places. First, the **vocabulary**: radius, chord, secant, and tangent sound alike, and a mix-up wrecks the whole problem. Second, **which angle rule to use**, because the rule changes depending on where the vertex sits (center, on the circle, inside, or outside). Third, the **halving**: most angle rules need half of an arc, and students forget to halve, or they add when they should subtract. This section fixes all three.

Core Ideas

Start with the parts of a circle. A **radius** goes from the center to the circle. A **chord** joins two points on the circle. A **diameter** is a chord through the center, so it is the longest chord and equals two radii. A **secant** is a line that cuts the circle at two points. A **tangent** is a line that touches the circle at exactly one point, called the point of tangency. An **arc** is a piece of the circle, and a full circle is 360 degrees.

Arc Measure and Central Angles

A **central angle** has its vertex at the center. Its measure equals the measure of the arc it cuts off.

If a central angle is 80° , then its arc is 80° . A semicircle is 180° and the whole circle is 360° .

Inscribed Angle Theorem

An **inscribed angle** has its vertex on the circle. Its measure is **half** of its intercepted arc:

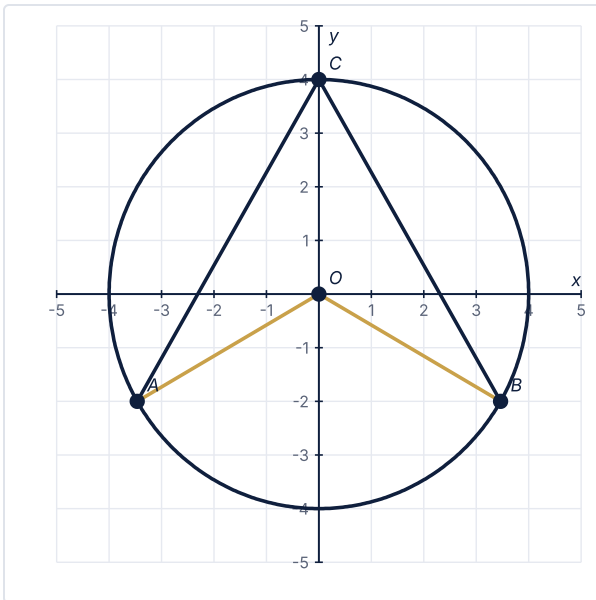
$$m\angle ACB = \frac{1}{2} m\widehat{AB}.$$

Two inscribed angles that cut off the same arc are equal. An angle inscribed in a semicircle is a right angle. Opposite angles of an inscribed quadrilateral add to 180° .

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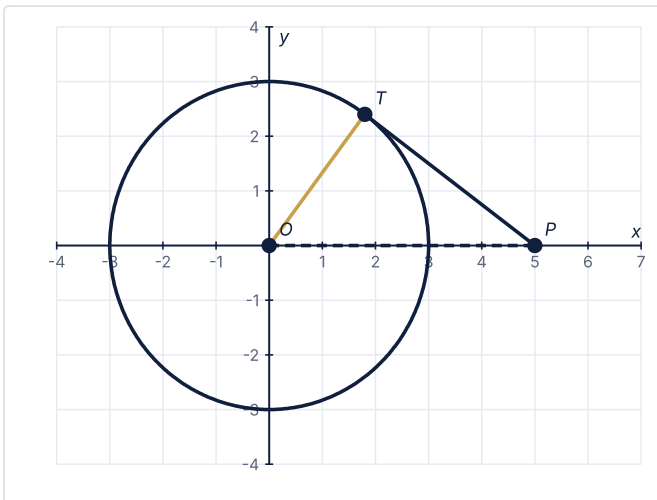
Arc AB (the lower arc) measures 120 degrees. The central angle AOB is 120 degrees, and the inscribed angle ACB is half of it, 60 degrees. Notice that the picture shows the key idea: the inscribed angle (at C) and the central angle (at O) look at the same arc. The center sees the arc "twice as wide." That is why you halve.

Tangent Rules

A tangent is **perpendicular** to the radius drawn to the point of tangency.

Two tangent segments drawn to a circle from the same outside point are **equal in length**.

The angle between a tangent and a chord equals **half** the intercepted arc.



The radius OT meets the tangent PT at a right angle. Here $OT = 3$, $OP = 5$, so $PT = 4$ (a 3-4-5 triangle).

Now the rules for angles and lengths that are not at the center or on the circle. Where the vertex sits tells you which rule to use.

Vertex location	Angle measure	Segment rule
Center	arc	radius = radius
On the circle	$\frac{1}{2}$ (arc)	tangent-chord uses the same half rule
Inside (two chords)	$\frac{1}{2}$ (arc + vertical arc)	$ab = cd$ (pieces of chords multiply)

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Vertex location	Angle measure	Segment rule
Outside (two secants, or tangent and secant, or two tangents)	$\frac{1}{2}$ (far arc – near arc)	whole $\times$ outside piece is the same for each secant; tangent ² = whole $\times$ outside

Chords and Secants

Chord-chord: if chords AB and CD meet inside the circle at E , then $AE \cdot EB = CE \cdot ED$.

Secant-secant: from an outside point P , $PA \cdot PB = PC \cdot PD$, where each product is (outside piece) $\times$ (whole secant).

Tangent-secant: $PT^2 = PA \cdot PB$, where PT is the tangent and PB is the whole secant.

Worked Examples

EXAMPLE 1: CENTRAL AND INSCRIBED ANGLES

In a circle, central angle $\angle AOB = 84^\circ$. Point C is on the major arc. Find $m\angle ACB$.

SOLUTION

The central angle equals its arc, so $m\widehat{AB} = 84^\circ$.

The inscribed angle ACB cuts off the same arc, so it is half: $84 \div 2 = 42$.

$m\angle ACB = 42^\circ$.

EXAMPLE 2: ANGLE IN A SEMICIRCLE

Segment AB is a diameter and C is on the circle. If $m\angle A = 37^\circ$, find $m\angle B$.

SOLUTION

A diameter cuts off a 180° arc, so the inscribed angle at C is 90° .

The angles of the triangle add to 180° , so $m\angle B = 180 - 90 - 37 = 53$.

$m\angle B = 53^\circ$.

EXAMPLE 3: TANGENT LENGTH

A circle has radius 7 and center O . Point P is 25 units from O , and PT is tangent at T . Find PT .

SOLUTION

The radius OT is perpendicular to the tangent, so triangle OTP has a right angle at T .

The hypotenuse is $OP = 25$ and a leg is $OT = 7$.

$PT^2 = 25^2 - 7^2 = 625 - 49 = 576$, so $PT = 24$.

$PT = 24$.

EXAMPLE 4: TWO CHORDS

Chords AB and CD meet at E inside a circle. $AE = 6$, $EB = 4$, $CE = 3$. Find ED .

SOLUTION

Use chord-chord: $AE \cdot EB = CE \cdot ED$.

$6 \cdot 4 = 3 \cdot ED$, so $24 = 3ED$.

$ED = 8$.

EXAMPLE 5: ANGLE OUTSIDE THE CIRCLE

Two secants meet at an outside point P . They cut off a far arc of 140° and a near arc of 40° . Find the angle at P .

SOLUTION

Outside angles use half the **difference** of the arcs.

$m\angle P = \frac{1}{2}(140 - 40) = \frac{1}{2}(100) = 50$.

$m\angle P = 50^\circ$.

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EXAMPLE 6: TANGENT AND SECANT

From P , tangent $PT = 12$ and a secant meets the circle at A (nearer) and B (farther). The outside piece is $PA = 8$. Find the chord AB .

SOLUTION

Use $PT^2 = PA \cdot PB$: $144 = 8 \cdot PB$, so $PB = 18$.

The chord is the whole secant minus the outside piece: $AB = PB - PA = 18 - 8 = 10$.

$AB = 10$. Check: $8 \cdot 18 = 144 = 12^2$.

Coach's Playbook

- 1 Name the vertex.** Ask: is the angle at the center, on the circle, inside, or outside? That picks your rule.
- 2 Mark the intercepted arc.** Shade the arc between the two sides. For outside angles mark both arcs, the far one and the near one.
- 3 Halve or not.** Center angle: no halving. Every other case halves something.
- 4 Sum or difference.** Inside the circle, add the two arcs. Outside the circle, subtract near from far.
- 5 Lengths: multiply, never add.** Chord, secant and tangent length rules are all products. Write the product equation first, then solve.
- 6 Check with a sketch.** A tangent length must be shorter than the whole secant from the same point, and an arc cannot be more than 360 degrees.

Common traps

1) Forgetting to halve an inscribed angle. 2) Adding the arcs for an outside angle (it must be a difference). 3) Treating the whole secant as the chord: the chord is the whole secant minus the outside piece. 4) Assuming a line that touches a circle near a point is tangent: only a line perpendicular to the radius is tangent.

Mistake Clinic**Spot the error 1**

Problem: Arc AB measures 100° and C is on the circle away from AB . Find $m\angle ACB$. Wrong work: "The angle cuts off a 100° arc, so the angle is 100° ." What went wrong: The angle at C is inscribed, not central. Inscribed angles are half of their arc. Right solution: $m\angle ACB = \frac{1}{2}(100) = 50^\circ$.

Spot the error 2

Problem: Two secants meet at P outside the circle with arcs 130° (far) and 50° (near). Find $\angle P$. Wrong work: " $\frac{1}{2}(130 + 50) = 90^\circ$." What went wrong: Adding the arcs is the rule for two chords crossing **inside** the circle. An outside vertex uses the difference. Right solution: $\frac{1}{2}(130 - 50) = 40^\circ$.

Spot the error 3

Problem: Tangent $PT = 6$. A secant from P has outside piece $PA = 3$. Find chord AB . Wrong work: " $6^2 = 3 \cdot AB$, so $AB = 12$." What went wrong: The rule uses the **whole** secant PB , not the chord AB . Right solution: $36 = 3 \cdot PB$, so $PB = 12$, and $AB = 12 - 3 = 9$.

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Real-World Connection

A circular stage has seats all around its edge. A chord of the circle marks the front of the stage, and its arc is 80° . Every seat on the far side of the circle sees the stage front at the same 40° angle, because all those inscribed angles cut off the same arc. Camera crews use this fact to place cameras that keep the same view.

Challenge Corner

Problem: Tangents from P touch a circle of radius 6 at A and B , and $\angle APB = 60^\circ$. Find PA and OP . Solution: Since $OA \perp PA$ and $OB \perp PB$, quadrilateral $OAPB$ has angles $90, 90, 60$, so $\angle AOB = 120^\circ$. The line OP bisects both angles, so $\angle OPA = 30^\circ$ and $\angle AOP = 60^\circ$. In the 30 - 60 - 90 triangle OAP with $OA = 6$ opposite 30° : $OP = 12$ and $PA = 6\sqrt{3}$. Check: $6^2 + (6\sqrt{3})^2 = 36 + 108 = 144 = 12^2$.

Practice**PRACTICE Circle Basics and Angle Relationships**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- An inscribed angle intercepts an arc of 130° . What is the measure of the angle?
(A) 130° (B) 65° (C) 260° (D) 50°
- Segment AB is a diameter of a circle and C is another point on the circle. If $m\angle A = 34^\circ$, what is $m\angle B$?
(A) 34° (B) 90° (C) 56° (D) 146°
- A circle has radius 9 and center O . A tangent from P touches the circle at T , and $OP = 15$. What is PT ?
(A) 6 (B) 12 (C) $\sqrt{306}$ (D) 24
- Tangent segments PA and PB are drawn from P to a circle. If $PA = 3x + 2$ and $PB = 5x - 10$, what is PA ?
(A) 20 (B) 6 (C) 40 (D) 12
- Chords AB and CD meet at E . $AE = 8$, $EB = 3$, $CE = 4$. What is ED ?
(A) 5 (B) 7 (C) 6 (D) 12
- Two chords cross inside a circle. One angle formed intercepts an arc of 70° , and its vertical angle intercepts an arc of 110° . What is the measure of that angle?
(A) 40° (B) 70° (C) 180° (D) 90°
- Two secants meet at an outside point and cut off a far arc of 150° and a near arc of 50° . What is the angle at the point?
(A) 25° (B) 100° (C) 200° (D) 50°
- Quadrilateral $ABCD$ is inscribed in a circle. $m\angle A = 2x + 10$ and $m\angle C = 3x - 20$ (opposite angles). What is $m\angle A$?
(A) 38° (B) 94° (C) 86° (D) 180°
- From point P , a tangent has length 10. A secant from P has an outside piece of 4. How long is the chord inside the circle?
(A) 6 (B) 25 (C) 29 (D) 21
- An inscribed angle measures $3x + 4$ degrees and its intercepted arc measures $(8x - 12)$ degrees. Find the measure of the arc.
- A tangent and a chord meet at the point of tangency and form an angle of 72° . Find the measure of the intercepted arc.
- Chords AB and CD meet at E . $AE = 3$, $EB = 8$, $CE = x$, $ED = x + 2$. Find x .
- From P , one secant has outside piece 5 and chord 7 inside the circle. A second secant has outside piece 4. Find the length of the chord of the second secant.
- Two tangents from P touch a circle with center O at A and B . If $\angle APB = 50^\circ$, find $\angle AOB$.
- A tangent and a secant meet at P . The secant cuts off a far arc $(5x + 20)^\circ$ and a near arc $(2x + 10)^\circ$, and the angle at P is 35° . Find the measure of the near arc.

19.2 Arc Length, Sectors, and Radians

Find arc lengths and sector areas as fractions of a circle, then learn radians, the angle measure that makes the formulas short and clean.

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Why This Matters

A sprinkler sweeps part of a lawn, a clock hand traces a piece of a circle, and a pizza slice is a wedge. In each case you need to measure either how far something travels along a curve (**arc length**) or how much area a wedge covers (**sector area**). Engineers use the same ideas for gears and wheels, and scientists use them for orbits and waves.

Students get stuck in three places. They forget that arc length and sector area are **fractions of the whole circle**. They mix up the circumference formula with the area formula. And **radians** feel strange at first, because angles are suddenly written with π . This section builds the fraction idea first, then radians as a faster way to do the same job.

Core Ideas

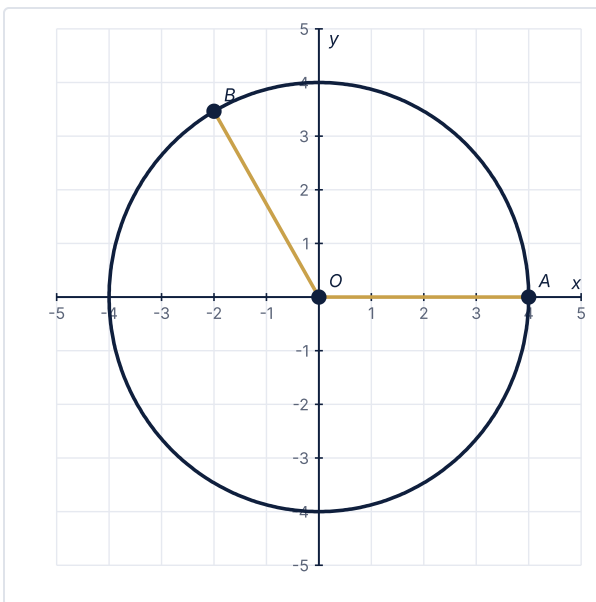
A full circle is 360° . A central angle of θ degrees takes up the fraction $\frac{\theta}{360}$ of the circle. So the arc is that fraction of the circumference, and the sector is that fraction of the area.

Arc Length and Sector Area (degrees)

For a circle of radius r and a central angle of θ degrees:

$$s = \frac{\theta}{360} \cdot 2\pi r \quad A = \frac{\theta}{360} \cdot \pi r^2.$$

Arc length is measured in length units (cm, m). Sector area is measured in square units.



A sector with radius 4 and central angle 120 degrees. Its arc AB is one third of the circle's circumference, and the sector is one third of the circle's area.

In the picture, the angle is 120° , so the fraction is $\frac{120}{360} = \frac{1}{3}$. The arc is $\frac{1}{3} \cdot 2\pi(4) = \frac{8\pi}{3}$ and the sector area is $\frac{1}{3} \cdot \pi(16) = \frac{16\pi}{3}$.

A **radian** is another way to measure an angle. One radian is the central angle whose arc is exactly one radius long. Because the whole circumference is $2\pi r$, a full turn is 2π radians.

Degrees and Radians

$$180^\circ = \pi \text{ radians.}$$

To change degrees to radians, multiply by $\frac{\pi}{180}$. To change radians to degrees, multiply by $\frac{180}{\pi}$.

Degrees	0	30	45	60	90	120	135	150	180	270	360
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π

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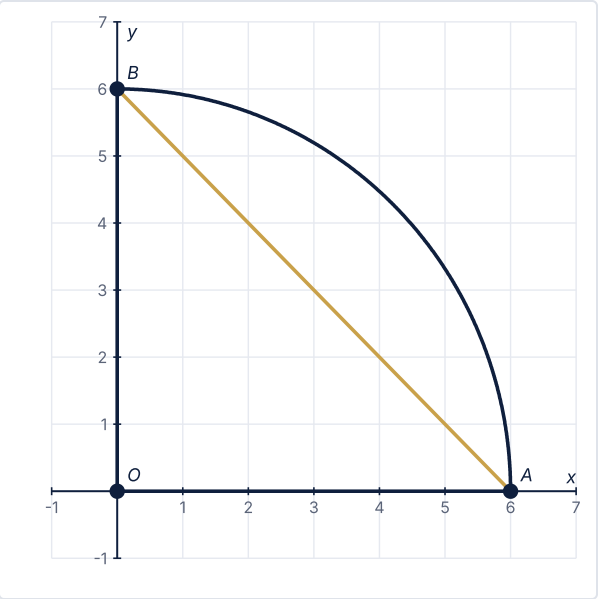
Arc Length and Sector Area (radians)

If θ is in radians:

$s = r\theta$ $A = \frac{1}{2}r^2\theta$.

These only work when θ is in radians. Also $A = \frac{1}{2}rs$.

Why do the radian formulas look so clean? With θ in radians, the fraction of the circle is $\frac{\theta}{2\pi}$. So $s = \frac{\theta}{2\pi} \cdot 2\pi r = r\theta$ and $A = \frac{\theta}{2\pi} \cdot \pi r^2 = \frac{1}{2}r^2\theta$. The π cancels.



A quarter circle of radius 6. A segment is a sector minus a triangle, here the region between chord AB and the arc. Here is a quick way to decide which tool to use. If a problem gives you an angle in degrees, think fraction of 360. If the angle has a π in it, think radians and use $s = r\theta$. If you have to go back and forth, remember the single anchor fact $180^\circ = \pi$ radians, and every other conversion follows by multiplying or dividing.

You know	You want	Use
radius, angle in degrees	arc length	$\frac{\theta}{360} \cdot 2\pi r$
radius, angle in degrees	sector area	$\frac{\theta}{360} \cdot \pi r^2$
radius, angle in radians	arc length	$r\theta$
radius, angle in radians	sector area	$\frac{1}{2}r^2\theta$
arc length and radius	angle in radians	$\theta = \frac{s}{r}$

Two more facts help with perimeter problems. The **perimeter of a sector** is the arc plus both radii, so $P = 2r + s$. The **area of a segment** (the region between a chord and its arc) is the sector area minus the triangle formed by the two radii and the chord. A triangle with two sides r and included angle θ has area $\frac{1}{2}r^2 \sin \theta$, which you will meet again in later sections.

Worked Examples

EXAMPLE 1: ARC LENGTH IN DEGREES

A circle has radius 9 cm. Find the length of an arc with a central angle of 40° .

SOLUTION

Fraction of the circle: $\frac{40}{360} = \frac{1}{9}$.
Circumference: $2\pi(9) = 18\pi$.
Arc length $= \frac{1}{9} \cdot 18\pi = 2\pi$.
 $s = 2\pi \approx 6.28$ cm.

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EXAMPLE 2: SECTOR AREA IN DEGREES

A sector has radius 6 m and central angle 150° . Find its area.

SOLUTION

Fraction: $\frac{150}{360} = \frac{5}{12}$.

Circle area: $\pi(6)^2 = 36\pi$.

Sector area = $\frac{5}{12} \cdot 36\pi = 15\pi$.

$A = 15\pi \text{ m}^2 \approx 47.1 \text{ m}^2$.

EXAMPLE 3: CONVERTING ANGLES

(a) Convert 225° to radians. (b) Convert $\frac{7\pi}{6}$ to degrees.

SOLUTION

(a) $225 \cdot \frac{\pi}{180} = \frac{225\pi}{180} = \frac{5\pi}{4}$ (divide top and bottom by 45).

(b) $\frac{7\pi}{6} \cdot \frac{180}{\pi} = 7 \cdot 30 = 210$.

(a) $\frac{5\pi}{4}$ radians. (b) 210° .

EXAMPLE 4: RADIAN FORMULAS

A circle has radius 8 and a central angle of $\frac{3\pi}{4}$ radians. Find the arc length and the sector area.

SOLUTION

Arc length: $s = r\theta = 8 \cdot \frac{3\pi}{4} = 6\pi$.

Sector area: $A = \frac{1}{2}r^2\theta = \frac{1}{2}(64)\frac{3\pi}{4} = 24\pi$.

Check with $A = \frac{1}{2}rs = \frac{1}{2}(8)(6\pi) = 24\pi$. It matches.

$s = 6\pi$, $A = 24\pi$.

EXAMPLE 5: FINDING THE ANGLE

On a circle of radius 12 cm, an arc has length 5π cm. Find the central angle in radians and in degrees.

SOLUTION

Use $s = r\theta$: $5\pi = 12\theta$, so $\theta = \frac{5\pi}{12}$ radians.

Convert: $\frac{5\pi}{12} \cdot \frac{180}{\pi} = 75$.

$\theta = \frac{5\pi}{12}$ **radians** = 75° .

EXAMPLE 6: REGION BETWEEN A CHORD AND AN ARC

In the quarter-circle figure, $r = 6$ and the central angle is 90° . Find the area between the chord AB and the arc (a circular segment).

SOLUTION

Sector area: $\frac{90}{360} \cdot \pi(36) = 9\pi$.

The chord and the two radii form a right triangle with legs 6 and 6, so its area is $\frac{1}{2}(6)(6) = 18$.

Segment area = sector – triangle = $9\pi - 18$.

$A = 9\pi - 18 \approx 10.27$ **square units**.

Coach's Playbook

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- 1 **Decide the unit first.** Is the angle in degrees or radians? Use the degree formulas with fractions of 360, or the radian formulas $s = r\theta$ and $A = \frac{1}{2}r^2\theta$.
- 2 **Think "fraction of a circle."** Write the fraction before anything else. If it simplifies (like $\frac{1}{3}$), the arithmetic becomes easy.
- 3 **Keep π exact** Leave answers in terms of π until the last step, then round only if asked.
- 4 **Label units.** Arc length is a length, sector area is square units. If your answer has the wrong kind of units, you used the wrong formula.
- 5 **Sanity check.** A 90° arc should be a quarter of the circumference. A sector with a small angle should have a small area.

Common traps

1) Using the area formula for arc length or the reverse. 2) Using $s = r\theta$ when θ is in degrees. 3) Converting degrees to radians by multiplying by $\frac{180}{\pi}$ (that goes the other way). 4) Forgetting the diameter-to-radius step: if a problem gives the diameter, halve it first.

Mistake Clinic

Spot the error 1

Problem: Find the arc length for a 60° angle on a circle of radius 6. Wrong work: " $s = r\theta = 6 \cdot 60 = 360$." What went wrong: The formula $s = r\theta$ needs radians, but 60 is in degrees. Right solution: $60^\circ = \frac{\pi}{3}$ radians, so $s = 6 \cdot \frac{\pi}{3} = 2\pi$. Check with the fraction method: $\frac{1}{6} \cdot 12\pi = 2\pi$.

Spot the error 2

Problem: Find the area of a sector with diameter 10 and angle 72° . Wrong work: "Area = $\frac{72}{360} \cdot \pi(10)^2 = 20\pi$." What went wrong: The number 10 is the diameter, not the radius. Right solution: $r = 5$, so area = $\frac{1}{5} \cdot \pi(25) = 5\pi$.

Spot the error 3

Problem: Convert $\frac{2\pi}{3}$ radians to degrees. Wrong work: " $\frac{2\pi}{3} \cdot \frac{\pi}{180} = \frac{2\pi^2}{540}$." What went wrong: To leave radians you must cancel π , so multiply by $\frac{180}{\pi}$. Right solution: $\frac{2\pi}{3} \cdot \frac{180}{\pi} = 120$, so the angle is 120° .

Real-World Connection

A lawn sprinkler reaches 15 m and sweeps back and forth through 120° . The watered region is a sector with area $\frac{1}{3} \cdot \pi(15)^2 = 75\pi \approx 235.6 \text{ m}^2$. The outer edge of the spray is an arc of length $\frac{1}{3} \cdot 30\pi = 10\pi \approx 31.4 \text{ m}$, which tells you how much fencing a garden border would need.

Challenge Corner

Problem: A sector of a circle has area 30π and arc length 5π . Find the radius, then the central angle in degrees.

Solution: Use $A = \frac{1}{2}rs$: $30\pi = \frac{1}{2}r(5\pi)$, so $60 = 5r$ and $r = 12$. Then $\theta = \frac{s}{r} = \frac{5\pi}{12}$ radians, which is 75° . Check: $\frac{75}{360} \cdot \pi(144) = 30\pi$.

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Practice

PRACTICE Arc Length, Sectors, and Radians

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- Which is 120° written in radians?
(A) $\frac{\pi}{3}$ (B) $\frac{3\pi}{2}$ (C) $\frac{2\pi}{3}$ (D) 120π
- What is $\frac{5\pi}{6}$ radians in degrees?
(A) 30° (B) 150° (C) 120° (D) 300°
- A circle has radius 10. What is the length of an arc with a central angle of 72° ?
(A) 2π (B) 8π (C) 40π (D) 4π
- A sector has radius 4 and central angle 90° . What is its area?
(A) 2π (B) 8π (C) 4π (D) 16π
- A circle has radius 5. An arc subtends a central angle of 2.4 radians. What is the arc length?
(A) 12 (B) $\frac{25}{12}$ (C) 7.4 (D) 12π
- A sector has radius 10 and central angle $\frac{\pi}{5}$ radians. What is its area?
(A) 20π (B) 10π (C) 5π (D) 50π
- On a circle of diameter 9, an arc has length 6π . What is the central angle?
(A) 60° (B) 180° (C) 240° (D) 120°
- A sector of a circle with radius 6 has area 12π . What is the central angle in radians?
(A) $\frac{\pi}{3}$ (B) $\frac{4\pi}{3}$ (C) 2π (D) $\frac{2\pi}{3}$
- The tip of an 8 cm minute hand moves for 25 minutes. How far does the tip travel?
(A) $\frac{20\pi}{3}$ cm (B) $\frac{10\pi}{3}$ cm (C) $\frac{40\pi}{3}$ cm (D) 8π cm
- A circle has radius 12. Find the exact length of an arc with central angle 105° .
- Convert $\frac{11\pi}{12}$ radians to degrees.
- A sector has radius 9 and central angle $\frac{4\pi}{3}$ radians. Find its exact area.
- Find the exact perimeter of a sector with radius 10 and central angle 72° (include both radii).
- A sector has radius 12 and central angle 60° . Find the exact area of the region between the chord joining the endpoints of the arc and the arc itself.
- A sector has arc length 9π and radius 18. Find its area in terms of π and the central angle in degrees.

19.3 Equations of Circles

Write and read circle equations, complete the square to find a center and radius, and solve problems with lines that cross or touch a circle.

Why This Matters

A circle is one of the few shapes you can describe completely with a short equation. That equation lets a computer draw a wheel, a game programmer decide whether a character has been hit by a circular blast, and a radio engineer decide which houses sit inside a tower's range. In later courses, circles lead straight to trigonometry, conic sections, and calculus.

Students struggle with three things. The **signs** in $(x - h)^2 + (y - k)^2 = r^2$ are backwards from what feels natural, so a center of $(2, -1)$ looks like $(x - 2)$ and $(y + 1)$. The **radius** is hidden: the equation holds r^2 , not r . And **completing the square** feels like a trick until you see it is just a way of un-expanding the squares. This section takes each idea in turn.

Core Ideas

A circle is the set of all points at a fixed distance r from a center. The distance formula turns that sentence into an equation. If the center is (h, k) and a point (x, y) is on the circle, then its distance to the center is r , so $\sqrt{(x - h)^2 + (y - k)^2} = r$. Square both sides and you get the standard form.

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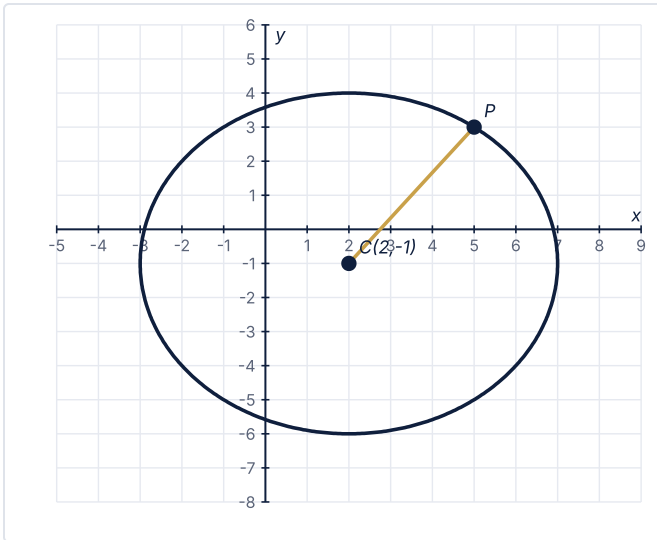
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Center-Radius Form

The circle with center (h, k) and radius $r > 0$ is

$$(x - h)^2 + (y - k)^2 = r^2.$$

With center at the origin this becomes $x^2 + y^2 = r^2$. The right side is the **square** of the radius, so if it equals 49, then $r = 7$.



The circle $(x-2)^2 + (y+1)^2 = 25$. Its center is $(2, -1)$ and every point is 5 units away, for example $P(5, 3)$, since $3^2 + 4^2 = 25$.

In the picture, the center is $(2, -1)$, so the equation has $(x - 2)$ and $(y + 1)$. Read the signs as "subtract the center coordinates." Subtracting -1 turns into adding 1.

Equation	Center	Radius
$x^2 + y^2 = 16$	$(0, 0)$	4
$(x - 3)^2 + (y - 5)^2 = 9$	$(3, 5)$	3
$(x + 4)^2 + y^2 = 20$	$(-4, 0)$	$\sqrt{20} = 2\sqrt{5}$
$(x - 1)^2 + (y + 6)^2 = 1$	$(1, -6)$	1

Inside, On, Outside

For the circle $(x - h)^2 + (y - k)^2 = r^2$ and a point $P = (a, b)$, compute $(a - h)^2 + (b - k)^2$.

If it equals r^2 , P is **on** the circle. If it is less, P is **inside**. If it is more, P is **outside**.

Sometimes a circle arrives in **general form**, like $x^2 + y^2 - 6x + 4y - 12 = 0$. You cannot read the center from that. To convert it, group the x terms and y terms, move the constant to the right side, and **complete the square** in each group. To complete the square on $x^2 + bx$, add $(\frac{b}{2})^2$. Remember to add the same amount to both sides.

Line and Circle Intersections

To find where a line meets a circle, **solve the line for y (or x) and substitute** into the circle equation. You get a quadratic. Its discriminant tells the story: positive means two intersection points (a secant), zero means one point (a **tangent**), negative means no points.

A tangent line is perpendicular to the radius at the point of tangency, so its slope is the **negative reciprocal** of the radius slope.

Worked Examples

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EXAMPLE 1: WRITE THE EQUATION

Write the equation of the circle with center $(-3, 2)$ and radius 4.

SOLUTION

Use $(x - h)^2 + (y - k)^2 = r^2$ with $h = -3$, $k = 2$, $r = 4$.

$$(x - (-3))^2 + (y - 2)^2 = 4^2.$$

$$(x + 3)^2 + (y - 2)^2 = 16.$$

EXAMPLE 2: INSIDE, ON, OR OUTSIDE

For the circle $(x - 1)^2 + (y + 2)^2 = 25$, classify $(4, 2)$, $(2, 1)$, and $(7, 0)$.

SOLUTION

$(4, 2)$: $(3)^2 + (4)^2 = 25$. Equal to 25, so **on** the circle.

$(2, 1)$: $(1)^2 + (3)^2 = 10 < 25$, so **inside**.

$(7, 0)$: $(6)^2 + (2)^2 = 40 > 25$, so **outside**.

EXAMPLE 3: COMPLETING THE SQUARE

Find the center and radius of $x^2 + y^2 - 6x + 4y - 12 = 0$.

SOLUTION

Group: $(x^2 - 6x) + (y^2 + 4y) = 12$.

Half of -6 is -3 , and $(-3)^2 = 9$. Half of 4 is 2 , and $2^2 = 4$. Add 9 and 4 to both sides.

$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 + 9 + 4, \text{ so } (x - 3)^2 + (y + 2)^2 = 25.$$

Center $(3, -2)$, radius 5.

EXAMPLE 4: DIAMETER ENDPOINTS

A circle has a diameter with endpoints $(-2, 1)$ and $(6, 7)$. Write its equation.

SOLUTION

The center is the midpoint: $\left(\frac{-2+6}{2}, \frac{1+7}{2}\right) = (2, 4)$.

The radius is half the diameter. Diameter length $= \sqrt{8^2 + 6^2} = 10$, so $r = 5$.

$$(x - 2)^2 + (y - 4)^2 = 25.$$

EXAMPLE 5: LINE MEETS CIRCLE

Find the points where the line $y = x + 1$ meets the circle $x^2 + y^2 = 25$.

SOLUTION

Substitute: $x^2 + (x + 1)^2 = 25$.

Expand: $2x^2 + 2x + 1 = 25$, so $2x^2 + 2x - 24 = 0$, so $x^2 + x - 12 = 0$.

Factor: $(x + 4)(x - 3) = 0$, so $x = -4$ or $x = 3$.

Find $y = x + 1$: the points are $(-4, -3)$ and $(3, 4)$.

Two points: $(-4, -3)$ and $(3, 4)$.

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EXAMPLE 6: TANGENT LINE

Find the tangent line to $(x - 1)^2 + (y - 2)^2 = 25$ at the point $T(4, 6)$.

SOLUTION

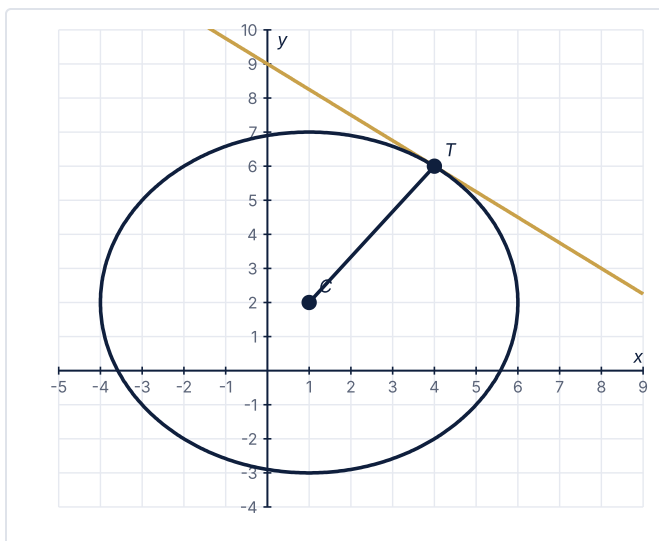
Check T : $3^2 + 4^2 = 25$. It is on the circle.

Radius slope from $C(1, 2)$ to $T(4, 6)$: $\frac{6-2}{4-1} = \frac{4}{3}$.

Tangent slope is the negative reciprocal: $-\frac{3}{4}$.

Point-slope: $y - 6 = -\frac{3}{4}(x - 4)$, so $4y - 24 = -3x + 12$, so $3x + 4y = 36$.

Tangent line: $3x + 4y = 36$. Check: distance from C to the line is $\frac{|3 + 8 - 36|}{5} = 5 = r$.



The line $3x + 4y = 36$ touches the circle $(x - 1)^2 + (y - 2)^2 = 25$ at $T(4, 6)$ and is perpendicular to the radius CT .

Coach's Playbook

- 1 Read the signs backwards.** In $(x - h)^2$, the center coordinate is h with the opposite sign of what you see. $(y + 7)^2$ means $k = -7$.
- 2 Square-root the right side last.** The number on the right is r^2 . Write down r separately so you do not forget the root.
- 3 Complete the square twice.** Handle x and y separately. Whatever you add on the left, add on the right.
- 4 Substitute, then use the discriminant.** For a line and a circle, one substitution creates a quadratic. Check the discriminant before you factor, to know how many answers to expect.
- 5 Use the radius for tangents.** The radius to the point of tangency gives you the slope fast. Perpendicular means negative reciprocal.

Common traps

1) Using the wrong sign for the center. 2) Calling the right side r instead of r^2 . 3) Forgetting to add the completed-square amounts to the right side as well. 4) Giving only x -values after solving a line-circle system, when the answer is points (x, y) .

Mistake Clinic

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Spot the error 1

Problem: Find the center and radius of $(x + 5)^2 + (y - 3)^2 = 49$. Wrong work: "Center (5, 3), radius 49." What went wrong: Two mistakes. The x -coordinate of the center is -5 because $x + 5 = x - (-5)$. And 49 is r^2 , not r . Right solution: Center $(-5, 3)$, radius $\sqrt{49} = 7$.

Spot the error 2

Problem: Complete the square: $x^2 + y^2 + 8x - 2y - 8 = 0$. Wrong work: " $(x^2 + 8x + 16) + (y^2 - 2y + 1) = 8$, so $(x + 4)^2 + (y - 1)^2 = 8$." What went wrong: The student added 16 and 1 on the left but not on the right. Right solution: $(x^2 + 8x + 16) + (y^2 - 2y + 1) = 8 + 16 + 1 = 25$, so $(x + 4)^2 + (y - 1)^2 = 25$. Center $(-4, 1)$, radius 5.

Spot the error 3

Problem: Find the tangent slope to $x^2 + y^2 = 25$ at $(3, 4)$. Wrong work: "The radius slope is $\frac{4}{3}$, and a tangent is perpendicular, so the slope is $-\frac{4}{3}$." What went wrong: Perpendicular slopes are **negative reciprocals**. The student flipped the sign but not the fraction. Right solution: Slope $= -\frac{3}{4}$. Check: the product $\frac{4}{3} \cdot (-\frac{3}{4}) = -1$.

Real-World Connection

A radio tower covers every point within 10 km of its location. Put the tower at the origin and its coverage zone is $x^2 + y^2 \leq 100$. A village at $(6, 9)$ has $36 + 81 = 117 > 100$, so it is outside. A village at $(6, 8)$ has $36 + 64 = 100$, so it sits exactly on the edge of the signal.

Challenge Corner

Problem: The line $y = x + k$ is tangent to the circle $x^2 + y^2 = 8$. Find the possible values of k . Solution: Substitute: $x^2 + (x + k)^2 = 8$, so $2x^2 + 2kx + k^2 - 8 = 0$. For a tangent there is exactly one solution, so the discriminant is zero: $(2k)^2 - 4(2)(k^2 - 8) = 0$. That gives $4k^2 - 8k^2 + 64 = 0$, so $k^2 = 16$ and $k = 4$ or $k = -4$. Check: the distance from the origin to $x - y + 4 = 0$ is $\frac{4}{\sqrt{2}} = 2\sqrt{2} = \sqrt{8}$.

Practice**PRACTICE Equations of Circles**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- Which is the equation of the circle with center $(2, -5)$ and radius 3?

(A) $(x + 2)^2 + (y - 5)^2 = 9$ (B) $(x - 2)^2 + (y + 5)^2 = 3$ (C) $(x - 2)^2 + (y + 5)^2 = 9$ (D) $(x + 2)^2 + (y - 5)^2 = 3$
- What is the radius of the circle $(x + 1)^2 + (y - 3)^2 = 64$?

(A) 8 (B) 64 (C) 16 (D) 4
- What is the center of $x^2 + y^2 - 8x + 6y = 0$?

(A) $(-4, 3)$ (B) $(8, -6)$ (C) $(4, 3)$ (D) $(4, -3)$
- Which point lies on the circle $(x - 3)^2 + (y + 1)^2 = 25$?

(A) $(6, 2)$ (B) $(6, 3)$ (C) $(0, -1)$ (D) $(3, 5)$

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5. A circle has a diameter with endpoints (1, 2) and (7, 10). Which is its equation?
 (A) $(x - 4)^2 + (y - 6)^2 = 100$ (B) $(x + 4)^2 + (y + 6)^2 = 25$ (C) $(x - 4)^2 + (y - 6)^2 = 25$
 (D) $(x - 4)^2 + (y - 6)^2 = 5$
6. How many points do the line $y = x + 10$ and the circle $x^2 + y^2 = 36$ have in common?
 (A) 0 (B) 1 (C) 2 (D) 3
7. What is the radius of the circle $x^2 + y^2 + 4x - 10y + 13 = 0$?
 (A) 16 (B) 13 (C) 2 (D) 4
8. The circle $x^2 + y^2 = 100$ passes through (6, 8). What is the slope of the tangent line there?
 (A) $\frac{4}{3}$ (B) $-\frac{3}{4}$ (C) $\frac{3}{4}$ (D) $-\frac{4}{3}$
9. For what positive value of k is the line $y = x + k$ tangent to the circle $x^2 + y^2 = 8$?
 (A) 2 (B) 8 (C) 4 (D) 16
10. Find the center and radius of $x^2 + y^2 + 6x - 10y + 18 = 0$.
11. Write the equation of the circle with center $(-1, 4)$ that passes through the point $(2, 0)$.
12. Find the two points where the line $y = x - 5$ meets the circle $(x - 1)^2 + (y + 2)^2 = 10$.
13. Find the tangent line to $x^2 + y^2 = 34$ at the point $(5, 3)$. Write it as $ax + by = c$.
14. Find all values of k so that the line $y = 2x + k$ is tangent to the circle $x^2 + y^2 = 5$.
15. A circle passes through $(0, 0)$, $(8, 0)$ and $(0, 6)$. Write its equation.

19.4 Coordinate Geometry Toolkit

Use the distance, midpoint, section and slope formulas to find lengths and points, write line equations, prove facts about figures, and compute polygon areas on a grid.

Why This Matters

Coordinate geometry puts algebra and geometry in the same room. Once a figure is placed on a grid, lengths, angles and areas become calculations with numbers. This is how maps, video games, and design software work: a computer does not "see" a triangle, it stores the coordinates of three points and calculates everything else.

Students tend to get stuck in four places: mixing up the **distance** and **midpoint** formulas, handling negative numbers when subtracting coordinates, remembering that **perpendicular slopes are negative reciprocals**, and turning a geometry claim ("this is a parallelogram") into a proof using coordinates. This section gives you one reliable tool for each.

Core Ideas

Distance and Midpoint

For $A(x_1, y_1)$ and $B(x_2, y_2)$:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}, \quad M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

The distance formula is the Pythagorean theorem on a grid: the horizontal change and vertical change are the two legs.

The **midpoint** averages the coordinates. Another way to see it: the midpoint is the point that moves exactly half of the way from A to B . That idea extends to any fraction of the way.

Section Formula

The point P that divides $\overline{AB}$ in the ratio $m : n$ (measured from A to B) is

$$P = \left(\frac{nx_1 + mx_2}{m + n}, \frac{ny_1 + my_2}{m + n} \right).$$

Equivalent and easier to remember: $P = A + \frac{m}{m+n}(B - A)$, taken coordinate by coordinate.

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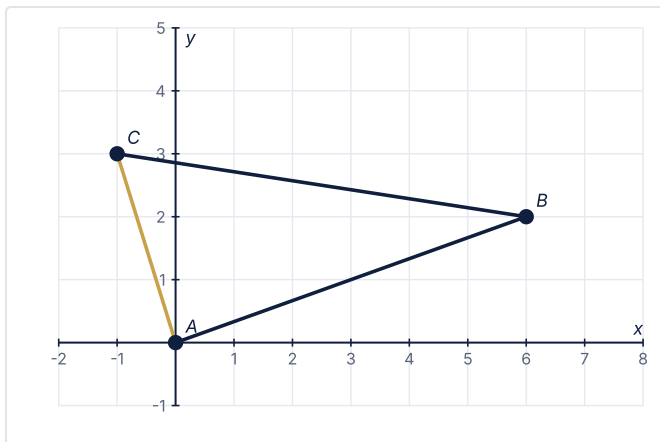
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Slopes

$$\text{Slope: } m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Parallel lines have equal slopes. **Perpendicular** lines have slopes whose product is -1 (negative reciprocals). A vertical line and a horizontal line are perpendicular even though a vertical line has no slope.

I want to show...	Use...	Evidence
two sides are equal	distance formula	same length
sides are parallel	slopes	equal slopes
angle is a right angle	slopes	product is -1
diagonals bisect each other	midpoint formula	same midpoint
a point is halfway or $\frac{1}{3}$ of the way	midpoint or section formula	coordinates



Triangle ABC. Slope of AB is $1/3$ and slope of AC is -3 . Their product is -1 , so the angle at A is a right angle.

A **coordinate proof** is just a plan: choose which facts to calculate, calculate them with the formulas above, and state the conclusion in words. For a parallelogram, you can show both pairs of opposite sides are parallel, or that the diagonals share a midpoint, or that one pair is both equal and parallel.

Area by Coordinates (Shoelace)

For a polygon with vertices $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ listed in order around the shape:

$$A = \frac{1}{2} \left| \sum_{i=1}^n (x_i y_{i+1} - x_{i+1} y_i) \right|,$$

where the last vertex connects back to the first.

Write the points in two columns, cross-multiply down and to the right, then down and to the left, subtract, and halve.

Worked Examples

EXAMPLE 1: DISTANCE AND MIDPOINT

Find the distance between $A(-2, 3)$ and $B(4, -5)$, and the midpoint of $\overline{AB}$.

SOLUTION

Changes: $\Delta x = 4 - (-2) = 6$ and $\Delta y = -5 - 3 = -8$.

$$AB = \sqrt{6^2 + (-8)^2} = \sqrt{100} = 10.$$

$$\text{Midpoint: } \left(\frac{-2 + 4}{2}, \frac{3 + (-5)}{2} \right) = (1, -1).$$

$AB = 10$, midpoint $(1, -1)$.

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EXAMPLE 2: DIVIDING A SEGMENT

Find the point P on $\overline{AB}$ that is $\frac{2}{5}$ of the way from $A(1, 2)$ to $B(11, 7)$.

SOLUTION

Change from A to B : $(10, 5)$.

Move $\frac{2}{5}$ of that: $(\frac{2}{5} \cdot 10, \frac{2}{5} \cdot 5) = (4, 2)$.

Add to A : $(1 + 4, 2 + 2) = (5, 4)$.

$P = (5, 4)$. (This divides AB in the ratio $2 : 3$.)

EXAMPLE 3: PARALLEL AND PERPENDICULAR LINES

Find the equation of the line through $(2, 1)$ that is (a) parallel and (b) perpendicular to $2x - 3y = 6$.

SOLUTION

Solve for y : $y = \frac{2}{3}x - 2$, so the slope is $\frac{2}{3}$.

(a) Parallel slope $\frac{2}{3}$: $y - 1 = \frac{2}{3}(x - 2)$, so $3y - 3 = 2x - 4$, so $2x - 3y = 1$.

(b) Perpendicular slope $-\frac{3}{2}$: $y - 1 = -\frac{3}{2}(x - 2)$, so $2y - 2 = -3x + 6$, so $3x + 2y = 8$.

(a) $2x - 3y = 1$. **(b)** $3x + 2y = 8$.

EXAMPLE 4: PROVE A RIGHT TRIANGLE

Show that the triangle with vertices $A(0, 0)$, $B(6, 2)$, $C(-1, 3)$ is a right triangle.

SOLUTION

Slope of $AB = \frac{2-0}{6-0} = \frac{1}{3}$. Slope of $AC = \frac{3-0}{-1-0} = -3$.

The product is $\frac{1}{3} \cdot (-3) = -1$, so $AB \perp AC$.

Double-check with lengths: $AB^2 = 40$, $AC^2 = 10$, $BC^2 = 49 + 1 = 50$, and $40 + 10 = 50$.

The angle at A is a right angle, so the triangle is right.

EXAMPLE 5: PROVE A PARALLELOGRAM

Show that $A(1, 1)$, $B(5, 2)$, $C(7, 6)$, $D(3, 5)$ are the vertices of a parallelogram.

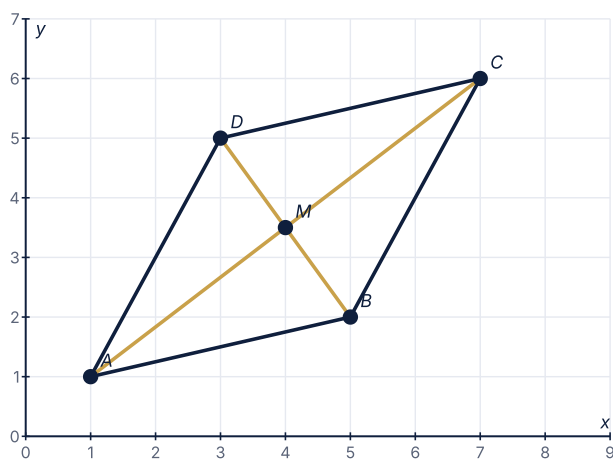
SOLUTION

Midpoint of $\overline{AC}$: $(\frac{1+7}{2}, \frac{1+6}{2}) = (4, 3.5)$.

Midpoint of $\overline{BD}$: $(\frac{5+3}{2}, \frac{2+5}{2}) = (4, 3.5)$.

The diagonals share a midpoint, so they bisect each other.

A quadrilateral whose diagonals bisect each other is a parallelogram. (Check: slope of AB and DC are both $\frac{1}{4}$; slope of AD and BC are both 2.)



The diagonals AC and BD of $ABCD$ share the midpoint $M(4, 3.5)$, which proves $ABCD$ is a parallelogram.

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EXAMPLE 6: AREA OF A POLYGON

Find the area of the quadrilateral with vertices $(1, 1)$, $(7, 2)$, $(6, 6)$, $(2, 5)$, listed in order.

SOLUTION

Compute $x_i y_{i+1} - x_{i+1} y_i$ for each edge, wrapping back to the start.

$$(1)(2) - (7)(1) = -5. \quad (7)(6) - (6)(2) = 30. \quad (6)(5) - (2)(6) = 18. \quad (2)(1) - (1)(5) = -3.$$

$$\text{Sum: } -5 + 30 + 18 - 3 = 40. \quad \text{Area} = \frac{1}{2}|40| = 20.$$

Area = 20 square units.

Coach's Playbook

- 1 Sketch the points.** A quick graph shows which sides are which, whether your answer is reasonable, and which order to list vertices for area.
- 2 Write each change in parentheses.** Compute $(x_2 - x_1)$ and $(y_2 - y_1)$ as separate lines. Subtracting a negative is where most arithmetic errors hide.
- 3 Choose the lightest proof.** To show a parallelogram, midpoints of diagonals is one calculation. To show a right angle, slopes are faster than lengths.
- 4 Say the conclusion.** End a proof with a sentence: "Because the slopes multiply to -1 , the sides are perpendicular."
- 5 Check the answer against the picture.** Midpoints lie between the endpoints, and areas should be about the size of the bounding box.

Common traps

1) Adding coordinates in the distance formula instead of subtracting. 2) Using the slope itself, not the negative reciprocal, for a perpendicular line. 3) Listing polygon vertices out of order for the area formula (the edges must go around the shape). 4) Confusing the ratio $m : n$ in the section formula, so check by sketching: a ratio of $1 : 3$ puts the point near A .

Mistake Clinic**Spot the error 1**

Problem: Find the distance from $(-1, 2)$ to $(3, -4)$. Wrong work: " $\sqrt{(3-1)^2 + (-4-2)^2} = \sqrt{4+36}$." What went wrong: The student wrote $3-1$ instead of $3-(-1)$, dropping the negative sign. Right solution: $\sqrt{(3-(-1))^2 + (-4-2)^2} = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$.

Spot the error 2

Problem: Are the lines $y = \frac{3}{4}x + 1$ and $y = \frac{4}{3}x - 5$ perpendicular? Wrong work: "The slopes are reciprocals, so yes." What went wrong: Perpendicular slopes must be **negative** reciprocals, so their product must be -1 . Right solution: $\frac{3}{4} \cdot \frac{4}{3} = 1$, not -1 . The lines are not perpendicular (and not parallel).

Spot the error 3

Problem: Find the area of the triangle $(0, 0)$, $(4, 0)$, $(0, 6)$ by the shoelace formula. Wrong work: " $(0 \cdot 0 + 4 \cdot 6 + 0 \cdot 0) - (0 \cdot 4 + 0 \cdot 0 + 6 \cdot 0) = 24$, so the area is 24." What went wrong: The student forgot the factor $\frac{1}{2}$. Right solution: The sum is 24, and the area is $\frac{1}{2}(24) = 12$. Check: $\frac{1}{2}(4)(6) = 12$.

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Real-World Connection

A delivery drone flies from a warehouse at $(2, 3)$ to a customer at $(14, 12)$ on a map where one unit is 100 m. The straight-line distance is $\sqrt{12^2 + 9^2} = 15$ units, or 1500 m. A battery swap station is placed halfway, at the midpoint $(8, 7.5)$.

Challenge Corner

Problem: Three vertices of a parallelogram $ABCD$ (in order) are $A(1, 1)$, $B(6, 2)$, $C(8, 6)$. Find D and the area. Solution: In a parallelogram the diagonals bisect each other, so the midpoint of AC equals the midpoint of BD . Therefore $A + C = B + D$, giving $D = A + C - B = (1 + 8 - 6, 1 + 6 - 2) = (3, 5)$. For the area, use $\overrightarrow{AB} = (5, 1)$ and $\overrightarrow{AD} = (2, 4)$: area = $|5 \cdot 4 - 1 \cdot 2| = 18$. Shoelace check: the sum is $(1 \cdot 2 - 6 \cdot 1) + (6 \cdot 6 - 8 \cdot 2) + (8 \cdot 5 - 3 \cdot 6) + (3 \cdot 1 - 1 \cdot 5) = -4 + 20 + 22 - 2 = 36$, half is 18.

Practice**PRACTICE** Coordinate Geometry Toolkit

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What is the distance between $(-1, 2)$ and $(4, 14)$?
(A) 17 (B) 13 (C) 169 (D) 7
- What is the midpoint of the segment with endpoints $(-6, 9)$ and $(10, -3)$?
(A) $(8, 3)$ (B) $(4, 6)$ (C) $(16, -12)$ (D) $(2, 3)$
- What is the slope of every line parallel to $4x - 2y = 9$?
(A) 2 (B) -2 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$
- A line passes through $(0, 3)$ and $(6, 6)$. What is the slope of a line perpendicular to it?
(A) $\frac{1}{2}$ (B) 2 (C) -2 (D) $-\frac{1}{2}$
- Point P is one third of the way from $A(-4, 1)$ to $B(8, 10)$. What are the coordinates of P ?
(A) $(4, \frac{11}{2})$ (B) $(0, 4)$ (C) $(4, 7)$ (D) $(1, 5)$
- What is the area of the triangle with vertices $(0, 0)$, $(8, 0)$ and $(3, 6)$?
(A) 48 (B) 30 (C) 12 (D) 24
- The triangle with vertices $(0, 0)$, $(4, 0)$ and $(2, 4)$ is best described as
(A) isosceles but not equilateral (B) equilateral
(C) a right triangle (D) scalene
- The points $(1, 2)$, $(3, 6)$ and $(5, y)$ lie on one line. What is y ?
(A) 8 (B) 12 (C) 10 (D) 14
- A point on the x -axis is equally distant from $(1, 3)$ and $(5, 1)$. What is its x -coordinate?
(A) 3 (B) 2 (C) 1 (D) 4
- Find the exact distance between $A(-1, 4)$ and $B(3, -2)$.
- The midpoint of a segment is $(3, -1)$ and one endpoint is $(-2, 4)$. Find the other endpoint.
- Find the equation of the perpendicular bisector of the segment from $A(-1, 1)$ to $B(5, 5)$. Write it as $ax + by = c$.
- Find the area of the quadrilateral with vertices $(0, 0)$, $(6, 0)$, $(8, 5)$, $(2, 7)$, listed in order.
- Find the point P on $\overline{AB}$ with $AP : PB = 3 : 2$, where $A(-5, -1)$ and $B(5, 9)$.
- Three vertices of parallelogram $ABCD$, in order, are $A(1, 1)$, $B(6, 2)$, $C(8, 6)$. Find the coordinates of D and the area of the parallelogram.

Answers**Answers and Explanations**

Complete solutions for every practice problem, numbered to match. Read the explanation even when your answer is right.

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19.1 Circle Basics and Angle Relationships

PRACTICE · CIRCLE BASICS AND ANGLE RELATIONSHIPS

1. B. An inscribed angle is half its arc: $130/2 = 65$. Choice A forgets to halve.
2. C. Angle C is in a semicircle, so $C = 90^\circ$. Then $B = 180 - 90 - 34 = 56$. Choice B gives only the right angle.
3. B. The radius is perpendicular to the tangent, so $PT^2 = 15^2 - 9^2 = 225 - 81 = 144$ and $PT = 12$. Choice C comes from adding the squares instead of subtracting.
4. A. Equal tangents: $3x + 2 = 5x - 10$, so $x = 6$ and $PA = 20$. Choice A is PA ; 6 is only x .
5. C. Chord-chord: $8 \cdot 3 = 4 \cdot ED$, so $ED = 6$. Choice B adds instead of multiplying.
6. D. Two chords crossing inside make an angle equal to half the sum of the intercepted arcs: $\frac{1}{2}(70 + 110) = 90$. Choice A is half the difference, the outside-angle rule.
7. D. Outside angle is half the difference: $\frac{1}{2}(150 - 50) = 50$. Choice B is half the sum (wrong rule for an outside point).
8. C. Opposite angles add to 180: $5x - 10 = 180$, $x = 38$, so $A = 2(38) + 10 = 86$. Choice A is x ; choice B is angle C .
9. D. $10^2 = 4 \cdot PB$ gives $PB = 25$; the chord is $25 - 4 = 21$. Choice B forgets to subtract the outside piece.
10. 68 degrees. Inscribed angle is half the arc: $3x + 4 = \frac{1}{2}(8x - 12) = 4x - 6$, so $x = 10$. The arc is $8(10) - 12 = 68$.
11. 144 degrees. A tangent-chord angle is half its arc, so the arc is $2(72) = 144$.
12. $x = 4$. Chord-chord: $3 \cdot 8 = x(x + 2)$, so $x^2 + 2x - 24 = 0$, $(x + 6)(x - 4) = 0$. Lengths are positive, so $x = 4$.
13. 11. First secant: $5(5 + 7) = 60$. Second: $4 \cdot w = 60$, so $w = 15$. The chord is $15 - 4 = 11$.
14. 130 degrees. Tangents to the circle at A and B make right angles with the radii, so the quadrilateral $OAPB$ gives $\angle AOB = 360 - 90 - 90 - 50 = 130$.
15. 50 degrees. $\frac{1}{2}[(5x + 20) - (2x + 10)] = 35$, so $3x + 10 = 70$, $x = 20$. Near arc $= 2(20) + 10 = 50$.

19.2 Arc Length, Sectors, and Radians

PRACTICE · ARC LENGTH, SECTORS, AND RADIAN

1. C. Multiply by $\frac{\pi}{180}$: $120 \cdot \frac{\pi}{180} = \frac{2\pi}{3}$. Choice A is 60° .
2. B. $\frac{5\pi}{6} \cdot \frac{180}{\pi} = 5 \cdot 30 = 150$. Choice C is $\frac{2\pi}{3}$.
3. D. Fraction $\frac{72}{360} = \frac{1}{5}$, circumference 20π , arc $= 4\pi$. Choice A uses the radius in place of the circumference.
4. C. Quarter of $\pi(4)^2 = 16\pi$ is 4π . Choice B is half the circle.
5. A. $s = r\theta = 5(2.4) = 12$. The angle is already in radians. Choice D multiplies by π for no reason.
6. B. $A = \frac{1}{2}r^2\theta = \frac{1}{2}(100)\frac{\pi}{5} = 10\pi$. Choice A forgets the one half.
7. C. The diameter is 9, so $r = 4.5$ and the circumference is 9π . The fraction is $\frac{6\pi}{9\pi} = \frac{2}{3}$ and $\frac{2}{3} \cdot 360 = 240$. Choice D comes from treating 9 as the radius.
8. D. $A = \frac{1}{2}r^2\theta$ gives $12\pi = \frac{1}{2}(36)\theta = 18\theta$, so $\theta = \frac{2\pi}{3}$. Choice A is what you get if you drop the one half.
9. A. 25 minutes is $\frac{25}{60} = \frac{5}{12}$ of a turn. The tip's circle has circumference 16π , so the distance is $\frac{5}{12} \cdot 16\pi = \frac{20\pi}{3}$. Choice C uses 16 as the radius.
10. 7π . Fraction $\frac{105}{360} = \frac{7}{24}$. Circumference $= 24\pi$. Arc $= \frac{7}{24} \cdot 24\pi = 7\pi$.
11. 165° . $\frac{11\pi}{12} \cdot \frac{180}{\pi} = 11 \cdot 15 = 165$.
12. 54π . $A = \frac{1}{2}r^2\theta = \frac{1}{2}(81)\frac{4\pi}{3} = 54\pi$.
13. $20 + 4\pi$. Arc $= \frac{1}{5} \cdot 20\pi = 4\pi$. Add the two radii: $10 + 10 + 4\pi$.
14. $24\pi - 36\sqrt{3}$. Sector $= \frac{1}{6}\pi(144) = 24\pi$. The triangle is equilateral with side 12, area $\frac{\sqrt{3}}{4}(144) = 36\sqrt{3}$. Subtract.
15. Area 81π , angle 90° . $A = \frac{1}{2}rs = \frac{1}{2}(18)(9\pi) = 81\pi$. $\theta = \frac{s}{r} = \frac{9\pi}{18} = \frac{\pi}{2}$ radians $= 90^\circ$.

19.3 Equations of Circles

PRACTICE · EQUATIONS OF CIRCLES

1. C. Center $(2, -5)$ gives $(x - 2)$ and $(y + 5)$, and the right side is $r^2 = 9$. Choice B forgets to square the radius.
2. A. The right side is $r^2 = 64$, so $r = \sqrt{64} = 8$. Choice B reports r^2 .
3. D. Complete the square: $(x - 4)^2 + (y + 3)^2 = 25$. The center is $(4, -3)$. Choice C has the wrong sign on y .
4. B. $(6 - 3)^2 + (3 + 1)^2 = 9 + 16 = 25$. The other points give 18, 9 and 36, none equal to 25.
5. C. Center is the midpoint $(4, 6)$; the diameter is $\sqrt{36 + 64} = 10$, so $r = 5$ and $r^2 = 25$. Choice A uses the diameter squared.
6. A. Substitute: $2x^2 + 20x + 64 = 0$, so $x^2 + 10x + 32 = 0$. The discriminant is $100 - 128 < 0$, so no real solutions. The line is $\frac{10}{\sqrt{2}} \approx 7.07$ from the center, more than 6.
7. D. $(x + 2)^2 + (y - 5)^2 = -13 + 4 + 25 = 16$, so $r = 4$. Choice A gives r^2 rather than r .
8. B. Radius slope is $\frac{8}{6} = \frac{4}{3}$. The tangent slope is the negative reciprocal, $-\frac{3}{4}$. Choice D flips the sign only.
9. C. Substitute: $2x^2 + 2kx + k^2 - 8 = 0$. Zero discriminant: $4k^2 - 8(k^2 - 8) = 0$ gives $k^2 = 16$, so $k = 4$. Choice A is $k = 2$, which gives two intersection points.
10. Center $(-3, 5)$, radius 4. Complete the square: $(x + 3)^2 + (y - 5)^2 = -18 + 9 + 25 = 16$.
11. $(x + 1)^2 + (y - 4)^2 = 25$. The radius squared is the squared distance from $(-1, 4)$ to $(2, 0)$: $3^2 + 4^2 = 25$.
12. $(0, -5)$ and $(4, -1)$. Substitute $y = x - 5$: $(x - 1)^2 + (x - 3)^2 = 10$, so $2x^2 - 8x = 0$, giving $x = 0$ or $x = 4$. Then $y = -5$ or -1 .

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13. $5x + 3y = 34$. The radius slope is $\frac{3}{5}$, so the tangent slope is $-\frac{5}{3}$. Then $y - 3 = -\frac{5}{3}(x - 5)$, which simplifies to $5x + 3y = 34$.
14. $k = 5$ or $k = -5$. Substitute: $5x^2 + 4kx + k^2 - 5 = 0$. A zero discriminant gives $16k^2 - 20(k^2 - 5) = 0$, so $k^2 = 25$.
15. $(x - 4)^2 + (y - 3)^2 = 25$. The angle at the origin is a right angle, so the hypotenuse from $(8, 0)$ to $(0, 6)$ is a diameter. Its midpoint is $(4, 3)$ and its length is 10, so $r = 5$.

19.4 Coordinate Geometry Toolkit

PRACTICE · COORDINATE GEOMETRY TOOLKIT

1. B. $\Delta x = 5$, $\Delta y = 12$, and $\sqrt{25 + 144} = 13$. Choice A adds the changes $(5 + 12)$; choice C forgets the square root.
2. D. Average the coordinates: $\frac{-6+10}{2} = 2$ and $\frac{9+(-3)}{2} = 3$. Choice B adds without halving.
3. A. Solve for y : $y = 2x - 4.5$, slope 2. Parallel lines share the slope. Choice B is the opposite sign.
4. C. The slope is $\frac{6-3}{6-0} = \frac{1}{2}$. A perpendicular slope is the negative reciprocal, -2 . Choice A is the slope itself.
5. B. Move $\frac{1}{3}$ of $(12, 9)$, which is $(4, 3)$, from A: $(-4 + 4, 1 + 3) = (0, 4)$. Choice A is the midpoint and choice C is two thirds of the way.
6. D. The base is 8 and the height is 6, so $\frac{1}{2}(8)(6) = 24$. Choice A forgets the factor $\frac{1}{2}$.
7. A. The sides are 4, $\sqrt{4 + 16} = \sqrt{20}$, and $\sqrt{20}$. Two sides are equal and the third differs, so it is isosceles. The slopes of the equal sides are 2 and -2 , whose product is -4 , not -1 , so it is not a right triangle.
8. C. The slope from $(1, 2)$ to $(3, 6)$ is 2. From $(3, 6)$ to $(5, y)$ the slope must be 2 as well: $\frac{y-6}{2} = 2$, so $y = 10$.
9. B. Let the point be $(x, 0)$. Then $(x - 1)^2 + 9 = (x - 5)^2 + 1$, so $-2x + 10 = -10x + 26$, so $8x = 16$ and $x = 2$. Check: both distances are $\sqrt{10}$.
10. $2\sqrt{13}$. $\Delta x = 4$, $\Delta y = -6$, distance $= \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}$.
11. $(8, -6)$. The midpoint formula gives $\frac{-2+x}{2} = 3$ and $\frac{4+y}{2} = -1$, so $x = 8$ and $y = -6$.
12. $3x + 2y = 12$. Midpoint is $(2, 3)$. Slope of AB is $\frac{4}{6} = \frac{2}{3}$, so the perpendicular slope is $-\frac{3}{2}$. Then $y - 3 = -\frac{3}{2}(x - 2)$ gives $3x + 2y = 12$.
13. 38. Shoelace terms: $0 \cdot 6 \cdot 5 - 8 \cdot 0 = 30$, $8 \cdot 7 - 2 \cdot 5 = 46$, $2 \cdot 0 - 0 \cdot 7 = 0$. Sum is 76, and half is 38.
14. $(1, 5)$. Move $\frac{3}{5}$ of the change $(10, 10)$, which is $(6, 6)$, from A: $(-5 + 6, -1 + 6) = (1, 5)$.
15. $D = (3, 5)$ and area = 18. The diagonals share a midpoint, so $D = A + C - B = (3, 5)$. Area $= |5 \cdot 4 - 1 \cdot 2| = 18$ from the vectors $(5, 1)$ and $(2, 4)$.

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