



Right-Triangle Trigonometry

Four focused lessons, worked examples, a mistake clinic, and 60 practice problems with full explanations.

In this booster

20.1 Special Right Triangles and Pythagorean Triples

20.2 Sine, Cosine, and Tangent Ratios

20.3 Applications: Angles of Elevation and Depression

20.4 The Laws of Sines and Cosines

Every section teaches the idea, walks through the most common problem types, flags the traps students fall into, and ends with fifteen practice problems. Full answers and explanations are at the back.

20.1 Special Right Triangles and Pythagorean Triples

Master the 45-45-90 and 30-60-90 patterns, recognize Pythagorean triples, and use similar right triangles to find missing lengths without a calculator.

Why This Matters

Right triangles hide inside almost everything you build or measure: the diagonal of a screen, the slope of a ramp, the brace on a shelf, the distance across a field. Two triangle shapes are so common that people memorize them: the **45-45-90** triangle (half of a square) and the **30-60-90** triangle (half of an equilateral triangle). Knowing them lets you skip the calculator and get exact answers.

Students get stuck on three things. They forget **which side is which** (the short leg, the long leg, or the hypotenuse). They mix up the $\sqrt{2}$ and $\sqrt{3}$ factors. And they try to use the Pythagorean theorem on every problem, which works but is slow and error-prone. This section gives you patterns to recognize and a method to check every answer.

Core Ideas

The **Pythagorean theorem** says that in a right triangle with legs a and b and hypotenuse c , $a^2 + b^2 = c^2$. Everything in this section comes from it.

45-45-90 Triangle

The legs are equal. If each leg is a , the hypotenuse is $a\sqrt{2}$.

To go from hypotenuse to leg, **divide by $\sqrt{2}$** : $\text{leg} = \frac{c}{\sqrt{2}} = \frac{c\sqrt{2}}{2}$.

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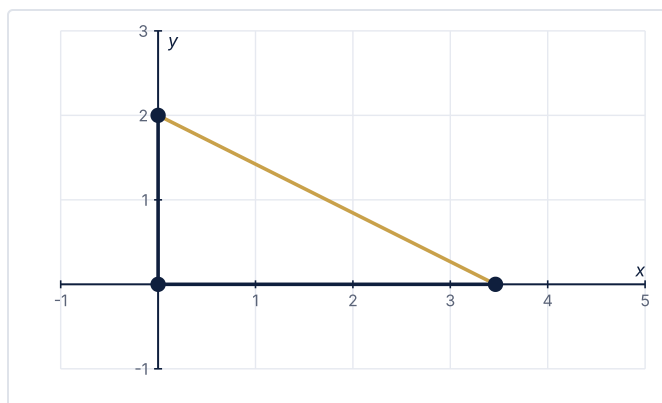


A 45-45-90 triangle with legs 4. The hypotenuse is 4 times the square root of 2, about 5.66.

30-60-90 Triangle

Let the **short leg** (opposite 30°) be a . Then the **long leg** (opposite 60°) is $a\sqrt{3}$ and the **hypotenuse** is $2a$.

Sides in order of size: $a, a\sqrt{3}, 2a$. The smallest side is opposite the smallest angle.



A 30-60-90 triangle. The short leg is 2 (opposite 30 degrees), the long leg is 2 times the square root of 3, about 3.46, and the hypotenuse is 4.

Why do these patterns work? Cut a square along its diagonal and you get a 45-45-90 triangle: with side 1, the diagonal is $\sqrt{1^2 + 1^2} = \sqrt{2}$. Cut an equilateral triangle of side $2a$ in half along its height and you get a 30-60-90 triangle: the hypotenuse is $2a$, the short leg is a , and the long leg is $\sqrt{(2a)^2 - a^2} = a\sqrt{3}$.

A **Pythagorean triple** is a set of three whole numbers with $a^2 + b^2 = c^2$. Multiples of a triple are also triples, because scaling a triangle keeps it right.

Basic triple	Times 2	Times 3	Times 5
3, 4, 5	6, 8, 10	9, 12, 15	15, 20, 25
5, 12, 13	10, 24, 26	15, 36, 39	25, 60, 65
8, 15, 17	16, 30, 34	24, 45, 51	40, 75, 85
7, 24, 25	14, 48, 50	21, 72, 75	35, 120, 125

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Similar Right Triangles and the Altitude

If the altitude to the hypotenuse of a right triangle has length h and splits the hypotenuse into pieces p and q then

$$h^2 = pq, \quad \text{leg}^2 = (\text{hypotenuse piece next to it}) \times (\text{whole hypotenuse}).$$

The altitude creates two smaller right triangles, each similar to the original triangle. Equal angles give proportional sides.

Worked Examples

EXAMPLE 1: ISOSCELES RIGHT TRIANGLE

(a) A 45-45-90 triangle has legs of length 7. Find the hypotenuse. (b) The hypotenuse is 10. Find each leg.

SOLUTION

(a) Hypotenuse = $7\sqrt{2}$.

(b) Leg = $\frac{10}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$.

(a) $7\sqrt{2} \approx 9.90$. (b) $5\sqrt{2} \approx 7.07$.

EXAMPLE 2: 30-60-90 TRIANGLE

Find the missing sides of a 30-60-90 triangle if (a) the short leg is 5, (b) the hypotenuse is 14, (c) the long leg is 9.

SOLUTION

(a) Long leg = $5\sqrt{3}$, hypotenuse = 10.

(b) Short leg = $\frac{14}{2} = 7$, long leg = $7\sqrt{3}$.

(c) Short leg = $\frac{9}{\sqrt{3}} = 3\sqrt{3}$, hypotenuse = $2(3\sqrt{3}) = 6\sqrt{3}$.

(a) $5\sqrt{3}$ and 10. (b) 7 and $7\sqrt{3}$. (c) $3\sqrt{3}$ and $6\sqrt{3}$.

EXAMPLE 3: SQUARES AND EQUILATERAL TRIANGLES

(a) A square has side 8. Find its diagonal. (b) An equilateral triangle has side 12. Find its height and area.

SOLUTION

(a) The diagonal splits the square into 45-45-90 triangles: diagonal = $8\sqrt{2}$.

(b) The height splits the triangle into 30-60-90 triangles with hypotenuse 12, short leg 6, so the height is $6\sqrt{3}$.

Area = $\frac{1}{2}(12)(6\sqrt{3}) = 36\sqrt{3}$.

(a) $8\sqrt{2}$. (b) height $6\sqrt{3} \approx 10.39$, area $36\sqrt{3} \approx 62.35$.

EXAMPLE 4: USING TRIPLES

(a) Is a triangle with sides 20, 21, 29 a right triangle? (b) A right triangle has a leg of 10 and hypotenuse 26. Find the other leg.

SOLUTION

(a) Test: $20^2 + 21^2 = 400 + 441 = 841 = 29^2$. Yes, a right triangle.

(b) 10 and 26 are twice 5 and 13, so the triple is 5-12-13 times 2.

The other leg is $2 \times 12 = 24$. Check: $100 + 576 = 676 = 26^2$.

(a) Yes. (b) 24.

EXAMPLE 5: ALTITUDE TO THE HYPOTENUSE

In a right triangle, the altitude to the hypotenuse splits it into pieces of length 4 and 9. Find the altitude and both legs.

SOLUTION

Altitude: $h^2 = 4 \cdot 9 = 36$, so $h = 6$.

The whole hypotenuse is $4 + 9 = 13$.

Leg next to the piece 4: $\sqrt{4 \cdot 13} = 2\sqrt{13}$. Leg next to the piece 9: $\sqrt{9 \cdot 13} = 3\sqrt{13}$.

Check: $52 + 117 = 169 = 13^2$.

$h = 6$, legs $2\sqrt{13}$ and $3\sqrt{13}$.

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EXAMPLE 6: TRAPEZOID WITH A 60 DEGREE ANGLE

An isosceles trapezoid has bases 10 and 22 and legs of length 12. The base angles are 60° . Find its height and area.

SOLUTION

Drop perpendiculars from the ends of the short base. Each side leaves an overhang of $\frac{22-10}{2} = 6$ on the long base.

Each end is a 30-60-90 triangle with hypotenuse 12 (the leg of the trapezoid), short side 6 (the overhang), and long side $6\sqrt{3}$ (the height).

$$\text{Area} = \frac{1}{2}(10 + 22)(6\sqrt{3}) = 16 \cdot 6\sqrt{3} = 96\sqrt{3}.$$

Height $6\sqrt{3}$, **area** $96\sqrt{3} \approx 166.3$.

Coach's Playbook

- 1 Name the triangle.** Are two angles 45° (or two sides equal)? That is 45-45-90. Is there a 30° or 60° angle? That is 30-60-90. Otherwise try the Pythagorean theorem or a triple.
- 2 Find the anchor.** In a 30-60-90 triangle, always find the short leg first. In a 45-45-90 triangle, find the leg.
- 3 Check the order.** The hypotenuse must be the longest side. In 30-60-90: short < long < hypotenuse. $a < 1.73a < 2a$.
- 4 Rationalize at the end.** $\frac{9}{\sqrt{3}} = 3\sqrt{3}$. Multiply top and bottom by the root.
- 5 Scan for triples.** If two sides look like multiples of 3-4-5 or 5-12-13, write the third side without squaring anything.

Common traps

1) Multiplying by $\sqrt{2}$ when you should divide (going from hypotenuse to leg). 2) Putting the $\sqrt{3}$ on the short leg instead of the long leg. 3) Assuming any triangle with a 45° angle is 45-45-90 (it must be a right triangle). 4) Calling 6-8-11 a triple because it "looks close"; always check $a^2 + b^2 = c^2$.

Mistake Clinic**Spot the error 1**

Problem: A 45-45-90 triangle has a hypotenuse of 12. Find a leg. Wrong work: "Leg = $12\sqrt{2}$." What went wrong: The student multiplied. A leg is always shorter than the hypotenuse, so divide by $\sqrt{2}$. Right solution: Leg = $\frac{12}{\sqrt{2}} = 6\sqrt{2} \approx 8.49$.

Spot the error 2

Problem: In a 30-60-90 triangle, the shortest side is 4. Find the other two sides. Wrong work: "Hypotenuse = $4\sqrt{3}$ and the other leg = 8." What went wrong: The factors are swapped. The hypotenuse is twice the short leg and the long leg gets the $\sqrt{3}$. Right solution: Hypotenuse = 8, long leg = $4\sqrt{3}$. Check that $4^2 + (4\sqrt{3})^2 = 16 + 48 = 64 = 8^2$.

Spot the error 3

Problem: A right triangle has legs 9 and 12. Find the hypotenuse. Wrong work: " $9^2 + 12^2 = 81 + 144 = 225$, so the hypotenuse is 225." What went wrong: The student stopped before taking the square root. Right solution: $c = \sqrt{225} = 15$. This is the 3-4-5 triple times 3.

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Real-World Connection

A TV advertises a 55-inch diagonal. A screen's diagonal is the hypotenuse of a right triangle. If a square monitor has side 20 inches, the diagonal is $20\sqrt{2} \approx 28.3$ inches. A carpenter bracing a square frame uses the same fact to cut a diagonal brace of the right length.

Challenge Corner

Problem: In rectangle $ABCD$, $AD = 6$. Point E lies on side $\overline{CD}$ so that $\angle DAE = 30^\circ$. Find DE , AE , and the area of triangle ADE . **Solution:** Triangle ADE has a right angle at D and a 30° angle at A , so it is a 30-60-90 triangle. The side $AD = 6$ is next to the 30° angle and opposite the 60° angle, so it is the long leg. The short leg is $DE = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ and the hypotenuse is $AE = 4\sqrt{3}$. Area = $\frac{1}{2}(6)(2\sqrt{3}) = 6\sqrt{3} \approx 10.39$.

Practice**PRACTICE Special Right Triangles and Pythagorean Triples**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- A 45-45-90 triangle has legs of length 6. What is the length of the hypotenuse?
(A) 6 (B) 12 (C) $6\sqrt{2}$ (D) $6\sqrt{3}$
- A 30-60-90 triangle has a short leg of 9. What is the hypotenuse?
(A) 18 (B) $9\sqrt{3}$ (C) 27 (D) $9\sqrt{2}$
- The long leg of a 30-60-90 triangle is $12\sqrt{3}$. What is the hypotenuse?
(A) $12\sqrt{3}$ (B) 36 (C) 12 (D) 24
- Which set of numbers can be the side lengths of a right triangle?
(A) 6, 8, 12 (B) 9, 40, 41 (C) 10, 24, 27 (D) 11, 60, 62
- The diagonal of a square is 10. What is the side length?
(A) $10\sqrt{2}$ (B) 5 (C) $5\sqrt{2}$ (D) 20
- An equilateral triangle has side length 10. What is its height?
(A) $5\sqrt{3}$ (B) $10\sqrt{3}$ (C) $5\sqrt{2}$ (D) 5
- The altitude to the hypotenuse of a right triangle splits the hypotenuse into pieces of length 3 and 12. What is the altitude?
(A) 9 (B) 36 (C) 7.5 (D) 6
- A rectangle has a diagonal of 25 and one side of 7. What is the other side?
(A) 18 (B) 24 (C) 26 (D) 32
- A 45-45-90 triangle has a hypotenuse of 8. What is its area?
(A) 32 (B) 8 (C) 16 (D) $16\sqrt{2}$
- A 30-60-90 triangle has a hypotenuse of 20. Find the lengths of the short leg and the long leg.
- A right triangle has a leg of 20 and a hypotenuse of 29. Find the other leg.
- An isosceles right triangle has a perimeter of $20 + 10\sqrt{2}$. Find its area.
- An equilateral triangle has an area of $25\sqrt{3}$. Find its height.
- The altitude to the hypotenuse of a right triangle has length 12, and one piece of the hypotenuse is 9. Find the length of the shorter leg of the triangle.
- An isosceles trapezoid has bases 8 and 20 and base angles of 45° . Find its area and its perimeter.

20.2 Sine, Cosine, and Tangent Ratios

Use SOH-CAH-TOA to find unknown sides and angles in right triangles, and see why sine and cosine of complementary angles are equal.

Why This Matters

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How tall is a flagpole you cannot climb? How steep is a ramp? How far away is a ship seen from a lighthouse? Trigonometry answers all of these using one idea: in a right triangle, the **ratio of two sides depends only on the angle**, not on how big the triangle is. A calculator stores those ratios, so one angle and one side are enough to find everything else.

Students get stuck in four places: **labeling the sides** (opposite and adjacent change when you switch to the other acute angle), **choosing the right ratio, putting the unknown in the denominator** (when you must divide instead of multiply), and **calculator settings** (degree mode, not radian mode). This section turns those into a short routine.

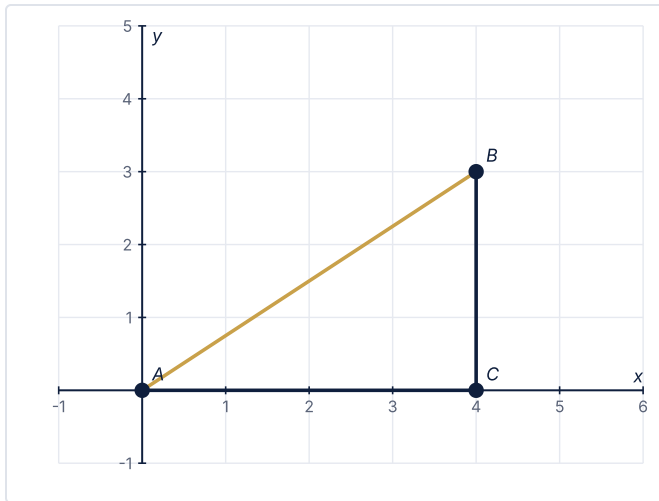
Core Ideas

Pick one acute angle in a right triangle, and call it θ . The **hypotenuse** is the side across from the right angle, always the longest. The **opposite** side is across from θ . The **adjacent** side touches θ and is not the hypotenuse.

The Three Ratios

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}.$$

Memory phrase: **SOH-CAH-TOA**. The ratios depend only on θ , and are the same for every right triangle with that angle.



Right triangle ABC with the right angle at C. For angle A: opposite = BC = 3, adjacent = AC = 4, hypotenuse = AB = 5. So $\sin A = 3/5$, $\cos A = 4/5$, $\tan A = 3/4$.

In the figure, angle A has opposite side 3 and adjacent side 4. If you look at angle B instead, the roles swap: opposite is now 4 and adjacent is 3. That is why you must always decide **which angle** before labeling.

Angle θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Finding Sides and Angles

To find a side: pick the ratio that links the known side, the unknown side, and the given angle. Write the equation, then solve. If the unknown is in the denominator, multiply both sides by it and divide.

To find an angle: use the inverse button: $\theta = \sin^{-1}\left(\frac{\text{opp}}{\text{hyp}}\right)$, $\theta = \cos^{-1}\left(\frac{\text{adj}}{\text{hyp}}\right)$, or $\theta = \tan^{-1}\left(\frac{\text{opp}}{\text{adj}}\right)$.

Complementary Angles

In a right triangle the two acute angles add to 90° , and

$$\sin \theta = \cos(90^\circ - \theta), \quad \tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \sin^2 \theta + \cos^2 \theta = 1.$$

The opposite side of one acute angle is the adjacent side of the other, which is why sine and cosine swap.

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The third formula comes from the Pythagorean theorem: divide $a^2 + b^2 = c^2$ by c^2 and you get $\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$. If you know one ratio, you can find the others by drawing the triangle and using the Pythagorean theorem for the missing side.

Worked Examples

EXAMPLE 1: WRITING THE RATIOS

A right triangle has legs 8 and 15 and hypotenuse 17. For the angle A opposite the side of length 8, write $\sin A$, $\cos A$ and $\tan A$.

SOLUTION

Opposite = 8, adjacent = 15, hypotenuse = 17.

$$\sin A = \frac{8}{17}, \cos A = \frac{15}{17}, \tan A = \frac{8}{15}.$$

Check: $\left(\frac{8}{17}\right)^2 + \left(\frac{15}{17}\right)^2 = \frac{64+225}{289} = 1$.

EXAMPLE 2: FINDING A SIDE WITH SINE

A ramp is 20 feet long and rises at an angle of 35° with the ground. How high does it rise? [Calculator, degree mode]

SOLUTION

The ramp is the hypotenuse (20) and the height is opposite the 35° angle.

$$\sin 35^\circ = \frac{h}{20}, \text{ so } h = 20 \sin 35^\circ.$$

$$h \approx 20(0.5736) = 11.47.$$

The ramp rises about 11.47 feet.

EXAMPLE 3: FINDING A SIDE WITH TANGENT

(a) The adjacent side is 9 and the angle is 52° . Find the opposite side. (b) The opposite side is 7 and the angle is 28° . Find the adjacent side and the hypotenuse.

SOLUTION

$$(a) \tan 52^\circ = \frac{x}{9}, \text{ so } x = 9 \tan 52^\circ \approx 11.52.$$

$$(b) \tan 28^\circ = \frac{7}{x}. \text{ The unknown is in the denominator, so } x = \frac{7}{\tan 28^\circ} \approx 13.17.$$

$$\text{For the hypotenuse use } \sin 28^\circ = \frac{7}{h}, \text{ so } h = \frac{7}{\sin 28^\circ} \approx 14.91.$$

(a) 11.52. (b) adjacent 13.17, hypotenuse 14.91.

EXAMPLE 4: FINDING AN ANGLE

A right triangle has legs 5 and 12. Find the smaller acute angle.

SOLUTION

The smaller angle is opposite the shorter leg 5, with adjacent leg 12.

$$\tan \theta = \frac{5}{12}, \text{ so } \theta = \tan^{-1}\left(\frac{5}{12}\right) \approx 22.62^\circ.$$

$$\theta \approx 22.62^\circ. \text{ The other acute angle is } 90 - 22.62 = 67.38^\circ.$$

EXAMPLE 5: COMPLEMENTARY ANGLES

(a) Write $\sin 20^\circ$ as a cosine. (b) Find x if $\sin(3x + 10)^\circ = \cos(2x + 5)^\circ$.

SOLUTION

$$(a) \sin 20^\circ = \cos(90^\circ - 20^\circ) = \cos 70^\circ.$$

$$(b) \text{ Sine equals cosine only when the angles add to } 90^\circ: (3x + 10) + (2x + 5) = 90.$$

$$5x + 15 = 90, \text{ so } x = 15. \text{ Check: } \sin 55^\circ = \cos 35^\circ.$$

(a) $\cos 70^\circ$. (b) $x = 15$.

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EXAMPLE 6: SOLVE THE TRIANGLE

A right triangle has hypotenuse 15 and one acute angle of 40° . Find the other acute angle and both legs.

SOLUTION

Other angle: $90 - 40 = 50^\circ$.

Opposite the 40° angle: $15 \sin 40^\circ \approx 9.64$.

Adjacent to the 40° angle: $15 \cos 40^\circ \approx 11.49$.

Check: $9.64^2 + 11.49^2 \approx 92.9 + 132.0 = 224.9 \approx 15^2$.

Angles $40^\circ, 50^\circ, 90^\circ$; legs 9.64 and 11.49.

Coach's Playbook

- 1 Draw and label.** Sketch the triangle. Mark the angle you are using, then label opposite, adjacent and hypotenuse relative to it.
- 2 Circle the two sides involved.** Whichever two sides (one known, one unknown) plus the angle appear in the problem choose the ratio: opposite and hypotenuse mean sine; adjacent and hypotenuse mean cosine; opposite and adjacent mean tangent.
- 3 Write the equation before the calculator.** For example $\sin 38^\circ = \frac{x}{12}$. Then solve for x .
- 4 Check the mode.** Your calculator must be in degree mode. If $\sin 30^\circ$ does not show 0.5, fix the mode.
- 5 Sanity check.** The hypotenuse must be the longest side. A leg opposite a small angle must be small.

Common traps

1) Using the wrong angle's opposite and adjacent sides. 2) Multiplying when the unknown is in the denominator, for example computing $7 \tan 28^\circ$ when you need $7 \div \tan 28^\circ$. 3) Leaving the calculator in radian mode. 4) Using $\tan^{-1}$ of the wrong ratio, such as adjacent over opposite.

Mistake Clinic**Spot the error 1**

Problem: In a right triangle with $\theta = 40^\circ$, the hypotenuse is 12. Find the side opposite θ . Wrong work: " $\cos 40^\circ = \frac{x}{12}$, so $x = 9.19$." What went wrong: Cosine pairs the adjacent side with the hypotenuse. The side opposite the angle needs sine. Right solution: $x = 12 \sin 40^\circ \approx 7.71$.

Spot the error 2

Problem: The opposite side is 6 and $\theta = 25^\circ$. Find the adjacent side. Wrong work: " $\tan 25^\circ = \frac{6}{x}$, so $x = 6 \tan 25^\circ = 2.80$." What went wrong: The student multiplied by the tangent when the unknown is in the denominator. A quick check shows the problem: the adjacent side should be longer than the opposite side for a small angle. Right solution: $x = \frac{6}{\tan 25^\circ} \approx 12.87$.

Spot the error 3

Problem: A right triangle has legs 5 and 12. Find the angle opposite the 5. Wrong work: " $\sin^{-1}(5/12) = 24.62^\circ$." What went wrong: The ratio $\frac{5}{12}$ is opposite over adjacent, which is tangent, not sine. Right solution: $\tan^{-1}(5/12) \approx 22.62^\circ$. Or use the hypotenuse 13: $\sin^{-1}(5/13) = 22.62^\circ$.

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Real-World Connection

A skateboard ramp must not exceed an angle of 15° with the ground. If the top is 0.9 m high, the ramp's ground length must be at least $\frac{0.9}{\tan 15^\circ} \approx 3.36$ m, and the sloped surface is $\frac{0.9}{\sin 15^\circ} \approx 3.48$ m long. Builders use exactly these two calculations.

Challenge Corner

Problem: In a right triangle, $\tan \theta = \frac{7}{24}$. Without finding θ , find $\sin \theta$ and $\cos \theta$ exactly. Solution: Draw a triangle with opposite 7 and adjacent 24. The hypotenuse is $\sqrt{49 + 576} = 25$. So $\sin \theta = \frac{7}{25}$ and $\cos \theta = \frac{24}{25}$. Check: $\frac{49+576}{625} = 1$, and $\frac{7/25}{24/25} = \frac{7}{24}$.

Practice**PRACTICE Sine, Cosine, and Tangent Ratios**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- In a right triangle, the side opposite angle A is 6, the adjacent side is 8, and the hypotenuse is 10. What is $\tan A$?
 (A) $\frac{3}{5}$ (B) $\frac{3}{4}$ (C) $\frac{4}{3}$ (D) $\frac{4}{5}$
- In right triangle ABC the right angle is at C , $AC = 12$, $BC = 5$ and $AB = 13$. What is $\sin A$?
 (A) $\frac{12}{13}$ (B) $\frac{5}{12}$ (C) $\frac{13}{5}$ (D) $\frac{5}{13}$
- Which angle x makes $\sin 25^\circ = \cos x$ true?
 (A) 65° (B) 25° (C) 75° (D) 155°
- A right triangle has hypotenuse 30 and an acute angle of 40° . How long is the leg adjacent to the 40° angle? (nearest hundredth)
 (A) 19.28 (B) 25.17 (C) 22.98 (D) 39.16
- The leg adjacent to a 55° angle is 12. How long is the leg opposite that angle? (nearest hundredth)
 (A) 8.40 (B) 17.14 (C) 9.83 (D) 20.92
- A 6 m ladder leans against a wall with its foot 1.5 m from the wall. What angle does the ladder make with the ground? (nearest hundredth)
 (A) 14.48° (B) 25.00° (C) 0.25° (D) 75.52°
- If $\sin(2x + 4)^\circ = \cos(3x + 1)^\circ$, what is x ?
 (A) 17 (B) 3 (C) 18 (D) 22
- In a right triangle, $\tan \theta = \frac{7}{24}$. What is $\sin \theta$?
 (A) $\frac{7}{24}$ (B) $\frac{24}{25}$ (C) $\frac{7}{25}$ (D) $\frac{25}{7}$
- In a right triangle, an acute angle is 62° and the opposite side is 14. What is the hypotenuse? (nearest hundredth)
 (A) 12.36 (B) 15.86 (C) 7.44 (D) 6.57
- In a right triangle, one acute angle is 37° and the adjacent leg is 20. Find the hypotenuse to the nearest hundredth.
- A right triangle has legs 9 and 4. Find the acute angle opposite the leg of length 4, to the nearest hundredth of a degree.
- A right triangle has hypotenuse 18 and an acute angle of 24° . Find the other acute angle and both legs to the nearest hundredth.
- An acute angle θ has $\sin \theta = \frac{8}{17}$. Find $\cos \theta$ and $\tan \theta$ exactly.
- Find x if $\sin(4x + 6)^\circ = \cos(2x + 12)^\circ$, and give both angle measures.
- An isosceles triangle has two sides of length 10 and base angles of 70° . Find the base, the height to the base, and the area, to the nearest hundredth.

20.3 Applications: Angles of Elevation and Depression

Turn word problems into right triangles to find heights, distances and directions using angles of elevation, angles of depression and bearings.

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Why This Matters

Surveyors, pilots, ship captains and builders all use right-triangle trigonometry to measure things they cannot reach with a tape measure. You can find the height of a tree from the ground, how far a boat is from a cliff, or the course a plane must fly. These are the problems students meet on homework and in physics, and the method is always the same: draw a right triangle, label it, and pick a ratio.

Students get stuck in three places: **drawing the diagram** from words, telling an **angle of elevation from an angle of depression**, and **multi-step problems** where one triangle gives a side that the next triangle needs. The fix is a careful picture and a plan that says which triangle to solve first.

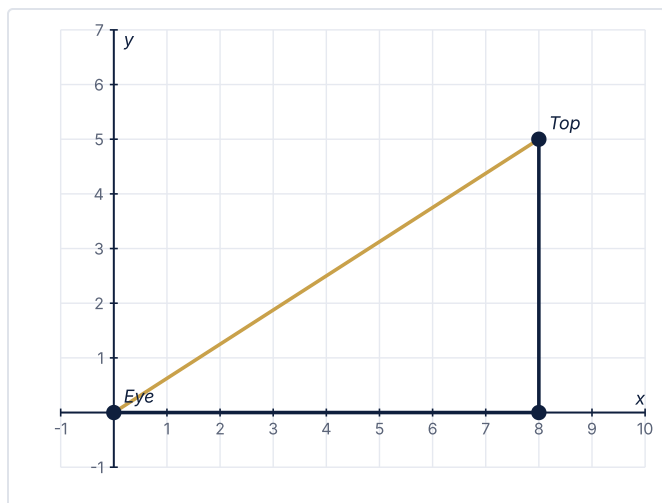
Core Ideas

Elevation and Depression

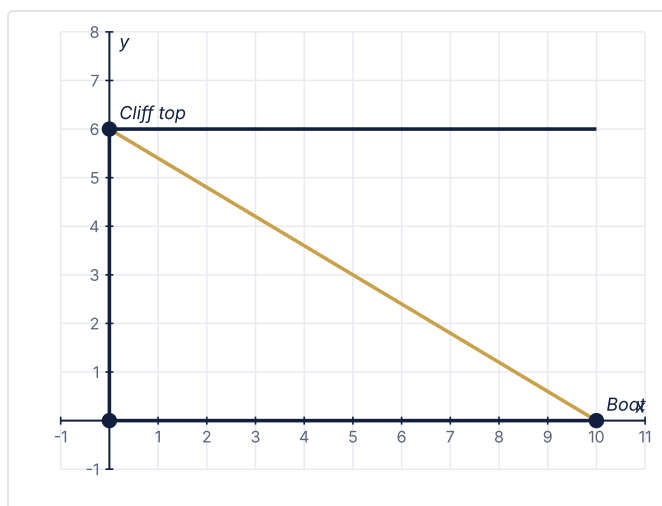
The **angle of elevation** is the angle between the horizontal and the line of sight **upward** to an object.

The **angle of depression** is the angle between the horizontal and the line of sight **downward** to an object.

Because horizontal lines are parallel, the angle of depression from A to B equals the angle of elevation from B to A (alternate interior angles).



Angle of elevation: the angle above the horizontal line when you look up at the top. Here $\tan(\text{angle}) = 5/8$, so the angle is about 32 degrees.



Angle of depression: the angle below the horizontal line when you look down. By alternate angles, it equals the angle of elevation from the boat up to the cliff top.

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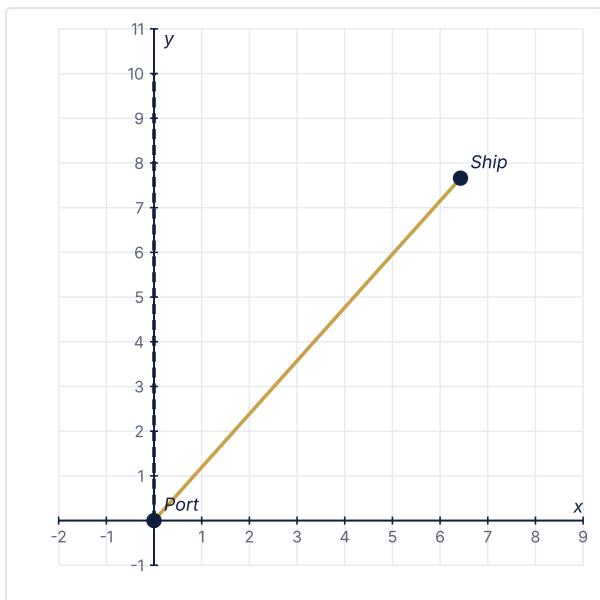
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Bearings

A **bearing** is a direction given as an angle measured **clockwise from north**, written with three digits, such as 040° or 135° .

If a ship travels d on bearing β : **east** distance $= d \sin \beta$ and **north** distance $= d \cos \beta$ (a negative result means west or south).



A bearing is measured clockwise from north. The ship sails on a bearing of 040° degrees, which is 40° degrees clockwise from the north line.

Direction	Bearing
North	000°
East	090°
South	180°
West	270°
North-east	045°
South-west	225°

A plan for any word problem: (1) Draw a picture and mark the right angle(s). (2) Mark the given angle and name the sides relative to it. (3) Choose sine, cosine or tangent based on the two sides involved. (4) Solve and add any extra height, for example your eye level above the ground. (5) Ask whether the answer is reasonable.

For **two-step problems**, such as two observers or two boats, use two right triangles that share a side. Let one unknown stand for the shared side, write one equation for each triangle, and solve. Often the shared side is a height.

Worked Examples

EXAMPLE 1: HEIGHT OF A TOWER

You stand 40 m from the base of a tower on level ground. Your eyes are 1.6 m above the ground. The angle of elevation to the top is 32° . How tall is the tower?

SOLUTION

The right triangle has horizontal leg 40 and the height above eye level as the opposite side.

$$\tan 32^\circ = \frac{x}{40}, \text{ so } x = 40 \tan 32^\circ \approx 24.99.$$

$$\text{Add the eye height: } 24.99 + 1.6 = 26.59.$$

The tower is about 26.59 m tall.

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EXAMPLE 2: ANGLE OF DEPRESSION

From the top of a 60 m cliff, the angle of depression to a boat is 25° . How far is the boat from the base of the cliff?

SOLUTION

The angle of elevation from the boat to the top is also 25° .

The cliff (60) is opposite that angle, and the distance d is adjacent: $\tan 25^\circ = \frac{60}{d}$.

$$d = \frac{60}{\tan 25^\circ} \approx 128.67.$$

The boat is about 128.67 m from the cliff.

EXAMPLE 3: A LADDER

An 8 m ladder leans against a wall and makes a 70° angle with the ground. How high up the wall does it reach, and how far is its foot from the wall?

SOLUTION

The ladder is the hypotenuse (8).

Height (opposite 70°): $8 \sin 70^\circ \approx 7.52$ m.

Distance (adjacent): $8 \cos 70^\circ \approx 2.74$ m.

The ladder reaches 7.52 m high; its foot is 2.74 m from the wall.

EXAMPLE 4: TWO OBSERVERS

From a point A on level ground the angle of elevation to the top of a building is 30° . After walking 40 m toward the building, the angle of elevation is 45° . Find the height of the building.

SOLUTION

Let the height be h and let d be the original distance from A to the base.

From A : $\tan 30^\circ = \frac{h}{d}$, so $h = \frac{d}{\sqrt{3}}$. From the new point: $\tan 45^\circ = 1 = \frac{h}{d-40}$, so $h = d - 40$.

Set equal: $\frac{d}{\sqrt{3}} = d - 40$, so $d \left(1 - \frac{1}{\sqrt{3}}\right) = 40$, so $d \approx 94.64$.

$h = d - 40 \approx 54.64$. Exact form: $h = 20(\sqrt{3} + 1)$.

The building is about 54.64 m tall.

EXAMPLE 5: BEARINGS

A ship leaves port and sails 20 km on a bearing of 040° . How far north and how far east of the port is it?

SOLUTION

North component: $20 \cos 40^\circ \approx 15.32$ km.

East component: $20 \sin 40^\circ \approx 12.86$ km.

Check: $\sqrt{15.32^2 + 12.86^2} \approx 20$.

About 15.32 km north and 12.86 km east.

EXAMPLE 6: TWO BOATS

From the top of a 30 m lighthouse, the angles of depression to two boats in a straight line from the base are 20° and 35° . How far apart are the boats?

SOLUTION

Farther boat: $d_1 = \frac{30}{\tan 20^\circ} \approx 82.42$ m.

Nearer boat: $d_2 = \frac{30}{\tan 35^\circ} \approx 42.84$ m.

Subtract: $82.42 - 42.84 = 39.58$.

The boats are about 39.58 m apart. (The smaller the depression angle, the farther the boat.)

Coch's Playbook

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- 1 **Make the picture big.** Draw a large diagram. Mark the right angle, the given angle, and the unknown. Write the given numbers on the sides.
- 2 **Move the angle to the triangle.** For depression, draw the horizontal first, then copy the angle to the other end of the sight line as an elevation.
- 3 **Add hidden heights last.** If your eye is 1.6 m above the ground, the triangle gives the height above eye level. Add 1.6 at the end.
- 4 **Two triangles, one shared side.** Name the shared side with a letter. Write one equation per triangle, then solve.
- 5 **Round at the end.** Keep the full calculator value in memory and round only the final answer.

Common traps

1) Measuring an angle of elevation from the vertical instead of the horizontal. 2) Forgetting to add the observer's eye height. 3) Using a bearing as an ordinary angle from the east axis, rather than clockwise from north. 4) Rounding the first triangle's answer before using it in the second.

Mistake Clinic

Spot the error 1

Problem: From a window 30 m up, the angle of depression to a car is 40° . How far is the car from the building? Wrong work: " $\tan 40^\circ = \frac{d}{30}$, so $d = 25.17$." What went wrong: The student put the horizontal distance over the vertical. The 30 m is opposite the 40° angle (at the car), and the distance is adjacent. Right solution: $\tan 40^\circ = \frac{30}{d}$, so $d = \frac{30}{\tan 40^\circ} \approx 35.75$ m.

Spot the error 2

Problem: You stand 25 m from a flagpole with your eyes 1.5 m high. The elevation to the top is 50° . How tall is the flagpole? Wrong work: " $25 \tan 50^\circ = 29.8$ m." What went wrong: The tangent gives only the height above eye level. Right solution: $29.8 + 1.5 = 31.3$ m (more precisely $25 \tan 50^\circ = 29.79$, so the pole is 31.29 m).

Spot the error 3

Problem: A hiker walks 10 km on a bearing of 120° . How far east is she? Wrong work: "East = $10 \cos 120^\circ = -5$ km, so she is 5 km west." What went wrong: The student swapped sine and cosine. For a bearing, east uses sine and north uses cosine. Right solution: East = $10 \sin 120^\circ \approx 8.66$ km. North = $10 \cos 120^\circ = -5$ km, meaning 5 km south.

Real-World Connection

A rescue pilot flying at 900 m sees a stranded boat at an angle of depression of 15° . The horizontal distance to the boat is $\frac{900}{\tan 15^\circ} \approx 3359$ m, and the line of sight is $\frac{900}{\sin 15^\circ} \approx 3477$ m long. Pilots and radar operators use the same triangles to estimate how far away a target is.

Challenge Corner

Problem: From a point on level ground, the angles of elevation to the top of a tower are 35° and, after walking 30 m closer, 50° . Find the height of the tower. Solution: Let d be the original distance. Then $h = d \tan 35^\circ$ and $h = (d - 30) \tan 50^\circ$. So $d \tan 35^\circ = (d - 30) \tan 50^\circ$, giving $d(\tan 50^\circ - \tan 35^\circ) = 30 \tan 50^\circ$ and $d \approx 72.74$. Then $h = 72.74 \tan 35^\circ \approx 50.93$ m. Check: $(72.74 - 30) \tan 50^\circ \approx 50.93$.

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Practice

PRACTICE Applications: Angles of Elevation and Depression

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- From a window, the angle of depression to a car is 35° . What is the angle of elevation from the car up to the window?
(A) 55° (B) 145° (C) 65° (D) 35°
- You stand 60 ft from a tree and the angle of elevation to its top is 28° (ignore your eye height). How tall is the tree to the nearest hundredth?
(A) 28.17 (B) 31.90 (C) 112.84 (D) 52.98
- From the top of a 100 m tower the angle of depression to a boat is 40° . How far is the boat from the base of the tower?
(A) 119.18 m (B) 64.28 m (C) 83.91 m (D) 155.57 m
- Which bearing points due south-west?
(A) 045° (B) 135° (C) 225° (D) 315°
- A plane flies 30 km on a bearing of 090° and then 40 km on a bearing of 000° . How far is it from its start?
(A) 70 km (B) 10 km (C) 35 km (D) 50 km
- A ramp rises 1.2 m over a horizontal distance of 10 m. What angle does the ramp make with the ground?
(A) 6.89° (B) 6.84° (C) 83.16° (D) 0.12°
- A kite string 80 m long makes a 55° angle with level ground and is pulled straight. How high is the kite?
(A) 65.53 m (B) 45.89 m (C) 97.66 m (D) 139.48 m
- A 12 m pole casts a 9 m shadow. What is the angle of elevation of the sun?
(A) 36.87° (B) 41.63° (C) 53.13° (D) 126.87°
- A cable runs from the top of a 24 m pole to the ground and makes a 60° angle with the ground. How long is the cable?
(A) 20.78 m (B) 41.57 m (C) 48.00 m (D) 27.71 m
- You stand 80 m from a building. Your eyes are 1.5 m above the ground, and the angle of elevation to the top is 38° . How tall is the building, to the nearest hundredth?
- From the top of a 45 m cliff, a boat is 150 m from the base of the cliff. Find the angle of depression to the boat, to the nearest hundredth of a degree.
- A ship sails 35 km on a bearing of 120° . How far east and how far south has it traveled, to the nearest hundredth?
- From a point 30 m from a tower the angle of elevation to the top is 50° . From a second point 70 m from the tower (in line) find the angle of elevation, to the nearest hundredth of a degree.
- From the top of a 50 m cliff, the angles of depression to two boats in line with the cliff are 18° and 32° . How far apart are the boats, to the nearest hundredth?
- A hiker looks at the top of a radio mast and measures an angle of elevation of 40° . After walking 25 m straight toward the mast, the angle of elevation is 55° . Find the height of the mast to the nearest hundredth (ignore eye height).

20.4 The Laws of Sines and Cosines

Solve triangles that have no right angle using the Law of Sines and Law of Cosines, handle the ambiguous case, and find areas with sine.

Why This Matters

Many real triangles have no right angle: the triangle formed by two ships and a port, the triangle a surveyor makes across a river, or the triangle between a camera and two points on a stage. The right-triangle ratios alone cannot solve them. Two more tools can: the **Law of Sines** and the **Law of Cosines**. Together they let you find every side and angle of any triangle from the right three pieces of information.

Students get stuck in three places: **deciding which law to use**, the **ambiguous case** where the data allows zero, one or two triangles, and **calculator slips** such as using radian mode or forgetting to take the square root at the end. This section provides a decision chart and a checklist so that you choose correctly each time.

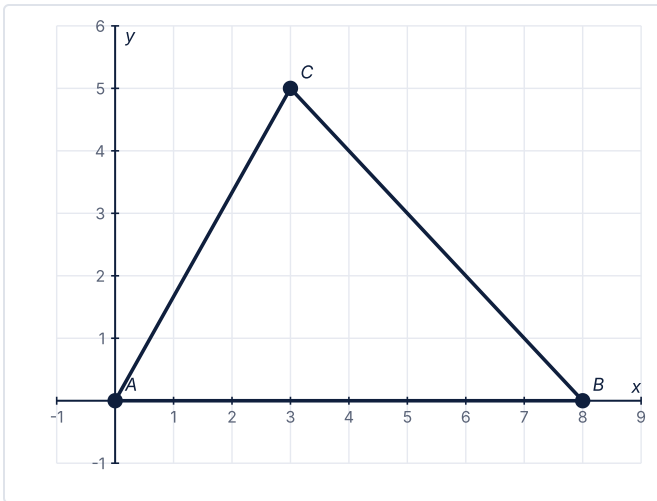
Core Ideas

In triangle ABC we label the side opposite angle A as a , opposite B as b , and opposite C as c . The three angles always add to 180° .

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Triangle ABC. By convention side a is opposite angle A , side b is opposite angle B , and side c is opposite angle C . In this picture $c = AB$, $a = BC$ and $b = CA$.

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

Use it when you know an angle and the side opposite it, plus one more angle or side (ASA, AAS, or SSA).

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cos C.$$

The same pattern works for any side: square the side you want, add the squares of the other two, and subtract 2·(those two sides) · cos(the angle between them). Use it for SAS (to find the third side) and SSS (to find an angle):

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}.$$

When $C = 90^\circ$, $\cos C = 0$ and the Law of Cosines becomes the Pythagorean theorem. The extra term $-2ab \cos C$ corrects for the angle not being right: if C is obtuse, $\cos C$ is negative, so c becomes longer than $\sqrt{a^2 + b^2}$.

Area with Sine

Area of a triangle = $\frac{1}{2}ab \sin C$, where C is the angle **between** sides a and b .

This works for any triangle and needs no height.

Given	Name	Use first
two angles and a side	ASA or AAS	Law of Sines
two sides and the angle between them	SAS	Law of Cosines
three sides	SSS	Law of Cosines
two sides and an angle not between them	SSA	Law of Sines (check for two answers)

The Ambiguous Case (SSA)

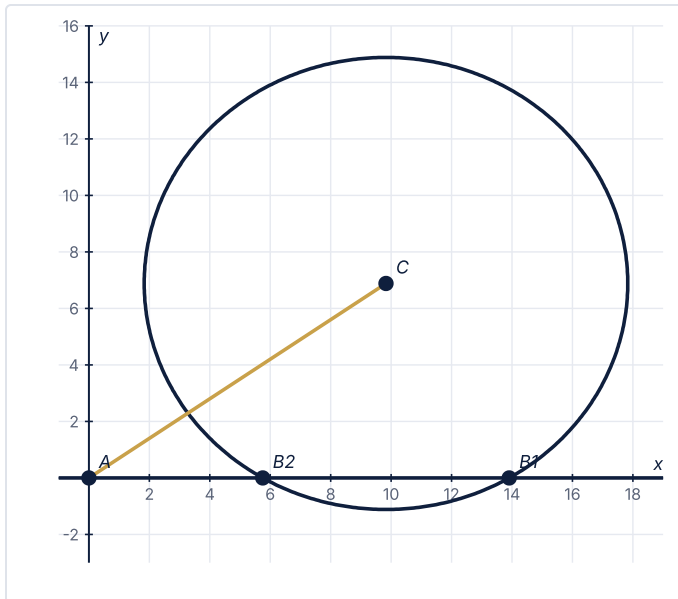
If you know angle A , side a (opposite A) and side b , the data might fit two triangles, one, or none. Compute the height $h = b \sin A$ and compare it with a .

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If A is acute and...	Number of triangles
$a < h$	0 (side a is too short to reach)
$a = h$	1 (a right triangle)
$h < a < b$	2
$a \geq b$	1



Ambiguous case: $A = 35^\circ$, $b = AC = 12$, $a = 8$. The circle of radius 8 around C meets the base line twice, at B_1 and B_2 , so there are two different triangles.

When there are two triangles, the Law of Sines gives $\sin B$, which has two angle solutions: the calculator's B and its supplement $180^\circ - B$. Check that each one together with A makes a sum less than 180° .

Worked Examples

EXAMPLE 1: LAW OF SINES (AAS)

In triangle ABC , $A = 40^\circ$, $B = 65^\circ$, and $a = 12$. Find C , b and c .

SOLUTION

$$C = 180 - 40 - 65 = 75^\circ.$$

$$\frac{b}{\sin 65^\circ} = \frac{12}{\sin 40^\circ}, \text{ so } b = \frac{12 \sin 65^\circ}{\sin 40^\circ} \approx 16.92.$$

$$c = \frac{12 \sin 75^\circ}{\sin 40^\circ} \approx 18.03.$$

$$C = 75^\circ, b \approx 16.92, c \approx 18.03.$$

EXAMPLE 2: LAW OF COSINES (SAS)

In triangle ABC , $a = 9$, $b = 14$, and $C = 48^\circ$. Find c and angle A .

SOLUTION

$$c^2 = 9^2 + 14^2 - 2(9)(14) \cos 48^\circ = 277 - 252(0.6691) \approx 108.38.$$

$$c \approx 10.41.$$

$$\text{Now use the Law of Sines: } \sin A = \frac{9 \sin 48^\circ}{10.41} \approx 0.6425, \text{ so } A \approx 39.98^\circ \text{ (it is acute because } a \text{ is not the longest side).}$$

$$c \approx 10.41, A \approx 39.98^\circ.$$

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EXAMPLE 3: LAW OF COSINES (SSS)

A triangle has sides 7, 9 and 12. Find its largest angle.

SOLUTION

The largest angle is opposite the longest side, 12. Call it C .

$$\cos C = \frac{7^2 + 9^2 - 12^2}{2(7)(9)} = \frac{49 + 81 - 144}{126} = \frac{-14}{126} \approx -0.1111.$$

$C \approx 96.38^\circ$ (obtuse, because the cosine is negative).

The largest angle is about 96.38° .

EXAMPLE 4: AREA WITH SINE

Two sides of a triangular garden are 10 m and 13 m, with an angle of 35° between them. Find its area.

SOLUTION

$$\text{Area} = \frac{1}{2}(10)(13) \sin 35^\circ = 65 \sin 35^\circ.$$

$$65(0.5736) \approx 37.28.$$

The area is about 37.28 m^2 .

EXAMPLE 5: THE AMBIGUOUS CASE

In triangle ABC , $A = 35^\circ$, $a = 8$, $b = 12$. How many triangles exist? Solve each.

SOLUTION

$h = 12 \sin 35^\circ \approx 6.88$. Since $6.88 < 8 < 12$, there are **two** triangles.

$$\sin B = \frac{12 \sin 35^\circ}{8} \approx 0.8604, \text{ so } B \approx 59.36^\circ \text{ or } B \approx 120.64^\circ.$$

$$\text{Triangle 1: } B = 59.36^\circ, C = 85.64^\circ, c = \frac{8 \sin 85.64^\circ}{\sin 35^\circ} \approx 13.91.$$

$$\text{Triangle 2: } B = 120.64^\circ, C = 24.36^\circ, c = \frac{8 \sin 24.36^\circ}{\sin 35^\circ} \approx 5.75.$$

Two triangles.

EXAMPLE 6: ACROSS A RIVER

Two surveyors stand at A and B on one bank, 120 m apart. A tree C is on the other bank. They measure $\angle A = 62^\circ$ and $\angle B = 54^\circ$. Find AC and the width of the river (the shortest distance from C to the line AB).

SOLUTION

$$C = 180 - 62 - 54 = 64^\circ.$$

$$\text{Law of Sines: } \frac{AC}{\sin 54^\circ} = \frac{120}{\sin 64^\circ}, \text{ so } AC \approx 108.01 \text{ m.}$$

The width is the height from C : $108.01 \sin 62^\circ \approx 95.37 \text{ m}$.

$AC \approx 108.01 \text{ m}$; river width $\approx 95.37 \text{ m}$.

Coach's Playbook

- 1 Sketch and label.** Draw the triangle, label each angle with a capital letter and the opposite side with the same lowercase letter.
- 2 Match the data to a pattern.** Use the chart: ASA and AAS use sines; SAS and SSS use cosines; SSA uses sines but check for two triangles.
- 3 Find the largest angle first with cosines.** In SSS, start with the largest angle. Then the other two are acute, so the sine rule gives a single answer.
- 4 Keep unrounded values.** Store intermediate answers in your calculator and round at the end.
- 5 Check with the angle sum and the biggest-side rule.** Angles must add to 180° , and the longest side must be opposite the largest angle.

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Common traps

1) Using the Law of Sines when you have SAS or SSS. 2) Forgetting the square root after computing c^2 . 3) Missing the second solution in the SSA case. 4) Computing $a^2 + b^2 - 2ab$ and then multiplying by $\cos C$ (the cosine multiplies only the $2ab$ part).

Mistake Clinic**Spot the error 1**

Problem: Sides $a = 5$, $b = 7$, and $C = 60^\circ$. Find c . Wrong work: " $c^2 = (5^2 + 7^2 - 2 \cdot 5 \cdot 7) \cos 60^\circ = (4)(0.5) = 2$, so $c = 1.41$." What went wrong: The cosine multiplies only $2ab$, not the whole expression. Right solution: $c^2 = 25 + 49 - 70 \cos 60^\circ = 74 - 35 = 39$, so $c = \sqrt{39} \approx 6.24$.

Spot the error 2

Problem: $A = 40^\circ$, $a = 9$, $b = 12$. Find angle B . Wrong work: " $\sin B = \frac{12 \sin 40^\circ}{9} = 0.857$, so $B = 58.99^\circ$. Done." What went wrong: Since $h = 12 \sin 40^\circ = 7.71 < 9 < 12$, there are two triangles. The supplement is also valid. Right solution: $B = 58.99^\circ$ or $B = 121.01^\circ$. Both fit because $40 + 121.01 < 180$.

Spot the error 3

Problem: A triangle has sides 8, 11 and 15. Find the largest angle. Wrong work: " $\cos C = \frac{8^2 + 11^2 - 15^2}{2(8)(11)} = \frac{-40}{176}$, so $C = \cos^{-1}(0.227) = 76.86^\circ$." What went wrong: The student dropped the negative sign. A negative cosine gives an obtuse angle. Right solution: $C = \cos^{-1}(-0.2273) \approx 103.14^\circ$.

Real-World Connection

Two hikers leave a trailhead on different paths that form a 70° angle. After a few hours they are 40 km and 55 km from the start. How far apart are they? By the Law of Cosines, $c^2 = 40^2 + 55^2 - 2(40)(55) \cos 70^\circ \approx 3120$, so they are about 55.86 km apart. Navigators use this to compute distances between ships, planes and landmarks.

Challenge Corner

Problem: A triangle has sides 7, 9 and 12. Find its area in two ways. Solution: From Example 3, $\cos C = -\frac{1}{9}$ for the angle opposite 12, so $\sin C = \sqrt{1 - \frac{1}{81}} = \frac{\sqrt{80}}{9}$. Area $= \frac{1}{2}(7)(9) \sin C = \frac{1}{2}(63) \frac{\sqrt{80}}{9} = \frac{7\sqrt{80}}{2} \approx 31.30$. Check with Heron's formula: $s = 14$, area $= \sqrt{14 \cdot 7 \cdot 5 \cdot 2} = \sqrt{980} \approx 31.30$.

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Practice

PRACTICE The Laws of Sines and Cosines

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- You know two sides of a triangle and the angle between them. Which should you use first to find the third side?
(A) Law of Sines (B) Law of Cosines (C) Pythagorean theorem
(D) Midpoint formula
- In triangle ABC , $A = 30^\circ$, $B = 45^\circ$ and $a = 10$. What is b to the nearest hundredth?
(A) 7.07 (B) 10.00 (C) 20.00 (D) 14.14
- A triangle has sides $a = 6$, $b = 8$, $c = 10$. What is $\cos A$?
(A) 0.80 (B) 0.60 (C) 0.75 (D) 1.25
- Two sides of a triangle are 8 and 11 and the angle between them is 30° . What is its area?
(A) 44 (B) 88 (C) 22 (D) 38.1
- In triangle ABC , $a = 5$, $b = 7$, $C = 60^\circ$. What is c to the nearest hundredth?
(A) 12.00 (B) 6.24 (C) 8.60 (D) 2.00
- In triangle ABC , $A = 50^\circ$, $a = 10$, $b = 12$. How many triangles are possible?
(A) 0 (B) 1 (C) 3 (D) 2
- Which information can produce two different triangles?
(A) $A = 30^\circ$, $a = 7$, $b = 10$ (B) $A = 30^\circ$, $a = 5$, $b = 10$ (C) $A = 30^\circ$, $a = 4$, $b = 10$
(D) $A = 30^\circ$, $a = 12$, $b = 10$
- In triangle ABC , $A = 52^\circ$, $B = 71^\circ$ and $c = 20$. What is a to the nearest hundredth?
(A) 16.67 (B) 22.55 (C) 18.79 (D) 20.45
- Two sides of a triangle are 15 and 20, and the angle between them is 70° . What is the area to the nearest hundredth?
(A) 281.91 (B) 140.95 (C) 51.30 (D) 150.00
- Two sides of a triangle are 12 and 15 and the angle between them is 110° . Find the third side to the nearest hundredth.
- A triangle has sides 8, 11 and 15. Find its largest angle to the nearest hundredth of a degree.
- In triangle ABC , $B = 62^\circ$, $C = 48^\circ$ and $b = 25$. Find c and a to the nearest hundredth.
- In triangle ABC , $A = 40^\circ$, $a = 9$ and $b = 12$. Find the two possible values of angle B to the nearest hundredth of a degree.
- A triangle has sides 6, 7 and 9. Find its area to the nearest hundredth by first finding the angle between the sides 6 and 7.
- Two ships leave a port. Ship A sails 40 km and ship B sails 55 km, and their courses form a 70° angle. A third ship wants to know the distance between A and B. Find it, to the nearest hundredth.

Answers Answers and Explanations

Complete solutions for every practice problem, numbered to match. Read the explanation even when your answer is right.

20.1 Special Right Triangles and Pythagorean Triples**PRACTICE · SPECIAL RIGHT TRIANGLES AND PYTHAGOREAN TRIPLES**

- C. Legs a give hypotenuse $a\sqrt{2} = 6\sqrt{2}$. Choice B doubles the leg; choice D uses the 30-60-90 factor.
- A. The hypotenuse is twice the short leg: $2(9) = 18$. Choice B is the long leg.
- D. The short leg is $\frac{12\sqrt{3}}{\sqrt{3}} = 12$, and the hypotenuse is $2(12) = 24$. Choice A is the long leg itself.
- B. $9^2 + 40^2 = 81 + 1600 = 1681 = 41^2$. The others fail: $36 + 64 \neq 144$, $100 + 576 = 676 \neq 729$, $121 + 3600 = 3721 \neq 3844$.
- C. Diagonal $= s\sqrt{2}$, so $s = \frac{10}{\sqrt{2}} = 5\sqrt{2}$. Choice A multiplies instead of dividing.
- A. The height is the long leg of a 30-60-90 triangle with hypotenuse 10 and short leg 5: $5\sqrt{3}$. Choice B uses the whole side as the short leg.
- D. $h^2 = 3 \cdot 12 = 36$, so $h = 6$. Choice C is the average of 3 and 12, which is not the rule; choice B forgets the square root.
- B. $\sqrt{25^2 - 7^2} = \sqrt{576} = 24$. This is the 7-24-25 triple. Choice A subtracts the legs instead (25 minus 7), and choice D adds them.
- C. The legs are $\frac{s}{\sqrt{2}} = 4\sqrt{2}$, so the area is $\frac{1}{2}(4\sqrt{2})^2 = \frac{1}{2}(32) = 16$. Choice A treats 8 as a leg.
- Short leg 10, long leg $10\sqrt{3}$. The short leg is half the hypotenuse, and the long leg is the short leg times $\sqrt{3}$.
21. The triple 20, 21, 29 works: $400 + 441 = 841 = 29^2$. Check: $29^2 - 20^2 = 441$.
50. Leg a has perimeter $a(2 + \sqrt{2})$. Set equal to $10(2 + \sqrt{2})$ to get $a = 10$. Area $= \frac{1}{2}(10)(10) = 50$.

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13. $5\sqrt{3}$. Area $\frac{\sqrt{3}}{4}s^2 = 25\sqrt{3}$ gives $s^2 = 100$, so $s = 10$. Height $= \frac{\sqrt{3}}{2}(10) = 5\sqrt{3}$.
14. 15. The other piece q satisfies $9q = 144$, so $q = 16$. The hypotenuse is 25. The leg next to the piece 9 is $\sqrt{9 \cdot 25} = 15$ (the other leg is 20).
15. Area 84, perimeter $28 + 12\sqrt{2}$. Each overhang is $\frac{20-8}{2} = 6$, so the height is 6 (45-45-90) and each leg is $6\sqrt{2}$. Area $= \frac{1}{2}(28)(6) = 84$. Perimeter $= 8 + 20 + 2(6\sqrt{2})$.

20.2 Sine, Cosine, and Tangent Ratios

PRACTICE · SINE, COSINE, AND TANGENT RATIOS

- B. Tangent is opposite over adjacent: $6/8 = 3/4$. Choice A is sine ($6/10$).
- D. Sine is opposite over hypotenuse. The side opposite A is $BC = 5$, so $\sin A = 5/13$. Choice A is $\cos A$.
- A. Sine and cosine of complementary angles are equal: $x = 90 - 25 = 65$. Choice B repeats the angle.
- C. The adjacent leg uses cosine: $30 \cos 40^\circ \approx 22.98$. Choice A uses sine; choice D divides instead of multiplies.
- B. $\tan 55^\circ = \frac{x}{12}$, so $x = 12 \tan 55^\circ \approx 17.14$. Choice A divides by tangent instead.
- D. $\cos \theta = \frac{1.5}{6} = 0.25$, so $\theta = \cos^{-1}(0.25) \approx 75.52^\circ$. Choice A uses $\sin^{-1}$, which finds the angle at the wall.
- A. The angles must sum to 90: $5x + 5 = 90$, so $x = 17$. Choice B comes from setting the angles equal ($2x + 4 = 3x + 1$).
- C. Draw opposite 7, adjacent 24, hypotenuse $\sqrt{49 + 576} = 25$. So $\sin \theta = 7/25$. Choice A repeats the tangent.
- B. $\sin 62^\circ = \frac{14}{h}$, so $h = \frac{14}{\sin 62^\circ} \approx 15.86$. Choice A multiplies by sine instead of dividing.
- 25.04. $\cos 37^\circ = \frac{20}{h}$, so $h = \frac{20}{\cos 37^\circ} \approx 25.04$.
- 23.96 degrees. $\tan \theta = \frac{4}{9}$, so $\theta = \tan^{-1}(4/9) \approx 23.96^\circ$.
- Other angle 66° ; opposite the 24° angle: $18 \sin 24^\circ \approx 7.32$; adjacent: $18 \cos 24^\circ \approx 16.44$.
- $\cos \theta = \frac{15}{17}$, $\tan \theta = \frac{8}{15}$. The missing side is $\sqrt{17^2 - 8^2} = 15$.
- $x = 12$; the angles are 54° and 36° . They must add to 90: $6x + 18 = 90$. Check: $\sin 54^\circ = \cos 36^\circ$.
- Base ≈ 6.84 , height ≈ 9.40 , area ≈ 32.14 . Split into two right triangles: half the base is $10 \cos 70^\circ = 3.42$, so the base is 6.84. Height $= 10 \sin 70^\circ = 9.40$. Area $= \frac{1}{2}(6.84)(9.40) \approx 32.14$.

20.3 Applications: Angles of Elevation and Depression

PRACTICE · APPLICATIONS: ANGLES OF ELEVATION AND DEPRESSION

- D. Horizontal lines are parallel, so alternate angles are equal: the elevation is also 35° . Choice A is the complement.
- B. $\tan 28^\circ = \frac{h}{60}$, so $h = 60 \tan 28^\circ \approx 31.90$. Choice A uses sine and choice C divides by the tangent.
- A. $\tan 40^\circ = \frac{100}{d}$, so $d = \frac{100}{\tan 40^\circ} \approx 119.18$. Choice C multiplies by the tangent instead of dividing.
- C. South is 180° and west is 270° , so south-west is halfway at 225° . Choice B is south-east.
- D. The legs are perpendicular (east, then north), so the distance is $\sqrt{30^2 + 40^2} = 50$. Choice A adds the lengths.
- B. $\tan \theta = \frac{1.2}{10} = 0.12$, so $\theta = \tan^{-1}(0.12) \approx 6.84^\circ$. Choice A uses sine, which would need the 10 m to be the hypotenuse.
- A. Height is opposite the angle and the string is the hypotenuse: $80 \sin 55^\circ \approx 65.53$. Choice B uses cosine.
- C. $\tan \theta = \frac{12}{9}$, so $\theta \approx 53.13^\circ$. Choice A flips the ratio.
- D. The pole is opposite the angle: $\sin 60^\circ = \frac{24}{L}$, so $L = \frac{24}{\sin 60^\circ} \approx 27.71$. Choice A multiplies by sine instead of dividing.
- 64.00 m. $80 \tan 38^\circ \approx 62.50$, plus 1.5 for the eye height gives 64.00.
- 16.70° . $\tan \theta = \frac{45}{150} = 0.3$, and $\theta = \tan^{-1}(0.3) \approx 16.70^\circ$.
- East about 30.31 km, south about 17.50 km. East $= 35 \sin 120^\circ = 30.31$. North $= 35 \cos 120^\circ = -17.5$, which is 17.5 south.
- 27.06° . Height $= 30 \tan 50^\circ \approx 35.75$. From the second point $\tan \theta = \frac{35.75}{70}$, so $\theta \approx 27.06^\circ$.
- 73.87 m. Distances: $\frac{50}{\tan 18^\circ} \approx 153.88$ and $\frac{50}{\tan 32^\circ} \approx 80.02$. Subtract.
- 50.86 m. With original distance d : $d \tan 40^\circ = (d - 25) \tan 55^\circ$, so $d = \frac{25 \tan 55^\circ}{\tan 55^\circ - \tan 40^\circ} \approx 60.61$. Then $h = d \tan 40^\circ \approx 50.86$.

20.4 The Laws of Sines and Cosines

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PRACTICE · THE LAWS OF SINES AND COSINES

1. B. SAS calls for the Law of Cosines, $c^2 = a^2 + b^2 - 2ab \cos C$. Choice A needs an angle and its opposite side.
2. $D. b = \frac{10 \sin 45^\circ}{\sin 30^\circ} \approx 14.14$. Choice A inverts the ratio.
3. $A. \cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{64 + 100 - 36}{160} = 0.80$. Choice B is $\cos B$.
4. $C. \text{Area} = \frac{1}{2}(8)(11) \sin 30^\circ = 44(0.5) = 22$. Choice A forgets the sine and choice D uses cosine.
5. $B. c^2 = 25 + 49 - 70 \cos 60^\circ = 74 - 35 = 39$, so $c \approx 6.24$. Choice C forgets the $-2ab \cos C$ term (it is $\sqrt{74}$).
6. $D. h = 12 \sin 50^\circ \approx 9.19$, and $9.19 < 10 < 12$, so there are 2 triangles. Choice B assumes SSA always gives one.
7. $A. \text{The height is } 10 \sin 30^\circ = 5$. Only choice A has $5 < a < 10$, which gives two triangles. Choice B gives a right triangle, choice C none, and choice D one because $a > b$.
8. $C. C = 57^\circ$, so $a = \frac{20 \sin 52^\circ}{\sin 57^\circ} \approx 18.79$. Choice B uses $\sin 71^\circ$ in the numerator (the wrong angle).
9. $B. \text{Area} = \frac{1}{2}(15)(20) \sin 70^\circ \approx 140.95$. Choice A forgets the factor $\frac{1}{2}$.
10. $22.18. c^2 = 144 + 225 - 360 \cos 110^\circ = 369 + 123.13 = 492.13$, so $c \approx 22.18$. The cosine of 110° is negative.
11. 103.14° . The largest angle is opposite 15: $\cos C = \frac{64 + 121 - 225}{176} = -0.2273$, so $C \approx 103.14^\circ$.
12. $c \approx 21.04$ and $a \approx 26.61$. $A = 70^\circ$. $c = \frac{25 \sin 48^\circ}{\sin 62^\circ}$ and $a = \frac{25 \sin 70^\circ}{\sin 62^\circ}$.
13. $B \approx 58.99^\circ$ or $B \approx 121.01^\circ$. $\sin B = \frac{12 \sin 40^\circ}{9} = 0.8571$, and the supplement also works since $40 + 121.01 < 180$.
14. $20.98. \cos C = \frac{36 + 49 - 81}{84} = \frac{1}{21}$, so $\sin C = \frac{\sqrt{440}}{21}$. $\text{Area} = \frac{1}{2}(6)(7) \frac{\sqrt{440}}{21} = \sqrt{440} \approx 20.98$.
15. $55.86 \text{ km. } c^2 = 40^2 + 55^2 - 2(40)(55) \cos 70^\circ = 4625 - 1504.9 = 3120.1$, so $c \approx 55.86$.

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