



Finance and Quantitative Reasoning for STEM and Business

Four focused lessons, worked examples, a mistake clinic, and 60 practice problems with full explanations.

In this booster

30.1 Percent Growth, Margins, and Break-Even Analysis

30.2 Interest, Present Value, and the Time Value of Money

30.3 Risk, Expected Value, and Decision Making

30.4 Quantitative Speed and Precision

Every section teaches the idea, walks through the most common problem types, flags the traps students fall into, and ends with fifteen practice problems. Full answers and explanations are at the back.

30.1 Percent Growth, Margins, and Break-Even Analysis

Compute percent change and growth factors, model revenue, cost, and profit with linear functions, find the break-even point, and keep markup and margin straight so you can analyze a small business.

Why This Matters

Business math is the most practical use of algebra. Every company, from a school fundraiser to a global manufacturer, asks the same questions: how much did sales grow, how much do we keep from each sale, and how many items must we sell before we stop losing money? The answers rest on percent change, linear revenue and cost functions, and the break-even point where they cross.

Students usually get stuck on four things. They divide by the wrong number when finding a percent change. They add percents when changes are applied one after another. They mix up markup and margin, which look alike but use different bases. And they forget that real quantities, such as the number of items sold, must be whole numbers. This section builds a clean routine for each of these, so you can model a business with a few lines of algebra.

Core Ideas

Start with percent change, the measure of how much something grew or shrank relative to where it started. The starting value is always the base, which is why a 20 percent raise and a 20 percent pay cut do not cancel.

Percent Change and Growth Factors

$$\text{percent change} = \frac{\text{new} - \text{old}}{\text{old}} \times 100\%.$$

A change of r (written as a decimal) multiplies the quantity by the **growth factor** $1 + r$. An increase of 8 percent has factor 1.08. A decrease of 15 percent has factor 0.85. Successive changes multiply their factors.

Because successive changes multiply, going up 20 percent and then down 20 percent gives $1.2 \times 0.8 = 0.96$, a net loss of 4 percent. To undo a change, divide by the factor, not by the percent. If a price after a 25 percent discount is 54 dollars, the original price is $54 \div 0.75 = 72$ dollars.

Next, the business model. Suppose a company sells x items at price p each. Its costs have two parts: **fixed costs** F (rent, equipment) that do not change with x , and a **variable cost** v per item (materials, labor). These give linear functions.

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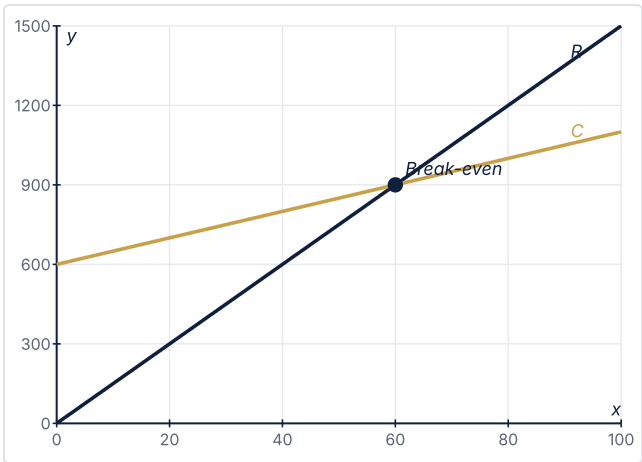
Revenue, Cost, and Profit

$$R(x) = px, \quad C(x) = F + vx, \quad \text{Profit } P(x) = R(x) - C(x) = (p - v)x - F.$$

The **break-even point** is where profit is 0, so revenue equals cost:

$$x = \frac{F}{p - v}.$$

The quantity $p - v$ is the **contribution margin** per item: the amount each sale contributes toward paying off the fixed cost.



Revenue $R = 15x$ and cost $C = 600 + 5x$ cross at $x = 60$ units, where both equal 900 dollars. To the right of that point the company earns a profit.

Look at the graph. Left of the crossing point the cost line is above the revenue line, so the company loses money. Right of the crossing point, revenue is higher and the gap is the profit. For selling more than the break-even quantity, each extra item adds exactly $p - v$ dollars of profit.

Markup versus Margin

Both compare profit to something, but the base differs.

$$\text{markup} = \frac{\text{price} - \text{cost}}{\text{cost}}, \quad \text{margin} = \frac{\text{price} - \text{cost}}{\text{price}}.$$

Because price is larger than cost, margin is always smaller than markup. A 50 percent markup is only a 33.3 percent margin.

Cost	Markup	Price	Profit per item	Margin
60	25%	75	15	20%
60	50%	90	30	33.3%
60	100%	120	60	50%
60	150%	150	90	60%

Two language habits prevent many errors. First, a change in a percentage is not a percent change. If a profit margin moves from 20 percent to 25 percent, it rose by 5 **percentage points**, but the margin itself grew by $\frac{5}{20} = 25\%$. Second, "percent of" means multiply, while "percent more than" means multiply by $1 + r$. Writing the factor first keeps you from working with the wrong number.

Break-even analysis also lets you compare plans. If plan A has fixed cost 500 and variable cost 3, and plan B has fixed cost 200 and variable cost 5, set $500 + 3x = 200 + 5x$ and solve to get $x = 150$. Below 150 items plan B is cheaper, and above 150 items plan A is cheaper. The crossing point of two cost lines is the key to the choice.

Worked Examples

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EXAMPLE 1: PERCENT CHANGE

A jacket's price rises from 40 dollars to 46 dollars. Later a different item falls from 80 dollars to 68 dollars. Find each percent change.

SOLUTION

First item: $\frac{46 - 40}{40} = \frac{6}{40} = 0.15$, an increase of **15 percent**.

Second item: $\frac{68 - 80}{80} = \frac{-12}{80} = -0.15$, a decrease of **15 percent**.

Each change is measured against its own starting price.

EXAMPLE 2: SUCCESSIVE CHANGES

A store raises a price by 20 percent and then discounts the new price by 20 percent. What is the net change?

SOLUTION

Factors: 1.20 and 0.80.

$$1.20 \times 0.80 = 0.96.$$

The price ends at 96 percent of the original: a net **decrease of 4 percent**, not 0.

EXAMPLE 3: WORKING BACKWARD

After a 25 percent discount, a bike costs 54 dollars. What was the original price?

SOLUTION

A 25 percent discount means the sale price is $75\% = 0.75$ of the original.

$$0.75x = 54, \text{ so } x = 54 \div 0.75 = 72.$$

The original price was **72 dollars**. Check: $72 \times 0.75 = 54$.

EXAMPLE 4: MARKUP AND MARGIN

A shop buys a lamp for 60 dollars and applies a 50 percent markup. Find the selling price and the margin.

SOLUTION

$$\text{Price} = 60(1.50) = 90 \text{ dollars.}$$

$$\text{Profit} = 90 - 60 = 30 \text{ dollars.}$$

$$\text{Margin} = \frac{30}{90} = \frac{1}{3} \approx 33.3\%.$$

EXAMPLE 5: BREAK-EVEN AND A PROFIT TARGET

A bakery sells cookies for 3.50 dollars each. Each cookie costs 1.10 dollars to make, and fixed costs are 1200 dollars. (a) Find the break-even quantity. (b) How many cookies must be sold to earn a profit of 600 dollars?

SOLUTION

Contribution per cookie: $3.50 - 1.10 = 2.40$ dollars.

$$(a) x = \frac{1200}{2.40} = 500 \text{ cookies.}$$

$$(b) \text{ Need } 2.40x - 1200 = 600, \text{ so } 2.40x = 1800 \text{ and } x = 750 \text{ cookies.}$$

EXAMPLE 6: A PRICE CHANGE

The bakery raises the price by 10 percent. How many cookies are now needed to break even?

SOLUTION

New price: $3.50(1.10) = 3.85$ dollars. New contribution: $3.85 - 1.10 = 2.75$ dollars.

$$x = \frac{1200}{2.75} \approx 436.4.$$

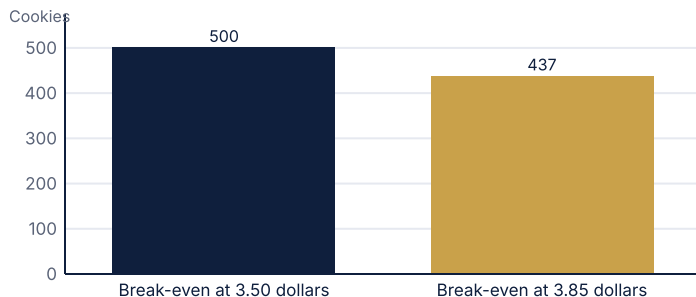
You cannot sell a fraction of a cookie, and selling 436 still loses a little money. Round **up: 437 cookies**.

A 10 percent price rise cut the break-even quantity by about 13 percent, because the contribution margin rose much more than 10 percent.

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A modest price increase lowers the number of cookies needed to break even from 500 to 437.

Coach's Playbook

- 1 Pick the base.** Before computing any percent, ask what the percent is "of". Percent change uses the old value. Margin uses the price. Markup uses the cost.
 - 2 Turn percents into factors.** Increase by r : multiply by $1 + r$. Decrease by r : multiply by $1 - r$. Chain changes by multiplying.
 - 3 Write the functions.** Define $R(x)$ and $C(x)$ in words first, then in symbols. Label units.
 - 4 Solve and interpret.** After finding x , say what it means and round to a sensible whole number.
 - 5 Check.** Plug your break-even into both R and C . They must match.
- ◆ To undo a percent change, divide by the factor.
 - ◆ To compare two plans, set their cost functions equal and solve.
 - ◆ A higher contribution margin lowers the break-even point.
 - ◆ If a problem asks for the minimum quantity to earn a profit, round up.

Common traps

1. Dividing by the new value instead of the old value when finding percent change. 2. Adding percents for successive changes. 3. Calling a 25 percent markup a 25 percent margin. 4. Forgetting that fixed costs are paid once, not per item.

Mistake Clinic

Spot the error 1

A student says: "A stock rose 50 percent on Monday and fell 50 percent on Tuesday, so it is back where it started."

Where is the error? The factors are 1.5 and 0.5, and $1.5 \times 0.5 = 0.75$. The stock ended 25 percent lower. The 50 percent drop is measured from the higher Monday value.

Spot the error 2

A student says: "A shirt costs 80 dollars after a 20 percent discount, so the original price was $80 \times 1.20 = 96$ dollars."

Where is the error? The 20 percent is taken off the original, not off the sale price. Divide by the factor: $80 \div 0.80 = 100$ dollars. Check: 20 percent off 100 dollars is 80 dollars.

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Spot the error 3

A student says: "I bought a game for 80 dollars and sold it for 100 dollars, a 25 percent markup, so my margin is also 25 percent." **Where is the error?** Markup uses cost as the base: $\frac{20}{80} = 25\%$. Margin uses the price: $\frac{20}{100} = 20\%$. They are different numbers.

Real-World Connection

A student club sells T-shirts. Printing setup costs 240 dollars, each shirt costs 6 dollars to print, and the club sells them for 14 dollars. Break-even is $\frac{240}{8} = 30$ shirts. The club knows it needs to sell at least 31 shirts to start raising money for its trip.

Challenge Corner

A store sells $x = 400 - 10p$ units per week when the price is p dollars, with a variable cost of 6 dollars per unit and weekly fixed costs of 1000 dollars. What price maximizes weekly profit? **Solution.** Profit $P(p) = (p - 6)(400 - 10p) - 1000 = -10p^2 + 460p - 3400$. This parabola opens downward, so the maximum is at the vertex $p = -\frac{460}{2(-10)} = 23$. At $p = 23$, demand is 170 units and profit is $17 \cdot 170 - 1000 = 1890$ dollars. The best price is **23 dollars**.

Practice**PRACTICE Percent Growth, Margins, and Break-Even Analysis**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- A price rises from 250 dollars to 290 dollars. What is the percent increase?
(A) 40% (B) 13.8% (C) 16% (D) 14%
- A quantity grows by 10 percent each year for 2 years. What is the total percent growth?
(A) 20% (B) 21% (C) 10% (D) 11%
- After a 30 percent discount, a coat costs 63 dollars. What was the original price?
(A) 90 dollars (B) 81.90 dollars (C) 44.10 dollars (D) 93 dollars
- A seller buys a toy for 80 dollars and sells it for 100 dollars. What is the profit margin (profit as a percent of price)?
(A) 25% (B) 80% (C) 120% (D) 20%
- A company has profit function $P(x) = 7x - 2100$, where x is the number of items sold. How many items give break-even?
(A) 2100 (B) 14700 (C) 300 (D) 150
- A company has $C(x) = 500 + 2x$ and $R(x) = 7x$. What is the revenue at the break-even point?
(A) 700 dollars (B) 100 dollars (C) 500 dollars (D) 1200 dollars
- A price is increased by 20 percent. By what percent must the new price be decreased to return to the original?
(A) about 16.7% (B) 20% (C) 25% (D) 12%
- A store buys an item for 36 dollars and wants a profit margin of 40 percent of the selling price. What price should it charge?
(A) 50.40 dollars (B) 76 dollars (C) 60 dollars (D) 90 dollars
- Fixed costs are 900 dollars, each item costs 5 dollars to make, and each sells for 12 dollars. What is the smallest whole number of items that earns a profit?
(A) 128 (B) 129 (C) 75 (D) 180
- A town's population grows from 5000 to 5650. What is the percent increase?
- A phone's price falls by 15 percent to 340 dollars. What was the original price in dollars?
- A shop buys an item for 45 dollars and applies a 60 percent markup. What is the profit margin as a percent of the selling price?

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13. Revenue is $R(x) = 18x$ and cost is $C(x) = 1350 + 6x$. How many items must be sold to avoid a loss? Give a whole number.
14. A company's sales grow 5 percent in year 1, grow 8 percent in year 2, and fall 2 percent in year 3. What is the net percent change over the three years? Round to one decimal place.
15. Fixed costs are 900 dollars and each item costs 4 dollars to make. The company breaks even when it sells 150 items. What price per item does it charge?

30.2 Interest, Present Value, and the Time Value of Money

Compare simple and compound interest, use the compound growth formula, discount future money to present value, convert rates to an effective annual rate, and solve for time with logarithms.

Why This Matters

A dollar today is worth more than a dollar next year, because today's dollar can be saved and earn interest. This simple fact, the **time value of money**, drives savings accounts, loans, car payments, retirement plans, and the price of every bond. Students usually get stuck on three things: choosing between simple and compound interest, using the compounding formula with the correct rate per period, and working backward from a future goal to the amount needed today.

The ideas here are also the first step into exponential functions in precalculus, into the number e , and into logarithms, which are the tool for asking "how long will it take?" If you can handle the formula $A = P(1 + \frac{r}{n})^{nt}$ and its inverse, you can compare any two financial offers fairly.

Core Ideas

Interest is the price of using money. If you deposit an amount P (the **principal**) at annual rate r , you earn interest. With **simple interest**, interest is paid only on the original principal, so growth is linear. With **compound interest**, interest is added to the balance and then earns interest itself, so growth is exponential.

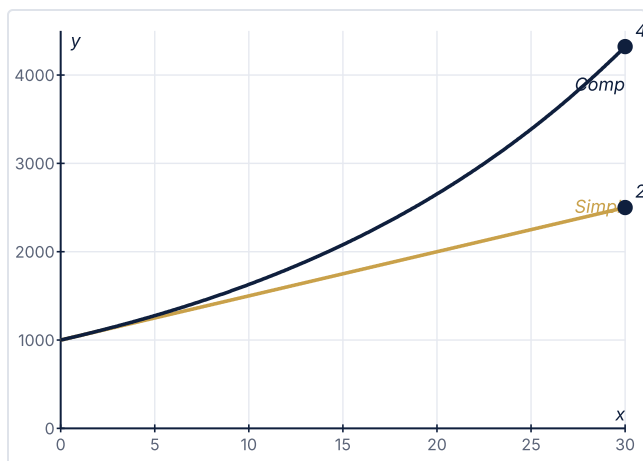
Simple and Compound Interest

Simple interest: $I = Prt$ and the balance is $A = P(1 + rt)$.

Compound interest, n compounding periods per year:

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

Here r is the annual rate as a decimal and t is in years. The rate per period is $\frac{r}{n}$ and the number of periods is nt .



1000 dollars at 5 percent. Simple interest grows along a line to 2500 dollars in 30 years. Compound interest curves upward to about 4322 dollars.

The more often interest is compounded, the more you earn, but the gain shrinks quickly. The table shows 1000 dollars at 6 percent for one year. Going from yearly to monthly compounding adds almost 2 dollars. Going from monthly to daily adds only 15 cents. The limit, reached by continuous compounding, is Pe^{rt} .

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Compounding	Periods per year	Balance after 1 year
Annually	1	1060.00
Semiannually	2	1060.90
Quarterly	4	1061.36
Monthly	12	1061.68
Daily	365	1061.83
Continuously	-	1061.84

Present Value and Effective Annual Rate

To find how much must be deposited **now** to reach a future value A :

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}} = A \left(1 + \frac{r}{n}\right)^{-nt}.$$

The **effective annual rate** (EAR) is the single yearly rate that gives the same growth as the compounded rate:

$$\text{EAR} = \left(1 + \frac{r}{n}\right)^n - 1.$$

Continuous compounding: $A = Pe^{rt}$.

Present value is simply compound growth run backward: instead of multiplying by the growth factor, you divide by it. The EAR lets you compare offers on equal footing. A rate of 6 percent compounded monthly sounds like 6 percent, but it behaves like 6.17 percent per year, so it beats a 6.1 percent account that compounds once a year.

When you need the **time** instead of the amount, the unknown sits in the exponent and you use logarithms. For $A = P(1 + i)^t$ with yearly compounding,

$$t = \frac{\ln(A/P)}{\ln(1 + i)}.$$

For a quick mental estimate of doubling time, the **rule of 72** says that money doubles in about $\frac{72}{\text{rate in percent}}$ years.

Where does the number $e \approx 2.71828$ come from? Imagine 1 dollar at 100 percent interest for one year. Annual compounding gives 2 dollars, monthly gives about 2.613, daily about 2.7146, and as the compounding becomes continuous the balance approaches e . That limit is why continuous compounding uses Pe^{rt} , and why e appears throughout science whenever growth is proportional to the current amount.

Year	Simple interest balance	Compound balance (5 percent)	Difference
0	1000	1000.00	0.00
10	1500	1628.89	128.89
20	2000	2653.30	653.30
30	2500	4321.94	1821.94

The table shows the central lesson. In the first few years the two methods look almost the same, so compounding seems unimportant. But the gap grows faster every year because compound growth builds on itself. This is why financial advisers urge people to start saving early: the last decade of growth is much larger than the first.

Be careful with vocabulary. A stated rate such as "6 percent compounded monthly" is often called the **nominal** rate or APR. The EAR is sometimes called APY. When two products use different compounding schedules, only the effective rates can be compared directly.

Worked Examples

EXAMPLE 1: SIMPLE INTEREST

Find the interest earned on 2500 dollars at 4 percent simple interest for 3 years.

SOLUTION

$$I = Prt = 2500(0.04)(3) = 300.$$

Interest is **300 dollars**, and the balance is 2800 dollars.

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EXAMPLE 2: ANNUAL COMPOUNDING

Find the balance after 10 years if 2000 dollars is invested at 5 percent compounded annually.

SOLUTION

$$A = 2000(1.05)^{10}.$$

$$1.05^{10} \approx 1.62889, \text{ so } A \approx 3257.79.$$

The balance is about **3257.79 dollars**. Simple interest would give only 3000.

EXAMPLE 3: QUARTERLY COMPOUNDING

5000 dollars is invested at 6 percent compounded quarterly for 4 years. Find the balance.

SOLUTION

Rate per period: $\frac{0.06}{4} = 0.015$. Number of periods: $4 \cdot 4 = 16$.

$$A = 5000(1.015)^{16} \approx 5000(1.26899) = 6344.93.$$

The balance is about **6344.93 dollars**.

EXAMPLE 4: PRESENT VALUE

How much should be deposited now at 4 percent compounded annually to have 10,000 dollars in 6 years?

SOLUTION

$$P = \frac{10000}{1.04^6}.$$

$$1.04^6 \approx 1.26532, \text{ so } P \approx 7903.15.$$

About **7903.15 dollars**. Check: $7903.15(1.04)^6 \approx 10000$.

EXAMPLE 5: COMPARING OFFERS WITH EAR

Bank A pays 9 percent compounded monthly. Bank B pays 9.2 percent compounded annually. Which pays more?

SOLUTION

Bank A: $\text{EAR} = \left(1 + \frac{0.09}{12}\right)^{12} - 1 = 1.0075^{12} - 1 \approx 0.0938$, or 9.38 percent.

Bank B: EAR is simply 9.2 percent.

Bank A pays more, **9.38 percent versus 9.2 percent**, even though its stated rate is lower.

EXAMPLE 6: SOLVING FOR TIME

How long does it take money to double at 6 percent compounded annually?

SOLUTION

Set $1.06^t = 2$.

$$t = \frac{\ln 2}{\ln 1.06} \approx \frac{0.6931}{0.05827} \approx 11.9 \text{ years.}$$

The rule of 72 predicts $\frac{72}{6} = 12$ years, very close.

Coach's Playbook

- 1 Identify the type.** Simple (linear) or compound (exponential)? Look for words such as "compounded".
- 2 Find the period rate.** Divide the annual rate by n , and multiply the years by n . Write both numbers before using the calculator.
- 3 Direction check.** Future value: multiply by the growth factor. Present value: divide by it. The future value must be larger than the present value.
- 4 Use EAR to compare.** Convert every offer to a yearly rate before ranking them.
- 5 Round at the end.** Keep full calculator precision until the final answer; round money to cents.

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- ◆ Use logarithms to solve for the time: take $\ln$ of both sides and divide.
- ◆ For quick estimates use the rule of 72, then confirm with the formula.
- ◆ Always write the rate as a decimal in the formula (0.05, not 5).
- ◆ Draw a timeline for multi-step problems: deposits and withdrawals at each date.

Common traps

1. Using the annual rate in every period without dividing by n . 2. Forgetting to multiply the exponent by n . 3. Multiplying instead of dividing when finding present value. 4. Treating a stated rate such as 12 percent compounded monthly as an effective rate of 12 percent.

Mistake Clinic

Spot the error 1

A student says: "1000 dollars at 6 percent compounded quarterly for 2 years is $1000(1.06)^8$." **Where is the error?** The 6 percent must be divided among the 4 quarters: the rate per quarter is 0.015. Correct: $1000(1.015)^8 \approx 1126.49$. The student's formula would give 1593.85, which is wildly high.

Spot the error 2

A student says: "To have 8000 dollars in 4 years at 5 percent, I need $8000(1 - 0.05 \cdot 4) = 6400$ dollars today." **Where is the error?** Compound growth runs backward by dividing by the growth factor, not by subtracting simple percents. Correct: $\frac{8000}{1.05^4} \approx 6581.62$ dollars.

Spot the error 3

A student says: "12 percent compounded monthly is the same as 12 percent per year." **Where is the error?** Compounding monthly earns interest on interest within the year. The EAR is $(1.01)^{12} - 1 \approx 12.68\%$, which is higher than 12 percent.

Real-World Connection

A student wants 6000 dollars for a car in 5 years. An account pays 4 percent compounded monthly. The needed deposit is $\frac{6000}{(1+0.04/12)^{60}} \approx 4914.02$ dollars. The earlier she starts saving, the less she has to deposit because time does the work.

Challenge Corner

You may take 5000 dollars today or receive 1200 dollars at the end of each of the next 5 years. If the money can earn 4 percent compounded annually, which is worth more today? **Solution.** Discount each payment: $\frac{1200}{1.04} + \frac{1200}{1.04^2} + \frac{1200}{1.04^3} + \frac{1200}{1.04^4} + \frac{1200}{1.04^5} \approx 1153.85 + 1109.47 + 1066.80 + 1025.77 + 986.31 = 5342.19$. The stream of payments is worth about **5342.19 dollars** today, so the payments are worth more than 5000 dollars.

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Practice

PRACTICE Interest, Present Value, and the Time Value of Money

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- What is the simple interest on 1200 dollars at 5 percent for 4 years?
 (A) 240 dollars (B) 1440 dollars (C) 60 dollars
 (D) 258.61 dollars
- Find the balance after 2 years on 800 dollars at 3 percent compounded annually.
 (A) 848.72 dollars (B) 848.00 dollars (C) 840.00 dollars
 (D) 824.00 dollars
- 1000 dollars is invested for 1 year at 8 percent. Which compounding schedule gives the largest balance?
 (A) Compounded annually (B) Compounded semiannually (C) Compounded monthly
 (D) Compounded daily
- How much must be deposited now at 6 percent compounded annually to have 5000 dollars in 3 years?
 (A) 5955.08 dollars (B) 4198.10 dollars (C) 4100.00 dollars
 (D) 4700.00 dollars
- What is the effective annual rate of 12 percent compounded quarterly?
 (A) 12.00% (B) 3.00% (C) 12.55% (D) 12.68%
- Which expression gives the balance on 1500 dollars at 8 percent compounded quarterly for 3 years?
 (A) $1500(1.02)^{12}$ (B) $1500(1.08)^3$ (C) $1500(1.02)^3$
 (D) $1500(1.08)^{12}$
- Use the rule of 72 to estimate how long it takes money to double at 4 percent per year.
 (A) 25 years (B) 12 years (C) 36 years (D) 18 years
- Solve $2000(1.05)^t = 3000$ for t to the nearest tenth of a year.
 (A) 8.3 years (B) 10.0 years (C) 7.0 years
 (D) 30.0 years
- 3000 dollars is invested at 5 percent compounded continuously for 6 years. What is the balance?
 (A) 4020.29 dollars (B) 4049.58 dollars (C) 3900.00 dollars
 (D) 4800.00 dollars
- Find the balance after 5 years if 4000 dollars is invested at 7 percent compounded annually. Round to the nearest cent.
- Find the balance after 2 years on 1500 dollars at 6 percent compounded monthly. Round to the nearest cent.
- How much must be deposited now at 5 percent compounded annually to have 20,000 dollars in 8 years? Round to the nearest cent.
- Find the effective annual rate of 6 percent compounded monthly. Give a percent to two decimal places.
- How many years does it take 1000 dollars to grow to 1800 dollars at 6 percent compounded annually? Round to the nearest tenth.
- A loan offer is 7.1 percent compounded annually. Another is 6.9 percent compounded monthly. Which has the lower effective cost, and what is the second offer's EAR to two decimal places?

30.3 Risk, Expected Value, and Decision Making

Use expected value, variance, and standard deviation to compare risky choices. Organize multi-stage uncertainty with probability trees and see why diversification lowers risk without lowering the average.

Why This Matters

Every money decision is a decision under uncertainty. Should a company launch a new product? Should a family buy insurance? Should an investor put everything into one stock or spread it around? Math gives a way to compare choices even when the outcome is unknown. The tools are expected value (the average payoff), variance and standard deviation (how spread out the payoffs are), and probability trees (a way to organize multi-stage uncertainty).

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Students usually struggle in three places. They look only at the average and ignore the risk. They forget to square the deviations when computing variance. And they multiply when they should add, or add when they should multiply, on a tree. By the end of this section you will know a clear routine for each, and you will understand why diversification, the idea of not putting all your eggs in one basket, actually works.

Core Ideas

Let X be an uncertain payoff, such as a one-year investment return, that takes values $x_1, \dots, x_k$ with probabilities $p_1, \dots, p_k$. Its **expected value** is the probability-weighted average, written μ .

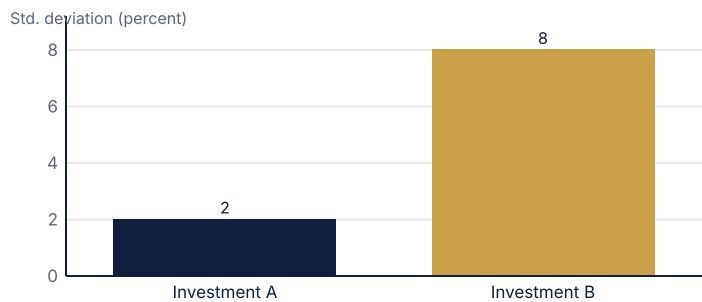
Expected Value, Variance, and Standard Deviation

$$\mu = E(X) = \sum x_i p_i, \quad \sigma^2 = \text{Var}(X) = \sum p_i (x_i - \mu)^2, \quad \sigma = \sqrt{\sigma^2}.$$

The expected value measures the center. The standard deviation σ measures typical distance from the center, in the same units as X . A larger σ means a riskier, more spread-out payoff.

Scaling: if you invest k times as much, both μ and σ are multiplied by k .

Two investments can have the same expected return and very different risk. Suppose Investment A returns 4 percent or 8 percent, each with probability one half, and Investment B returns -2 percent or 14 percent, each with probability one half. Both average 6 percent, but B swings far more widely.



Both investments average 6 percent, but B's standard deviation is four times A's, so B is much riskier.

Probability Trees

Multiply along a branch to get the probability of that whole path. Add the probabilities of all the paths that belong to the event. For a first stage with outcomes A_1, A_2 and a second stage with outcome E :

$$P(E) = P(A_1)P(E | A_1) + P(A_2)P(E | A_2).$$

Then $P(A_1 | E) = \frac{P(A_1)P(E | A_1)}{P(E)}.$

Diversification means spreading money across assets that do not move in lockstep. When one asset has a bad year, another may have a good year, so the ups and downs partly cancel. If two assets are independent, each with standard deviation σ , then a portfolio with half in each has variance

$$\left(\frac{1}{2}\right)^2 \sigma^2 + \left(\frac{1}{2}\right)^2 \sigma^2 = \frac{1}{2} \sigma^2,$$

so its standard deviation is $\frac{\sigma}{\sqrt{2}} \approx 0.707\sigma$. The expected return stays the same, but the risk drops by about 29 percent. That is the whole idea: same average, less variation.

Number of independent equal assets	Portfolio standard deviation
1	σ
2	0.707σ
4	0.5σ
9	0.333σ
100	0.1σ

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How do people choose between a safe option and a risky one? Expected value is a start, but it is not the whole story. A person who is **risk averse** will take a sure 900 dollars over a coin flip worth 2000 dollars or nothing (expected value 1000), because the possible loss of everything matters more than the extra 100 dollars on average. A person who is **risk neutral** simply picks the larger expected value. Neither is wrong; they weigh risk differently. What math adds is a clear statement of both numbers, so the choice is made with open eyes.

One simple way to compare investments with different means and spreads is to look at the return you get per unit of risk, $\frac{\mu}{\sigma}$. An investment with mean 6 percent and standard deviation 2 percent scores 3, while one with mean 6 percent and standard deviation 8 percent scores 0.75. This is not a perfect measure, but it turns the comparison into a single number and shows why equal averages do not mean equal quality.

Worked Examples

EXAMPLE 1: EXPECTED VALUE OF A PROJECT

A startup project costs nothing to enter. With probability 0.4 it earns 50 thousand dollars, and with probability 0.6 it loses 10 thousand dollars. Find the expected profit.

SOLUTION

$$E = 0.4(50) + 0.6(-10).$$

$$E = 20 - 6 = 14 \text{ thousand dollars.}$$

The project is worth 14,000 dollars on average, though any single try is either a gain of 50 thousand or a loss of 10 thousand.

EXAMPLE 2: COMPARING RISK

Investment A returns 4 percent or 8 percent with probability 0.5 each. Investment B returns -2 percent or 14 percent with probability 0.5 each. Compare mean and standard deviation.

SOLUTION

$$\text{Means: A: } 0.5(4) + 0.5(8) = 6. \text{ B: } 0.5(-2) + 0.5(14) = 6.$$

$$\text{Variance A: } 0.5(4 - 6)^2 + 0.5(8 - 6)^2 = 0.5(4) + 0.5(4) = 4, \text{ so } \sigma_A = 2.$$

$$\text{Variance B: } 0.5(-8)^2 + 0.5(8)^2 = 64, \text{ so } \sigma_B = 8.$$

Same average (6 percent), but B is four times as spread out. A cautious investor prefers A.

EXAMPLE 3: A PROBABILITY TREE

A company thinks the market is strong with probability 0.6 and weak with probability 0.4. A product succeeds with probability 0.8 in a strong market and 0.3 in a weak market. Find the probability of success.

SOLUTION

$$\text{Strong and success: } 0.6 \cdot 0.8 = 0.48.$$

$$\text{Weak and success: } 0.4 \cdot 0.3 = 0.12.$$

$$\text{Add the two paths: } P(\text{success}) = 0.48 + 0.12 = 0.60.$$

EXAMPLE 4: UPDATING WITH A TREE

Using Example 3, given that the product succeeded, what is the probability the market was strong?

SOLUTION

$$P(\text{strong} \mid \text{success}) = \frac{0.48}{0.60} = 0.8.$$

The share of successes coming from the strong branch is 80 percent.

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EXAMPLE 5: INSURANCE AS EXPECTED VALUE

A plan costs 120 dollars per year. Without it, there is a 5 percent chance of a 2000 dollar loss. Compare the premium to the expected loss.

SOLUTION

Expected loss: $0.05(2000) = 100$ dollars.

The premium (120) exceeds the expected loss (100) by 20 dollars, which pays the insurer's costs.

Many people still buy insurance because a 2000 dollar loss would be hard to absorb. They accept a lower average to remove a large risk.

EXAMPLE 6: MEAN AND SPREAD IN DOLLARS

A portfolio of 10,000 dollars earns -10% with probability 0.2, 5% with probability 0.5, and 20% with probability 0.3. Find the expected gain and standard deviation in dollars.

SOLUTION

Mean return: $0.2(-10) + 0.5(5) + 0.3(20) = -2 + 2.5 + 6 = 6.5\%$.

Variance: $0.2(-16.5)^2 + 0.5(-1.5)^2 + 0.3(13.5)^2 = 54.45 + 1.125 + 54.675 = 110.25$, so $\sigma = 10.5\%$.

In dollars: expected gain $0.065(10000) = 650$ and standard deviation $0.105(10000) = 1050$.

Outcome	Return	Probability	Return x Probability
Bad year	-10%	0.2	-2.0
Normal	5%	0.5	2.5
Good year	20%	0.3	6.0
Total		1.0	6.5

Coach's Playbook

- 1 List outcomes and probabilities.** Put them in a table. Probabilities must sum to 1.
 - 2 Find the mean.** Multiply each outcome by its probability and add.
 - 3 Find the spread.** Subtract the mean, **square**, weight by probability, add. Then take the square root for σ .
 - 4 Compare choices.** Higher mean is better for reward. Lower σ is better for safety. State both.
 - 5 Use a tree for stages.** Multiply along branches, add across branches that make your event.
- ◆ A variance of 0 means no risk at all: every outcome equals the mean.
 - ◆ The deviations $x_i - \mu$ always sum to zero when weighted, so you must square them.
 - ◆ In a tree, check that the probabilities leaving each node sum to 1.
 - ◆ Expected value tells you the long-run average, not what will happen on one try.

Common traps

1. Averaging the outcomes without using probabilities. 2. Forgetting to square the deviations, which produces a variance of 0. 3. Adding probabilities along a branch instead of multiplying, or multiplying across different branches instead of adding. 4. Choosing only by the highest mean without asking how risky it is.

Mistake Clinic

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Spot the error 1

A student says: "The returns are 1 percent, 5 percent, and 10 percent with probabilities 0.6, 0.3, and 0.1. The expected return is the average, 5.33 percent." **Where is the error?** The outcomes were not weighted by probability. Correct: $0.6(1) + 0.3(5) + 0.1(10) = 3.1$ percent. Most of the time the return is small, so the true mean is lower.

Spot the error 2

A student says: "Returns are 0 percent and 10 percent with probability 0.5 each, so the mean is 5. The variance is $0.5(0 - 5) + 0.5(10 - 5) = 0$, so there is no risk." **Where is the error?** The deviations were not squared. Correct: $0.5(25) + 0.5(25) = 25$, so $\sigma = 5$ percent. The deviations cancel unless squared.

Spot the error 3

A student says: "The probability of rain is 0.3 and the chance of being late is 0.5 if it rains and 0.1 if not. So $P(\text{late}) = 0.3(0.5) \cdot 0.7(0.1) = 0.0105$." **Where is the error?** The two paths to being late are separate ways the event can happen, so they are **added**, not multiplied: $0.15 + 0.07 = 0.22$.

Real-World Connection

A school fundraiser can hold an outdoor event that earns 3000 dollars if the weather is good and loses 500 dollars if not, or an indoor event that earns a sure 1200 dollars. If good weather has probability 0.7, the outdoor event has an expected value of $0.7(3000) + 0.3(-500) = 1950$ dollars. The outdoor event has the higher average, but the indoor event has no risk. The right choice depends on how much risk the group can tolerate.

Challenge Corner

Two independent assets have standard deviations 10 percent and 20 percent. A portfolio puts a fraction w in the first and $1 - w$ in the second. What value of w gives the smallest portfolio variance? **Solution.** $\text{Var}(w) = w^2(100) + (1 - w)^2(400) = 500w^2 - 800w + 400$. This parabola opens upward, so the minimum is at the vertex $w = \frac{800}{2 \cdot 500} = 0.8$. Then $\text{Var} = 100(0.64) + 400(0.04) = 64 + 16 = 80$, so $\sigma \approx 8.94\%$, which is less than both 10 percent and 20 percent. Diversification beats either asset alone.

Practice**PRACTICE Risk, Expected Value, and Decision Making**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- A bet wins 100 dollars with probability 0.2 and loses 20 dollars with probability 0.8. What is the expected value?
 (A) 4 dollars (B) 80 dollars (C) 20 dollars
 (D) -4 dollars
- Two investments both have a mean return of 7 percent. Their standard deviations are 3 percent and 9 percent. Which statement is true?
 (A) The second has the higher average
 (B) The first has the same average but less risk
 (C) The first is riskier because its standard deviation is smaller
 (D) They carry exactly the same risk
- A payoff is 2 or 6, each with probability 0.5. What is its variance?
 (A) 2 (B) 16 (C) 4 (D) 8

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4. The probability of rain is 0.3. If it rains, a bus is late with probability 0.5. If not, it is late with probability 0.1. What is the probability the bus is late?
 (A) 0.60 (B) 0.22 (C) 0.15 (D) 0.015
5. A policy has a 3 percent chance of paying 4000 dollars. What is the expected payout?
 (A) 4000 dollars (B) 1200 dollars (C) 12 dollars (D) 120 dollars
6. Why does diversification lower the risk of a portfolio?
 (A) It raises the expected return of every asset
 (B) It guarantees that no asset ever loses money
 (C) Assets do not always rise and fall together, so gains in some offset losses in others
 (D) It makes all assets move in the same direction
7. Investment A returns 10 percent or 2 percent with probability 0.5 each. Investment B returns 12 percent with probability 0.3 and 3 percent with probability 0.7. Which statement about expected return is true?
 (A) B has the higher expected return
 (B) They have the same expected return
 (C) Neither can be compared without the standard deviations
 (D) A has the higher expected return
8. An 8000 dollar investment has a mean return of 5 percent and a standard deviation of 10 percent. What is the standard deviation of the dollar gain?
 (A) 80 dollars (B) 800 dollars (C) 400 dollars (D) 8000 dollars
9. A launch succeeds with probability 0.5. The probability that the market is strong and the launch succeeds is 0.3. The market is strong with probability 0.4. Given that the launch succeeded, what is the probability the market was strong?
 (A) 0.3 (B) 0.75 (C) 0.6 (D) 0.12
10. A raffle ticket pays 50 dollars with probability $\frac{1}{10}$ and 10 dollars with probability $\frac{3}{10}$, and nothing otherwise. Find the expected payout in dollars.
11. A return is -4 percent, 2 percent, or 8 percent with probabilities 0.25, 0.5, and 0.25. Find the mean and the variance.
12. Machine A makes 60 percent of the parts and has a 2 percent defect rate. Machine B makes 40 percent and has a 5 percent defect rate. What is the probability that a randomly chosen part is defective?
13. Using the factory above, a part is found defective. What is the probability it came from Machine B? Give a fraction.
14. Two independent assets each have a standard deviation of 12 percent. A portfolio puts half its money in each. Find the portfolio's standard deviation to two decimal places.
15. Option X pays 700 dollars for sure. Option Y pays 1500 dollars with probability p and nothing otherwise. What is the smallest value of p for which Option Y has an expected value at least as large as Option X? Give a fraction.

30.4 Quantitative Speed and Precision

Estimate before you calculate, use scientific notation and orders of magnitude, convert units by canceling, solve Fermi problems, and check answers quickly under time pressure.

Why This Matters

In school, you get plenty of time and a calculator. In a meeting, a lab, or an interview, you often need a number now, and a good estimate beats a perfect answer that arrives too late. Quantitative speed is not about being fast at arithmetic. It is about knowing roughly how big an answer should be, catching nonsense before it reaches a report, and tracking units so you do not add meters to seconds.

Students usually get stuck for three reasons: they trust a calculator without checking the size of the result, they lose track of powers of ten, and they mix up units while converting. This section teaches four habits that fix all three: estimate first, use scientific notation, carry your units, and check every answer in at least one independent way. Engineers, scientists, analysts, and business managers use these habits every day.

Core Ideas

Estimation replaces each number with a nearby friendly number, usually one or two nonzero digits, then computes with the friendly numbers. The goal is the right size, not the exact digits. To estimate 37×52 , round to $40 \times 50 = 2000$. You can also trap the answer between bounds: $30 \times 50 = 1500$ is too low and $40 \times 60 = 2400$ is too high.

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Bounds, estimate, and exact value for 37×52 . The estimate 2000 is within 4 percent of the exact value 1924.

Scientific Notation and Order of Magnitude

A number is in scientific notation when written as $a \times 10^n$ with $1 \leq a < 10$ and n an integer.

$$(a \times 10^m)(b \times 10^n) = ab \times 10^{m+n}, \quad \frac{a \times 10^m}{b \times 10^n} = \frac{a}{b} \times 10^{m-n}.$$

The **order of magnitude** of a quantity is its power of ten. Saying one thing is "three orders of magnitude" larger than another means about $10^3 = 1000$ times larger. Multiply the leading numbers, add the exponents, then renormalize.

Prefix	Symbol	Power of ten	Example
kilo	k	10^3	1 kilometer = 1000 meters
centi	c	10^{-2}	100 centimeters = 1 meter
milli	m	10^{-3}	1000 millimeters = 1 meter
mega	M	10^6	1 megabyte = 1,000,000 bytes
giga	G	10^9	1 gigabyte = 1,000,000,000 bytes

Unit analysis (also called dimensional analysis) treats units like algebra symbols that cancel. To convert, multiply by a fraction equal to 1 whose top and bottom are the same quantity in different units, arranged so the unwanted unit cancels.

Unit Conversion Method

Write the starting quantity with its units. Multiply by conversion fractions equal to 1, such as $\frac{1000 \text{ m}}{1 \text{ km}}$ or $\frac{1 \text{ h}}{3600 \text{ s}}$. Cancel units until only the target unit remains.

$$72 \frac{\text{km}}{\text{h}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} = 20 \frac{\text{m}}{\text{s}}.$$

If the units do not cancel to the target, the setup is wrong.

A **Fermi problem** is a question about a quantity that is hard to measure exactly, such as how many heartbeats occur in a lifetime. You break it into factors that you can estimate, multiply them, and aim for the correct order of magnitude. Errors above and below tend to cancel, so a chain of rough estimates is often surprisingly close.

How accurate must an estimate be? That depends on the decision. If you are checking whether a calculator result has a misplaced decimal point, being within a factor of 2 is plenty. If you are ordering materials for a building, you may need to be within a few percent, and you should round **up** so you do not run short. Always ask what error you can afford before you decide how much work an estimate deserves.

Significant figures help you avoid false precision. If a measurement is given as 3.2 meters, an answer of 7.384591 square meters claims accuracy that the data cannot support. Report about as many significant digits as the least precise input. In a Fermi problem, one or two digits is enough, because every input is itself a rough guess.

Finally, build a checking routine and use it every time. First, compare the size of the answer to your estimate. Second, check the units. Third, ask whether the sign, the direction, and the range make sense. Fourth, substitute the answer back into the original problem if you can. A check that takes ten seconds can save a whole problem.

Worked Examples

EXAMPLE 1: ESTIMATE, THEN COMPUTE

Estimate 4872×0.52 , then find the exact value.

SOLUTION

Round: $4872 \approx 5000$ and $0.52 \approx 0.5$.

Estimate: $5000 \times 0.5 = 2500$.

Exact: $4872 \times 0.52 = 2533.44$.

The estimate is within about 1 percent. If a calculator had shown 25,334.4 or 253.34, you would know a decimal slipped.

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EXAMPLE 2: SCIENTIFIC NOTATION

Compute (a) $(3 \times 10^5)(4 \times 10^{-2})$ and (b) $\frac{6 \times 10^8}{2 \times 10^3}$. Write each in scientific notation.

SOLUTION

(a) Multiply leading numbers: $3 \cdot 4 = 12$. Add exponents: $5 + (-2) = 3$. So 12×10^3 , which is 1.2×10^4 .

(b) Divide leading numbers: $6 \div 2 = 3$. Subtract exponents: $8 - 3 = 5$. So 3×10^5 .

EXAMPLE 3: UNIT CONVERSION

Convert 72 kilometers per hour to meters per second.

SOLUTION

$$72 \frac{\text{km}}{\text{h}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} = \frac{72000 \text{ m}}{3600 \text{ s}}.$$

= 20 meters per second.

Shortcut worth remembering: divide km/h by 3.6 to get m/s.

EXAMPLE 4: A FERMI ESTIMATE

About how many times does a heart beat in an 80-year life, at 70 beats per minute?

SOLUTION

Beats per year: $70 \cdot 60 \cdot 24 \cdot 365$ (minutes per hour, hours per day, days per year).

Over 80 years: $70 \cdot 60 \cdot 24 \cdot 365 \cdot 80 = 2,943,360,000$.

Rounded: about 3×10^9 , or three billion beats.

Quick check by rounding: $70 \cdot 60 \approx 4000$ per hour, $\times 24 \approx 10^5$ per day, $\times 365 \approx 4 \times 10^7$ per year, $\times 80 \approx 3 \times 10^9$.

EXAMPLE 5: RATES AND UNITS

A printer prints 24 pages per minute and each page costs 1.5 cents of ink and paper. For a 5000-page job, find (a) the cost in dollars and (b) the time in hours.

SOLUTION

$$(a) 5000 \text{ pages} \cdot \frac{1.5 \text{ cents}}{1 \text{ page}} = 7500 \text{ cents} = 75 \text{ dollars}.$$

$$(b) 5000 \text{ pages} \cdot \frac{1 \text{ min}}{24 \text{ pages}} \approx 208.3 \text{ min, and } \frac{208.3}{60} \approx 3.47 \text{ hours}.$$

Notice how the units "pages" cancel, leaving cents or minutes.

EXAMPLE 6: MENTAL MATH AND A CHECK

Find 18 percent of 2450 quickly, and check the result.

SOLUTION

Break it up: 10 percent is 245. 8 percent is $245 \times 0.8 = 196$.

Add: $245 + 196 = 441$.

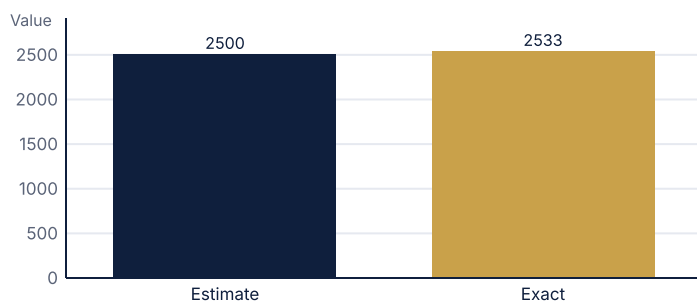
Check by estimation: 20% of 2500 is 500, and 18 percent of 2450 should be a bit smaller. 441 fits.

Second check: $441 \div 2450 = 0.18$.

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Rounding the numbers in Example 1 gives an estimate within about 1 percent of the exact answer.

Coach's Playbook

- 1 Estimate first.** Before using a calculator, write a rough answer. Compare the final answer to it.
 - 2 Write units on every number.** Carry them through each line. They tell you what operation makes sense.
 - 3 Use powers of ten.** Convert to scientific notation, handle the leading numbers and exponents separately, then recombine.
 - 4 Check a second way.** Reverse the operation, round differently, or plug the answer back into the problem.
 - 5 Know your reasonableness checks.** A mean must lie between the smallest and largest value. A percent decrease cannot exceed 100. A probability lies between 0 and 1.
- ◆ Round to one or two significant digits and keep track of whether you rounded up or down, so you know the direction of the error.
 - ◆ Use friendly fractions: 25 percent is one fourth, 12.5 percent is one eighth, 33 percent is one third.
 - ◆ Under time pressure, do the part you can do exactly (powers of ten, units), then estimate the rest.
 - ◆ Say the units aloud: "dollars per hour times hours gives dollars."

Common traps

1. Multiplying exponents instead of adding them when multiplying powers of ten. 2. Converting speeds the wrong direction (multiplying by 3.6 instead of dividing, or the reverse). 3. Ignoring units, so that hours and minutes are mixed in one product. 4. Rounding both numbers in the same direction so the error accumulates.

Mistake Clinic

Spot the error 1

A student says: " $(4 \times 10^3)(5 \times 10^4) = 20 \times 10^{12} = 2 \times 10^{13}$." **Where is the error?** The exponents were multiplied. When you multiply powers of ten, you add the exponents: $3 + 4 = 7$. Correct: $20 \times 10^7 = 2 \times 10^8$.

Spot the error 2

A student says: "To convert 120 km/h to m/s, multiply by 3.6, so the speed is 432 m/s." **Where is the error?** A speed of 432 meters per second is faster than a jet. Reasonableness check failed. Since $1 \text{ m/s} = 3.6 \text{ km/h}$, dividing by 3.6 turns km/h into m/s. Correct: $\frac{120}{3.6} \approx 33.3 \text{ m/s}$.

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Spot the error 3

A student says: "A worker earns 15 dollars per hour and works 90 minutes, so the pay is $15 \times 90 = 1350$ dollars." **Where is the error?** The units are mismatched: dollars per hour times minutes is not dollars. Convert first: $90 \text{ min} = 1.5 \text{ h}$, so the pay is $15 \times 1.5 = 22.50$ dollars.

Real-World Connection

A tip of 18 percent on a 47 dollar meal: 10 percent is 4.70 dollars, 8 percent is about 3.76 dollars, and together that is about 8.46 dollars. A quick estimate of 20 percent of 50 dollars is 10 dollars, so 8.46 dollars is reasonable. The same break-it-up method works for taxes, discounts, and fuel costs.

Challenge Corner

A town of 25,000 people uses 150 liters of water per person per day. (a) How many cubic meters per year does the town use? (b) An Olympic pool holds about 2500 cubic meters. About how many pools is that? **Solution.** (a) $25000 \cdot 150 \cdot 365 = 1,368,750,000$ liters, and since 1000 liters is 1 cubic meter, that is 1,368,750 cubic meters, about 1.4×10^6 . (b) $\frac{1,368,750}{2500} = 547.5$, so about **550 pools** per year.

Practice**PRACTICE Quantitative Speed and Precision**

Show your work on paper. Use a calculator only when a problem says so, or if it is allowed at your level.

- Which is the best estimate of 49×31 ?
(A) 1500 (B) 150 (C) 15000 (D) 800
- Compute $(2 \times 10^4)(3 \times 10^3)$.
(A) 6×10^7 (B) 5×10^7 (C) 6×10^{12} (D) 6×10^1
- Compute $\frac{9 \times 10^6}{3 \times 10^2}$.
(A) 3×10^3 (B) 3×10^8 (C) 3×10^4 (D) 27×10^8
- How many seconds are in 5 hours?
(A) 300 (B) 3600 (C) 1800 (D) 18000
- A cyclist rides at 15 meters per second. What is that speed in kilometers per hour?
(A) 4.17 (B) 54 (C) 900 (D) 15000
- Which is closest to the number of minutes in one week?
(A) 10^4 (B) 10^3 (C) 10^5 (D) 10^6
- A car gets 25 miles per gallon. About how many gallons does it use on a 150-mile trip?
(A) 3750 (B) 0.17 (C) 6 (D) 175
- Estimate $\frac{7.9 \times 12.1}{3.9}$.
(A) 2.4 (B) 240 (C) 96 (D) 24
- A person takes about 16 breaths per minute. Which is the best estimate of the number of breaths in one day?
(A) 2×10^4 (B) 2×10^3 (C) 2×10^5 (D) 2×10^6
- Estimate 6100×0.49 to the nearest thousand without a calculator.
- Compute $(5 \times 10^{-3})(8 \times 10^7)$ and write the answer in scientific notation.
- Convert 90 miles per hour to feet per second. (There are 5280 feet in a mile.)
- A school has 1200 students. Each uses 3 sheets of paper per day for 180 school days. A ream has 500 sheets. How many reams does the school use?
- A tap delivers 2.5 liters per minute. How long does it take to fill a tank of 1 cubic meter (1000 liters)? Give the answer in hours and minutes.

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15. A jacket costs 80 dollars. It is discounted 15 percent, and then 8 percent sales tax is added to the sale price. What is the final cost in dollars?

Answers and Explanations

Complete solutions for every practice problem, numbered to match. Read the explanation even when your answer is right.

30.1 Percent Growth, Margins, and Break-Even Analysis

PRACTICE · PERCENT GROWTH, MARGINS, AND BREAK-EVEN ANALYSIS

1. C. $\frac{290-250}{250} = \frac{40}{250} = 0.16$. Choice B divides by the new price instead of the old.
2. B. $1.10^2 = 1.21$. Choice A just adds the percents, ignoring that year 2 grows from the larger base.
3. A. $63 \div 0.70 = 90$. Choice B multiplies by 1.30, which treats 30 percent as an increase of the sale price.
4. D. Profit is 20 dollars and margin uses the price: $\frac{20}{100} = 20\%$. Choice A is the markup, which divides by the cost.
5. C. Set $7x - 2100 = 0$, so $x = 300$. Choice A is the fixed cost, not a quantity.
6. A. Break-even: $500 + 2x = 7x$, so $x = 100$. Revenue is $7(100) = 700$. Choice B (100) is the quantity, not the revenue.
7. A. The new price is $1.2p$. To return to p , multiply by $\frac{1}{1.2} = 0.8333$, a decrease of 16.7%. Choice B (20%) ignores the larger base.
8. C. Cost is 60 percent of the price, so $p = 36 \div 0.60 = 60$. Choice A is a 40 percent markup on cost, a different goal.
9. B. Break-even is $\frac{900}{7} \approx 128.6$. At 128 items the company still has a small loss, so round up to 129. Choice A (128) rounds down.
10. 13%. $\frac{5650-5000}{5000} = \frac{650}{5000} = 0.13$.
11. 400 dollars. $340 \div 0.85 = 400$.
12. 37.5%. Price = $45(1.6) = 72$. Profit = 27. Margin = $\frac{27}{72} = 0.375$.
13. 113. Break-even $\frac{1350}{12} = 112.5$. Since you cannot sell half an item and 112 would still lose money, round up to 113.
14. 11.1%. $1.05 \times 1.08 \times 0.98 = 1.11132$, an increase of about 11.1 percent.
15. 10 dollars. Break-even $150 = \frac{900}{p-4}$ gives $p - 4 = 6$, so $p = 10$.

30.2 Interest, Present Value, and the Time Value of Money

PRACTICE · INTEREST, PRESENT VALUE, AND THE TIME VALUE OF MONEY

1. A. $I = Prt = 1200(0.05)(4) = 240$. Choice B is the final balance, not the interest. Choice D is the compound interest.
2. A. $800(1.03)^2 = 800(1.0609) = 848.72$. Choice C is simple interest, which ignores interest earned on interest.
3. D. More frequent compounding means more interest on interest, so daily is largest. The differences are small but real.
4. B. Present value divides by the growth factor: $\frac{5000}{1.06^3} \approx 4198.10$. Choice A multiplies instead, which is the future value of 5000 dollars.
5. C. $(1.03)^4 - 1 \approx 0.1255$. Choice D is the EAR for monthly compounding.
6. A. Rate per quarter $0.08/4 = 0.02$, and $4 \cdot 3 = 12$ quarters. Choice B ignores the quarterly compounding. Choice D forgets to divide the rate by 4.
7. D. $\frac{72}{4} = 18$ years. Choice A is a typical guess from $100 \div 4$, which is the answer for simple interest on the original deposit.
8. A. $(1.05)^t = 1.5$, so $t = \frac{\ln 1.5}{\ln 1.05} \approx 8.31$. Choice B comes from dividing 50 percent growth by 5 percent per year, which is correct only for simple interest.
9. B. $A = 3000e^{0.05 \cdot 6} = 3000e^{0.3} \approx 4049.58$. Choice A is annual compounding, which is a little smaller. Choice C is simple interest.
10. 5610.21 dollars. $4000(1.07)^5 \approx 4000(1.40255) = 5610.21$.
11. 1690.74 dollars. Rate per month 0.005, 24 months: $1500(1.005)^{24} \approx 1690.74$.
12. 13536.79 dollars. $\frac{20000}{1.05^5} \approx \frac{20000}{1.47746} = 13536.79$.
13. 6.17%. $(1.005)^{12} - 1 \approx 0.061678$.
14. 10.1 years. $1.06^t = 1.8$, so $t = \frac{\ln 1.8}{\ln 1.06} \approx 10.09$.
15. 7.1 percent compounded annually is cheaper. The monthly offer has EAR $(1 + 0.069/12)^{12} - 1 \approx 7.12\%$, which is higher than 7.10 percent.

30.3 Risk, Expected Value, and Decision Making

PRACTICE · RISK, EXPECTED VALUE, AND DECISION MAKING

1. A. $0.2(100) + 0.8(-20) = 20 - 16 = 4$. Choice B subtracts 20 from 100 without using the probabilities. Choice D has the sign wrong.
2. B. Equal means; the smaller standard deviation means less spread, so less risk. Choice C reverses the meaning of standard deviation.
3. C. Mean is 4. Variance = $0.5(2 - 4)^2 + 0.5(6 - 4)^2 = 0.5(4) + 0.5(4) = 4$. Choice A (2) is the standard deviation, the square root.
4. B. Add the two paths: $0.3(0.5) + 0.7(0.1) = 0.15 + 0.07 = 0.22$. Choice B adds 0.5 and 0.1 directly, ignoring the weights.
5. D. $0.03(4000) = 120$. Choice C moves the decimal point the wrong way.
6. C. Imperfectly related assets partly cancel each other's swings, so the total varies less. It does not guarantee profit, and it does not change any single asset's mean.
7. D. A: $0.5(10) + 0.5(2) = 6$. B: $0.3(12) + 0.7(3) = 3.6 + 2.1 = 5.7$. The large 12 percent outcome is unlikely, so it does not dominate.
8. B. $0.10(8000) = 800$. Choice C (400) is the expected gain, not the spread.
9. C. $P(\text{strong} | \text{success}) = \frac{0.3}{0.5} = 0.6$. Choice B (0.75) is $\frac{0.3}{0.4}$, the chance of success given a strong market, which is the reverse question.
10. 8 dollars. $50 \cdot \frac{1}{10} + 10 \cdot \frac{3}{10} = 5 + 3 = 8$.

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11. Mean 2 percent, variance 18. Mean: $-1 + 1 + 2 = 2$. Variance: $0.25(-6)^2 + 0.5(0)^2 + 0.25(6)^2 = 9 + 0 + 9 = 18$. The standard deviation is about 4.24 percent.
12. 0.032 . $0.6(0.02) + 0.4(0.05) = 0.012 + 0.020 = 0.032$.
13. $\frac{5}{8}$ (0.625). $P(B \mid \text{defective}) = \frac{0.020}{0.032} = \frac{5}{8}$.
14. 8.49 percent. Variance $= 0.25(144) + 0.25(144) = 72$, so $\sigma = \sqrt{72} \approx 8.49$.
15. $\frac{7}{15}$. Need $1500p \geq 700$, so $p \geq \frac{700}{1500} = \frac{7}{15} \approx 0.467$.

30.4 Quantitative Speed and Precision

PRACTICE · QUANTITATIVE SPEED AND PRECISION

1. A. $49 \approx 50$ and $31 \approx 30$, so $50 \times 30 = 1500$. Choice B (150) is off by a factor of 10.
2. A. Multiply $2 \cdot 3 = 6$ and add exponents: $4 + 3 = 7$. Choice B adds the leading numbers. Choice C multiplies the exponents.
3. C. $9 \div 3 = 3$ and subtract exponents: $6 - 2 = 4$. Choice B adds the exponents instead of subtracting.
4. D. $5 \cdot 60 \cdot 60 = 18000$. Choice A (300) is the number of minutes.
5. B. $15 \cdot 3600 = 54000$ meters per hour, which is 54 kilometers per hour (multiply by 3.6). Choice A divides by 3.6, the wrong direction.
6. A. $7 \cdot 24 \cdot 60 = 10080 \approx 10^4$. Choice B is too small by a factor of 10.
7. C. Miles divided by miles per gallon: $150 \div 25 = 6$ gallons. Choice A multiplies, producing units of miles squared per gallon.
8. D. Round to $\frac{8 \times 12}{4} = 24$. The exact value is about 24.5. Choice A and B are off by powers of ten.
9. A. $16 \cdot 60 \cdot 24 = 23040 \approx 2 \times 10^4$. Choice B is too small by a factor of 10.
10. 3000. Round to $6000 \times 0.5 = 3000$. The exact value is 2989, so the estimate is very close.
11. 4×10^5 . $5 \cdot 8 = 40$ and $-3 + 7 = 4$, so $40 \times 10^4 = 4 \times 10^5$.
12. 132 feet per second. $90 \cdot \frac{5280}{3600} = 132$.
13. 1296 reams. Sheets: $1200 \cdot 3 \cdot 180 = 648000$. Reams: $648000 \div 500 = 1296$.
14. 6 hours 40 minutes. $1000 \div 2.5 = 400$ minutes, and $400 = 6 \cdot 60 + 40$.
15. 73.44 dollars. $80(0.85)(1.08) = 68(1.08) = 73.44$. Quick estimate: 80 minus 12 is 68, plus about 5.40 is 73.40.

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