



SCHOLAR COACH

Boutique Academic Coaching & Tutoring

THE DIGITAL SAT · READING AND WRITING · MATH

SAT English and Math Mastery.

Strategy guides for every question type, original math lessons for every tested skill, and three full-length practice exams with complete answers and explanations.

INSIDE

Words in Context · Inferences · Command of Evidence · Central Ideas and Details · Function of a Sentence · Algebra · Advanced Math · Problem-Solving and Data Analysis · Geometry and Trigonometry · Three Full-Length Practice Exams with Explanations



Table of Contents.

Reading and Writing strategy, every math skill, and three full-length practice exams.

Contents

Part One teaches the Reading and Writing question types. Part Two teaches every math skill. Part Three holds three full-length practice exams with explanations.

PART ONE: SAT READING AND WRITING

Words in Context: The Complete Method	5
Ch 1 The Philosophy of SAT Lexical Testing	5
Ch 2 The Core Taxonomy of Contextual Vocabulary Questions	5
Ch 3 The Five Structural Equations of Textual Constraints	6
Ch 4 The Core Protocol for Words in Context	6
Ch 5 Advanced Deep-Dive — Secondary and Tertiary Meanings	7
Ch 6 Step-by-Step Walkthroughs of Master-Class Sample Questions	8
Ch 7 Practice Problem Set (10 Complete SAT-Style Items)	11
Ch 8 Answer Keys and Comprehensive Explanations	14
Inferences	18
Ch 1 The Core Architecture of SAT Inferences	18
Ch 2 The Algebraic Inference Protocol	18
Ch 3 Walkthroughs of Real SAT-Style Sample Items	19
Ch 4 The 750+ Elimination Playbook for Inferences	20
Command of Evidence	22
Ch 1 Textual Evidence Selection ("Which quotation best supports...")	22
Ch 2 Quantitative Evidence ("Which finding supports or weakens...")	23
Ch 3 Student Notes Questions (Rhetorical Synthesis)	24
Ch 4 Elite Elimination Strategies for 750+ Scorers	25
Central Ideas and Details	27
Ch 1 The Anatomy of Central Idea, Purpose, and Summary Questions	27
Ch 2 The Core Blueprints and Structural Protocols	27
Ch 3 Walkthroughs of Master-Class Sample Items	29
Ch 4 Elite Elimination Strategies for 750+ Scorers	30
Words in Context: Advanced Practice	32
Ch 1 The Core Architecture of Words in Context Items	32
Ch 2 The Five-Step Algebraic Extraction Protocol	32
Ch 3 Decoding Official SAT Vocabulary Jargon	33
Ch 4 Walkthroughs of Real SAT-Style Sample Items	33
Ch 5 Top Mistakes Made by High-Scoring Students	35
Function of a Sentence	36
Ch 1 The Core Architecture of the SAT Paragraph	36
Ch 2 The Step-by-Step Elimination Protocol	37
Ch 3 Decoding the Test Maker's Jargon	37
Ch 4 Walkthroughs of Real SAT-Style Sample Items	38
Ch 5 Top Mistakes Made by High-Scoring Students	40

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

PART TWO: SAT MATH

How the Math Section Works	42
Algebra	44
M1 Linear Equations in One Variable	44
M2 Linear Functions	48
M3 Linear Equations in Two Variables	52
M4 Systems of Linear Equations	55
M5 Linear Inequalities in One or Two Variables	59
Key Answers and Explanations	62
Advanced Math	65
M6 Equivalent Expressions	65
M7 Nonlinear Equations and Systems	68
M8 Nonlinear Functions	72
Key Answers and Explanations	76
Problem-Solving and Data Analysis	78
M9 Ratios, Rates, Proportional Relationships, and Units	78
M10 Percentages	82
M11 One-Variable Data: Distributions and Measures of Center and Spread	86
M12 Two-Variable Data: Models and Scatterplots	90
M13 Probability and Conditional Probability	95
M14 Inference from Sample Statistics and Margin of Error	98
M15 Evaluating Statistical Claims: Observational Studies and Experiments	101
Key Answers and Explanations	104
Geometry and Trigonometry	107
M16 Area and Volume	107
M17 Lines, Angles, and Triangles	110
M18 Right Triangles and Trigonometry	114
M19 Circles	118
Key Answers and Explanations	123
PART THREE: FULL-LENGTH PRACTICE EXAMS	
Practice Exam 1	125
Exam 1 Reading and Writing: Module 1	125
Exam 1 Reading and Writing: Module 2	132
Exam 1 Math: Module 1	139
Exam 1 Math: Module 2	143
Key Practice Exam 1: Answers and Explanations	145
Practice Exam 2	157
Exam 2 Reading and Writing: Module 1	157
Exam 2 Reading and Writing: Module 2	164
Exam 2 Math: Module 1	171
Exam 2 Math: Module 2	174
Key Practice Exam 2: Answers and Explanations	176
Practice Exam 3	189
Exam 3 Reading and Writing: Module 1	189
Exam 3 Reading and Writing: Module 2	196
Exam 3 Math: Module 1	203
Exam 3 Math: Module 2	206
Key Practice Exam 3: Answers and Explanations	209

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



Welcome to SAT Mastery.

Everything you need in one place: the methods, the lessons, the practice, and the exams.

Welcome How to Use This Book

One book for the whole digital SAT: strategy guides for Reading and Writing, original math lessons for every tested topic, and three full-length practice exams with complete explanations.

What is inside

Part One, SAT Reading and Writing. Step-by-step methods for the highest-scoring question types: Words in Context, Inferences, Command of Evidence, Central Ideas and Details, and Function of a Sentence. Each guide teaches the structure of the question, a repeatable protocol, worked walkthroughs, practice items, and explanations.

Part Two, SAT Math. Nineteen original chapters that cover every skill in the four math domains: Algebra, Advanced Math, Problem-Solving and Data Analysis, and Geometry and Trigonometry. Each chapter has core ideas, worked examples, strategy, common traps, and eight practice problems with answers.

Part Three, Practice Exams. Three full-length tests built in the digital format, each with two Reading and Writing modules and two Math modules, plus an answer key, a skill map, and an explanation for every question.

Digital SAT section	Modules	Questions	Time
Reading and Writing	2	27 per module (54 total)	32 minutes per module
Math	2	22 per module (44 total)	35 minutes per module

A routine that works

- 1. Learn the method.** Read the guide for a question type, then work its practice items before reading the explanations.
- 2. Drill the skill.** In math, work every example yourself, then do the practice problems with the on-screen calculator open.
- 3. Test under real conditions.** Take each practice exam in one sitting, module by module, using the module times above.
- 4. Review with the skill map.** After each exam, use the skill map to find the topics that cost you points and return to that chapter.

About adaptive testing

On test day, the second module of each section adapts to your performance in the first. In these printed exams, Module 1 mixes easy, medium, and hard questions, and Module 2 is set at the harder level so you practice the questions that decide top scores.

SAT is a registered trademark of the College Board, which is not affiliated with this book.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



Words in Context: The Complete Method

To achieve a perfect score on the digital SAT Reading and Writing section, you must completely abandon the traditional concept of a "vocabulary test." The College Board does not award points for rote memorization of dictionary entries.

Ch 1 The Philosophy of SAT Lexical Testing

To achieve a perfect score on the digital SAT Reading and Writing section, you must completely abandon the traditional concept of a "vocabulary test." The College Board does not award points for rote memorization of dictionary entries. Instead, the Craft and Structure: Words in Context domain evaluates your ability to perform real-time text engineering and semantic analysis.

The question stems consistently use three explicit formulas:

As used in the passage, what does the word X "most nearly mean"?

Which choice completes the text with the most "logical and precise" phrase?

The author's use of the phrase X primarily serves to convey what [tonal/figurative] implication?

To master these items, you must view every words-in-context question as an algebraic system. The missing or targeted word is a precise variable (x), held in place by surrounding semantic constraints, tonal polarities, and syntactic structures. The text itself provides the exact mathematical proofs required to isolate the correct option. This textbook section breaks down the entire taxonomy of these questions, detailing the structures, traps, and execution frameworks necessary for a flawless performance.

Ch 2 The Core Taxonomy of Contextual Vocabulary Questions

The College Board splits Words in Context items into four specific functional categories. Every question you encounter on test day will match one of these structural blueprints.

SAT Words in Context Taxonomy

Secondary Definitions

Precise Definitions

Connotation & Tone

Figurative Language

1. The Secondary/Tertiary Definition Pattern

The test makers intentionally choose common, everyday words (e.g., nature, dynamic, scale, document, compromise) and use them in their rare, academic, or professional capacities. If you read the sentence and apply the word's primary definition (the one used in daily conversation), the sentence will become logically incoherent.

2. The Precise Definition (Domain-Specific Register) Pattern

These questions evaluate whether you can distinguish between near-synonyms based on the highly specific academic domain of the text (such as organic chemistry, macroeconomics, or literary criticism). While two answer choices might both mean "to change," one will be exclusively reserved for changing scientific data (rectify), while the other is reserved for changing an artistic medium (mutate).

3. The Connotation and Tone Implication Pattern

Here, the literal definition of the word is secondary to its emotional or evaluative charge. The text establishes a rigid positive, negative, or clinical perspective, and the correct answer must perfectly align with that psychological frequency without overshooting into extreme bias.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

4. The Figurative Language Interpretation Pattern

These questions evaluate non-literal prose, often found in 19th-century historical documents or literary humanities passages. The author uses a metaphor or physical analogy to represent an abstract intellectual or socio-political concept. Your job is to strip away the poetic imagery and map it directly to a cold, analytical statement.

Ch 3 The Five Structural Equations of Textual Constraints

Your ear is your greatest enemy on the SAT. If you read a sentence and "plug in" the answer choices to see which one "sounds right," you are falling directly into a trap designed by psychometricians. Decoy choices are meticulously engineered to sound grammatically pleasant and natural.

To bypass the ear and engage the analytical mind, you must identify which of the Five Structural Equations governs the passage.

Equation 1: The Punctuation / Appositive Equivalence ($A = x$)

The text introduces a term or blank and immediately follows it with a punctuation mark or grammatical structure that introduces a direct definition.

The Indicators: Colons (:), Em-dashes (—), Parentheses (()), or Appositive commas (, which is ,).

The Strategy: The text following the punctuation is the exact definition of the target variable. Locate it, strip away the fluff, and find its direct synonym in the options.

Equation 2: The Reversal / Polarity Shift ($A \neq x$)

The text establishes a specific baseline condition or behavior, then uses a structural pivot to state that the blank or target word is the absolute opposite of that baseline.

◆ **The Indicators:** However, conversely, although, while, whereas, despite, nonetheless, far from being.

◆ **The Strategy:** Identify the baseline trait. The correct answer must be the exact logical inversion or negation of that trait.

Equation 3: The Cross-Sentence Synchronic Thread ($A_1 \equiv A_2$)

The passage contains two distinct sentences that describe the exact same process or action. Sentence 1 will use an explicit descriptive phrase; Sentence 2 will contain the target word or blank.

The Indicators: Indeed, furthermore, tracking this trend, in doing so, subsequently.

The Strategy: The target word must act as a precise linguistic shorthand for the long descriptive phrase used in the parallel sentence.

Equation 4: The Internal Cause-and-Effect Matrix ($A \rightarrow x$)

The targeted word or blank describes an action or state that is the inescapable physical or logical consequence of an explicit cause stated earlier in the phrase.

The Indicators: Because, by doing so, thereby, since, consequently, as a result.

The Strategy: Ask yourself: What is the mandatory, physical outcome of the cause provided? If the cause is "extreme deep-sea atmospheric pressure," the mechanical outcome (x) must mean compressed or fortified, not simply "dense."

Equation 5: The Domain Match / Collocation Constraint

The targeted blank must form a natural, idiomatically precise pairing with a specific noun or scientific process mentioned in the phrase.

The Indicators: Academic subjects, data tracking verbs, specialized research methods.

The Strategy: Eliminate options that, while definitionally correct in a general sense, are never used by professionals within that specific field of study.

Ch 4 The Core Protocol for Words in Context

Follow this strict protocol on every single lexical item to guarantee a perfect conversion rate. Do not deviate from these steps under any circumstance.

[Step 1: Isolate & Variable-ize] → [Step 2: Hunt the Structural Equation] → [Step 3: Extract the Textual Anchor] → [Step 4: Formulate the Algebraic Prediction] → [Step 5: Run the Strict Elimination Filter]

Step 1: Isolate and Variable-ize

Read the passage smoothly to comprehend the macro-topic. Then, completely delete the target word or blank from your mind. Replace it with the variable x . Do not let your pre-existing familiarity with the word dictate your reading.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Step 2: Hunt the Structural Equation

Analyze the syntax of the paragraph. Are there contrast words? Is there an explanatory colon? Is there a parallel sentence structure? Identify which of the five structural equations is holding x in place.

Step 3: Extract the Textual Anchor

Underline the exact word, phrase, or sentence that dictates the value of x . This is your textual anchor. On the SAT, there are no "open" interpretations. The correct answer is always forced into existence by an explicit anchor on the page. If you cannot point your finger directly at the anchor, do not look at the answer choices.

Step 4: Formulate the Algebraic Prediction

Based strictly on the relationship between your anchor and your structural equation, write down or mentally lock in a simple "dummy word." This word does not need to be fancy or academic; terms as basic as "bad," "stop," "copy," or "hidden" are excellent placeholders.

Step 5: Run the Strict Elimination Filter

- Compare your algebraic prediction against choices A, B, C, and D.
- Eliminate choices that have the wrong valence (positive vs. negative charge).
- Eliminate choices that are too extreme or emotionally charged for an academic text.
- Eliminate choices that represent primary definitions when the text explicitly demands a secondary, contextual meaning.

Ch 5

Advanced Deep-Dive — Secondary and Tertiary Meanings

To secure a perfect score, you must familiarize yourself with the academic and official secondary meanings that the College Board uses to target unprepared students. The table below outlines the precise lexical transitions you must master.

Vocabulary Word	Primary Meaning (Everyday Use)	SAT Secondary/Tertiary Meaning (Academic Register)
Arrest	To handcuff a criminal and take them to jail.	To abruptly stop, halt, or check a natural progression or growth cycle.
Check	To look at something to verify its accuracy.	To limit, restrain, control, or hold back an encroaching force.
Climactic	Exciting or dramatic (often confused with climatic).	Pertaining to the peak, zenith, or ultimate point of a structural progression.
Compromise	To reach an agreement by making mutual concessions.	To dangerously expose, weaken, or undermine the structural integrity of something.
Conviction	A formal declaration that someone is guilty of a crime.	A deeply held, unshakeable belief or philosophical certainty.
Cow	A domesticated milk-producing farm animal.	To intimidate, frighten, or bully someone into absolute submission.
Design	A drawing, plan, or aesthetic pattern for an object.	A deliberate, highly calculated intention or underlying motive ("by design").
Document	A paper file, certificate, or official manuscript.	To meticulously record, verify, or provide empirical evidence for a phenomenon.
Economy	The financial system of a country or region.	Restraint, absolute efficiency, or the minimization of waste ("economy of movement").
Excellent	Exceptionally good or high-performing.	Merely possessing distinct, objective features that can be categorized ("an excellent trait").
Fabric	Cloth, textiles, or material used to make clothing.	The foundational structural framework or underlying weave of a system or society.
Facility	A physical building, plant, or research installation.	An innate, effortless aptitude or talent for executing a complex task.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Vocabulary Word	Primary Meaning (Everyday Use)	SAT Secondary/Tertiary Meaning (Academic Register)
Grave	A physical burial plot in a cemetery.	Possessing intense weight, critical seriousness, or highly dangerous stakes.
Informed	Having broad knowledge or access to up-to-date facts.	Shaped, dictated, structurally influenced, or deeply guided by an external factor.
Nature	Trees, wildlife, and the outdoor natural world.	The fundamental, unalterable baseline characteristics or essence of a concept.
Plastic	A synthetic polymer material used to make consumer goods.	Highly malleable, adaptable, flexible, or capable of being physically reshaped.
Qualify	To pass a test or meet conditions to enter a contest.	To add specific limitations, reservations, or conditions to a broad claim.
Read	To scan words on a page to comprehend their meaning.	To interpret the underlying significance, intent, or hidden patterns of a behavior.
Relish	A condimental food spread made from pickled cucumbers.	To experience deep, intense intellectual or emotional appreciation for something.
Scale	A device used to measure weight; a flake on a fish.	To ascend, climb over, or systematically surmount a steep structural barrier.
Scrape	To scratch a surface or injure skin via friction.	An uncomfortable, highly restrictive, or structurally difficult socio-economic situation.
Store	A retail establishment where goods are purchased.	A reserve, supply, accumulation, or hidden cache of essential resources.
Sustain	To keep something alive, operational, or going over time.	To experience or suffer a severe structural blow, injury, or loss ("sustained damage").
Temper	A person's state of mind or tendency to become angry.	To moderate, soften, dilute, mitigate, or reduce the intensity of an element.
Uniform	A matching outfit worn by members of an organization.	Remaining entirely constant, unvarying, identical, and homogeneous over time.
Want	To desire or wish for something you do not have.	An absolute, localized deficiency, destitution, or complete absence of an essential resource.
Weather	Atmospheric conditions like rain, sun, or wind.	To endure, survive, or successfully withstand a severe crisis or period of hardship.

Ch 6 Step-by-Step Walkthroughs of Master-Class Sample Questions

EXAMPLE 1 The Secondary Definition Problem

In her comprehensive analysis of agrarian labor shifts in late-feudal Bohemia, historian Marketa Závratká documents how local bailiffs would routinely _____ the traditional grazing privileges of peasantry during periods of market volatility. Far from being random acts of malice, these calculated administrative actions were designed to stabilize the estate's central revenue by systematically checking the economic autonomy of tenant farmers.

Which choice completes the text with the most logical and precise phrase?

- (A) compromise
- (B) negotiate
- (C) document
- (D) relish

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Step-by-Step Structural Execution:

- 1 **Isolate and Variable-ize:** The text states that local bailiffs would routinely x the traditional grazing privileges of peasantry.
- 2 **Hunt the Structural Equation:** Look at the second sentence. It begins with the phrase "Far from being random acts of malice, these calculated administrative actions were designed to..." This sets up a parallel explanation. The sentence concludes by stating these actions were designed to save revenue by "systematically checking the economic autonomy of tenant farmers." This is an instance of Equation 3: The Cross-Sentence Synchronic Thread.
- 3 **Extract the Textual Anchor:** The exact behavioral anchor is "systematically checking [limiting/restraining] the economic autonomy." Therefore, doing x to the privileges must mean to limit, weaken, or undermine those privileges to restrict autonomy.
- 4 **Formulate the Algebraic Prediction:** Our dummy word for x is "weaken," "undermine," or "restrict."

Run the Strict Elimination Filter:

- (A) compromise: High-scoring students often look past this word because they only think of "reaching a mutual agreement." However, the secondary definition of compromise is to dangerously expose, weaken, or undermine the structural integrity of something. If you compromise privileges, you undermine them. This perfectly matches our dummy word. Keep this.
- (B) negotiate: This implies a cooperative, back-and-forth discussion to reach an agreement. The text emphasizes that these were unilateral administrative actions designed to check autonomy, making cooperation contextually impossible. Eliminate.
- (C) document: This is a repetitive trap. The first sentence already uses the word "documents" to describe the historian's work ("historian Marketa Závratná documents..."). The test makers are trying to lure your ear into repeating a word that makes no sense when applied to what the bailiffs did to grazing privileges. Eliminate.
- (D) relish: This means to enjoy or appreciate. While the bailiffs might have enjoyed their power, "relishing privileges" does not structurally align with the functional mechanical action of checking autonomy. Eliminate.

CORRECT ANSWER: A**EXAMPLE 2 The Precise Definition / Domain-Specific Problem**

While the fundamental crystal lattices of igneous basalt remain highly resistant to mechanical weathering under ambient atmospheric conditions, prolonged exposure to acidic groundwater can radically _____ the mineral's structural integrity over deep geological time. This subterranean chemical transformation slowly breaks down the rigid silicate frameworks, eventually converting the dense rock into highly malleable kaolinite clay.

Which choice completes the text with the most logical and precise phrase?

- (A) rectify
- (B) mutate
- (C) alter
- (D) degenerate

Step-by-Step Structural Execution:

- 1 **Isolate and Variable-ize:** The sentence states that acidic groundwater can radically x the mineral's structural integrity.
- 2 **Hunt the Structural Equation:** The paragraph opens with the subordinating conjunction "While," establishing a clear Equation 2: Reversal / Polarity Shift. The first clause states that the basalt remains "highly resistant to mechanical weathering" (meaning it stays the same). The contrast clause containing our variable x must state that groundwater causes it to not stay the same.
- 3 **Extract the Textual Anchor:** The second sentence provides the precise chemical anchor: it describes a "subterranean chemical transformation" that breaks down structures and "eventually converting the dense rock into highly malleable kaolinite clay." Therefore, x must mean to transform, change, or convert the physical state of the mineral.
- 4 **Formulate the Algebraic Prediction:** Our dummy word is "change" or "transform."

Want a plan built around *your* student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Run the Strict Elimination Filter:

- (A) rectify: In academic domains, rectify means to correct an error, fix a mistake, or refine a mathematical value. Acidic groundwater is a blind natural force; it cannot "correct" a mineral. Eliminate.
- (B) mutate: This is a domain-specific trap. Mutate means to undergo a structural alteration, but it is strictly reserved for the domain of genetics and biology (DNA mutating). It is never used in geology to describe the chemical degradation of inanimate silicates. Eliminate.
- (C) alter: This is a clean, precise, and objective academic verb that means to change or transform the characteristics of an object. It fits the geological domain flawlessly without introducing unauthorized biological or intentional connotations. Keep this.
- (D) degenerate: This word carries a heavy pejorative, moral, or biological connotation meaning to decline into a lower, corrupt, or less functional moral state. In physical science, we describe the rock as changing or weathering, but rocks do not possess moral or biological standing to "degenerate" in this context. Eliminate.

CORRECT ANSWER: C**EXAMPLE 3** Connotation and Tone Implication Problem

Throughout his early essays, cultural critic Lionel Abel frequently adopted a highly _____ rhetorical posture, routinely dismissing contemporary theatrical productions as entirely devoid of intellectual merit and openly ridiculing the artistic choices of mainstream directors. However, archival discoveries of his personal journals suggest that this public display of harsh judgment was largely an performance calculated to mask his deep anxiety regarding his own creative limitations.

Which choice completes the text with the most logical and precise phrase?

- (A) adversarial
- (B) malevolent
- (C) diplomatic
- (D) hypercritical

Step-by-Step Structural Execution:

- 1 Isolate and Variable-ize: The text states that Abel adopted a highly x rhetorical posture.
- 2 **Hunt the Structural Equation:** The target word x is immediately followed by a trailing phrase that acts as an explicit definition, utilizing Equation 1: Punctuation / Appositive Equivalence. The phrase after the comma describes exactly what the posture looked like.
- 3 **Extract the Textual Anchor:** The definition anchor is: "routinely dismissing contemporary theatrical productions as entirely devoid of intellectual merit and openly ridiculing the artistic choices..." This represents a highly intense, negative, and judgmental posture.
- 4 Formulate the Algebraic Prediction: Our dummy word is "harshly judgmental" or "fault-finding."

Run the Strict Elimination Filter:

- (A) adversarial: This means involving conflict, opposition, or hostility (like two lawyers in a courtroom). While Abel was mean, he wasn't necessarily locked in a formal, two-way structural conflict; he was simply handing down one-way, brutal judgments of plays. Let's hold this but look for a more precise match for "fault-finding."
- (B) malevolent: This is an extreme emotional trap. Malevolent means possessing a deep, vicious desire to cause severe physical or psychological evil and suffering to others. Abel was a mean theater critic, but writing harsh reviews does not rise to the level of pure evil or malice. This overshoots the academic tone. Eliminate.
- (C) diplomatic: This means marked by tact, negotiation, and careful conciliation. This is the exact opposite of "openly ridiculing" artists. Eliminate.
- (D) hypercritical: This is a perfect domain-specific and tonal match. The prefix hyper- means excessively or overly, and critical means inclined to find fault or judge harshly. This perfectly operationalizes the anchor of "routinely dismissing productions as entirely devoid of merit." It fits the tone without overshooting into the moral extremity of malevolence. Keep this.

CORRECT ANSWER: D**Want a plan built around *your* student?**

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 4 Figurative Language Interpretation

In his 1854 address on constitutional jurisprudence, statesman Gideon Vance cautioned that the foundational laws of a republic must never be treated as gilded armor—impressive to behold in times of civic tranquility, yet utterly incapable of insulating the body politic when the fierce winds of factional crisis begin to howl. Rather, he argued, the constitution must function as a living, flexible shield, capable of adapting its defensive contours to protect the changing vulnerabilities of the state.

As used in the passage, the author's use of the phrase "gilded armor" primarily serves to emphasize that a law is useless if it is:

- (A) manufactured from materials that are prone to rapid environmental corrosion.
- (B) visually magnificent but fundamentally lacking in practical, protective utility during a crisis.
- (C) designed by aristocratic elites who lack a deep connection to the common populace.
- (D) altered too frequently to allow citizens to develop a firm understanding of its tenets.

Step-by-Step Structural Execution:

- 1 Isolate and Variable-ize:** The prompt targets the metaphor "gilded armor." It asks what trait this phrase emphasizes when a law is described this way.
- 2 Hunt the Structural Equation:** The text uses a direct physical description immediately following the em-dash to unpack the metaphor, deploying Equation 1: Punctuation / Appositive Equivalence.
- 3 Extract the Textual Anchor:** The explicit explanation anchor is: "impressive to behold in times of civic tranquility, yet utterly incapable of insulating the body politic when the fierce winds of factional crisis begin to howl." 4. Formulate the Algebraic Prediction: Translate the literal physical description into a conceptual principle. "Impressive to behold" = looks great/ornamental. "Utterly incapable of insulating during a crisis" = lacks real utility or protective value when things go wrong. Our prediction is: "looks great but doesn't actually work when needed."

Run the Strict Elimination Filter:

- A manufactured from materials that are prone to rapid environmental corrosion.**
Literalism trap. The text is using armor as an analogy for law. The author does not care about actual metal rusting or corroding in the real world. Eliminate.
- B visually magnificent but fundamentally lacking in practical, protective utility during a crisis.**
Flawless conceptual translation. "Visually magnificent" maps to "impressive to behold/gilded," and "lacking in practical, protective utility" maps to "incapable of insulating when the crisis winds howl." Keep this.
- C designed by aristocratic elites who lack a deep connection to the common populace.**
Socio-political projection trap. While "gilded" implies wealth, the text never discusses who designed the laws or their relationship to the common people. This is an unwarranted extrapolation. Eliminate.
- D altered too frequently to allow citizens to develop a firm understanding of its tenets.**
Misplaced target. The second half of the passage discusses a "flexible shield" that can adapt and change, but the phrase "gilded armor" is used to describe the opposite issue—something rigid that fails to protect during a storm. Eliminate.

CORRECT ANSWER: B

Ch 7 Practice Problem Set (10 Complete SAT-Style Items)

Apply the protocol to the following ten practice problems. Each item has been engineered to mimic the exact syntactic difficulty, vocabulary distribution, and distractor mechanics of the top-tier digital SAT.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

PROBLEM 1

Long before the advent of modern isotopic analysis, nineteenth-century mineralogists could only estimate the composition of deep-crustal deposits by observing the _____ crystal faces of sheared specimens. Because these external surface features were frequently distorted by superficial tectonic pressures, such visual assessments were notoriously unreliable and often led to highly contested classifications.

Which choice completes the text with the most logical and precise phrase?

- (A) internal
- (B) primary
- (C) gross
- (D) plastic

PROBLEM 2

In his 1924 treatise on experimental sociology, Arthur Pendelton warns that a researcher must never confuse a community's superficial compliance with state mandates for a true, deep-seated ideological alignment. Compliance, Pendelton demonstrates, is frequently a highly _____ behavior, adopted solely to evade immediate punitive economic sanctions and discarded the precise moment state surveillance relaxes.

Which choice completes the text with the most logical and precise phrase?

- (A) transient
- (B) uniform
- (C) conscientious
- (D) chronic

PROBLEM 3

Although the initial critical reception of choreographing savant Maya Lin's debut performance was marked by widespread public confusion, avant-garde theorists quickly realized that her seemingly erratic, disconnected movements possessed a highly refined internal _____. Every sudden halt and physical contortion was meticulously timed to mirror the mathematical syncopations of the accompanying staccato score.

Which choice completes the text with the most logical and precise phrase?

- (A) economy
- (B) friction
- (C) volatility
- (D) facility

PROBLEM 4

The rapid proliferation of marine microplastics has introduced unprecedented analytical hurdles for oceanic toxicologists seeking to measure baseline chemical concentrations. Because synthetic polymers absorb organic pollutants at highly erratic rates depending on shifting ambient wave energies, researchers find that their localized tissue samples are consistently _____ by unpredictable external contaminations, rendering broad environmental generalizations impossible.

Which choice completes the text with the most logical and precise phrase?

- (A) documented
- (B) compromised
- (C) qualified
- (D) sustained

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

PROBLEM 5

For decades, historians of science assumed that the early astronomical charts of the Joseon Dynasty were created purely for agricultural forecasting. However, archivist Ji-Yeon Park recently uncovered a series of imperial edicts revealing that these highly detailed stellar maps actually possessed an intensely _____ character, deliberately engineered to legitimize the sovereign's political authority by aligning court rituals with celestial events.

Which choice completes the text with the most logical and precise phrase?

- (A) calculated
- (B) random
- (C) generic
- (D) transparent

PROBLEM 6

While the mainstream economic advisors of the era advocated for aggressive, sweeping interventions to stimulate industrial manufacturing, central bank governor Silas Vance maintained a notably _____ stance. Vance repeatedly stated that any premature influx of state capital would inevitably trigger hyperinflation, arguing instead that the market should be allowed to naturally correct its own structural imbalances over a multi-year cycle.

Which choice completes the text with the most logical and precise phrase?

- (A) adversarial
- (B) conservative
- (C) diplomatic
- (D) erratic

PROBLEM 7

Behavioral ecologists studying urban avian populations have noted that the aggressive foraging strategies typical of *Passer domesticus* are rarely innate; rather, these highly assertive competitive habits are structurally _____ by the localized density of anthropogenic food waste, with birds in resource-scarce concrete environments demonstrating significantly higher levels of territorial defense than their rural counterparts.

Which choice completes the text with the most logical and precise phrase?

- (A) terminated
- (B) checked
- (C) informed
- (D) qualified

PROBLEM 8

In her literary retrospective on the gothic prose of the Brontë sisters, critic Evelyn Thorne argues that the desolate Yorkshire moors must never be interpreted as a mere passive setting; instead, the unforgiving landscape operates as an active, oppressive architect of isolation—a malevolent physical force that systematically breaks down the psychological boundaries of the characters, driving them into states of desperate emotional dependency.

As used in the passage, the author's use of the bolded phrase "architect of isolation" primarily serves to emphasize that the landscape:

- (A) was deliberately designed by the authors to reflect 19th-century real estate developments.
- (B) actively shapes and enforces the characters' profound state of social and emotional detachment.
- (C) physically prevents the characters from building permanent stone shelters on the moors.
- (D) represents a beautiful artistic achievement that provides comfort to the characters in times of grief.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

PROBLEM 9

In many high-alpine ecosystems, the rapid expansion of invasive shrub species can severely _____ the growth of native perennial herbs. By forming dense, sprawling mats across the thin topsoil, these encroaching woody plants effectively block incoming solar radiation and monopolize localized nitrogen stores, leaving the native flora completely destitute of essential life-sustaining resources.

Which choice completes the text with the most logical and precise phrase?

- (A) arrest
- (B) sanction
- (C) temper
- (D) weather

PROBLEM 10

While many political commentators celebrated the newly ratified international treaty as an unassailable triumph of modern global diplomacy, constitutional scholar Marcus Vance offered a highly _____ assessment. Vance pointed out that Section 4 of the document contained a series of ambiguous sub-clauses that effectively exempted the world's largest manufacturing economies from compliance, thereby rendering the pact's environmental enforcement mechanisms functionally toothless.

Which choice completes the text with the most logical and precise phrase?

- (A) enthusiastic
- (B) qualified
- (C) indifferent
- (D) superficial

Ch 8

Answer Keys and Comprehensive Explanations

PROBLEM 1 EXPLANATION

CORRECT ANSWER: C (GROSS)

- ◆ **Structural Analysis:** The text uses Equation 4: Internal Cause-and-Effect Matrix. The mineralogists were forced to estimate composition before modern microscopic methods, relying solely on observing the external surface features of the crystal faces.
- ◆ **Anchor Verification:** The second sentence explicitly references these features as "external surface features." * Definitional Tracking: The primary definition of gross is unpleasant or vulgar. However, its academic secondary definition means total, overarching, or visible to the naked eye without magnification (e.g., gross anatomy). Therefore, "gross crystal faces" means the external, visible macroscopic features.
- ◆ **Distractor Breakdown:** A is a direct contradiction (they couldn't see internal structures). B is an unwarranted assumption. D (plastic) means malleable, which doesn't fit the visual descriptive anchor.

PROBLEM 2 EXPLANATION

CORRECT ANSWER: A (TRANSIENT)

- ◆ **Structural Analysis:** This item deploys Equation 1: Punctuation / Appositive Equivalence. The targeted behavior is set off by a trailing appositive comma phase describing how it operates.
- ◆ **Anchor Verification:** The phrase after the comma states the behavior is "adopted solely to evade immediate punitive economic sanctions and discarded the precise moment state surveillance relaxes." * Definitional Tracking: A behavior that is adopted temporarily and immediately discarded when conditions change is short-lived, temporary, or fleeting. Transient is the precise academic synonym for short-lived.
- ◆ **Distractor Breakdown:** B is wrong because the text describes how it changes over time, not how it stays identical. C implies a moral or deeply principled dedication, which contradicts the idea of faking it to avoid a fine. D (chronic) means long-lasting, the exact opposite of transient.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

PROBLEM 3 EXPLANATION**CORRECT ANSWER: A (ECONOMY)**

- ◆ **Structural Analysis:** This problem utilizes Equation 2: Reversal / Polarity Shift, triggered by the contrast word "Although." The initial clause notes that public observers saw "widespread confusion" and "erratic, disconnected movements." The contrast clause must show that the movements actually had a tight, purposeful, non-wasteful structure.
- ◆ **Anchor Verification:** The final sentence reveals that "Every sudden halt and physical contortion was meticulously timed to mirror the mathematical syncopations..." This shows an extreme absence of waste or randomness.
- ◆ **Definitional Tracking:** The secondary definition of economy is restraint, absolute efficiency, or the minimization of excess or waste. A routine where every single micro-movement is perfectly calculated possesses an internal economy.
- ◆ **Distractor Breakdown:** B is a literalist trap matching physical movement. C means instability and chaotic change, which matches the public's incorrect impression. D means ease of execution, which doesn't address the structural organization of the movements.

PROBLEM 4 EXPLANATION**CORRECT ANSWER: B (COMPROMISED)**

- ◆ **Structural Analysis:** This sentence tracks an Equation 4: Internal Cause-and-Effect Matrix, signaled by the subordinating conjunction "Because." The cause is that synthetic polymers absorb pollutants at highly erratic rates based on unpredictable wave energies.
- ◆ **Anchor Verification:** The consequence is that the samples are struck by "unpredictable external contaminations, rendering broad environmental generalizations impossible." * **Definitional Tracking:** When a scientific sample is contaminated, its absolute purity and scientific validity have been undermined, weakened, or damaged. The precise academic word for undermining structural or analytical integrity is compromise.
- ◆ **Distractor Breakdown:** A is wrong because contamination prevents accurate documentation. C (qualified) means to limit a claim, but an external contaminant physically ruins a sample; it doesn't limit an argument. D (sustained) means kept alive or suffered, but "suffered by contamination" is grammatically incorrect phrasing.

PROBLEM 5 EXPLANATION**CORRECT ANSWER: A (CALCULATED)**

- ◆ **Structural Analysis:** This text relies on Equation 2: Reversal / Polarity Shift, initiated by the contrast marker "However." The initial baseline assumption was that the charts were created "purely for agricultural forecasting" (a simple, utilitarian, non-political function). The new discovery must contradict this simplicity.
- ◆ **Anchor Verification:** The text states the maps were "deliberately engineered to legitimize the sovereign's political authority." * **Definitional Tracking:** An action that is deliberately engineered, highly planned, and precisely executed to achieve a specific socio-political outcome is a calculated move. This targets the secondary definition of calculated (deliberate/schemed) rather than its mathematical primary use.
- ◆ **Distractor Breakdown:** B is a direct contradiction of "deliberately engineered." C means basic or unoriginal, which ignores the highly specific political function. D means easy to see through, which does not capture the element of planning.

PROBLEM 6 EXPLANATION**CORRECT ANSWER: B (CONSERVATIVE)**

- ◆ **Structural Analysis:** This item uses Equation 2: Reversal / Polarity Shift, anchored by the word "While." The mainstream advisors wanted "aggressive, sweeping interventions" (large-scale, rapid state spending changes). Vance's stance must be the logical opposite.
- ◆ **Anchor Verification:** Vance argued that spending capital was "premature" and that the market should be left to "naturally correct its own structural imbalances over a multi-year cycle." This is a cautious approach that favors minimal intervention.
- ◆ **Definitional Tracking:** In economic and political domains, a conservative stance is one characterized by caution, a preference for gradual organic change, and a profound reluctance to deploy aggressive state interventions.
- ◆ **Distractor Breakdown:** A overshoots the clinical tone; Vance disagreed, but his stance itself is an economic philosophy, not a personal act of hostility. C means tactful or peacekeeping, but Vance was openly telling them their plans would cause hyperinflation. D means random and changing, which contradicts his steady multi-year philosophy.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

PROBLEM 7 EXPLANATION

CORRECT ANSWER: C (INFORMED)

- ◆ **Structural Analysis:** This item features an instance of Equation 3: Cross-Sentence Synchronic Thread. The first sentence uses a semicolon and the phrase "rather, these highly assertive competitive habits are structurally [blank] by the localized density of anthropogenic food waste." The second sentence provides the parallel data thread.
- ◆ **Anchor Verification:** The second sentence states that birds in resource-scarce environments show significantly higher territorial defense than rural birds. This means the environment directly shapes, guides, influences, and dictates the behavior.
- ◆ **Definitional Tracking:** The secondary academic definition of the verb inform means to give form to, structurally influence, guide, or serve as the defining characteristic of something (e.g., "The author's childhood experiences inform her writing"). Therefore, the habits are informed by the food waste density.
- ◆ **Distractor Breakdown:** A is a direct contradiction; the waste starts or increases the behavior, it doesn't end it. B (checked) means limited, but the waste increases aggression in cities. D means limited by conditions, which doesn't capture the active shaping force of the environment.

PROBLEM 8 EXPLANATION

CORRECT ANSWER: B (ACTIVELY SHAPES AND ENFORCES THE CHARACTERS' PROFOUND STATE OF SOCIAL AND EMOTIONAL DETACHMENT.)

- ◆ **Structural Analysis:** This is a Figurative Language Interpretation item. The prompt isolates the metaphor "architect of isolation." You must use Equation 1: Punctuation / Appositive Equivalence to locate the literal definition following the em-dash.
- ◆ **Anchor Verification:** The text defines the architect as: "a malevolent physical force that systematically breaks down the psychological boundaries of the characters, driving them into states of desperate emotional dependency."
- ◆ **Definitional Tracking:** An "architect" is something that actively builds, designs, or shapes a structure. "Isolation" means detachment. Therefore, the phrase emphasizes that the moors actively build and enforce that detachment. Choice B maps to this conceptual link flawlessly.
- ◆ **Distractor Breakdown:** A is a literalist trap focused on actual real estate or architecture. C is a literalist trap focused on physical building materials. D introduces a completely incorrect positive valence ("beautiful," "comfort") that directly violates the negative anchor words "oppressive" and "malevolent."

PROBLEM 9 EXPLANATION

CORRECT ANSWER: A (ARREST)

- ◆ **Structural Analysis:** This text deploys an Equation 4: Internal Cause-and-Effect Matrix. The second sentence begins with the phrase "By forming dense, sprawling mats across the thin topsoil..." which sets up the physical cause.
- ◆ **Anchor Verification:** The mechanical consequence is that the shrubs "effectively block incoming solar radiation and monopolize localized nitrogen stores, leaving the native flora completely destitute." If plants have no light and no nitrogen, their growth will completely stop.
- ◆ **Definitional Tracking:** The primary definition of arrest is to place someone in police custody. The secondary academic definition of arrest is to completely stop, halt, check, or disrupt a natural development or physiological progression. Blocking light and nitrogen arrests the herbs' growth.
- ◆ **Distractor Breakdown:** B (sanction) means to officially permit or reward, the exact opposite of what we need. C (temper) means to moderate or dilute, which is too weak; the plants are left "completely destitute," meaning growth is halted, not just slowed down. D (weather) means to endure and survive a crisis, which is what the herbs are failing to do.

PROBLEM 10 EXPLANATION

CORRECT ANSWER: B (QUALIFIED)

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- ◆ **Structural Analysis:** This final item utilizes Equation 2: Reversal / Polarity Shift, triggered by the contrast conjunction "While." The mainstream commentators viewed the treaty as an "unassailable triumph" (a statement of absolute perfection with zero flaws). Vance's assessment must be the logical opposite: an assessment that points out specific flaws, limits, or conditions.
- ◆ **Anchor Verification:** The second sentence notes that Vance "pointed out that Section 4 contained a series of ambiguous sub-clauses that effectively exempted the world's largest manufacturing economies from compliance." Vance is adding a massive condition or limitation to the treaty's success.
- ◆ **Definitional Tracking:** The secondary definition of the verb qualify is to limit, modify, or add specific conditions, reservations, or restrictions to a broad statement or assessment. A qualified assessment is one that says, "This is good, but only under certain conditions." This perfectly fits Vance's critique.
- ◆ **Distractor Breakdown:** A is a direct contradiction of his negative review. C means possessing absolutely no emotional care or interest, but Vance was deeply interested enough to analyze Section 4 and argue against its efficacy. D means skin-deep or looking only at the surface; Vance did the exact opposite by digging deep into the specific sub-clauses of Section 4.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



Inferences

In the digital SAT Reading and Writing section, Inference questions represent the peak of the Information and Ideas domain. These questions present stems such as:

In the digital SAT Reading and Writing section, Inference questions represent the peak of the Information and Ideas domain. These questions present stems such as:

- ◆ Which choice most logically completes the text?
- ◆ It can reasonably be concluded from the passage that...
- ◆ Based on the text, the author would most likely agree with which statement?

To earn a perfect score, you must entirely redefine what the word "inference" means in the context of the SAT. In everyday conversation, an inference is an educated guess or a creative reading between the lines. On the SAT, an inference is a strict, inescapable logical deduction. The correct answer must be 100% provable based only on the explicit premises provided in the text. If an option requires even a small leap of faith, a speculative projection, or a real-world assumption, it is mathematically wrong.

Ch 1 The Core Architecture of SAT Inferences

SAT inference texts are designed like incomplete formal logic proofs. The test makers provide premises ($A \rightarrow B$ and $B \rightarrow C$) and purposefully stop short of drawing the final boundary. Your job is to supply the missing link ($A \rightarrow C$).

There are three primary structural prototypes for these questions.

Prototype 1: The Condition/Constraint Overlap Pattern

- ◆ **The Mechanism:** The text establishes a set of strict criteria or physical conditions required for a certain phenomenon to occur (e.g., Stellar formation requires a gas density above Threshold X and a temperature below Y). It then introduces a specific subject that meets or fails to meet those conditions.
- ◆ **The Inference:** The correct answer applies the criteria directly to the subject to state a definitive outcome.

Prototype 2: The Typology Contrast Pattern

- ◆ **The Mechanism:** The text categorizes a subject into two or more distinct types, species, or historical periods, explicitly contrasting their traits (e.g., Group A feeds exclusively during daylight, while Group B hunts solely during new moon cycles).
- ◆ **The Inference:** The correct answer will flip the contrast or apply a limitation to one group based on the traits of the other.

Prototype 3: The Methodological Flaw/Variable Isolation Pattern

The Mechanism: A scientist sets up an experiment to isolate Variable X by holding Variable Y constant. However, an unexpected environmental shift introduces an unmonitored change to Variable Y .

The Inference: The data collected cannot be safely attributed to Variable X alone.

Ch 2 The Algebraic Inference Protocol

Do not read the paragraph and think about what "makes sense" as a next sentence. Treat the text as a mathematical proof.

[Step 1: Extract Explicit Premises] → [Step 2: Map the Overlap Variable] → [Step 3: Define the Inescapable Boundary] → [Step 4: Reject Extrapolations]

Step 1: Extract the Explicit Premises

Read the text and list the absolute facts. Strip away the descriptive adjectives and reduce the sentences to core assertions.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Step 2: Identify the Overlap Variable

Look for the common denominator that links the sentences together. If Sentence 1 talks about "soil acidity" and Sentence 2 talks about "soil acidity affecting root depth," soil acidity is your overlap variable.

Step 3: Write Your Own "Must-Be-True" Conclusion

Before looking at the answer choices, bridge the overlap to state what must happen. Keep your prediction incredibly boring, repetitive, and literal.

Step 4: Run the Filter (Eliminate Extrapolations)

Compare the options to your prediction. The correct answer will often feel like a redundant echo of what was already stated.

Ch 3 Walkthroughs of Real SAT-Style Sample Items

Let's apply this algebraic inference protocol to master class problems.

GUIDED WALKTHROUGH 1 The Condition/Constraint Pattern

Deep-sea hydrothermal vent communities rely entirely on chemosynthetic bacteria, which synthesize organic molecules using dissolved sulfide compounds streaming from the Earth's crust. These bacteria require ambient water temperatures between 2°C and 35°C to metabolize compounds efficiently; if temperatures exceed 40°C, the metabolic enzymes denature instantly, halting production. In a 2025 expedition to the Mariana Trench, oceanographer Maya Lin observed a newly opened hydrothermal vent field where the immediate outflow zone maintained a sustained temperature of 52°C.

Which choice most logically completes the text?

- (A) The chemosynthetic bacteria in this newly discovered vent field will rapidly evolve to tolerate temperatures above 52°C.
- (B) Chemosynthetic bacteria located directly within the immediate outflow zone of the new vent field will be unable to efficiently synthesize organic molecules.
- (C) Deep-sea communities surrounding the new vent field will thrive at a higher rate than those located near cooler vents.
- (D) Oceanographer Maya Lin's equipment likely suffered thermal damage due to the extreme heat of the immediate outflow zone.

Step-by-Step Execution:

Extract Premises:

Premise 1: Vent communities need chemosynthetic bacteria.

Premise 2: These bacteria require temperatures between 2°C and 35°C to function.

Premise 3: Temperatures above 40°C halt their production instantly.

Premise 4: A new vent's immediate outflow zone is at 52°C.

- ◆ **Map the Overlap Variable:** The overlap variable is temperature and bacterial function. 52°C is well above the absolute limit of 40°C.
- ◆ **Define the Inescapable Boundary:** Since 52°C > 40°C, the bacteria in that immediate outflow zone will have their enzymes denatured and their production halted.

Evaluate the Choices:

- A** ...will rapidly evolve to tolerate...
Speculative extrapolation trap. The text says they denature instantly. It gives zero evidence about evolutionary adaptation timelines. Eliminate.
- B** Chemosynthetic bacteria located directly within the immediate outflow zone of the new vent field will be unable to efficiently synthesize organic molecules.
Flawless, boring, completely forced deduction. Because 52°C is past the 40°C threshold, they cannot metabolize or synthesize. Keep this.
- C** Deep-sea communities surrounding the new vent field will thrive at a higher rate...
Out of bounds. If the bacteria fail, the communities will likely suffer, not thrive. This contradicts the premises. Eliminate.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

D Maya Lin's equipment likely suffered thermal damage...

Real-world plausibility trap. While equipment can melt, the text provides absolutely zero parameters regarding the melting point of Lin's tools. Eliminate.

CORRECT ANSWER: B**GUIDED WALKTHROUGH 2** The Typology Contrast Pattern

Paleontologists classify ancient archosaurs into two distinct lineages based on ankle joint morphology: Crurotarsi, which possess a flexible, interlocking joint that allows for a semi-erect, sprawling gait, and Avemetatarsalia, which feature a rigid, hinge-like joint restricted to a purely erect, vertical plane of motion. The flexible gait of the Crurotarsi enabled efficient navigation through dense, swampy terrains but capped their maximum running velocity. Conversely, the rigid vertical alignment of Avemetatarsalia maximized linear acceleration on open plains, though it severely compromised lateral stability on uneven ground. A fossil trackway discovered in an ancient, wide-open floodplain deposit consists entirely of deeply impressed, straight-line footprints demonstrating an exceptionally high linear velocity and minimal lateral deviation.

Based on the text, what can reasonably be concluded about the trackway?

- A** It was most likely produced by an archosaur belonging to the Crurotarsi lineage.
- B** It preserves evidence of a predator-prey chase occurring across an ancient floodplain.
- C** It was most likely produced by an archosaur belonging to the Avemetatarsalia lineage.
- D** It demonstrates that some Crurotarsi developed rigid ankle joints to adapt to open plain environments.

Step-by-Step Execution:**Extract Premises:**

Crurotarsi = Flexible joint → sprawling gait → good for swamps, low max velocity.

Avemetatarsalia = Rigid joint → erect/vertical motion → high linear velocity on open plains, poor lateral stability.

Discovery = Open floodplain, straight-line footprints, high linear velocity, minimal lateral deviation.

- ◆ **Map the Overlap Variable:** The fossil trackway matches the exact diagnostic traits of the Avemetatarsalia lineage (high linear velocity, open plains, straight-line/minimal lateral deviation).
- ◆ **Define the Inescapable Boundary:** The creature that made the trackway must have possessed the traits of Avemetatarsalia, not Crurotarsi.

Evaluate the Choices:

- A** It was most likely produced by an archosaur belonging to the Crurotarsi lineage.
Direct contradiction of the velocity and terrain data. Eliminate.
- B** It preserves evidence of a predator-prey chase...
Creative writing trap. High velocity does not explicitly prove a chase; the animal could have just been running alone. Eliminate.
- C** It was most likely produced by an archosaur belonging to the Avemetatarsalia lineage.
Simple, direct match. The physical properties of the trackway mirror the functional profile assigned exclusively to Avemetatarsalia. Keep this.
- D** It demonstrates that some Crurotarsi developed rigid ankle joints...
Contradicts the initial typology definition. Crurotarsi are defined by their flexible joints. Eliminate.

CORRECT ANSWER: C**Ch 4** The 750+ Elimination Playbook for Inferences

When navigating inference choices, look for these three diagnostic flaws to execute instant eliminations.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

1. The "Speculative Future" Flaw

If an answer choice predicts what will happen next in a timeline or how a population will adjust to a crisis over generations (e.g., will eventually become dominant, will adapt, will cause scientists to shift funding), it is almost certainly wrong. The SAT cannot prove the future; it can only prove the logical limits of the current text.

2. The Unauthorized Superlative/Comparison

Watch out for choices that claim a variable is the most effective, least common, highest yielding, or faster than all others. Unless the text explicitly uses a superlative to rank variables, any option introducing a comparison or ranking is an illegal extrapolation.

3. The "Sounds Reasonable in the Real World" Decoy

The test makers intentionally write distractors that align with common sense or high school science concepts (e.g., Deforestation causes carbon dioxide levels to rise). If that specific mechanism is not detailed in the provided block of text, you cannot select it. You must pretend you know absolutely nothing about the universe outside of the 4-sentence window on your screen.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



Command of Evidence

On the digital SAT, Command of Evidence questions make up a massive portion of the Information and Ideas domain.

On the digital SAT, Command of Evidence questions make up a massive portion of the Information and Ideas domain. These questions aren't just testing whether you understand what an author is saying; they are testing your ability to evaluate the structural integrity of an argument. You are being asked to step into the role of a peer reviewer or data analyst.

These questions generally fall into three distinct types:

- ◆ **Textual Evidence Selection:** Choosing the exact quotation or text excerpt that supports a specific hypothesis.
- ◆ **Quantitative/Graphical Evidence (Supports or Weakens):** Evaluating whether empirical data or research findings validate, invalidate, or complicate a scientific conclusion.
- ◆ **Rhetorical Synthesis (Student Notes):** Synthesizing a bulleted list of notes to accomplish a highly specific communicative or rhetorical goal.

Ch 1 Textual Evidence Selection ("Which quotation best supports...")

The Core Architecture

In these items, you are presented with a scientific hypothesis or a literary/historical claim. The text will contain a background passage, followed by a specific claim. Your job is to select the quotation that provides direct, unassailable proof for that specific claim.

The Core Trap

The "On-Topic" Mirage: The test makers will fill the wrong answer choices with quotations that discuss the exact same species, historical figure, or chemical compound mentioned in the prompt. However, these decoy quotes will describe a completely different relationship than the one specified by the hypothesis.

The Step-by-Step Protocol

[Step 1: Isolate the Targeted Claim] → [Step 2: Turn Claim into an Algebraic Equation] → [Step 3: Filter Quotes for BOTH Variables]

- ◆ **Isolate the Target Claim:** Do not read for the overall summary of the passage. Read to find the exact, specific claim the prompt asks you to support. Underline it or state it in your own words.
- ◆ **Turn the Claim into an Algebraic Equation:** Break the claim down into two interacting variables: Cause (A) → Effect (B). For example, if a researcher claims that high canopy density prevents the growth of ferns due to solar blockages, your equation is: High Canopy Density (A) → Fewer Ferns (B).
- ◆ **Filter Options with the Equation:** Evaluate each quote strictly. The correct answer must explicitly mention or show a direct relationship between both variable A and variable B . If a quote only talks about canopy density (A) but doesn't mention ferns or growth rates (B), it is structurally incomplete and wrong.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

PRACTICE QUESTION 1 Textual Evidence

A literature professor is analyzing the work of 19th-century poet Clara Vance. The professor claims that despite Vance's public reputation for writing verse that celebrated urban industrials, her private correspondences reveal a deep, underlying anxiety regarding the environmental toll of rapid factory expansion.

Which quotation from Vance's private letters best supports the professor's claim?

- (A) "The skyline of Manchester grows grander by the year, as the great brick chimneys reach toward the clouds like monuments to human ambition."
- (B) "I spent the morning crafting a sonnet on the weaving looms, marveling at how thousands of threads can be brought into such precise order by the power of steam."
- (C) "Though the productivity of the newly opened foundry near our estate is undeniable, I find myself mournfully contemplating the thick, grey soot that now permanently cloaks our ancient valley oaks."
- (D) "My editors insist that the public desires more stanzas celebrating the forward march of the locomotive, yet I grow weary of writing to satisfy the marketplace."

Step-by-Step Explanation

- ◆ **Isolate the Claim:** The claim is that Vance's private letters show a deep, underlying anxiety regarding the environmental toll of rapid factory expansion.
- ◆ **Turn the Claim into an Equation:** Factory Expansion (A) → Anxiety about Environmental Damage (B).

Filter Options:

Choice A mentions chimneys and factories (A) but praises them as grand monuments. It completely lacks anxiety or environmental damage (B). Eliminate.

Choice B describes marveling at steam looms (A) but expresses admiration for order, not environmental anxiety (B). Eliminate.

Choice C mentions the newly opened foundry (A, factory expansion) and explicitly shows anxiety about the environmental toll: "mournfully contemplating the thick, grey soot that now permanently cloaks our ancient valley oaks" (B, environmental toll). This hits both variables perfectly. Keep this.

Choice D shows anxiety about her editors and the marketplace, not about the environmental toll of factories. Eliminate.

CORRECT ANSWER: C

Ch 2 Quantitative Evidence ("Which finding supports or weakens...")**The Core Architecture**

These questions present a scientific experiment or a social science study accompanied by data or research findings. You must determine which finding either supports (strengthens) or weakens (refutes) the researchers' stated conclusion.

The Core Trap

Misreading Graph/Text Alignment: Choosing an answer that accurately states a fact from the data, but completely flips the logical relationship required by the conclusion (e.g., choosing a finding that supports when the question explicitly asked for a finding that weakens).

The Step-by-Step Protocol

- ◆ **Locate the Conclusion:** Find the final sentence of the text. Identify what the researchers conclude their data actually shows.
- ◆ **Determine the Logical Charge (+ or -):** * If the prompt asks you to support, you are looking for a finding that says: When Cause happens, Effect happens exactly as predicted.

If the prompt asks you to weaken, you are looking for a finding that says: When Cause happens, Effect does NOT happen, or the exact opposite happens.

Verify the Variables: Ensure the finding compares the correct experimental groups (e.g., comparing the control group to the experimental group rather than two irrelevant sub-variables).

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

PRACTICE QUESTION 2 Weakening a Conclusion

In a 2024 study, ecologist Liam O'Connor investigated whether the introduction of the invasive kudzu vine negatively impacts local soil biodiversity. O'Connor measured the microbial species richness (the total number of distinct bacterial strains per cubic meter) in plot soils that had been heavily colonized by kudzu for over five years, comparing them to adjacent, uncolonized control plots containing only native flora. O'Connor concluded that kudzu colonization systematically degrades soil health by drastically reducing microbial diversity.

Which finding, if true, would most directly weaken O'Connor's conclusion?

- (A) The uncolonized control plots exhibited a slight increase in microbial species richness over the course of the five-year study period.
- (B) The plots heavily colonized by kudzu demonstrated an average microbial species richness that was virtually identical to or marginally higher than that of the uncolonized control plots.
- (C) Microbial species richness was found to vary wildly across both colonized and uncolonized plots depending on the level of seasonal rainfall.
- (D) Plots that had been colonized by kudzu for only one year possessed significantly lower microbial species richness than plots colonized for five years.

Step-by-Step Explanation

- ◆ **Locate the Conclusion:** O'Connor concludes that kudzu colonization degrades soil health by drastically reducing microbial diversity.
- ◆ **Determine the Logical Charge:** The question asks to weaken. Therefore, we need a finding that shows kudzu colonization does not reduce microbial diversity. (Kudzu plots $\geq$ Native plots in diversity).

Filter Options:

Choice A tells us the control plots increased in diversity. This doesn't tell us how they compare to the kudzu plots, so it doesn't directly weaken the conclusion. Eliminate.

Choice B explicitly states that the kudzu plots had a microbial richness that was virtually identical to or marginally higher than the control plots. If kudzu plots have equal or more diversity than native plots, O'Connor's claim that kudzu "drastically reduces" diversity is completely falsified. This perfectly weakens the conclusion. Keep this.

Choice C introduces a confounding variable (rainfall) but doesn't directly challenge the core comparison between the two plot types. Eliminate.

Choice D shows that longer colonization actually has higher diversity than short-term colonization, which accidentally supports the idea that kudzu plots hold onto or build diversity over time, but it doesn't directly neutralize the baseline control comparison as cleanly as Choice B. Eliminate.

CORRECT ANSWER: B

Ch 3 Student Notes Questions (Rhetorical Synthesis)**The Core Architecture**

These questions are a unique feature of the digital SAT. You are presented with a bulleted list of 4 to 6 notes compiled by a hypothetical student. The prompt will give you an explicit goal (e.g., "The student wants to emphasize a difference between the two architectural styles. Which choice most effectively uses information from the notes to accomplish this goal?").

The Core Trap

The "Good Summary" Distractor: Test makers will write an answer choice that is a beautiful, flawless summary of all the notes provided. Students choose it because it feels like a great conclusion. However, if that choice does not specifically accomplish the narrow goal stated in the prompt (e.g., emphasizing a difference), it is a failing grade.

The Step-by-Step Protocol

[Step 1: IGNORE THE NOTES] → [Step 2: Isolate the Rhetorical Goal] → [Step 3: Filter Choices for Goal Execution]

- ◆ **CRITICAL STEP: IGNORE THE NOTES FIRST.** Do not waste time reading the bullet points deeply. The notes are always factually true, and all answer choices will pull accurate facts from them. The notes are not where the trap lies.
- ◆ **Isolate the Rhetorical Goal:** Read the prompt sentence carefully. Pinpoint the exact linguistic task requested.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

If it asks to emphasize a contrast, look for transition words like *whereas*, *while*, or *unlike*.

If it asks to introduce a specific discovery, look for the choice that states the name of the discovery and its key feature.

Filter Choices by Strict Execution: Evaluate the choices based solely on whether they do the job requested. If a choice lists a similarity when the prompt asked for a difference, eliminate it immediately without checking the notes.

PRACTICE QUESTION 3 Student Notes

While preparing a research paper, a student took the following notes:

The Great Sphinx of Giza is a massive limestone statue located in Egypt, carved around 2500 BCE during the Old Kingdom.

It features the body of a lion and the head of a human pharaoh.

The Olmec Colossal Heads are a collection of seventeen massive stone sculptures located in Mexico, carved between 900 BCE and 400 BCE.

They feature realistic, distinct human faces carved from basalt boulders.

Both monuments were created by ancient civilizations to project political and spiritual authority.

The student wants to emphasize a contrast in the primary materials used to construct the two monuments. Which choice most effectively uses information from the notes to accomplish this goal?

- (A) Ancient civilizations constructed monuments like the Great Sphinx of Giza and the Olmec Colossal Heads to project political and spiritual authority over their territories.
- (B) While both monuments are massive ancient stone sculptures, the Great Sphinx of Giza was carved around 2500 BCE, whereas the Olmec Colossal Heads were carved much later, between 900 BCE and 400 BCE.
- (C) The Great Sphinx of Giza features the blended anatomy of a lion and a human pharaoh, but the Olmec Colossal Heads depict entirely realistic human faces.
- (D) The Great Sphinx of Giza was carved out of native limestone strata, whereas the Olmec Colossal Heads were sculpted from massive basalt boulders.

Step-by-Step Explanation

Ignore the Notes: Skip straight to the prompt goal.

Isolate the Rhetorical Goal: The goal is to emphasize a contrast in the primary materials used.

Filter Choices:

Choice A describes a similarity in purpose (political/spiritual authority). It completely ignores materials and contrast. Eliminate.

Choice B emphasizes a contrast in time periods/chronology (2500 BCE vs. 900–400 BCE), not materials. Eliminate.

Choice C emphasizes a contrast in artistic subject matter/anatomy (lion/human blend vs. realistic human faces). Eliminate.

Choice D uses a contrast indicator ("*whereas*") to explicitly contrast the primary materials: "limestone strata" versus "basalt boulders". This fulfills the goal with absolute precision.

CORRECT ANSWER: D

Ch 4 Elite Elimination Strategies for 750+ Scorers

When moving through Command of Evidence items under tight time constraints, you can confidently accelerate your pace by executing these three rules of elimination.

1. Eliminate Quotes that are "Pure Description"

In Textual Evidence questions, any answer choice that merely describes scenery, provides a definition, or lists raw biographical facts about a person can be discarded immediately. The correct quote must always show a dynamic relationship or mechanistic link between variables.

2. Spot the "No Comparison" Flaw in Data

In Quantitative Evidence questions, if a conclusion makes a comparative claim (e.g., Group X performed better than Group Y), any answer choice that only gives you data about Group X in isolation without showing its relationship to Group Y cannot support or weaken the claim. A baseline requires a comparative data point.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

3. Reject Truths that Miss the Target

In Student Notes questions, always remind yourself that every single choice is factually accurate to the notes. If you find yourself thinking, "But Choice C is completely true based on bullet point 2," stop yourself. Ask: "Did the prompt ask me to state a true fact, or did it ask me to execute a specific rhetorical goal?" Treat the notes as raw materials; the question stem is your blueprint.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



Central Ideas and Details

In the digital SAT Reading and Writing section, Information and Ideas questions account for a significant portion of your score.

In the digital SAT Reading and Writing section, Information and Ideas questions account for a significant portion of your score. This category contains several distinct question types that evaluate your ability to comprehend, synthesize, and track explicit or implicit arguments made by authors:

Central Idea / Main Idea

Primary Purpose

Summary Completion

Best Statement of the Passage's Central Claim

Supporting Detail Questions

To achieve a perfect 800, you must realize that the College Board does not evaluate these tasks subjectively. Just like vocabulary or sentence function questions, reading comprehension on the SAT is completely literal. The correct answer is an exact, unassailable translation of text directly on the page, wrapped in academic language.

Ch 1 The Anatomy of Central Idea, Purpose, and Summary Questions

While these question stems look slightly different, they all evaluate a single macro-skill: hierarchical reading. You must separate the author's primary argument (the structural canopy) from the evidence used to construct it (the leaves).

1. Central Idea vs. Primary Purpose vs. Central Claim

Students frequently confuse these three concepts, but they assess two different axes of a text:

- ◆ **Central Idea / Central Claim:** This answers WHAT the text is saying. It is an objective distillation of the overarching argument or discovery.
- ◆ **Primary Purpose:** This answers WHY the text was written. It focuses on the author's rhetorical goal, usually beginning with an active verb (e.g., to advocate, to analyze, to challenge, to contextualize).
- ◆ **Summary Completion:** This demands a comprehensive synthesis of BOTH the central claim and its foundational structural shifts from the beginning to the end of the passage.

Ch 2 The Core Blueprints and Structural Protocols

SAT texts for these categories are drawn directly from rigorous academic domains: humanities, science, and social sciences. Let's break down the step-by-step framework to master each question type.

Protocol 1: Central Idea & Central Claim Questions

- ◆ **The Trap:** High-scoring students are frequently tricked by choices that are true details mentioned directly in the paragraph but are too narrow to count as the central idea.
- ◆ **The Blueprint:** The correct option must cover the entire span of the text, acting as a structural umbrella.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Step-by-Step Execution:

- 1 **Locate the Core Thesis Shift:** Read the text specifically hunting for the author's turning point. This is often marked by structural pivots like however, conversely, a recent study reveals, or strikingly.
- 2 **Apply the "Umbrella Test":** Look at your chosen answer and ask: Does this statement encompass the evidence presented in the beginning, middle, and end of the text, or does it just describe one sentence?
- 3 **Eliminate Out-of-Bounds and Half-Right Choices:** The correct answer will never assume, extrapolate, or expand past the literal bounds of the paragraph.

Protocol 2: Primary Purpose Questions

- ◆ **The Trap:** Verbs that are rhetorically inaccurate. For example, a text that simply reports data from an experiment is not "advocating" for a policy change; it is merely "presenting findings."
- ◆ **The Blueprint:** Pay meticulous attention to the first word of each answer choice. The verb must align with the tone and execution of the text.

[Step 1: Track tone & structure] → [Step 2: Select precise rhetorical verb] → [Step 3: Match trailing text to thesis]

Step-by-Step Execution:

- 1 **Identify Textual Dynamics:** Is the author neutral, analytical, or evaluative? If the text is a neutral summary of historical facts, immediately eliminate options beginning with aggressive verbs like refute, challenge, criticize, or lobby.
- 2 **Evaluate the Verb-Object Match:** Break the answer choices into two halves: the rhetorical action (the initial verb) and the object of that action (the rest of the choice). Both components must match perfectly.

Protocol 3: Summary Completion Questions

- ◆ **The Trap:** Choppy choices that jump over the vital narrative or argument transition in the text, or focus entirely on the final data point.
- ◆ **The Blueprint:** A true summary represents the chronological or logical progression of the whole passage.

Step-by-Step Execution:

- 1 **Block the Paragraph into Phases:** Divide the text into its natural segments (e.g., Phase 1: Old historical perspective → Phase 2: Discovery of new artifact → Phase 3: New interpretation).
- 2 **Check for Chronological Fidelity:** The correct answer must move in structural alignment with the passage, preserving the relationship between the premises and the final conclusion.

Protocol 4: Supporting Detail Questions

The Trap: Paraphrases that introduce unauthorized modifiers. A choice that says an effect occurs "exclusively in cold climates" is wrong if the text states it occurs "predominantly in cold climates."

The Blueprint: Absolute literalism. These questions are open-book search-and-match operations.

Step-by-Step Execution:

- 1 **Rephrase the Prompt:** Turn the question stem into a specific search query (e.g., What did the researchers look for in the soil?).
- 2 **Scan for Key Locators:** Skim the text for matching technical nouns, dates, or names to anchor your target sentence.
- 3 **Perform Synchronic Translation:** Find the sentence containing the answer, and match it to the choice that uses precise, non-extreme synonyms to restate that exact fact.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Ch 3 Walkthroughs of Master-Class Sample Items**GUIDED WALKTHROUGH 1** Best Statement of the Central Claim

For decades, architectural preservationists have assumed that ancient Roman concrete maintained its remarkable structural integrity over two millennia solely due to the meticulous sourcing of volcanic ash from the Pozzuoli region. However, material scientist Aris Thorne and colleagues recently utilized high-resolution electron microscopy to examine elemental cross-sections of structures in Ostia. Their analysis revealed that macroscopic lime clasts—previously dismissed by historians as mere evidence of sloppy mixing practices—actually functioned as a self-healing mechanism. When micro-cracks form within the concrete matrix, water infiltrates the lime clasts, recrystallizing as calcium carbonate to seal the fissures. This internal chemical reaction, rather than regional ingredient purity alone, accounts for the durability of the ancient infrastructure.

Which choice best states the main idea of the text?

- (A) Recent structural analysis demonstrates that macroscopic lime clasts provided a chemical self-healing mechanism that contributed significantly to the longevity of ancient Roman concrete.
- (B) Historians have long undervalued the importance of volcanic ash sourced from Pozzuoli in the formulation of highly durable Roman building materials.
- (C) High-resolution electron microscopy has disproved the long-standing architectural theory that ancient Roman concrete was poorly or carelessly mixed during construction.
- (D) The durability of Roman concrete varies depending on whether structures were built with volcanic ash from Pozzuoli or lime clasts from Ostia.

Step-by-Step Execution:

- 1 **Locate the Core Thesis Shift:** Look for the pivot. The text starts with an old assumption (Pozzuoli ash alone caused durability). Sentence 2 pivots: "However, material scientist Aris Thorne and colleagues recently utilized..." Their discovery reveals that lime clasts act as a "self-healing mechanism."
- 2 **Apply the Umbrella Test:** The central claim must unify the shift from the old ash theory to the new discovery of lime-clast healing kinetics.

Filter the Choices:

- (A) **Recent structural analysis demonstrates that macroscopic lime clasts provided a chemical self-healing mechanism...**
Perfect match. It captures the discovery that alters the entire historical narrative and summarizes the ultimate takeaway of the text. Keep this.
- (B) **Historians have long undervalued the importance of volcanic ash...**
Reverse trap. The text says historians overvalued or exclusively relied on the ash theory, ignoring the lime clasts. Eliminate.
- (C) **High-resolution electron microscopy has disproved the long-standing architectural theory that ancient Roman concrete was poorly or carelessly mixed during construction.**
True-but-narrow detail trap. While the text notes that lime clasts were previously dismissed as sloppy mixing, the purpose of the microscope was not simply to defend the Romans' mixing skills; it was to discover why the concrete survived (self-healing). Eliminate.
- (D) **The durability of Roman concrete varies depending on whether structures were built with volcanic ash... or lime clasts...**
Misinterpretation. The text implies they are both parts of the concrete, not that structures were divided into two isolated types. Eliminate.

CORRECT ANSWER: A

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

GUIDED WALKTHROUGH 2 Primary Purpose

In a 2021 study, evolutionary biologist Chloe Dupont evaluated the long-standing hypothesis that the vivid warning coloration of certain toxic nudibranch species serves primarily to deter visual predators such as reef fish. Dupont presented varying color morphs of both conspicuous and cryptic nudibranchs to local fish populations in controlled aquatic enclosures. Strikingly, the rate of predatory strikes did not differ significantly between brightly colored specimens and those that blended into the substrate; instead, predators avoided both groups equally if they had previously consumed any unpalatable nudibranch. This suggests that chemical defense efficacy, rather than optical salience, governs predatory avoidance behaviors in these marine ecosystems.

Which choice best describes the primary purpose of the text?

- (A) To challenge a prevailing methodology used by biologists to categorize marine visual predators.
- (B) To present research findings that complicate an established explanation for a biological trait.
- (C) To argue against the deployment of artificial aquatic enclosures in evolutionary biology experiments.
- (D) To reconcile conflicting theoretical models regarding how marine organisms synthesize chemical defenses.

Step-by-Step Execution:

- 1 **Identify Textual Dynamics:** The text presents a study by Dupont testing a "long-standing hypothesis" about warning coloration. The results were unexpected ("Strikingly... did not differ significantly"). It concludes by modifying our understanding of what governs predatory avoidance. The tone is objective and analytical.
- 2 **Evaluate the Verb-Object Match:** The text is presenting a study's results that cut against an old hypothesis.

Filter the Choices:

- (A) **To challenge a prevailing methodology...**
Misplaced target. Dupont challenged an explanation (warning coloration functions as a primary deterrent), not the methodology (the way they do experiments). Eliminate.
- (B) **To present research findings that complicate an established explanation for a biological trait.**
Excellent academic match. The "biological trait" is warning coloration, the "established explanation" is visual predator deterrence, and Dupont's findings "complicate" it by showing chemical efficacy matters more. Keep this.
- (C) **To argue against the deployment of artificial aquatic enclosures...**
Absurd extreme option. Dupont used these enclosures successfully; the text doesn't criticize them. Eliminate.
- (D) **To reconcile conflicting theoretical models...**
Jargon trap. The text doesn't bring two opposing views together to find middle ground (reconcile); it highlights data that pushes against one view. Eliminate.

CORRECT ANSWER: B

Ch 4 Elite Elimination Strategies for 750+ Scorers

When filtering options for Central Ideas and Details, look for these three distinct flaws to execute immediate eliminations.

1. The "Too Narrow" (Detail) Distractor

This is the single most common trap for high-scoring students. An answer choice will state an indisputable fact written directly in sentence 3 of the text. However, if the question asks for the central claim, and that choice fails to capture the main overarching pivot, it is functionally wrong. A true detail does not make a main idea.

2. The Extreme Modifier Shift

Pay absolute attention to limiting words within the choices: all, every, always, exclusively, unique, never, or fundamentally. If an SAT text says that a phenomenon is "seldom observed," an answer choice stating that the phenomenon "never occurs" or is "impossible to track" is an immediate structural failure.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

3. The Unwarranted Extrapolation

Perfect-score seekers often think too deeply. They look at the scientific findings in the passage and think, "Well, if this is true, then it must logically mean that in the future, this policy change will happen." Stop. If the author did not explicitly state that future projection on the page, the choice is invalid. The SAT rewards rigorous literalism, not creative extension.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



Words in Context: Advanced Practice

In the Reading and Writing section of the SAT, vocabulary is no longer tested through rote memorization of obscure flashcards. Instead, the digital SAT evaluates your mastery of vocabulary via Words in Context questions.

In the Reading and Writing section of the SAT, vocabulary is no longer tested through rote memorization of obscure flashcards. Instead, the digital SAT evaluates your mastery of vocabulary via Words in Context questions.

Earning a perfect score requires moving past the misconception that these questions test your knowledge of definitions. The College Board explicitly states that these items assess your ability to "determine the meaning of a word or phrase in context" and "use vocabulary effectively to accomplish a rhetorical purpose." To achieve absolute accuracy, you must treat these questions as linguistic algebra problems. The missing word is a variable (x), and the surrounding text contains a series of non-negotiable semantic equations that dictate exactly what x must be.

Ch 1 The Core Architecture of Words in Context Items

Every Words in Context question consists of an information-dense, 3-to-5 sentence academic or literary excerpt containing a blank. The test makers intentionally select answer choices that are closely related in meaning, but only one option will satisfy the precise structural constraints of the paragraph.

The College Board builds these semantic constraints using three primary structural patterns.

Prototype 1: The Cross-Sentence Synchronic Pattern (Direct Redundancy)

- ◆ **The Mechanism:** The text introduces a concept or trait in one sentence, and then re-references or applies that exact same concept in the sentence containing the blank.
- ◆ **The Key:** The missing word is simply a stylistic or direct synonym of an explicit clue word found elsewhere in the text. The test makers are testing your ability to track a single thread of logic across boundaries.

Prototype 2: The Structural Contrast Pattern (Antonym Constraints)

- ◆ **The Mechanism:** The text establishes a baseline expectation, behavior, or physical state, and then shifts direction using a contrast coordinator or subordinating conjunction (e.g., however, whereas, while, by contrast, nevertheless).
- ◆ **The Key:** The missing word must act as the exact logical opposite or negation of the baseline state. The constraint is binary: if the baseline is complex, the blank must mean simple; if the baseline is public, the blank must mean private.

Prototype 3: The Internal Collocation & Colleague Pattern (Rhetorical Purpose)

- ◆ **The Mechanism:** The blank is embedded within a sentence that describes a specific scientific, mathematical, or literary process.
- ◆ **The Key:** The correct answer must match the domain-specific register of the sentence. The choice must accurately reflect how researchers, artists, or historians manipulate variables. For example, a scientist does not "fix" an error in a data set; they rectify it. A historian does not "show" a trend; they substantiate it.

Ch 2 The Five-Step Algebraic Extraction Protocol

Never read the answer choices and "plug them in" to see which one sounds best to your ear. Your ear is easily fooled by familiar phrasing or words that are grammatically correct but rhetorically imprecise. Follow this rigorous step-by-step protocol.

[Step 1: Locate & Target] → [Step 2: Identify structural signals] → [Step 3: Extract textual proof] → [Step 4: Formulate "Your Word"] → [Step 5: Filter options]

Step 1: Isolate the Blank (x)

Read the text smoothly to understand the broad topic. Then, treat the blank as a blank slate. Do not jump to conclusions based on the first half of the sentence.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Step 2: Identify the Structural Signals

Scan the text for structural roadmaps. Highlight or mentally note:

- ◆ **Punctuation Marks:** Colons (:) and em-dashes (—) almost always signal that an explanation or definition is immediately following.
- ◆ **Transition Words:** Words like indeed or furthermore signal continuation (synonym match). Words like conversely or although signal reversal (antonym match).

Step 3: Extract the Textual Proof (The Anchor)

Underline the exact phrase or word in the text that dictates the value of the blank. Every correct answer on the SAT is supported by an unambiguous textual anchor. If you cannot point your finger at a specific word or phrase that forces the blank to have a certain meaning, you are guessing, not analyzing.

Step 4: Write Your Own "Dummy Word"

Before looking at choices A, B, C, or D, write down or mentally formulate a simple, blunt word that fits the blank perfectly based on the anchor. Even a word as simple as "good," "bad," "hidden," or "clear" works perfectly as an algebraic placeholder.

Step 5: Filter the Choices (The Synchronic Match)

Compare your dummy word to the answer choices. Eliminate words that are too extreme, words that have the wrong positive/negative charge, and words that fit grammatically but fail to match the precise definition required by the textual anchor.

Ch 3 Decoding Official SAT Vocabulary Jargon

The digital SAT frequently features words that have multiple meanings. The correct answer is often a secondary or tertiary definition of a common word, chosen to match the academic register of the text.

Common Word	Primary Meaning (Everyday Use)	SAT Contextual Meaning (Academic/Official)
Calculate	To compute a number mathematically.	To deliberately design an action to achieve a specific effect ("a calculated risk").
Compelling	Exciting or highly entertaining.	Having powerful, non-negotiable force or validity ("compelling evidence").
Concede	To admit defeat in a game or election.	To acknowledge the validity of an opposing point or limitation.
Qualify	To meet the requirements for a position.	To limit, modify, or add conditions to a previous statement or claim.
Sanction	To punish a country economically.	To officially approve, allow, or give permission for an action.

Ch 4 Walkthroughs of Real SAT-Style Sample Items

Let's apply this algebraic extraction protocol to master class contextual vocabulary items.

GUIDED WALKTHROUGH 1 The Cross-Sentence Synchronic Pattern

Macroeconomist Elena Rostova argues that national fiscal interventions during market corrections are rarely spontaneous; rather, such monetary policies are typically _____ years in advance by central banking committees tracking structural indicators. Indeed, historical archives show that the sweeping banking reforms of 1934 were the result of nearly a decade of deliberate, methodical drafting.

Which choice completes the text with the most logical and precise phrase?

- (A) abandoned
- (B) anticipated
- (C) orchestrated
- (D) accelerated

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Step-by-Step Execution:

- 1 **Isolate the Blank:** The sentence states that monetary policies are typically [blank] years in advance by central banking committees.
- 2 **Identify Structural Signals:** The transition word "Indeed" opens the second sentence, signaling that the second sentence will provide a direct, parallel example that validates the claim made in the first sentence.
- 3 **Extract the Textual Proof (The Anchor):** The second sentence describes the banking reforms as the result of "nearly a decade of deliberate, methodical drafting." Therefore, the first sentence must say that these policies are created via deliberate, methodical drafting years in advance. Additionally, the phrase "rarely spontaneous; rather..." sets up a contrast with spontaneity. The blank must mean the opposite of spontaneous and the synonym of deliberately drafted.
- 4 **Formulate a Dummy Word:** Let's use "planned" or "carefully prepared."

Filter the Choices:

- (A) abandoned: Opposite of what we need. The reforms were carried out, not thrown away. Eliminate.
- (B) anticipated: This means to look forward to or expect. It does not capture the active, hands-on, deliberate creation process described by "methodical drafting." Eliminate.
- (C) orchestrated: This means to arrange, plan, or direct elements to achieve a desired result. This perfectly aligns with "deliberate, methodical drafting" and stands as the exact opposite of a "spontaneous" action. Keep this.
- (D) accelerated: This means to speed up. The text notes that the process took nearly a decade, which contradicts the idea of speeding things up. Eliminate.

CORRECT ANSWER: C**GUIDED WALKTHROUGH 2 The Structural Contrast Pattern**

Although many architectural historians have characterized early colonial timber structures as purely utilitarian assets built solely for immediate survival, recent archaeological excavations of foundations in coastal settlements reveal that these early longhouses frequently featured highly _____ geometric layouts designed to project civic authority and aesthetic sophistication.

Which choice completes the text with the most logical and precise phrase?

- (A) rudimentary
- (B) elaborate
- (C) transient
- (D) uniform

Step-by-Step Execution:

- 1 **Isolate the Blank:** The text describes the longhouses as featuring highly [blank] geometric layouts.
- 2 **Identify Structural Signals:** The paragraph opens with the subordinating conjunction "Although," establishing a binary contrast between how historians used to view the structures and what recent excavations actually reveal.
- 3 **Extract the Textual Proof (The Anchor):** The old view characterized the structures as "purely utilitarian assets built solely for immediate survival." The new view must contradict this. Furthermore, the text states these layouts were designed to project "civic authority and aesthetic sophistication." The blank modifies "geometric layouts" and must contrast with "purely utilitarian/survival-based" while aligning with "aesthetic sophistication."
- 4 **Formulate a Dummy Word:** Let's use "complex" or "fancy/sophisticated."

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Filter the Choices:

- (A) rudimentary: This means basic, primitive, or undeveloped. This is a synonym for "purely utilitarian," which matches the old view, not the new contrast view required after "Although." Eliminate.
- (B) elaborate: This means involved, detailed, or intricate. This perfectly contrasts with "purely utilitarian" and logically supports the idea of projecting "aesthetic sophistication." Keep this.
- (C) transient: This means temporary or short-lived. This aligns with the idea of "immediate survival," failing to capture the shift to permanent sophistication. Eliminate.
- (D) uniform: This means remaining the same or identical. While a geometric layout can be uniform, uniformity does not inherently convey the "aesthetic sophistication" or contrast with utility required by the anchor. Eliminate.

CORRECT ANSWER: B**Ch 5 Top Mistakes Made by High-Scoring Students**

Avoid these three traps to keep your vocabulary score perfect.

Trap 1: The "Looks Good in Isolation" Trap

The test makers love to place a word in the choices that fits beautifully into the sentence if you only read the three words before and after the blank. Always verify your choice against the entire paragraph. A word is only correct if it matches the anchor located in a completely separate sentence.

Trap 2: Seduction by High-Level Vocabulary

Do not select an answer choice simply because it is the most complex, academic-sounding word in the list. The SAT frequently uses simple words (e.g., share, clear, match) as the correct answers because they are the only words that fit the exact mathematical logic of the context.

Trap 3: Ignoring Negative and Positive Valence

If your textual anchor indicates that a process is failing or limited, the missing word must carry a negative valence (charge). Track the mathematical polarity (+ or -) of the sentence structures. A single word like rarely, scarcely, or belied can flip the necessary valence of the blank from positive to negative. Always calculate the net charge of the context before selecting your final answer.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



Function of a Sentence

In the Reading and Writing section of the SAT, questions that ask you to describe the function of a specific sentence (often underlined or referenced directly by the question stem) are not testing your general reading comprehension.

In the Reading and Writing section of the SAT, questions that ask you to describe the function of a specific sentence (often underlined or referenced directly by the question stem) are not testing your general reading comprehension. They are testing your mastery of textual architecture.

To earn a perfect score, you must stop reading sentences as isolated statements of fact and start reading them as functional gears in a machine. Every sentence in a highly structured SAT paragraph has a job to do: it introduces a problem, provides context, shifts direction, generalizes, or illustrates a previous claim.

This comprehensive guide breaks down the precise structural anatomy of these questions and provides a step-by-step framework to solve them with absolute certainty.

Ch 1 The Core Architecture of the SAT Paragraph

SAT paragraphs are meticulously engineered. They are extracted from academic journals, condensed into tight 3-to-5 sentence blocks, and heavily edited to follow rigid logical patterns. There is zero wasted space.

When a question asks, "Which choice best describes the function of the [underlined/referenced] sentence in the text as a whole?" it is asking you to identify the logical relationship between that specific sentence and the sentences surrounding it.

To predict the correct answer before looking at the choices, you must track structural landmarks. There are four primary prototypes for these questions. Let's break down their exact blueprints.

Prototype 1: The Research Disruption/Limitation Pattern

- ◆ **Sentence 1 (The Baseline Consensus):** Establishes a scientific fact, a point of general agreement, or an area of absolute confidence.
- ◆ **Sentence 2 (The Disruption/The Target):** Introduces an critical unknown variable, a limitation to current knowledge, or a specific problem that prevents further progress. (This is typically your target sentence).
- ◆ **Sentence 3 (The Attempted Solution):** Introduces a specific study, methodology, or research team that tried to resolve this exact unknown variable.
- ◆ **Sentence 4 (The Negative Result):** Concludes that the method failed to sufficiently resolve the variable, leaving the original problem intact.

Prototype 2: The Evolutionary/Historical Generalization Pattern

- ◆ **Sentence 1 (Historical Baseline):** Establishes a natural or historical state of affairs for a variable in Location A (e.g., Species X preys on Species Y in Asia).
- ◆ **Sentence 2 (The Shift/Dislocation):** Explains how one variable was moved to Location B, separating the interconnected elements.
- ◆ **Sentence 3 (The Core Thesis/The Target):** States a broad, conceptual ecological or historical principle/generalization (e.g., "But deep biological dependencies can survive across centuries and continents.").
- ◆ **Sentence 4 & 5 (The Empirical Proof):** Re-introduces the second variable into Location B and provides rapid, empirical statistics proving that the broad generalization stated in Sentence 3 is entirely correct.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Prototype 3: The Behavioral Mechanics Pattern

- ◆ **Sentence 1 (The General Behavioral Phenomenon):** Introduces an unusual or counterintuitive animal or natural behavior (e.g., some birds don't build nests).
- ◆ **Sentence 2 (Subgroup Focus):** Narrows the scope to a specific sub-species or group that performs this behavior.
- ◆ **Sentence 3 (The Step-by-Step Mechanics/The Target):** Strips away abstract theory to describe the exact, immediate physical or chemical actions taken to execute the behavior.
- ◆ **Sentence 4 (The Response/Aftermath):** Describes how other organisms or the surrounding environment react to the completed action.

Prototype 4: The Extreme Analogue Pattern

- ◆ **Sentence 1 (The Extreme Local Setting):** Describes a harsh, unforgiving, or unusual geographic environment on Earth.
- ◆ **Sentence 2 (The Research Rationale/The Target):** Explicitly explains why scientists travel to this miserable location (because it serves as a perfect chemical, geological, or atmospheric analogue for space or ancient Earth).
- ◆ **Sentence 3 (The Local Organisms):** Identifies the specific microbes or features surviving in this terrestrial setting.
- ◆ **Sentence 4 (The Global/Universal Goal):** Outlines the overarching, future-oriented mission objectives (e.g., using this data to build better tools or look for alien life).

Ch 2 The Step-by-Step Elimination Protocol

Do not read the answer choices first. The test makers are experts at writing "distractor" choices—options that use exact words from the text but describe the wrong logical relationship. Follow this strict four-step workflow to isolate the correct answer.

[Step 1: Locate & Bracket Target] → [Step 2: Assign Structural Label] → [Step 3: Define Context Link] → [Step 4: Filter Choices]

Step 1: Locate and Separate the Target

Read the paragraph once for basic context. Then, mentally bracket the exact sentence the question stem asks about. Read it completely isolated from the rest of the text. Ask yourself: What is the bare-bones subject and verb of this single sentence?

Step 2: Assign a Functional Label

Before looking back at the rest of the paragraph, look at how the target sentence operates internally. Does it contain a contrast word like however, but, or yet? Is it a broad declaration devoid of numbers, or is it dense with specific measurements?

If it contains contrast + uncertainty → Label it a Limitation/Problem.

If it is a broad statement devoid of specific species names or data → Label it a Generalization/Premise.

If it is a step-by-step physical breakdown of an action → Label it a Mechanical Detail/Execution.

If it contains words like because, replicates, or mirrors → Label it a Rationale/Explanation.

Step 3: Map the Contextual Connectors

Look immediately at the sentence before and the sentence after your target.

How does the target sentence change the direction of the text?

Is the sentence after it providing data to prove it?

Is the sentence after it introducing a group that tries to fix it?

Write down a one-line prediction of its job. For example: "It introduces a problem that a team of scientists then tries to fix with mixed results."

Step 4: Run the Filter on the Options

Evaluate the answer choices based only on your structural prediction. Eliminate any choice that fails to match the logical "job" of the sentence, even if the information in the choice is true.

Ch 3 Decoding the Test Maker's Jargon

To get a perfect score, you must learn to translate the abstract academic language used in correct SAT answer choices. The test makers intentionally replace simple words with precise, high-level vocabulary.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Simple Structural Concept	SAT Answer Choice Translation
It tells you a general rule.	"It presents a generalization..." / "It establishes a theoretical premise..."
It gives an example of something.	"...that is exemplified by the discussion..." / "...is illustrated by..."
It shows why scientists went there.	"It indicates why astrobiologists choose to conduct research..." / "It states a rationale..."
It points out what we don't know yet.	"It identifies the problem..." / "It notes a key analytical limitation..." / "It defines an uncertainty..."
It shows exactly how the animal did it.	"It offers a detail about how [the organism] carries out the behavior..." / "It details a mechanism..."

Ch 4 Walkthroughs of Real SAT-Style Sample Items

Let's apply this exact textbook framework to master class problems from each prototype.

GUIDED WALKTHROUGH 1 The Disruption Pattern

Glaciologists are certain that the massive Thwaites Glacier will eventually breach its subglacial ridge and accelerate into the ocean. They are much less certain, however, about the precise decade this collapse will initiate, since that depends on the underlying topography of the seabed that is largely unmapped. Geophysicist Tariq Al-Mansoor and colleagues recently investigated whether satellite-based radar interferometry could be used to chart these deep marine trenches but concluded that this method could not sufficiently resolve the seafloor's contours to allow the glacier's precise timeline to be firmly fixed.

Which choice best describes the function of the second sentence in the overall structure of the text?

Step-by-Step Execution:

- 1 Locate & Separate:** The target sentence is the bolded text. It notes that scientists are "much less certain" about the decade because the seabed is "largely unmapped."
- 2 Assign a Functional Label:** It contains the contrast marker however and the phrase less certain. This is a statement of uncertainty/limitation.
- 3 Map the Connectors:** Sentence 1 says what they know. Sentence 2 (our target) says what they don't know. Sentence 3 introduces a team (Al-Mansoor) trying to map it, followed by a negative result ("could not sufficiently resolve"). Therefore, the target sentence's job is to introduce the exact problem/unknown variable that Al-Mansoor tried to solve.

Evaluate the Choices:

- A It introduces the core analytical problem that Al-Mansoor and colleagues sought to address.**
Perfect match. The "unmapped seabed" is the precise analytical problem Al-Mansoor's radar study sought to address. Keep this.
- B It summarizes the primary empirical data collected by Al-Mansoor and colleagues.**
Wrong. Al-Mansoor's data isn't mentioned until Sentence 3. Sentence 2 describes an existing gap in knowledge, not new data. Eliminate.
- C It outlines a major methodological drawback inherent to satellite-based radar.**
Distractor. The text does mention a drawback to radar, but that happens in Sentence 3 ("could not sufficiently resolve"). Sentence 2 doesn't mention radar at all. Eliminate.
- D It details how the glaciological community responded to Al-Mansoor's findings.**
Wrong. The text does not mention the community's response or reception. Eliminate.

CORRECT ANSWER: A

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

GUIDED WALKTHROUGH 2 The Generalization Pattern

The agave plant diversified in Mesoamerica, where the snout weevil *Scyphophorus acupunctatus* specialized in boring into its stalks. In the nineteenth century, ornamental agaves were exported to southern Europe, far from any native weevils. But ecological dependencies between specialist herbivores and their hosts can endure across vast geographical divides and historical eras. Around 2012, *S. acupunctatus* was accidentally introduced to Spain near where ecologist Elena Santos and colleagues had been surveying native flora. Within two seasons, nearly 85 percent of the agave populations had been colonized by the insects.

Which choice best describes the function of the third sentence in the overall structure of the text?

Step-by-Step Execution:

- 1 Locate & Separate:** Target: "But ecological dependencies between specialist herbivores and their hosts can endure across vast geographical divides and historical eras."
- 2 Assign a Functional Label:** Notice that this sentence contains zero specific names, dates, or numbers. It discusses "specialist herbivores" and "hosts" in general. This is a broad conceptual generalization or thesis statement.
- 3 Map the Connectors:** Sentences 1 and 2 give a specific historical backdrop (agaves and weevils separated). Sentence 3 states a universal rule: separations don't break ecological bonds forever. Sentences 4 and 5 act as immediate, terrifying proof of that rule by showing what happened when they met again in Spain. Therefore, the target sentence provides a general principle that the rest of the text illustrates.

Evaluate the Choices:

- A** It proposes a competing explanation for the rapid decline of agave plants observed by Santos and colleagues.
Wrong. The sentence is not a competing explanation; it is the framework that explains the entire situation. Eliminate.
- B** It establishes a theoretical premise that is illustrated by the specific case of the agaves and weevils.
High-level match. "Theoretical premise" perfectly describes the broad generalization, and "illustrated by the specific case" perfectly describes how the text uses the Spain data to prove it. Keep this.
- C** It clarifies the environmental factors that enabled both species to migrate across continents.
Distractor. The text mentions how they migrated (exported ornamentally / introduced accidentally), but the target sentence discusses the endurance of ecological dependencies, not migration mechanics. Eliminate.
- D** It defines the explicit hypothesis that Santos and colleagues originally intended to test in Spain.
Wrong. The passage indicates Santos and colleagues were simply "surveying native flora"—they weren't hunting for weevils deliberately until the infestation occurred. Eliminate.

CORRECT ANSWER: B

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

GUIDED WALKTHROUGH 3 The Extreme Analogue Pattern

Antarctica's McMurdo Dry Valleys comprise one of the coldest, most hyper-arid zones on Earth. Geochemist Marcus Vance and other paleobiologists search for biochemical signatures in this frozen landscape because the environment closely mirrors the archaic conditions of early Mars. The unique endolithic cyanobacteria isolated inside the porous rocks of the Dry Valleys may offer clues about Martian life. By studying how these resilient strains survive such nutrient-depleted states on Earth, Vance's team hopes to determine whether similar life might have existed on Mars and to develop the best tools to look for evidence of it.

Which choice best describes the function of the underlined sentence in the text as a whole?

Step-by-Step Execution:

- 1 Locate & Separate:** Target: The second sentence, explaining that Vance searches this landscape because it mirrors early Mars.
- 2 Assign a Functional Label:** The word because signals a cause, motivation, or rationale. The phrase closely mirrors establishes it as an environmental analogue.
- 3 Map the Connectors:** Sentence 1 sets up a harsh Earth environment. Sentence 2 explains why scientists choose to study it. Sentences 3 and 4 show what they found there and what their ultimate goals are. The target sentence bridges the terrestrial location with the planetary research objectives.

Evaluate the Choices:

- A To delineate the structural constraints of studying microbial life in frozen regions.**
Wrong. The sentence doesn't focus on constraints or what makes the research hard; it explains the purpose behind it. Eliminate.
- B To argue that early Mars conditions were significantly colder than those of modern Antarctica.**
Misinterpretation. The text says the environments mirror (resemble) each other, not that one was colder than the other. Eliminate.
- C To indicate why paleobiologists choose to conduct research in the McMurdo Dry Valleys.**
Simple, perfectly precise match. It explains the exact rationale ("because it mirrors Mars") for selecting this fieldwork site. Keep this.
- D To account for the total absence of multi-cellular organisms in hyper-arid landscapes.**
Out of bounds. The paragraph focuses on micro-organisms (cyanobacteria), not explaining why larger animals are absent. Eliminate.

CORRECT ANSWER: C**Ch 5** Top Mistakes Made by High-Scoring Students

Even students tracking at a 750+ score level fall into classic traps on these specific questions. Memorize these three structural pitfalls to preserve your perfect score.

Trap 1: The "True-But-Wrong" Option

The test makers love to write an answer choice that accurately summarizes information found elsewhere in the paragraph, but completely misstates the function of the targeted sentence. Always double-check your bracketing. Ensure the answer choice describes what the underlined sentence is doing, not what the last sentence is doing.

Trap 2: Seduction by Synonyms

Do not select an option simply because it matches vocabulary words from the paragraph. If the target sentence says "cyanobacteria," an answer choice that states "It outlines the morphological classification of cyanobacteria" looks incredibly tempting. However, if the sentence's actual logical job was to act as an environmental rationale, the option is structurally wrong. Function trumps topic every single time.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Trap 3: Over-interpreting Tone

SAT texts are objective, academic excerpts. Avoid any answer choices that use overly dramatic verbs like "It attacks an established consensus," "It disproves a popular myth," or "It laments a failure." Look for neutral, clinical terms like "It identifies an uncertainty," "It offers a detail," or "It presents a generalization."

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



How the Math Section Works

The structure, pacing, calculator strategy, and formulas behind every math question.

The Format at a Glance

The digital SAT Math section has two modules of 22 questions each, 44 questions in all, with 35 minutes per module. Roughly three out of four questions are multiple choice with four answer choices. The rest are student-produced responses, where you type your answer instead of choosing one. You may use a calculator on every question, and a built-in graphing calculator is available on screen.

Domain	Share of questions	Skills tested
Algebra	About 35%	Linear equations in one variable, linear functions, linear equations in two variables, systems of two linear equations, linear inequalities
Advanced Math	About 35%	Equivalent expressions, nonlinear equations in one variable and systems, nonlinear functions
Problem-Solving and Data Analysis	About 15%	Ratios, rates, proportional relationships and units; percentages; one-variable data; two-variable data; probability; inference and margin of error; statistical claims
Geometry and Trigonometry	About 15%	Area and volume; lines, angles, and triangles; right triangles and trigonometry; circles

How the Modules Work

Within each module, questions run roughly from easier to harder inside each domain. Your performance in Module 1 decides whether Module 2 is the easier or the harder version. A strong Module 1 unlocks the harder Module 2, which is where the highest scores are earned. Every question is worth the same, so never sink three minutes into one problem. Mark it, move on, and come back.

Pacing Rule

You have about 95 seconds per question on average. Aim to finish easy questions in under a minute so you can bank time for the hard ones.

If you have not found a path after about 90 seconds, mark the question, make your best guess, and move on.

Calculator Strategy

- ◆ **Graph it.** For any equation, system, or function question, graph the expressions and read intersections, intercepts, and vertices directly.
- ◆ **Plug in the answer choices.** When the choices are numbers, test them in the problem. Start with the middle values.
- ◆ **Pick numbers.** When a problem has variables and no actual values, choose simple numbers, compute, and compare with the choices.
- ◆ **Check, do not trust.** After solving by hand, confirm with the calculator. A two-second check catches most sign errors.

Student-Produced Responses

- ◆ Enter a positive or negative number, a fraction, or a decimal. Fractions such as $\frac{3}{4}$ and decimals such as 0.75 are both accepted.
- ◆ If your answer is a repeating decimal, enter at least four digits after the decimal point, or use the fraction.
- ◆ If the question asks for a percent, enter only the number, without the percent sign.
- ◆ There is no partial credit and no penalty for guessing, so always enter something.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Formulas Worth Knowing

The test provides a short reference sheet, but you work faster when these are memorized. Each is used again in the chapters that follow.

Topic	Formula
Rectangle	$A = \ell w$
Triangle	$A = \frac{1}{2}bh$
Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$
Circle	$A = \pi r^2$ and $C = 2\pi r$
Pythagorean theorem	$a^2 + b^2 = c^2$
Special right triangles	45°-45°-90°: sides $x, x, x\sqrt{2}$. 30°-60°-90°: sides $x, x\sqrt{3}, 2x$
Rectangular solid	$V = \ell wh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}\ell wh$
Slope	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Circle in the plane	$(x - h)^2 + (y - k)^2 = r^2$
Arc length and sector	Arc $= \frac{\theta}{360} \cdot 2\pi r$, sector area $= \frac{\theta}{360} \cdot \pi r^2$

How to use the chapters that follow

Each chapter opens with what the SAT asks, teaches the core ideas, walks through worked examples from easy to hard, and ends with a strategy section, a list of common traps, and eight practice problems. Do the practice problems with the calculator open, then read every explanation, even the ones you got right.

Want a plan built around **your** student?
Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



Algebra

5 chapters of original lessons with worked examples, then 40 practice problems with full answers and explanations.

M1 Linear Equations in One Variable

Master solving linear equations, including fractions and decimals, equations with no or infinitely many solutions, and constant-finding questions. These show up on every test and reward careful, organized algebra.

What the SAT Asks

Linear equations in one variable are the foundation of the whole Algebra domain. You will see them in plain form ("What is the solution to the equation?"), in disguise as word problems ("how many hours until the two plans cost the same?"), and in a favorite trick form that asks about a number of solutions or a missing constant ("For what value of k does the equation have no solution?"). A typical test has about 1 to 4 of these questions across the two math modules, and some of them are the quickest points on the whole test.

Difficulty ranges from one-step warm-ups to multi-step equations with fractions, decimals and a parameter. The hard versions are rarely about harder algebra. They test whether you understand what an equation is: a statement that is true for some values (one solution), no values (no solution), or every value (infinitely many solutions). Once you can read an equation that way, the parameter questions become routine.

Core Ideas

An equation is a balance scale. Whatever you do to one side you must do to the other, so the two sides stay equal. The goal is to **isolate the variable**: undo operations in reverse order using inverse operations (add and subtract, then multiply and divide). Three habits make this reliable. First, **distribute** before anything else: $3(x - 4) = 3x - 12$, and a minus sign in front of a parenthesis flips every sign inside it. Second, **combine like terms** on each side. Third, **collect variable terms on one side** and constants on the other.

Solving a Linear Equation

Simplify each side (distribute, combine like terms), move all x terms to one side and all constants to the other, then divide by the coefficient of x .

Always check by substituting your answer into the original equation. On the SAT this takes ten seconds and catches sign errors.

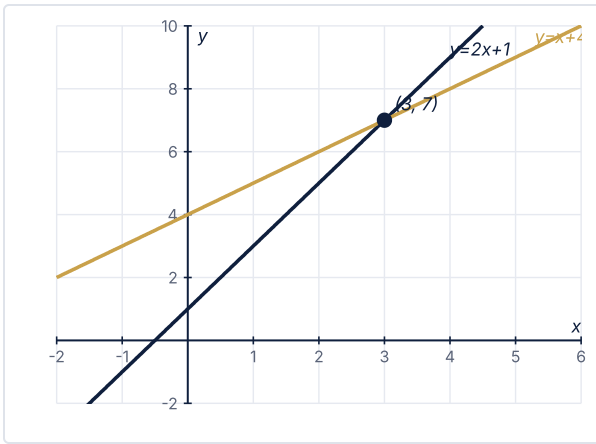
Fractions. Multiply both sides by the least common denominator (LCD) to clear every fraction at once. For $\frac{x}{3} + \frac{x}{4} = 7$ the LCD is 12, so $4x + 3x = 84$ and $x = 12$. Be sure to multiply every term, including whole numbers, and wrap numerators with more than one term in parentheses: $\frac{x-1}{2}$ times 6 becomes $3(x - 1)$, not $3x - 1$. **Decimals.** Multiply by 10, 100 or 1000 to make every coefficient a whole number, or just work with the decimals if the calculator is open.

How many solutions? Simplify until the equation looks like $ax + b = cx + d$ and compare the pieces. The picture is useful: each side is a line, and a solution is an x -value where the two lines meet.

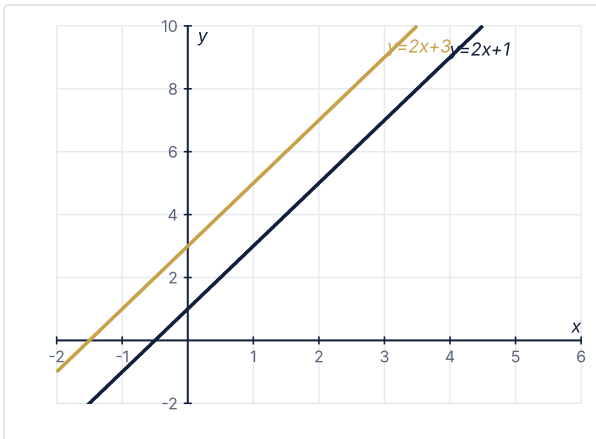
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



The equation $2x + 1 = x + 4$ has exactly one solution, the x -coordinate of the crossing point: $x = 3$.



The equation $2x + 1 = 2x + 3$ has no solution: the lines have equal slopes and different intercepts, so they never meet.

Number of Solutions of $ax + b = cx + d$

One solution when $a \neq c$ (different slopes). Solve normally: $x = \frac{d-b}{a-c}$.

No solution when $a = c$ and $b \neq d$. The variables cancel and you are left with a false statement such as $4 = 9$.

Infinitely many solutions when $a = c$ and $b = d$. The variables cancel and you are left with a true statement such as $5 = 5$.

Parameter questions. When a problem says "for what value of k " the equation has no solution (or infinitely many), expand both sides, set the x -coefficients equal to find k , then confirm the constants differ (no solution) or match (infinitely many). The constants decide which case you are in, so never skip that last comparison.

Translating words. Convert phrases one piece at a time: "is" means $=$, "more than" means add, "less than" reverses order ("5 less than x " is $x - 5$), "twice a number" is $2x$, and a rate times a time is an amount. Define your variable in a sentence first, then build the equation.

Words	Algebra
a number increased by 7	$x + 7$
7 less than a number	$x - 7$
twice the sum of a number and 3	$2(x + 3)$
a fee of 20 plus 6 per hour for h hours	$20 + 6h$
three-fourths of a number is 18	$\frac{3}{4}x = 18$

Why the constants decide. Suppose an equation simplifies to $7x + 2 = 7x + 2$. Every value of x works, because both sides are literally the same expression. If it simplifies to $7x + 2 = 7x + 5$, the left side is always 3 less than the right side, so they can never be equal. Thinking in terms of "always equal", "never equal" and "equal once" is faster than any formula and it is exactly how the parameter questions are built.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Answers that are not whole numbers. A solution such as $x = \frac{5}{2}$ or $x = -0.75$ is perfectly normal. On student-produced response questions you may enter a fraction or a decimal, so there is no need to round unless the question says so. If the grid shows a negative sign, include it.

Worked Examples

EXAMPLE 1 (WARM-UP)

Solve $3(x - 4) + 5 = 2x + 9$.

SOLUTION

Distribute: $3x - 12 + 5 = 2x + 9$, so $3x - 7 = 2x + 9$.

Subtract $2x$ from both sides: $x - 7 = 9$.

Add 7: $x = 16$.

Check: left side $3(12) + 5 = 41$, right side $2(16) + 9 = 41$. The answer is **16**.

EXAMPLE 2 (MULTIPLE CHOICE)

What is the solution to $\frac{2x+1}{3} - \frac{x-1}{2} = 2$?

- A. 3
- B. 7
- C. 11
- D. 17

SOLUTION

The LCD is 6. Multiply every term by 6: $2(2x + 1) - 3(x - 1) = 12$.

Distribute, remembering the minus sign hits both terms in the second parenthesis: $4x + 2 - 3x + 3 = 12$.

Combine: $x + 5 = 12$, so $x = 7$. The answer is **B**.

The tempting choice C comes from writing $-3x - 1$ instead of $-3x + 3$, which gives $x + 1 = 12$.

EXAMPLE 3 (STUDENT-PRODUCED RESPONSE)

In the equation $4(kx - 3) = 12x + 20$, k is a constant. If the equation has no solution, what is the value of k ?

SOLUTION

Expand the left side: $4kx - 12 = 12x + 20$.

For the x terms to cancel, the coefficients must match: $4k = 12$, so $k = 3$.

Then the equation becomes $12x - 12 = 12x + 20$, which simplifies to $-12 = 20$, a false statement. So no solution is confirmed.

The answer is **3**.

EXAMPLE 4 (SHORTCUT)

If $4x + 3 = 19$, what is the value of $8x + 5$?

SOLUTION

Do not rush to find x first, though that works: $x = 4$, so $8(4) + 5 = 37$.

The shortcut: $8x + 5$ is built from $4x + 3$. Notice $8x + 6 = 2(4x + 3) = 38$, so $8x + 5 = 38 - 1 = 37$.

The answer is **37**. Watch for this pattern: the SAT often asks for an expression that is a multiple of the given one.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 5 (WORD PROBLEM)

Gym A charges a 45 dollar joining fee plus 28 dollars per month. Gym B charges a 15 dollar joining fee plus 33 dollars per month. After how many months will the total cost at the two gyms be the same?

SOLUTION

Let m be the number of months. Gym A costs $45 + 28m$ and Gym B costs $15 + 33m$.

Set them equal: $45 + 28m = 15 + 33m$.

Subtract $28m$ and 15: $30 = 5m$, so $m = 6$.

Check: $45 + 168 = 213$ and $15 + 198 = 213$. The answer is **6 months**.

SAT Strategy

- 1 Plug in answer choices.** When the equation is messy, test the choices starting with the middle value. Choices are listed in order, so one test tells you whether to go higher or lower.
- 2 Use the on-screen calculator.** Type the left side as one graph and the right side as another. The x -coordinate of the intersection is the solution. For a parameter problem, enter the equation with a slider for k and watch when the lines become parallel (no solution) or merge (infinitely many).
- 3 Look for the target expression.** If the question asks for $6x - 9$ or $x + 5$ instead of x , check whether it is a multiple or a shift of the given equation before solving.
- 4 Clear fractions and decimals first.** Multiply by the LCD (or by 10 or 100) to work with whole numbers, which cuts down on arithmetic slips.
- 5 Always read the last line of the question.** Some ask for x , others for $2x$, for $x - 3$ or for the number of months. Underline what is being asked before you start.

Common traps

(1) Distributing to only the first term: $3(x - 4)$ is $3x - 12$, not $3x - 4$. (2) Dropping the sign change when subtracting a parenthesis: $-(x - 1) = -x + 1$. (3) Forgetting to multiply every term, including constants, by the LCD when clearing fractions. (4) Finding k for "no solution" but not checking the constants; if they also match, the equation has infinitely many solutions instead. (5) Solving for x when the question asks for an expression in x .

Practice**PRACTICE Linear Equations in One Variable**

Choose the best answer or enter your answer. Calculator allowed.

1. What is the solution to $5(x - 2) = 3x + 8$?
(A) 5 (B) 9 (C) 18 (D) -1
2. What value of x satisfies $0.4x + 1.2 = 0.1x + 3$?
(A) 3.6 (B) 14 (C) 6 (D) 18
3. In the equation $3(2x - 5) = kx + 7$, k is a constant. If the equation has no solution, what is the value of k ?
(A) 3 (B) 12 (C) -6 (D) 6
4. If $5x - 2 = 18$, what is the value of $15x - 6$?
(A) 54 (B) 18 (C) 60 (D) 12
5. Maya has read $\frac{3}{5}$ of a novel and has 80 pages left to read. How many pages are in the novel?
(A) 32 (B) 200 (C) 120 (D) 400
6. What is the solution to $\frac{5x-3}{4} = \frac{x+9}{2}$?
7. In the equation $2(3x - 4) + k = 6x + 11$, k is a constant. If the equation has infinitely many solutions, what is the value of k ?
8. In the equation $\frac{kx-2}{3} = x + \frac{k}{2}$, k is a constant. If $x = 2$ is a solution, what is the value of k ?

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

M2 Linear Functions

Learn slope, intercepts and function notation, then use them to read tables and graphs, write equations from two points, interpret rates in context, and handle parallel and perpendicular lines.

What the SAT Asks

Linear functions are everywhere on the digital SAT. Expect questions such as "What is the slope of the line?", "Which equation defines f ?", "What does the number 12 represent in the function $h(t) = 12t + 40$?" and "Line m is perpendicular to line ℓ ; what is the slope of m ?" A typical test has about 2 to 5 questions that are mainly about linear functions, and many more questions quietly use them (tables, graphs and word problems).

The easy questions ask you to evaluate $f(5)$ or read a slope from a graph. The hard ones give you two points or a table and ask you to build the equation, interpret a constant in context, or use parallel and perpendicular conditions with an unknown. All of them reduce to two numbers: the **rate of change** (slope) and the **starting value** (intercept).

Core Ideas

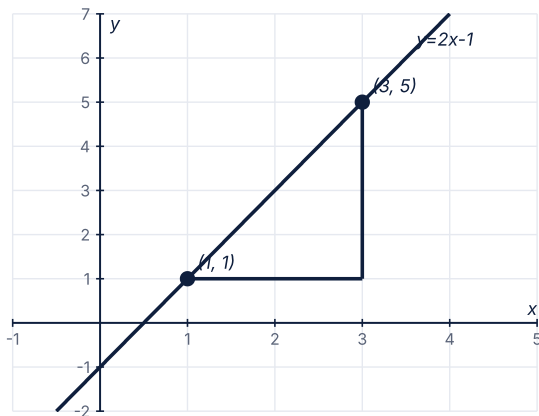
A **function** assigns exactly one output to each input. The notation $f(x)$ is read " f of x " and means "the output when the input is x ." It is not multiplication. If $f(x) = 4x - 7$, then $f(5) = 4(5) - 7 = 13$, and the sentence " $f(x) = 21$ " is an equation to solve: $4x - 7 = 21$ gives $x = 7$. A function is **linear** when it changes by the same amount for every one-unit step in the input.

Slope

The slope of the line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}.$$

Subtract in the same order on top and bottom. Positive slope rises left to right, negative slope falls, zero slope is horizontal, and a vertical line has undefined slope.



Run of 2 and rise of 4 give slope $\frac{4}{2} = 2$. The line crosses the y -axis at -1 .

Slope-Intercept Form

$$f(x) = mx + b$$

m is the slope (the change in output for each 1-unit increase in input) and b is the y -intercept, the output when the input is 0, so the graph passes through $(0, b)$.

Point-slope form $y - y_1 = m(x - x_1)$ is handy when you know one point and the slope.

Building an equation from two points. First find m . Then substitute either point into $y = mx + b$ and solve for b . For $(2, 5)$ and $(8, 23)$: $m = \frac{23-5}{8-2} = 3$, then $5 = 3(2) + b$ gives $b = -1$, so $y = 3x - 1$. A faster check: plug in the other point to be sure.

Reading rates from tables and words. If a table has equally spaced inputs, divide the change in output by the change in input. A rate phrase such as "7 dollars per hour" or "decreases by 3 liters each minute" is the slope, and a phrase such as "initial", "starting", "flat fee" or "at time 0" is the intercept.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

t (minutes)	0	5	10	15
$h(t)$ (liters left in a tank)	120	105	90	75

In the table above the output drops by 15 every time the input rises by 5, so the slope is -3 liters per minute and the value at $t = 0$ is 120. The function is $h(t) = -3t + 120$.

Interpreting $f(x) = mx + b$ in Context

Slope m : the amount the output changes for each 1-unit increase in the input, with units "output units per input unit."

Intercept b : the output when the input is 0 (the starting amount).

Parallel lines have equal slopes. **Perpendicular lines** have slopes that are negative reciprocals, so $m_1 m_2 = -1$. For slope $\frac{3}{4}$ the perpendicular slope is $-\frac{4}{3}$.

Reading a graph. To find the slope from a graph, pick two points that sit exactly on grid intersections, count the rise and the run, and attach a sign. The y -intercept is where the line crosses the vertical axis; the x -intercept is where $f(x) = 0$, found by solving $mx + b = 0$ to get $x = -\frac{b}{m}$. On the SAT, graphs are often drawn with different scales on the two axes, so read the labels before you count squares.

Special lines. A horizontal line has equation $y = c$ and slope 0, so it is a constant function. A vertical line $x = c$ has undefined slope and is not the graph of a function, because one input would have many outputs. A line through the origin has $b = 0$, which means $f(x) = mx$ and the output is **proportional** to the input.

Same line, different looks. The same linear relationship can be given as an equation, a table, a graph, or a sentence, and the SAT likes to switch between them. A strong habit is to convert whatever you are given into $y = mx + b$ first. From there every question (slope, intercept, parallel, value at some input) is a one-line calculation.

Worked Examples

EXAMPLE 1 (WARM-UP)

The function f is defined by $f(x) = 4x - 7$. What is the value of x for which $f(x) = 21$?

SOLUTION

Replace $f(x)$ with 21: $4x - 7 = 21$.

Add 7: $4x = 28$.

Divide by 4: $x = 7$. Check: $f(7) = 28 - 7 = 21$. The answer is **7**.

Be careful: $f(5)$ would ask for an output (13), while $f(x) = 21$ asks for an input.

EXAMPLE 2 (MULTIPLE CHOICE)

A line passes through the points (2, 5) and (8, 23). Which equation defines the line?

A. $y = 3x + 5$

B. $y = 3x - 1$

C. $y = \frac{1}{3}x + \frac{13}{3}$

D. $y = 6x - 7$

SOLUTION

Slope: $\frac{23-5}{8-2} = \frac{18}{6} = 3$. That already eliminates C and D.

Intercept: $5 = 3(2) + b$ gives $b = -1$, so $y = 3x - 1$.

Choice A reuses the y -value 5 as the intercept, which is only valid at $x = 0$. Check B at the second point: $3(8) - 1 = 23$. The answer is **B**.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 3 (IN CONTEXT)

A tank drains at a constant rate. The table shows the liters $h(t)$ left after t minutes.

t (minutes)	0	5	10	15
$h(t)$ (liters)	120	105	90	75

SOLUTION

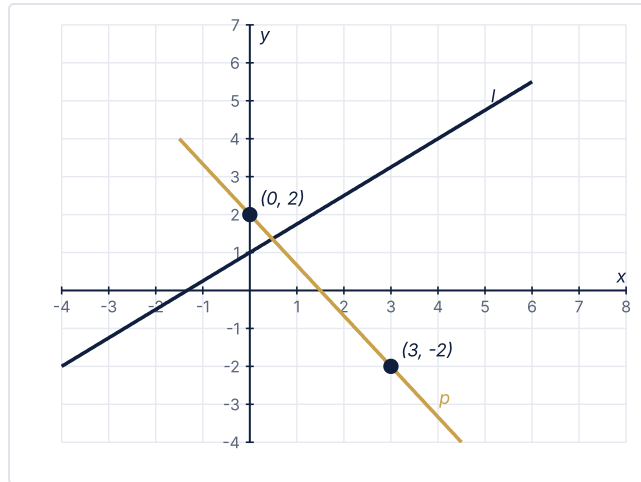
Using the table above: from $t = 0$ to $t = 5$ the output changes by $105 - 120 = -15$, so the slope is $\frac{-15}{5} = -3$.

The intercept is $h(0) = 120$, so $h(t) = -3t + 120$.

Meaning: the tank holds **120 liters at the start** and loses **3 liters each minute**. It is empty when $-3t + 120 = 0$, at $t = 40$ minutes.

EXAMPLE 4 (PERPENDICULAR LINES)

Line ℓ is given by $y = \frac{3}{4}x + 1$. Line p is perpendicular to ℓ and passes through $(3, -2)$. What is the y -intercept of line p ? (Student-produced response)

**SOLUTION**

The slope of ℓ is $\frac{3}{4}$, so the slope of p is the negative reciprocal, $-\frac{4}{3}$.

Point-slope form: $y + 2 = -\frac{4}{3}(x - 3)$, so $y = -\frac{4}{3}x + 4 - 2 = -\frac{4}{3}x + 2$.

The y -intercept is **2**. The graph confirms that p crosses the y -axis at 2 and meets ℓ at a right angle.

EXAMPLE 5 (UNKNOWN IN A POINT)

A line passes through $(1, k)$ and $(5, 10)$ and is parallel to the line $y = 2x - 3$. What is the value of k ?

SOLUTION

Parallel lines have equal slopes, so the slope of the line through the two points must be 2.

Set up: $\frac{10-k}{5-1} = 2$, so $10 - k = 8$.

Therefore $k = 2$. Check: the slope from $(1, 2)$ to $(5, 10)$ is $\frac{8}{4} = 2$. The answer is **2**.

SAT Strategy

- 1 Name the slope and the intercept first.** In any context problem, say in words what changes at a constant rate (slope) and what the starting value is (intercept) before you look at the choices.
- 2 Test points.** If you are given two points and four equations, plug each point into the choices. The right equation satisfies both, and one failed point eliminates a choice immediately.
- 3 Graph it.** Type the equation or the two points into the on-screen calculator. Use the graph to confirm slope, intercepts, and whether lines are parallel or perpendicular.
- 4 Use a table of values.** For $f(x) = k$ questions, enter $y = f(x)$ and $y = k$ and read the intersection, or substitute the answer choices as inputs.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 5 Keep units attached.** The slope has units of output per input, such as dollars per hour. Choices that swap the roles of slope and intercept are common.

Common traps

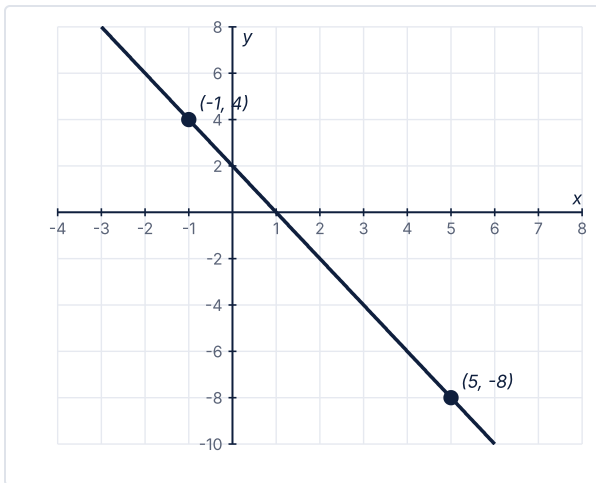
(1) Subtracting coordinates in different orders, such as $\frac{y_2 - y_1}{x_1 - x_2}$, which flips the sign of the slope. (2) Confusing $f(x) = 21$ (solve for the input) with $f(21)$ (find the output). (3) For perpendicular lines, flipping the fraction but not the sign, or the sign but not the fraction. (4) Reading the first table value as the intercept when the table does not start at an input of 0. (5) Calling the slope a total amount instead of a rate.

Practice

PRACTICE Linear Functions

Choose the best answer or enter your answer. Calculator allowed.

- The function f is defined by $f(x) = -2x + 9$. What is the value of $f(-3)$?
 (A) 3 (B) 12 (C) 15 (D) 6
- The graph of a line is shown. What is the slope of the line?



- (A) -2 (B) 2 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$
- The table shows values of a linear function f . Which equation defines f ?

x	0	2	4	6
$f(x)$	5	11	17	23

 (A) $f(x) = 6x + 5$ (B) $f(x) = 5x + 3$ (C) $f(x) = 3x + 11$
 (D) $f(x) = 3x + 5$
- A candle is lit and burns at a constant rate. Its height is 14 centimeters after 3 hours and 4 centimeters after 8 hours. Which statement about the candle is true?
 (A) The candle was 14 centimeters tall when it was lit.
 (B) The candle's height decreases by 2 centimeters each hour.
 (C) The candle's height decreases by 5 centimeters each hour.
 (D) The candle burns out 20 hours after it is lit.
- Line ℓ is defined by $y = \frac{2}{3}x - 4$. Which equation defines the line that is perpendicular to ℓ and passes through $(6, 1)$?
 (A) $y = \frac{3}{2}x - 8$ (B) $y = -\frac{2}{3}x + 5$ (C) $y = -\frac{3}{2}x + 10$ (D) $y = -\frac{3}{2}x + 1$
- The linear function f satisfies $f(2) = 5$ and $f(5) = 17$. What is the value of $f(8)$?
- The linear function f satisfies $f(0) = -4$ and $f(6) = 14$. For what value of x does $f(x) = 32$?
- Line r passes through $(-2, 1)$ and $(4, k)$ and is perpendicular to the line $y = 2x + 5$. What is the value of k ?

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

M3 Linear Equations in Two Variables

Work with standard form $Ax + By = C$: find intercepts and slope, test points, write equations from two points, model two-item situations, and interpret every number in context.

What the SAT Asks

Many SAT linear relationships are written in **standard form**, $Ax + By = C$, instead of $y = mx + b$. You will be asked to find an intercept or slope from such an equation, decide whether a point lies on the line, write the equation of a line through two points, or explain what a number in the equation means in a real situation. A typical test has about 2 to 4 questions in this area, and most of them are word problems with two kinds of items being bought, mixed or counted.

The difficulty runs from "is this point on the line?" up to questions where the equation is hidden in a story and you must decide what x , y and each constant mean. The strongest tool is not a formula but a habit: **substitute a simple value** (like 0) into the equation and say in words what happens.

Core Ideas

A linear equation in two variables has infinitely many solutions, each an ordered pair (x, y) . All of those pairs lie on one straight line. A point is on the line exactly when its coordinates make the equation true, so to test a point you substitute both coordinates and see whether the two sides match.

Standard Form and What It Tells You

$$Ax + By = C$$

x-intercept: set $y = 0$, giving $x = \frac{C}{A}$. **y-intercept:** set $x = 0$, giving $y = \frac{C}{B}$.

Slope: solve for y to get $y = -\frac{A}{B}x + \frac{C}{B}$, so the slope is $-\frac{A}{B}$.

For $3x - 4y = 24$: setting $y = 0$ gives $x = 8$; setting $x = 0$ gives $y = -6$; and the slope is $-\frac{3}{-4} = \frac{3}{4}$. Two intercepts are enough to draw the whole line, which makes standard form the fastest form to graph. Plot the two intercepts and join them.

Form	Equation	Best for
Slope-intercept	$y = mx + b$	reading slope and y -intercept
Standard	$Ax + By = C$	intercepts, two-quantity models
Point-slope	$y - y_1 = m(x - x_1)$	building a line from a point and a slope

Writing an equation from two points. Find the slope $\frac{y_2 - y_1}{x_2 - x_1}$, then use point-slope form with either point and rearrange to the form the question asks for. If the choices are in standard form, move the x term to the left and clear fractions. To check a choice, substitute both points; a correct equation passes both.

Parallel and perpendicular lines. Parallel lines have equal slopes, so in standard form the coefficients of x and y are proportional: $3x + 4y = 8$ and $3x + 4y = 16$ are parallel. Perpendicular lines have slopes whose product is -1 . For $Ax + By = C$ a perpendicular line has the form $Bx - Ay = D$, with the same two coefficients swapped and one sign changed.

Meaning of the Pieces in a Model

In $ax + by = c$ where x and y count two kinds of items: a and b are the **price (or amount) per item** and c is the **total**.

The term ax is the total value contributed by the x items. A point such as $(0, \frac{c}{b})$ means "all of the total is spent on the y items and none on the x items."

Modeling two quantities. If tickets cost 6 dollars for students and 10 dollars for adults and the total is 840 dollars, the equation is $6s + 10a = 840$. Check units on every term: $6s$ is dollars, $10a$ is dollars, and 840 is dollars. If the units on the two sides do not match, the model is wrong. Intercepts have real meanings here: they are the extreme cases where only one kind of item is used.

Constraints with whole numbers. In a context, only some solutions make sense. You cannot sell 3.5 tickets, and a negative quantity is impossible. When a question asks which pair of values is possible, test the choices in the equation and also check that they are non-negative whole numbers.

Worked Examples

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 1 (WARM-UP, MULTIPLE CHOICE)

Which point lies on the line $4x + 5y = 2$?

- A. $(1, -2)$
- B. $(3, -2)$
- C. $(-3, 2)$
- D. $(2, -1)$

SOLUTION

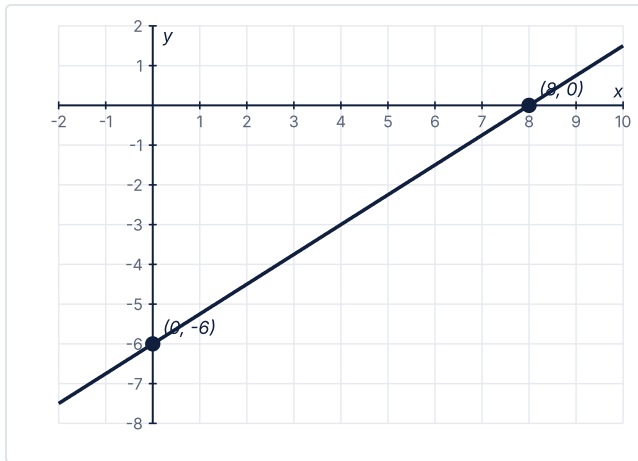
Substitute each point. A: $4 + (-10) = -6$. B: $12 + (-10) = 2$, which matches.

C: $-12 + 10 = -2$. D: $8 - 5 = 3$.

Only B works. The answer is **B**. Choice C has the right numbers with the signs swapped, a common near-miss.

EXAMPLE 2 (STUDENT-PRODUCED RESPONSE)

The graph of $3x - 4y = 24$ is shown. What is the slope of the line?

**SOLUTION**

Method 1: solve for y . Subtract $3x$: $-4y = -3x + 24$. Divide by -4 : $y = \frac{3}{4}x - 6$.

Method 2: slope = $-\frac{A}{B} = -\frac{-3}{-4} = \frac{3}{4}$.

The graph agrees: from $(0, -6)$ to $(8, 0)$ the rise is 6 and the run is 8, giving $\frac{6}{8} = \frac{3}{4}$. The answer is **3/4** (or 0.75).

EXAMPLE 3 (INTERPRETATION, MULTIPLE CHOICE)

A school fundraiser sells student tickets for 6 dollars each and adult tickets for 10 dollars each. The equation $6s + 10a = 840$ represents the situation where s is the number of student tickets and a is the number of adult tickets sold, and the total is 840 dollars. What does the point $(0, 84)$ represent?

- A. 84 student tickets and no adult tickets were sold.
- B. 84 adult tickets and no student tickets were sold.
- C. The price of an adult ticket is 84 dollars.
- D. A total of 84 tickets was sold at 10 dollars each.

SOLUTION

The point $(0, 84)$ means $s = 0$ and $a = 84$. Check: $6(0) + 10(84) = 840$, so it satisfies the equation.

The first coordinate is student tickets, so there are no student tickets, and the second is adult tickets, so there are 84 of them.

The answer is **B**. Choice A reverses the roles of the coordinates.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 4 (TWO POINTS)

A line passes through $(-2, 7)$ and $(4, -5)$. The line can be written as $2x + By = C$, where B and C are constants. What is the value of C ?

SOLUTION

The slope is $\frac{-5-7}{4-(-2)} = \frac{-12}{6} = -2$.

Point-slope with $(4, -5)$: $y + 5 = -2(x - 4)$, so $y = -2x + 3$.

Move the x term: $2x + y = 3$. So $B = 1$ and $C = 3$.

Check with $(-2, 7)$: $2(-2) + 7 = 3$. The answer is **3**.

EXAMPLE 5 (PERPENDICULAR WITH AN UNKNOWN)

The line $kx + 5y = 20$ is perpendicular to the line $x - 5y = 3$. What is the value of k ?

SOLUTION

Slope of the second line: $-\frac{1}{-5} = \frac{1}{5}$. Slope of the first line: $-\frac{k}{5}$.

Perpendicular slopes multiply to -1 : $-\frac{k}{5} \cdot \frac{1}{5} = -1$.

So $\frac{k}{25} = 1$ and $k = 25$. The answer is **25**.

Check: the first line is $y = -5x + 4$ with slope -5 , and $-5 \cdot \frac{1}{5} = -1$.

SAT Strategy

- 1 Substitute 0 for each variable.** This gives both intercepts in seconds and often reveals what a context equation means.
- 2 Test points in the choices.** For "which equation passes through these points" or "which point is on the line", substitute instead of deriving. Start with the choice that has the nicest numbers.
- 3 Graph in the on-screen calculator.** Standard form can be typed directly. Zoom to the intercepts, then tap the points to read the slope or check which choice matches.
- 4 Say the meaning in words.** For interpretation questions, replace each symbol with its meaning before reading the choices. Eliminate any choice that confuses the number of items with the price or the total.
- 5 Check the signs of the slope.** If A and B have the same sign in $Ax + By = C$, the slope is negative. If they have opposite signs, the slope is positive.

Common traps

(1) Using $\frac{A}{B}$ as the slope instead of $-\frac{A}{B}$. (2) Mixing up the intercepts: the x -intercept uses $y = 0$, not $x = 0$. (3) Reversing the coordinates of a point in a context problem, so that $(0, 84)$ is read as 84 of the wrong item. (4) Forgetting that a perpendicular line needs both the flip and the sign change. (5) Choosing a solution that fits the equation but is not a whole, non-negative count.

Practice**PRACTICE Linear Equations in Two Variables**

Choose the best answer or enter your answer. Calculator allowed.

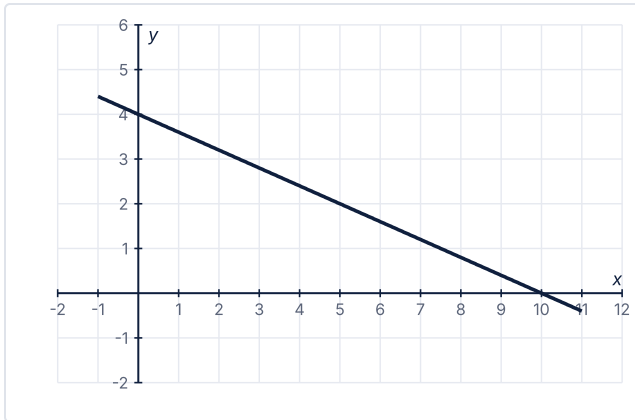
- Which point lies on the graph of $2x - 3y = 12$ in the xy -plane?
 (A) $(0, 4)$ (B) $(4, 0)$ (C) $(3, 2)$ (D) $(9, 2)$
- The graph of $5x + 2y = 30$ in the xy -plane has a y -intercept at $(0, b)$. What is the value of b ?
 (A) 6 (B) 15 (C) 30 (D) $\frac{5}{2}$
- What is the slope of the line with equation $6x - 9y = 18$?
 (A) $-\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) 2

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

4. A food truck sells wraps for 8 dollars each and smoothies for 5 dollars each. In one day the truck made 1,260 dollars, represented by $8w + 5s = 1260$, where w is the number of wraps and s is the number of smoothies sold. Which of the following is the best interpretation of $5s$ in this context?
- (A) The revenue, in dollars, from smoothies sold
(B) The number of smoothies sold
(C) The price of one smoothie, in dollars
(D) The total revenue, in dollars, for the day
5. Which equation defines the line that is parallel to $3x + 4y = 8$ and passes through the point $(4, 1)$?
- (A) $3x + 4y = 8$ (B) $4x - 3y = 13$ (C) $3x - 4y = 8$ (D) $3x + 4y = 16$
6. Maya buys notebooks that cost 3 dollars each and pens that cost 1.50 dollars each. She spends exactly 30 dollars, represented by $3n + 1.5p = 30$, where n is the number of notebooks and p is the number of pens. If she buys 6 notebooks, how many pens does she buy?
7. A line passes through the points $(1, -5)$ and $(4, 1)$. What is the x -coordinate of the x -intercept of the line?
8. The graph of $2x + 5y = 20$ and the two coordinate axes form a triangle, as shown. What is the area of the triangle?



M4 Systems of Linear Equations

Solve systems by substitution and elimination, read solutions from graphs, decide when a system has no or infinitely many solutions, and use shortcuts for expressions like $x + y$.

What the SAT Asks

A system of linear equations is two (sometimes three) equations that must be true at the same time. The SAT asks you to solve for x , y or an expression like $x + y$; to decide how many solutions a system has; to find the value of a constant that makes a system have no solution or infinitely many; and to model a word problem with two unknown quantities. A typical test has about 2 to 4 of these questions.

Difficulty ranges from a system you can solve in two lines to a parameter question that depends on seeing how the coefficients compare. Most hard questions have a shortcut: you do not need both variables, or you can read the answer from the structure of the equations. This chapter teaches the methods first and then the shortcuts.

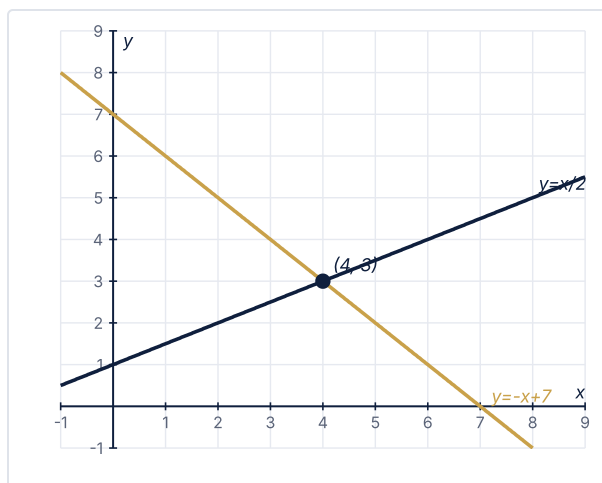
Core Ideas

A **solution** of a system is an ordered pair (x, y) that satisfies every equation. Geometrically, each equation is a line, and a solution is a point where the lines meet. Two lines in a plane can cross once (one solution), be parallel and distinct (no solution), or be the same line (infinitely many solutions).

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



The solution of the system $y = \frac{1}{2}x + 1$ and $y = -x + 7$ is the intersection point $(4, 3)$.

Substitution

When one equation already gives a variable (like $y = 2x - 1$) or is easy to solve for a variable, replace that variable in the other equation.

Steps: isolate a variable, substitute into the other equation, solve, then substitute back to find the second variable.

Elimination (Addition)

Multiply one or both equations so that a variable has opposite coefficients, then add the equations to cancel it.

Example: $3x + 2y = 16$ and $5x - 2y = 16$ add to $8x = 32$, so $x = 4$. Elimination is usually faster when both equations are in standard form.

Use whichever method needs fewer steps. If an equation is already solved for y , substitute. If both are in the form $Ax + By = C$, eliminate. Always substitute your final pair into **both** original equations to check.

How Many Solutions?

For $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ compare the ratios of coefficients:

One solution: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (the slopes differ).

No solution: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (same slope, different lines).

Infinitely many solutions: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (the equations describe the same line).

Case	Graph	After solving
One solution	lines cross	$x = 3, y = 5$
No solution	parallel lines	false statement, like $0 = 6$
Infinitely many	same line	true statement, like $0 = 0$

Shortcuts for expressions. If the question asks for $x + y$ or $x - y$, try adding or subtracting the equations before solving. When the x and y coefficients add up to the same number in the two equations, adding gives a multiple of $x + y$ directly. For $6x + 2y = 26$ and $2x + 6y = 30$, adding gives $8x + 8y = 56$, so $x + y = 7$ without finding x or y .

Word problems. Choose one variable for each unknown quantity and write one equation for each relationship the story gives. Typical pairs are a **count equation** (adult tickets plus child tickets equals the total number of tickets) and a **value equation** (price times count, added up, equals the total money). Say what each variable means in a sentence before you write the equations, and finish by answering what was asked, which might be a count, a price or a sum.

Parameters. If a system has a constant such as k and the question gives the number of solutions, use the ratio rules above. Make the x and y coefficients proportional to find k , then check the constants to see whether you get no solution or infinitely many.

Worked Examples

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 1 (WARM-UP)

If $y = 2x - 1$ and $3x + y = 14$, what is the value of y ?

SOLUTION

Substitute $2x - 1$ for y in the second equation: $3x + (2x - 1) = 14$.

Combine: $5x - 1 = 14$, so $5x = 15$ and $x = 3$.

Back-substitute: $y = 2(3) - 1 = 5$. Check in the second equation: $9 + 5 = 14$.

The answer is **5**. Notice the question asks for y , not x .

EXAMPLE 2 (MULTIPLE CHOICE)

Which ordered pair (x, y) is the solution to the system $3x + 2y = 16$ and $5x - 2y = 16$?

- A. (2, 4)
- B. (4, 2)
- C. (6, -1)
- D. (0, 8)

SOLUTION

The y terms already have opposite coefficients, so add the equations: $8x = 32$ and $x = 4$.

Substitute into the first equation: $12 + 2y = 16$, so $y = 2$. The pair is (4, 2).

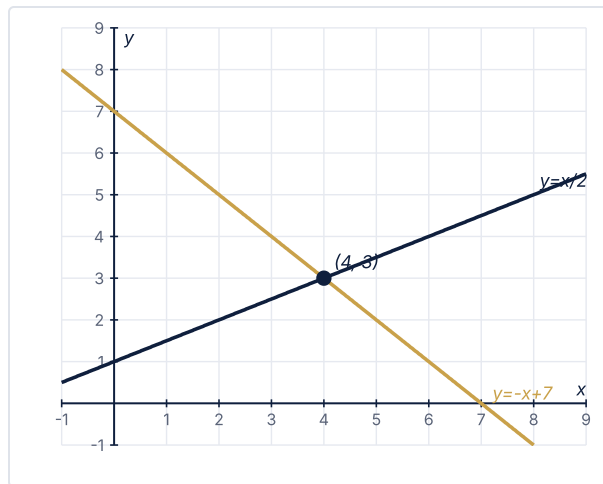
Check the second equation: $20 - 4 = 16$. The answer is **B**.

The other choices are traps: (6, -1) and (0, 8) satisfy only the first equation, and A reverses the coordinates.

EXAMPLE 3 (READING A GRAPH)

The graph shows the system $y = \frac{1}{2}x + 1$ and $y = -x + 7$. Which ordered pair is the solution?

- A. (0, 7)
- B. (3, 4)
- C. (4, 3)
- D. (0, 1)

**SOLUTION**

A solution must lie on both lines, so it is the intersection point.

The lines cross at $x = 4$: the first gives $y = 3$ and the second gives $y = -4 + 7 = 3$.

The answer is **C**, (4, 3). Choices A and D are the y -intercepts of the lines, each on only one of them.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 4 (SHORTCUT)

If $6x + 2y = 26$ and $2x + 6y = 30$, what is the value of $x + y$?

SOLUTION

Add the equations: $8x + 8y = 56$.

Divide by 8: $x + y = 7$.

Check by solving fully: $x = 3$ and $y = 4$ satisfy both equations ($18 + 8 = 26$ and $6 + 24 = 30$) and $3 + 4 = 7$. The answer is **7**.

The shortcut saved the whole elimination process.

EXAMPLE 5 (PARAMETER)

In the system $2x + 3y = 9$ and $kx + 6y = 20$, k is a constant. If the system has no solution, what is the value of k ?

SOLUTION

No solution means the lines are parallel, so the coefficients of x and y are proportional: $\frac{2}{k} = \frac{3}{6}$.

Cross-multiply: $12 = 3k$, so $k = 4$.

Check the constants: with $k = 4$, double the first equation to get $4x + 6y = 18$, which conflicts with $4x + 6y = 20$. The lines are parallel but different, so there is no solution. The answer is **4**.

If the second constant had been 18, the same k would give infinitely many solutions.

SAT Strategy

- 1 Look at what is asked.** If it is $x + y$, $x - y$ or $2x + y$, try adding or subtracting (or scaling) the equations before solving.
- 2 Choose the faster method.** A variable already isolated: substitute. Both equations in standard form: eliminate.
- 3 Use the on-screen calculator.** Type both equations and tap the intersection. This is the quickest way to solve and to verify an algebra answer. For a parameter problem, add a slider for k and see when the lines become parallel.
- 4 Plug in answer choices.** For "which ordered pair" questions, test each pair in both equations. A pair that fails one equation is out.
- 5 Compare coefficients for counting solutions.** Divide corresponding coefficients. If x and y ratios match, the answer is no solution or infinitely many, depending on the constants.

Common traps

- (1) Subtracting an equation but not changing the sign of every term, such as $5x - (3x - 2y) = 2x - 2y$ instead of $2x + 2y$.
- (2) Solving for x and stopping when the question asks for y or $x + y$.
- (3) Equal x and y ratios do not guarantee no solution; if the constants also match, it is infinitely many.
- (4) Checking an ordered pair in only one equation.
- (5) Reversing the coordinates of the pair, or mixing up which quantity is x in a word problem.

Practice**PRACTICE Systems of Linear Equations**

Choose the best answer or enter your answer. Calculator allowed.

- What is the solution (x, y) to the system $y = 3x - 2$ and $2x + y = 13$?
 (A) (7, 3) (B) (3, 7) (C) (5, 3) (D) (1, 11)
- The solution to the system $2x + 3y = 12$ and $4x - 3y = 6$ is (x, y) . What is the value of y ?
 (A) 1 (B) 3 (C) 2 (D) 6
- How many solutions does the system $4x - 6y = 10$ and $6x - 9y = 14$ have?
 (A) No solutions (B) Exactly one solution
 (C) Exactly two solutions (D) Infinitely many solutions
- If $5x + 3y = 31$ and $3x + 5y = 25$, what is the value of $x + y$?
 (A) 8 (B) 14 (C) 56 (D) 7

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

5. Taylor buys 3 notebooks and 2 binders for 19 dollars. Another day, Taylor buys 2 notebooks and 3 binders for 21 dollars. If prices are the same both days, what is the combined cost, in dollars, of 1 notebook and 1 binder?
 (A) 3 (B) 5 (C) 8 (D) 40
6. A school club sold 140 raffle tickets. Single tickets cost 2 dollars each and bundle tickets cost 5 dollars each. The club collected 400 dollars in all. How many bundle tickets did the club sell?
7. In the system $kx + 4y = 10$ and $3x + 2y = 5$, k is a constant. If the system has infinitely many solutions, what is the value of k ?
8. In the system $ax + 3y = 11$ and $2x - by = 7$, a and b are constants. If the solution to the system is $(2, -1)$, what is the value of $a + b$?

M5 Linear Inequalities in One or Two Variables

Learn to solve and graph inequalities, handle compound inequalities, test points in systems, and turn "at least" and "no more than" word problems into the right whole-number answer.

What the SAT Asks

Inequalities show up whenever a situation has a limit: a budget, a weight limit, a minimum score, a deadline. The test uses them in three main ways. Some questions ask you to **solve** an inequality and pick the matching solution set. Some give a word problem and ask you to **write** the inequality or find the greatest or least whole number that works. Others show a **graph** (a shaded region in the coordinate plane) and ask whether a point is a solution of a system.

Expect about 1 to 3 questions per module on this skill, ranging from easy one-step solving to a hard question about a parameter such as "for what value of k does the inequality have no solution?" The skills are the same as for equations, with exactly one new rule you must never forget.

Core Ideas

An **inequality** compares two quantities with $<$, $>$, $\leq$ or $\geq$. A **solution** is any value that makes it true, and usually there are infinitely many, so the answer is a set of values described by an inequality like $x < -5$ or drawn on a number line. On a number line, an **open circle** means the endpoint is not included ($<$ or $>$) and a **closed circle** means it is included ($\leq$ or $\geq$).

The Flip Rule

You may add or subtract the same number on both sides, or multiply or divide both sides by the same **positive** number, without changing the direction.

If you multiply or divide both sides by a **negative** number, the inequality sign **flips**: $-3x > 12$ becomes $x < -4$.

Why does it flip? Because $2 < 5$ is true, but multiplying by -1 gives -2 and -5 , and $-2 > -5$. Negation reverses order. A habit that never fails: whenever you divide by a negative, circle the sign and reverse it right then.

A **compound inequality** such as $-5 < 2x - 1 \leq 9$ has three parts. Whatever you do, do to **all three** parts. Here, add 1 to get $-4 < 2x \leq 10$, then divide by 2 to get $-2 < x \leq 5$. The answer means x is greater than -2 **and** at most 5. Written with intervals, this is $(-2, 5]$.

Words in the problem	Symbol
at least, minimum of, no less than	$\geq$
at most, maximum of, no more than, cannot exceed	$\leq$
more than, greater than, exceeds	$>$
fewer than, less than, under	$<$

Graphing a Linear Inequality in the Plane

Graph the boundary line $y = mx + b$. Make it **dashed** for $<$ or $>$ and **solid** for $\leq$ or $\geq$.

Shade **above** the line for $y > mx + b$ or $y \geq mx + b$, and **below** for $y < mx + b$ or $y \leq mx + b$.

For a system, the solution is the overlap of all the shaded regions. A point is a solution only if it makes **every** inequality true.

The fastest test of a point is substitution: plug in the x and y values and check each inequality. Remember that a point lying exactly on a dashed boundary is **not** a solution, because the inequality is strict. In word problems, a quantity that is a count of people or objects must be a whole number, so after solving you round **down** for a maximum (you cannot exceed the limit) and **up** for a minimum (you must reach the requirement).

Worked Examples

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 1 (EASY, MULTIPLE CHOICE)

Which inequality is equivalent to $-3x + 7 > 22$? A) $x > -5$ B) $x < -5$ C) $x > 5$ D) $x < 5$

SOLUTION

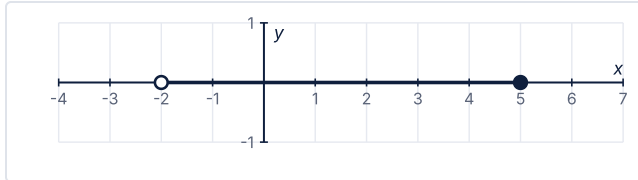
Subtract 7 from both sides: $-3x > 15$.

Divide both sides by -3 and **flip** the sign: $x < -5$.

The answer is **B**. Choice A forgot to flip, and choices C and D made a sign error on 15 divided by -3 .

EXAMPLE 2 (COMPOUND, STUDENT-PRODUCED RESPONSE)

How many integers x satisfy $-5 < 2x - 1 \leq 9$?



Solution set $-2 < x \leq 5$.

SOLUTION

Add 1 to all three parts: $-4 < 2x \leq 10$.

Divide all parts by 2: $-2 < x \leq 5$.

The number line shows an open circle at -2 and a closed circle at 5 . The integers are $-1, 0, 1, 2, 3, 4, 5$, which is **7** integers. Do not count -2 , because the circle is open.

EXAMPLE 3 (WORD PROBLEM)

Mia has 90 dollars. Concert tickets cost 12 dollars each and she must also pay a 15 dollar parking fee. What is the greatest number of tickets she can buy?

SOLUTION

Let t be the number of tickets. Total cost is $12t + 15$, and it must be **at most** 90: $12t + 15 \leq 90$.

Subtract 15: $12t \leq 75$. Divide by 12: $t \leq 6.25$.

Tickets are whole numbers, and she cannot exceed the limit, so round **down**: **6** tickets. (Check: $12(6) + 15 = 87 \leq 90$, but $12(7) + 15 = 99 > 90$.)

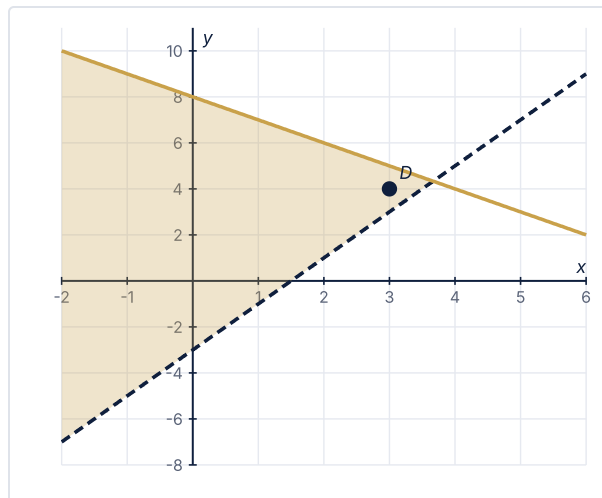
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 4 (SYSTEM IN THE PLANE)

The shaded region shows the solutions of $y > 2x - 3$ and $x + y \leq 8$. Which point is in the solution set? A) $(0, -3)$ B) $(2, 7)$ C) $(4, 4)$ D) $(3, 4)$



Region above the dashed line and on or below the solid line.

SOLUTION

Test each point in **both** inequalities.

A) $-3 > 2(0) - 3$ is $-3 > -3$, false (strict, so a point on the dashed line fails).

B) $x + y = 9 \leq 8$ is false. C) $4 > 2(4) - 3 = 5$ is false.

D) $4 > 2(3) - 3 = 3$ is true, and $3 + 4 = 7 \leq 8$ is true. The answer is **D**.

EXAMPLE 5 (HARD: A PARAMETER)

For what value of k does the inequality $3(x - 2) > kx + 5$ have **no** solution?

SOLUTION

Distribute and move everything to one side: $3x - 6 - kx - 5 > 0$, so $(3 - k)x - 11 > 0$.

If $k \neq 3$, you can divide by $3 - k$ (flipping if negative) and always get a solution, so no solution is impossible.

If $k = 3$, the x terms cancel and you get $-11 > 0$, which is false for every x . So the answer is $k = 3$.

SAT Strategy

- 1 Solve it like an equation first.** Then check the direction. Test one value from your answer in the original inequality; if it fails, you forgot to flip.
- 2 Plug in answer choices.** For "which point is a solution" and "which value satisfies" questions, substitute the choices into every inequality. This is often faster than graphing.
- 3 Use the on-screen graphing calculator.** Type each inequality; the calculator shades the region. Click the intersection or zoom to see which labeled point sits inside the overlap.
- 4 Translate the words carefully.** Underline the limit word (at most, at least), name the variable, then write cost or total on the left and the limit on the right.
- 5 Round the right way.** For a maximum count, round down; for a minimum count, round up, then verify with the whole number.

Common traps

(1) Forgetting to flip the sign when dividing or multiplying by a negative. (2) Treating "at most" as $<$ instead of $\leq$, which changes whether the endpoint counts. (3) Rounding a maximum count up (for example answering 7 tickets when 6.25 is the limit). (4) Calling a point on a dashed line a solution. (5) In compound inequalities, changing only the middle part.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

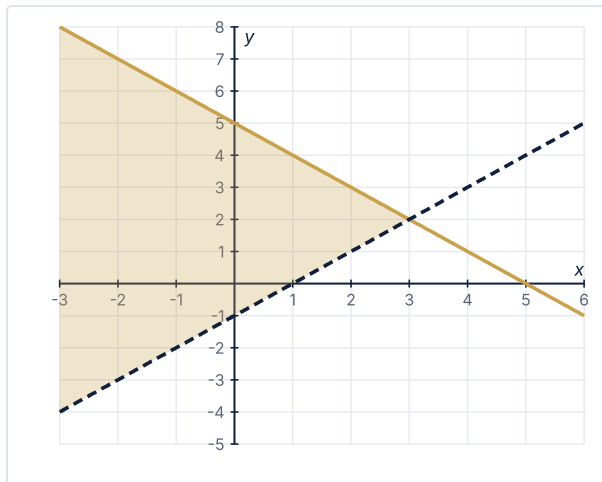
BOOK NOW →

Practice

PRACTICE Linear Inequalities in One or Two Variables

Choose the best answer or enter your answer. Calculator allowed.

- Which inequality is equivalent to $-4x + 9 \geq 25$?
 (A) $x \geq -4$ (B) $x \leq -4$ (C) $x \leq 4$ (D) $x \geq 4$
- Which of the following is the solution set of $3 < 2x + 1 \leq 11$?
 (A) $1 \leq x < 5$ (B) $2 < x \leq 12$ (C) $4 < x \leq 12$ (D) $1 < x \leq 5$
- A taxi charges a 3 dollar fee plus 2.50 dollars per mile. Marcus can spend no more than 28 dollars. Which inequality gives the possible numbers of miles m ?
 (A) $3 + 2.5m \leq 28$ (B) $3 + 2.5m \geq 28$ (C) $2.5 + 3m \leq 28$ (D) $3 + 2.5m < 28$
- The shaded region shows the solutions of $y \leq -x + 5$ and $y > x - 1$. Which point is a solution of the system?

Boundaries $y = -x + 5$ (solid) and $y = x - 1$ (dashed).

- (A) $(1, 5)$ (B) $(4, 1)$ (C) $(0, -1)$ (D) $(2, 2)$
- Plan A costs 25 dollars per month plus 5 dollars per class. Plan B costs 10 dollars per month plus 8 dollars per class. For which numbers of classes c in a month is Plan A cheaper than Plan B?
 (A) $c < 5$ (B) $c > 15$ (C) $c > 5$ (D) $c > 3$
- What is the greatest integer x that satisfies $2(3x - 4) < 5x + 9$?
- A delivery van can carry at most 1,000 pounds in total. The driver weighs 160 pounds, the van's fixed equipment weighs 140 pounds, and each box weighs 18 pounds. What is the greatest number of boxes the van can carry?
- The inequality $ax + 3 < 5x - 9$ has solution set $x > 3$, where a is a constant. What is the value of a ?

Answers Algebra: Answers and Explanations

Solutions for every practice problem, numbered to match.

M1 Linear Equations in One Variable**PRACTICE · LINEAR EQUATIONS IN ONE VARIABLE**

- B. Distribute: $5x - 10 = 3x + 8$. Subtract $3x$: $2x - 10 = 8$. Add 10: $2x = 18$, so $x = 9$. Choice A comes from forgetting to distribute the -2 properly; choice C forgets to divide by 2.
- C. Subtract $0.1x$ and 1.2 : $0.3x = 1.8$, so $x = 6$. Choice B adds 1.2 instead of subtracting it ($0.3x = 4.2$); choice D divides by 0.1 instead of 0.3.
- D. Expand: $6x - 15 = kx + 7$. For no solution the x coefficients must match, so $k = 6$. Then $-15 = 7$ is false, so there is no solution. Choice A halves the 6 and choice C flips the sign.
- A. Since $15x - 6 = 3(5x - 2)$, the value is $3(18) = 54$. Choice B forgets to multiply by 3, and choice C uses $15x$ alone ($x = 4$ gives 60).
- B. Let n be the pages. $\frac{2}{5}n = 80$ (the unread part), so $n = 80 \cdot \frac{5}{2} = 200$. Choice A multiplies 80 by $\frac{2}{5}$ and choice C uses the read fraction on 200 pages.
7. Multiply by 4: $5x - 3 = 2(x + 9) = 2x + 18$. Then $3x = 21$, so $x = 7$. Check: $\frac{35-3}{4} = 8$ and $\frac{16}{2} = 8$.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

7. 19. Expand: $6x - 8 + k = 6x + 11$. The x terms cancel, and for infinitely many solutions the constants must match: $-8 + k = 11$, so $k = 19$.
8. 16. Substitute $x = 2$: $\frac{2k-2}{3} = 2 + \frac{k}{2}$. Multiply by 6: $2(2k - 2) = 12 + 3k$, so $4k - 4 = 12 + 3k$ and $k = 16$. Check: $\frac{30}{3} = 10$ and $2 + 8 = 10$.

M2 Linear Functions

PRACTICE · LINEAR FUNCTIONS

- C. $f(-3) = -2(-3) + 9 = 6 + 9 = 15$. Choice A comes from treating $-2(-3)$ as -6 .
- A. Both labeled points are on the line, so the slope is $\frac{-8-4}{5-(-1)} = \frac{-12}{6} = -2$. The line falls from left to right, so the slope is negative. Choice B has the wrong sign and choices C and D invert the fraction.
- D. The output rises by 6 when the input rises by 2, so the slope is 3, not 6. The value at $x = 0$ is 5, so $f(x) = 3x + 5$. Choice A uses the change over 2 units as the slope, and choice C uses $f(2)$ as the intercept.
- B. The slope is $\frac{4-14}{8-3} = -2$, so the height falls 2 cm per hour. Then the initial height is $14 + 2(3) = 20$ cm (so A is false), and the candle is gone at $t = 10$ hours, not 20 (so D is false). Choice C uses the 5-hour time difference as the rate.
- C. The slope of ℓ is $\frac{2}{3}$, so the perpendicular slope is $-\frac{3}{2}$. Using $(6, 1)$: $1 = -\frac{3}{2}(6) + b$ gives $b = 10$. Choice A has the right size but the wrong sign, choice B only negates, and choice D uses 1 as the intercept.
29. The slope is $\frac{17-5}{9-2} = 4$, so $f(8) = f(5) + 4(3) = 17 + 12 = 29$.
12. The slope is $\frac{14-(-4)}{6} = 3$ and the intercept is -4 , so $f(x) = 3x - 4$. Solve $3x - 4 = 32$: $3x = 36$, so $x = 12$.
- 2. The perpendicular slope is $-\frac{1}{2}$. Then $\frac{k-1}{4-(-2)} = -\frac{1}{2}$, so $k - 1 = -3$ and $k = -2$.

M3 Linear Equations in Two Variables

PRACTICE · LINEAR EQUATIONS IN TWO VARIABLES

- D. Substitute: $(0, 4)$ gives -12 ; $(4, 0)$ gives 8 ; $(3, 2)$ gives 0 ; $(9, 2)$ gives $18 - 6 = 12$, which matches. Only D works.
- B. Set $x = 0$: $2y = 30$, so $y = 15$. Choice A is the x -intercept ($5x = 30$), and choice C forgets to divide by 2.
- C. Solve for y : $-9y = -6x + 18$, so $y = \frac{6}{9}x - 2 = \frac{2}{3}x - 2$. The slope is $\frac{2}{3}$. Choice A forgets the sign change from dividing by -9 , and choice B inverts the fraction.
- A. Each smoothie costs 5 dollars and s is the number sold, so $5s$ is the money earned from smoothies. Choice B is the value of s , choice C is the number 5, and choice D is the whole left side.
- D. A parallel line keeps the same left side, $3x + 4y$. Substituting $(4, 1)$ gives $12 + 4 = 16$, so $3x + 4y = 16$. Choice A is the original line, and choice B is the perpendicular line through $(4, 1)$.
8. Substitute $n = 6$: $3(6) + 1.5p = 30$, so $18 + 1.5p = 30$, $1.5p = 12$ and $p = 8$.
- 3.5. The slope is $\frac{1-(-5)}{4-1} = 2$. Then $y = 2x + b$ with $1 = 2(4) + b$ gives $b = -7$. Set $y = 0$: $2x = 7$, so $x = 3.5$ (or $\frac{7}{2}$).
20. The intercepts are $x = \frac{20}{2} = 10$ and $y = \frac{20}{5} = 4$. The triangle has legs 10 and 4, so the area is $\frac{1}{2}(10)(4) = 20$.

M4 Systems of Linear Equations

PRACTICE · SYSTEMS OF LINEAR EQUATIONS

- B. Substitute $y = 3x - 2$: $2x + 3x - 2 = 13$, so $5x = 15$ and $x = 3$, then $y = 7$. Choice A reverses the coordinates; choices C and D satisfy only the second equation.
- C. Add the equations: $6x = 18$, so $x = 3$. Substitute into $2x + 3y = 12$: $6 + 3y = 12$, so $y = 2$. Choice B is the value of x , not y .
- A. The ratio of x coefficients is $\frac{4}{6} = \frac{2}{3}$ and the ratio of y coefficients is $\frac{-6}{9} = -\frac{2}{3}$, so the lines are parallel. The ratio of constants is $\frac{10}{14} = \frac{5}{7}$, which differs, so the lines are distinct and there is no solution. Multiplying the first equation by 1.5 gives $6x - 9y = 15$, not 14.
- D. Add the equations: $8x + 8y = 56$, so $x + y = 7$. Choice C stops before dividing by 8, and choice A is the coefficient total. (Check: $x = 5, y = 2$.)
- C. Add the equations: $5n + 5b = 40$, so $n + b = 8$. No need to find each price (the notebook costs 3 dollars and the binder 5 dollars). Choices A and B are the individual prices, and choice D forgets to divide by 5.
40. Let s be single tickets and b bundles. $s + b = 140$ and $2s + 5b = 400$. Substitute $s = 140 - b$: $280 - 2b + 5b = 400$, so $3b = 120$ and $b = 40$ (then $s = 100$).
6. Multiply the second equation by 2: $6x + 4y = 10$. For the first equation to be the same line, $k = 6$, and the constants match (10 and 10), so the lines coincide.
10. Substitute $(2, -1)$: $2a - 3 = 11$ gives $a = 7$; $4 + b = 7$ gives $b = 3$. So $a + b = 10$.

M5 Linear Inequalities in One or Two Variables

PRACTICE · LINEAR INEQUALITIES IN ONE OR TWO VARIABLES

- B. Subtract 9: $-4x \geq 16$. Divide by -4 and flip: $x \leq -4$. Choice A forgot to flip.
- D. Subtract 1 from all three parts: $2 < 2x \leq 10$. Divide by 2: $1 < x \leq 5$. Choice C forgot to divide, and choice D added 1 instead of subtracting it.
- A. "No more than" means $\leq$, and the fee is added once while the rate multiplies m . Choice D uses a strict sign, which would exclude spending exactly 28 dollars.
- D. Test each point. $(2, 2)$: $2 \leq 3$ true and $2 > 1$ true. $(1, 5)$ fails $5 \leq 4$. $(4, 1)$ fails $1 > 3$. $(0, -1)$ is on the dashed line, and $-1 > -1$ is false. The answer is $(2, 2)$.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

5. C. Plan A is cheaper when $25 + 5c < 10 + 8c$. Subtract $5c$ and 10: $15 < 3c$, so $c > 5$. Choice B has the sign reversed.
6. 16. Distribute: $6x - 8 < 5x + 9$. Subtract $5x$ and add 8: $x < 17$. The inequality is strict, so 17 is not allowed and the greatest integer is 16.
7. 38. Solve $18b + 140 + 160 \leq 1000$, so $18b \leq 700$ and $b \leq 38.9$. Round down because boxes are whole and the limit cannot be exceeded: 38. (Check: 39 boxes give 1,002 pounds.)
8. 1. Rearrange: $(a - 5)x < -12$. For the answer to be $x > 3$, the sign must flip, so $a - 5 < 0$ and $x > \frac{-12}{a-5} = \frac{12}{5-a}$. Set $\frac{12}{5-a} = 3$ to get $a = 1$.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



Advanced Math

3 chapters of original lessons with worked examples, then 24 practice problems with full answers and explanations.

M6 Equivalent Expressions

Expand, factor and simplify with confidence: special products, exponent and radical rules, rational expressions, and polynomial division, plus the pick-a-number trick for "which expression is equivalent" questions.

What the SAT Asks

Many Advanced Math questions give you an expression and ask, "Which of the following is equivalent?" Two expressions are **equivalent** when they have the same value for every allowed input, even though they look different. The stems are short, but the algebra underneath can be long, so you need clean technique and a few shortcuts.

Typical tasks: expand a product, factor a trinomial, simplify a fraction, apply exponent and radical rules, divide polynomials, or combine algebraic fractions. A test usually has 2 to 4 of these questions across both modules, from easy (combine like terms) to hard (add rational expressions with different denominators). They are often the quickest points on the test if you know the patterns.

Core Ideas

Expanding means multiplying out. Distribute every term of one factor to every term of the other (FOIL for two binomials), then **combine like terms**, which are terms with the same variable part: $3x^2 - 5x^2 = -2x^2$, but $3x^2$ and $3x$ cannot be combined. **Factoring** is expanding in reverse. Always look first for a greatest common factor: $6x^3y - 9x^2y^2 = 3x^2y(2x - 3y)$.

Special Products and Factoring Patterns

Difference of squares: $a^2 - b^2 = (a - b)(a + b)$.

Perfect-square trinomials: $a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - 2ab + b^2 = (a - b)^2$.

Trinomial factoring: $x^2 + bx + c = (x + p)(x + q)$ where $p + q = b$ and $pq = c$.

A sum of squares $a^2 + b^2$ does **not** factor over the real numbers, and $(a + b)^2 \neq a^2 + b^2$.

Patterns can hide. For example, $x^4 - 13x^2 + 36$ looks quartic but behaves like a quadratic in x^2 : it factors as $(x^2 - 4)(x^2 - 9)$ and then, using difference of squares twice, as $(x - 2)(x + 2)(x - 3)(x + 3)$. This is called **rewriting to reveal structure**. A perfect-square check is also useful: $16x^2 + bx + 25$ is a perfect square when $b = \pm 2(4)(5) = \pm 40$.

Exponent and Radical Rules

$x^m \cdot x^n = x^{m+n}$, $\frac{x^m}{x^n} = x^{m-n}$, $(x^m)^n = x^{mn}$, $(xy)^n = x^n y^n$.

$x^{-n} = \frac{1}{x^n}$ and $x^0 = 1$ for $x \neq 0$.

Rational exponents: $x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$, so $x^{1/2} = \sqrt{x}$ and $x^{2/3} = \sqrt[3]{x^2}$.

With a fractional exponent, take the root of the number first, then the power: $8^{2/3} = (\sqrt[3]{8})^2 = 4$. Apply the exponent to every factor inside parentheses, including the coefficient: $(27x^9)^{2/3} = 27^{2/3}x^6 = 9x^6$.

A **rational expression** is a fraction of polynomials. To simplify, **factor** the numerator and denominator, then cancel common **factors** (never common terms). To add or subtract, find a common denominator, rewrite each fraction, and combine the numerators: $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$. When the numerator has degree at least as high as the denominator, use **polynomial division** to write the result as a quotient plus a remainder over the divisor.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Division and the Remainder Form

If dividing $p(x)$ by $d(x)$ gives quotient $q(x)$ and remainder r , then

$$\frac{p(x)}{d(x)} = q(x) + \frac{r}{d(x)}.$$

Check by multiplying: $p(x) = q(x)d(x) + r$.

Worked Examples

EXAMPLE 1 (EASY, MULTIPLE CHOICE)

Which expression is equivalent to $(2x - 3)^2 - (x - 5)(x + 5)$? A) $3x^2 - 12x + 34$ B) $3x^2 - 12x - 16$ C) $5x^2 - 12x + 34$ D) $3x^2 + 12x + 34$

SOLUTION

Square the binomial: $(2x - 3)^2 = 4x^2 - 12x + 9$.

The second product is a difference of squares: $(x - 5)(x + 5) = x^2 - 25$.

Subtract the whole second expression: $4x^2 - 12x + 9 - x^2 + 25 = 3x^2 - 12x + 34$. The answer is **A**.

Choice B forgot to change the sign of -25 . Choice C added the x^2 terms instead of subtracting.

EXAMPLE 2 (STUDENT-PRODUCED RESPONSE)

The expression $(3x + a)(x - 5)$ is equivalent to $3x^2 + bx - 20$, where a and b are constants. What is the value of b ?

SOLUTION

Expand: $(3x + a)(x - 5) = 3x^2 - 15x + ax - 5a = 3x^2 + (a - 15)x - 5a$.

Match the constant term: $-5a = -20$, so $a = 4$.

Then $b = a - 15 = 4 - 15 = -11$.

EXAMPLE 3 (EXPONENTS, MULTIPLE CHOICE)

Which expression is equivalent to $(27x^9y^{-3})^{2/3}$, for positive x and y ? A) $\frac{18x^6}{y^2}$ B) $\frac{9x^6}{y^2}$ C) $9x^6y^2$ D) $\frac{9x^{27/2}}{y^2}$

SOLUTION

Apply the $2/3$ power to each factor: $27^{2/3} \cdot x^{9 \cdot 2/3} \cdot y^{-3 \cdot 2/3}$.

$27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9$. Next, x^6 and $y^{-2} = \frac{1}{y^2}$.

The result is $\frac{9x^6}{y^2}$, choice **B**. Choice A multiplied 27 by $2/3$ instead of raising it to that power.

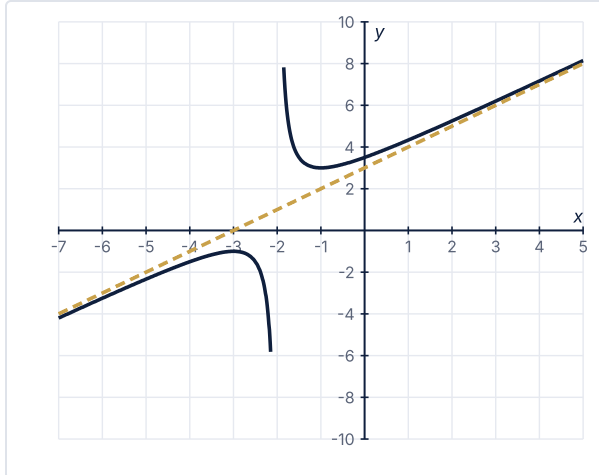
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 4 (POLYNOMIAL DIVISION)

Which expression is equivalent to $\frac{x^2 + 5x + 7}{x + 2}$ for $x \neq -2$? A) $x + 3$ B) $x + 3 + \frac{1}{x + 2}$ C) $x + 5 + \frac{7}{x + 2}$ D) $x + \frac{5x + 7}{2}$



The curve and its slant line $y = x + 3$.

SOLUTION

Divide: x^2 divided by x is x . Multiply $x(x + 2) = x^2 + 2x$ and subtract to get $3x + 7$.

$3x$ divided by x is 3 . Multiply $3(x + 2) = 3x + 6$ and subtract to get a remainder of 1 .

So the expression equals $x + 3 + \frac{1}{x + 2}$, choice **B**. Quick check with $x = 1$: the original is $\frac{13}{3}$, and $1 + 3 + \frac{1}{3} = \frac{13}{3}$.

The graph confirms this: it approaches the slant line $y = x + 3$ away from the vertical asymptote at $x = -2$.

EXAMPLE 5 (HARD, ALGEBRAIC FRACTIONS)

Which expression is equivalent to $\frac{2}{x - 3} - \frac{x + 1}{x^2 - 9}$ for $x \neq \pm 3$? A) $\frac{x + 5}{x^2 - 9}$ B) $\frac{x + 1}{x^2 - 9}$ C) $\frac{1 - x}{x - 3}$ D) $\frac{x + 3}{x^2 - 9}$

SOLUTION

Factor the second denominator: $x^2 - 9 = (x - 3)(x + 3)$. The common denominator is $(x - 3)(x + 3)$.

Rewrite the first fraction: $\frac{2}{x - 3} = \frac{2(x + 3)}{(x - 3)(x + 3)} = \frac{2x + 6}{x^2 - 9}$.

Subtract numerators, using parentheses around the second one: $\frac{2x + 6 - (x + 1)}{x^2 - 9} = \frac{x + 5}{x^2 - 9}$. The answer is **A**.

Watch the minus sign: it is $2x + 6 - x - 1$, not $2x + 6 - x + 1$.

SAT Strategy

- 1 Pick a number.** Choose an easy value like $x = 2$ (avoid values that make a denominator zero), compute the original expression, then test each choice. The one that matches is correct. If two match, try a second number such as $x = 3$.
- 2 Use the on-screen graphing calculator.** Enter the original expression and one choice at a time. If the graphs lie exactly on top of each other, they are equivalent. You can also compare values in a table.
- 3 Look at the answer choices first.** If they are all factored, factor the original. If they are all expanded, expand. The form of the choices tells you the direction.
- 4 Match coefficients.** When an expression is stated to equal another with unknown constants, expand and set the matching coefficients equal.
- 5 Check the constant and leading terms.** Multiplying only the first terms and the last terms of a product often eliminates two choices quickly.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Common traps

(1) Writing $(a + b)^2 = a^2 + b^2$ and dropping the middle term $2ab$. (2) Cancelling terms instead of factors, such as turning $\frac{x+4}{x}$ into 4. (3) Forgetting to distribute a subtraction sign over a whole numerator. (4) Multiplying the exponent by the coefficient, as in $(27x^9)^{2/3}$ becoming $18x^6$. (5) Adding fractions by adding numerators and denominators.

Practice**PRACTICE Equivalent Expressions**

Choose the best answer or enter your answer. Calculator allowed.

- Which expression is equivalent to $(x + 4)^2 - (x - 4)^2$?
 (A) $16x$ (B) 32 (C) $2x^2 + 32$ (D) $8x$
- Which expression is equivalent to $6x^3y - 9x^2y^2$?
 (A) $3x^2y(2x - 3)$ (B) $3xy(2x^2 - 3y)$ (C) $15x^2y(x - y)$
 (D) $3x^2y(2x - 3y)$
- Which expression is equivalent to $\left(\frac{a^{-2}b^3}{a^4b}\right)^{-1}$, for positive a and b ?
 (A) $\frac{b^2}{a^6}$ (B) $\frac{a^2}{b^4}$ (C) $\frac{a^6}{b^2}$ (D) a^6b^2
- Which expression is equivalent to $\frac{x^2 - x - 12}{x^2 - 9}$ for $x \neq \pm 3$?
 (A) $\frac{x - 4}{x - 3}$ (B) $\frac{x - 12}{-9}$ (C) $\frac{x + 4}{x + 3}$ (D) $\frac{x - 4}{x + 3}$
- Which expression is equivalent to $\frac{3}{x + 2} + \frac{4}{x - 1}$?
 (A) $\frac{7}{2x + 1}$ (B) $\frac{7x + 5}{x^2 + x - 2}$ (C) $\frac{7x + 5}{2x + 1}$
 (D) $\frac{7x - 5}{x^2 + x - 2}$
- The expression $16x^2 + bx + 25$ is a perfect-square trinomial, where b is a positive constant. What is the value of b ?
- For $x \neq 2$, the expression $\frac{2x^2 - 3x + 5}{x - 2}$ is equivalent to $2x + 1 + \frac{r}{x - 2}$, where r is a constant. What is the value of r ?
- The equation $\frac{a}{x - 1} + \frac{b}{x + 2} = \frac{5x + 4}{x^2 + x - 2}$ is true for all $x \neq 1, -2$, where a and b are constants. What is the value of ab ?

M7 Nonlinear Equations and Systems

Solve quadratics four ways, use the discriminant, catch extraneous solutions in rational and radical equations, handle exponential and absolute value equations, and master line-curve systems and circles.

What the SAT Asks

Linear equations are only the start. In Advanced Math you will solve quadratics, equations with fractions or square roots, exponential equations, and equations with absolute value. You will also meet systems made of a line and a parabola or circle, and equations of circles. Some stems ask for the solution, some for the **number** of solutions, and some for a constant that creates a certain number of solutions.

Typical tests have about 4 to 6 such questions, from a routine factoring problem to a multi-step item that mixes algebra with a graph. The big ideas are few, and the on-screen graphing calculator is a powerful backup, so the goal is to know when to use which tool.

Core Ideas

A **quadratic equation** $ax^2 + bx + c = 0$ can be solved four ways. (1) **Factor** and set each factor to zero: $2x^2 - 5x - 12 = (2x + 3)(x - 4) = 0$ gives $x = -\frac{3}{2}$ or $x = 4$. (2) **Square roots** when there is no middle term: $(x - 3)^2 = 16$ gives $x = 3 \pm 4$. (3) **Completing the square**, which rewrites $x^2 + bx$ as $(x + \frac{b}{2})^2 - \frac{b^2}{4}$. (4) The **quadratic formula**, which always works.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Quadratic Formula and the Discriminant

The solutions of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The **discriminant** is $D = b^2 - 4ac$. If $D > 0$ there are two real solutions; if $D = 0$ there is exactly one; if $D < 0$ there are none.

The discriminant is a favorite on the SAT: "For what value of k does the equation have exactly one solution?" means set $D = 0$. A tangent line to a parabola or circle also corresponds to exactly one intersection, which means a discriminant of zero after substitution.

Equations that need a check. In a rational equation, multiply both sides by the common denominator, solve, then reject any value that makes a denominator zero. Such a value is an **extraneous solution**. In a radical equation, isolate the radical, square both sides, solve, then plug every answer back in, because squaring can create false solutions. For an **absolute value** equation $|u| = c$ with $c > 0$, solve both $u = c$ and $u = -c$. For an **exponential equation**, rewrite both sides with the same base and set the exponents equal: $4^{x-1} = 8^{x+1}$ becomes $2^{2x-2} = 2^{3x+3}$, so $2x - 2 = 3x + 3$.

Systems with a Line and a Curve

Substitute the line into the other equation to get one equation in one variable, solve it, then find the matching y values with the line.

The number of solutions of the system equals the number of intersection points: two, one (tangent) or none, matching $D > 0$, $D = 0$, $D < 0$.

Equation of a Circle

A circle with center (h, k) and radius r has equation

$$(x - h)^2 + (y - k)^2 = r^2.$$

If the equation is given in expanded form, group the x terms and y terms and **complete the square** for each to read the center and radius.

Worked Examples

EXAMPLE 1 (EASY, MULTIPLE CHOICE)

What are the solutions of $2x^2 - 5x - 12 = 0$? A) $x = -\frac{3}{2}$ and $x = 4$ B) $x = \frac{3}{2}$ and $x = -4$ C) $x = -3$ and $x = \frac{4}{2}$ D) $x = \frac{5 \pm \sqrt{121}}{2}$

SOLUTION

Factor: find two numbers with product $2(-12) = -24$ and sum -5 : they are -8 and 3 . Split the middle term: $2x^2 - 8x + 3x - 12 = 2x(x - 4) + 3(x - 4) = (2x + 3)(x - 4)$.

Set each factor to zero: $x = -\frac{3}{2}$ or $x = 4$. The answer is **A**.

Check with the formula: $\frac{5 \pm \sqrt{25 + 96}}{4} = \frac{5 \pm 11}{4}$, which is 4 or $-\frac{3}{2}$. Choice D forgot to divide by $2a = 4$.

EXAMPLE 2 (DISCRIMINANT, STUDENT-PRODUCED RESPONSE)

The equation $3x^2 - 12x + k = 0$ has exactly one real solution. What is the value of k ?

SOLUTION

Exactly one solution means the discriminant is zero: $b^2 - 4ac = 0$.

Here $a = 3$, $b = -12$, $c = k$: $144 - 12k = 0$.

So $k = 12$. (Check: $3x^2 - 12x + 12 = 3(x - 2)^2$, a perfect square with the single root $x = 2$.)

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 3 (EXTRANEOUS SOLUTION)

What is the solution set of $\frac{x^2}{x-4} = \frac{16}{x-4} + 3$? A) $\{-1, 4\}$ B) $\{4\}$ C) $\{-1\}$ D) $\{-4, 1\}$

SOLUTION

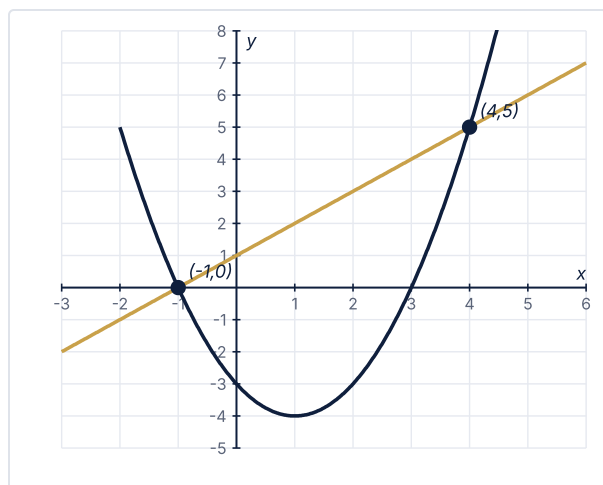
Multiply both sides by $x - 4$ (with $x \neq 4$): $x^2 = 16 + 3(x - 4) = 3x + 4$.

Rearrange: $x^2 - 3x - 4 = 0$, so $(x - 4)(x + 1) = 0$ and $x = 4$ or $x = -1$.

But $x = 4$ makes the denominators zero, so it is extraneous. Only $x = -1$ remains: check $\frac{1}{-5} = \frac{16}{-5} + 3 = \frac{-1}{5}$. The answer is **C**. Choice A is the classic trap: it keeps the extraneous value.

EXAMPLE 4 (LINE AND PARABOLA)

The system $y = x^2 - 2x - 3$ and $y = x + 1$ has two solutions (x_1, y_1) and (x_2, y_2) . What is $y_1 + y_2$?



A parabola and a line meeting at two points.

SOLUTION

Substitute: $x^2 - 2x - 3 = x + 1$, so $x^2 - 3x - 4 = 0$ and $(x - 4)(x + 1) = 0$.

The x values are 4 and -1 . Use $y = x + 1$: the points are $(4, 5)$ and $(-1, 0)$.

So $y_1 + y_2 = 5 + 0 = 5$. The graph shows the line crossing the parabola exactly at these two points.

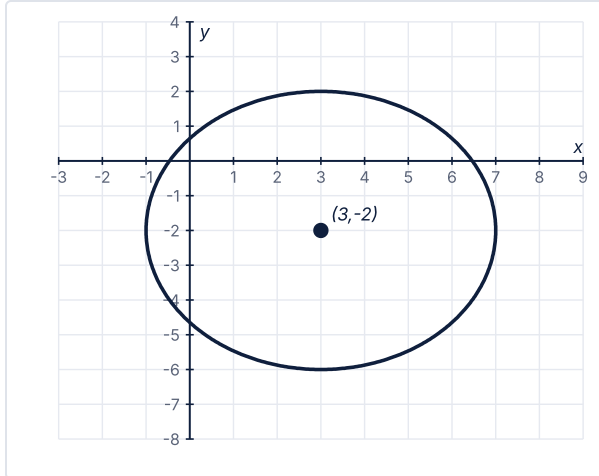
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 5 (HARD, CIRCLES)

A circle in the xy -plane has equation $x^2 + y^2 - 6x + 4y - 3 = 0$. What are its center and radius? A) center $(3, -2)$, radius 4 B) center $(-3, 2)$, radius 4 C) center $(3, -2)$, radius 16 D) center $(3, -2)$, radius $\sqrt{3}$



The circle $(x - 3)^2 + (y + 2)^2 = 16$.

SOLUTION

Group terms: $(x^2 - 6x) + (y^2 + 4y) = 3$.

Complete each square by adding 9 and 4 to **both sides**: $(x^2 - 6x + 9) + (y^2 + 4y + 4) = 3 + 9 + 4$.

This gives $(x - 3)^2 + (y + 2)^2 = 16$. Compare with $(x - h)^2 + (y - k)^2 = r^2$: center $(3, -2)$ and $r^2 = 16$, so $r = 4$. The answer is **A**.

Choice B flips the signs of the center, and choice C reports r^2 instead of r .

SAT Strategy

- 1 Graph it.** For "how many solutions" or "find the solution" questions, type each side of the equation as its own graph in the on-screen graphing calculator. The x -coordinates of the intersection points are the solutions. For systems, the intersections are the answers.
- 2 Plug in the answer choices.** Radical, rational and exponential equations are quickly tested this way, and it also protects you from extraneous solutions.
- 3 Know the quadratic shortcuts.** The sum of the solutions of $ax^2 + bx + c = 0$ is $-\frac{b}{a}$ and the product is $\frac{c}{a}$, so you can answer "sum of the solutions" without solving.
- 4 Use the discriminant for parameters.** When a constant k is chosen to make one, two or no solutions, rearrange to $ax^2 + bx + c = 0$ and use $D = b^2 - 4ac$.
- 5 Complete the square for circles.** Add half the coefficient squared to both sides, once for x and once for y , before you read off h , k and r .

Common traps

(1) Keeping an extraneous solution after multiplying by a denominator or squaring both sides. (2) Writing $\sqrt{x^2} = x$ or squaring only one term of $(x + 2)^2$. (3) For an absolute value equation, finding only the positive case. (4) Reading the center of $(x + 4)^2 + (y - 1)^2 = 25$ as $(4, -1)$ instead of $(-4, 1)$, or reporting $r^2 = 25$ as the radius. (5) Forgetting to add the completing-the-square numbers to both sides.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Practice

PRACTICE Nonlinear Equations and Systems

Choose the best answer or enter your answer. Calculator allowed.

- What are the solutions of $x^2 + 2x - 15 = 0$?
 (A) $x = -3$ and $x = 5$ (B) $x = 3$ and $x = 5$ (C) $x = 3$ and $x = -5$
 (D) $x = -3$ and $x = -5$
- What is the sum of all solutions of $|2x - 5| = 9$?
 (A) 5 (B) 7 (C) -2 (D) 14
- What is the solution of $4^{x-1} = 8^{x+1}$?
 (A) $x = 5$ (B) $x = -5$ (C) $x = -1$ (D) $x = 1$
- What is the solution of $\sqrt{2x+7} = x+2$?
 (A) $x = -3$ (B) $x = 1$ and $x = -3$ (C) $x = 1$
 (D) $x = 3$
- The equation of a circle is $x^2 + y^2 + 8x - 2y - 8 = 0$. What are its center and radius?
 (A) center $(4, -1)$, radius 5 (B) center $(-4, 1)$, radius 25 (C) center $(4, -1)$, radius 25
 (D) center $(-4, 1)$, radius 5
- In the xy -plane, the line $y = 4x - 3$ and the parabola $y = x^2 + 2x + k$ intersect at exactly one point, where k is a constant. What is the value of k ?
- What is the positive solution of $\frac{1}{x} + \frac{1}{x+3} = \frac{1}{2}$?
- For a positive constant k , the line $y = x + k$ is tangent to the circle $x^2 + y^2 = 18$. What is the value of k ?

M8 Nonlinear Functions

Read and build quadratic, polynomial and exponential functions: vertex, standard and factored forms, zeros and end behavior, the remainder theorem, growth and decay models, transformations and composition.

What the SAT Asks

This skill covers functions whose graphs are not straight lines: **quadratic**, **polynomial** (cubic and higher) and **exponential** functions, plus how graphs move under transformations and how functions combine. The stems usually ask you to identify a feature (vertex, maximum, zero, intercept, growth rate) or to choose an equation that matches a graph, table or description.

Expect about 4 to 6 questions in this family across the test. Easy ones read a feature from a form that is already given. Harder ones ask you to convert between forms, use the factor or remainder theorem, interpret the constants in an exponential model, or find the effect of a transformation. The on-screen graphing calculator can check almost all of them.

Core Ideas

A **quadratic function** has three useful forms, and each form shows a different feature at a glance. The graph is a **parabola** that opens up if $a > 0$ (it has a **minimum**) and down if $a < 0$ (it has a **maximum**). The **zeros** (also called roots or x -intercepts) are the inputs where $f(x) = 0$.

Form	Equation	What it shows
Standard	$f(x) = ax^2 + bx + c$	y -intercept c ; axis of symmetry $x = -\frac{b}{2a}$
Vertex	$f(x) = a(x - h)^2 + k$	vertex (h, k) ; max or min value k
Factored	$f(x) = a(x - p)(x - q)$	zeros p and q ; vertex at $x = \frac{p+q}{2}$

Quadratic Features

The vertex of $f(x) = ax^2 + bx + c$ has x -coordinate $-\frac{b}{2a}$. The parabola is symmetric about the vertical line through its vertex, so the vertex lies exactly halfway between the two zeros.

The maximum (if $a < 0$) or minimum (if $a > 0$) value of f is the y -coordinate of the vertex, and it occurs at the x -coordinate of the vertex.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

A **polynomial function** is a sum of terms $a_n x^n$. Its zeros are where its graph meets the x -axis. Its **end behavior** depends on the leading term: for odd degree the two ends go in opposite directions, for even degree they go the same direction, and a negative leading coefficient flips the picture. An even-multiplicity zero, like $(x - 2)^2$, makes the graph touch the axis and bounce back.

Factor Theorem and Remainder Theorem

When a polynomial $p(x)$ is divided by $x - c$, the remainder is $p(c)$.

Therefore $(x - c)$ is a factor of $p(x)$ if and only if $p(c) = 0$, and c is then a zero.

An **exponential function** has the form $f(t) = a \cdot b^t$. The constant a is the **initial value** (the value at $t = 0$). The constant b is the **growth factor** per unit of t : if $b > 1$ the quantity grows, and if $0 < b < 1$ it decays. A rate of r per period gives $b = 1 + r$ (growth of 5% means $b = 1.05$; decay of 8% means $b = 0.92$). If a quantity doubles every d units, then $f(t) = a \cdot 2^{t/d}$.

Transformations and Composition

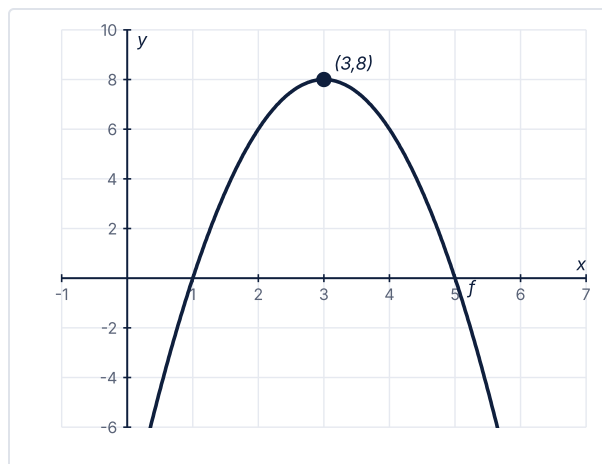
$f(x - h) + k$ shifts the graph of f right h and up k . $-f(x)$ reflects over the x -axis, $f(-x)$ over the y -axis, and $c f(x)$ stretches vertically by c .

The composition $f(g(x))$ means: put $g(x)$ into f . Work from the inside out, and note that $f(g(x))$ and $g(f(x))$ usually differ.

Worked Examples

EXAMPLE 1 (EASY, MULTIPLE CHOICE)

The function $f(x) = -2(x - 3)^2 + 8$ is graphed. What is the maximum value of f , and where does it occur? A) 8 at $x = 3$ B) 8 at $x = -3$ C) 3 at $x = 8$ D) -2 at $x = 3$



Graph of $f(x) = -2(x - 3)^2 + 8$.

SOLUTION

The function is in vertex form $a(x - h)^2 + k$ with $a = -2$, $h = 3$, $k = 8$.

Because $a < 0$, the parabola opens down and the vertex $(3, 8)$ is the highest point.

The maximum value is **8**, occurring at $x = 3$: choice **A**. Choice B uses the wrong sign for h , and choice C swaps the coordinates.

EXAMPLE 2 (STUDENT-PRODUCED RESPONSE)

The quadratic function $g(x) = a(x - 2)(x - 8)$ passes through the point $(1, 14)$. What is the minimum value of g ?

SOLUTION

Substitute the point: $14 = a(1 - 2)(1 - 8) = 7a$, so $a = 2$. Since $a > 0$, the parabola opens up and has a minimum.

The zeros are 2 and 8, so the vertex has $x = \frac{2+8}{2} = 5$.

Evaluate: $g(5) = 2(3)(-3) = -18$.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 3 (REMAINDER THEOREM)

When $p(x) = 2x^3 - 5x^2 + x - 7$ is divided by $x - 3$, what is the remainder? A) 5 B) -5 C) -109 D) 17

SOLUTION

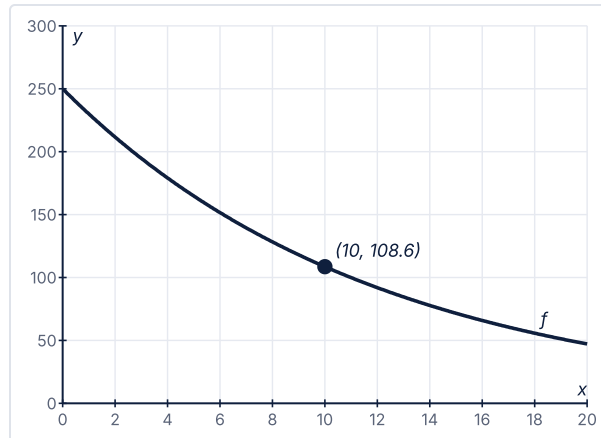
By the remainder theorem, the remainder equals $p(3)$.

$p(3) = 2(27) - 5(9) + 3 - 7 = 54 - 45 + 3 - 7 = 5$. The answer is **A**.

Choice C is $p(-3)$, the value you get if you divide by $x + 3$ or use the wrong sign of the root. Long division would give the same remainder, but it takes much longer.

EXAMPLE 4 (EXPONENTIAL MODEL)

The mass of a sample is modeled by $f(t) = 250(0.92)^t$ grams, where t is the number of days. What is the mass after 10 days, to the nearest tenth of a gram?



Exponential decay with initial value 250.

SOLUTION

The constant 250 is the starting mass at $t = 0$. The factor $0.92 = 1 - 0.08$ means the sample loses 8% of its mass each day.

Substitute $t = 10$: $f(10) = 250(0.92)^{10}$.

$0.92^{10} \approx 0.4344$, so $f(10) \approx 108.6$. The answer is **108.6** grams. (The graph decays quickly at first and then levels off, never reaching zero.)

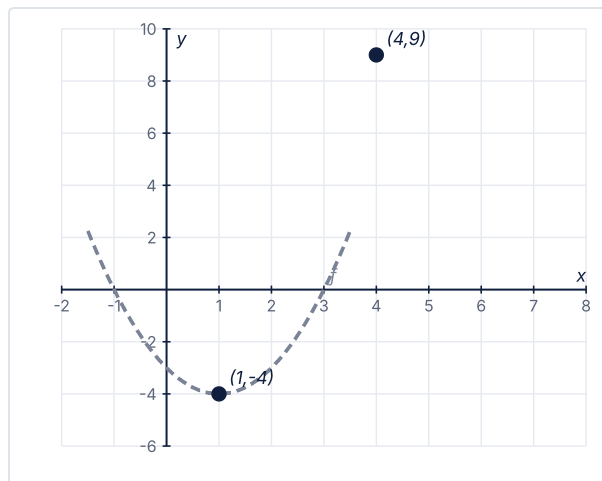
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 5 (HARD, TRANSFORMATIONS)

Let $f(x) = x^2 - 2x - 3$ and $h(x) = -f(x - 3) + 5$. At what point does h reach its maximum? A) (4, 9) B) (-2, 9) C) (4, 1) D) (2, 1)



The dashed parabola f is moved by h .

SOLUTION

Find the vertex of f : $x = -\frac{-2}{2} = 1$ and $f(1) = 1 - 2 - 3 = -4$, so the vertex is $(1, -4)$ and the graph opens up.

Shift right 3: the vertex moves to $(4, -4)$. The negative sign reflects over the x -axis, so it becomes $(4, 4)$ and the parabola opens down. Finally add 5: the vertex is $(4, 9)$.

Check by expanding: $h(x) = -x^2 + 8x - 7$, and $h(4) = -16 + 32 - 7 = 9$. The answer is **A**. Choice B shifts left, and choice C forgets that the reflection also changes the sign of -4 .

SAT Strategy

- 1 Match the form to the question.** If it asks for the vertex or maximum, use vertex form. If it asks for zeros, use factored form. If it asks for the y -intercept, read c from standard form.
- 2 Graph it.** In the on-screen graphing calculator, enter the function. Tap the vertex, zeros and intercepts to read them directly, and enter answer choices to see which one matches a given graph.
- 3 Test points.** For "which equation matches this table or graph", plug in two or three points from the table into each choice. A wrong choice usually fails the very first point.
- 4 Use theorems, not long division.** For remainders and factors, evaluate $p(c)$. For a missing coefficient, set $p(c) = 0$ and solve.
- 5 Read exponentials by their pieces.** Find a from $t = 0$, find b by dividing consecutive outputs, and convert percent change to a factor using $1 \pm r$.

Common traps

(1) Reading the vertex of $a(x - 3)^2 + 8$ as $(-3, 8)$. (2) Treating 0.92 as a 92% decay rate when it is an 8% decay rate (a factor of 0.92). (3) Computing $f(g(x))$ when the problem asks for $g(f(x))$. (4) Using $x - c$ with the wrong sign of c in the remainder theorem. (5) Choosing a maximum value when the question asks where it occurs, or the reverse.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Practice

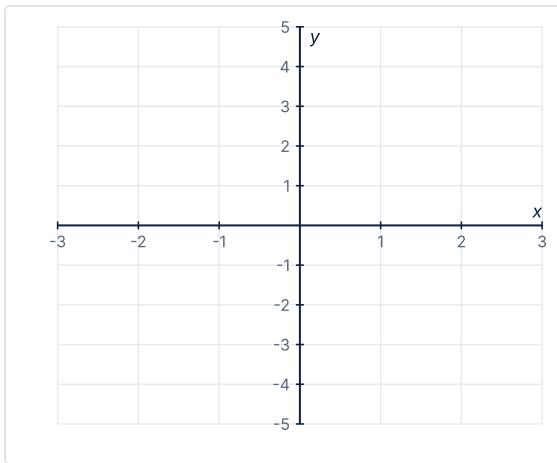
PRACTICE Nonlinear Functions

Choose the best answer or enter your answer. Calculator allowed.

- The function $f(x) = x^2 - 6x + 5$ can be written in vertex form. Which of the following is equivalent?
 (A) $f(x) = (x - 3)^2 - 4$ (B) $f(x) = (x + 3)^2 - 4$ (C) $f(x) = (x - 3)^2 + 14$
 (D) $f(x) = (x - 6)^2 - 31$
- The table gives values of the quadratic function f . Which equation defines f ?

x	0	1	2	3	4
$f(x)$	3	0	-1	0	3

 (A) $f(x) = x^2 + 4x + 3$ (B) $f(x) = x^2 - 4x - 3$ (C) $f(x) = x^2 - 4x + 3$
 (D) $f(x) = x^2 - 2x + 3$
- The population of a town is 12,000 and grows by 3% each year. Which function gives the population P after t years?
 (A) $P(t) = 12000(0.03)^t$ (B) $P(t) = 12000(1.03)^t$ (C) $P(t) = 12000(3)^t$
 (D) $P(t) = 12000 + 0.03t$
- The graph of the cubic polynomial p is shown. Which of the following could define $p(x)$?



Graph of the cubic p .

- Let $f(x) = 3x - 2$ and $g(x) = x^2 + 1$. What is the value of $f(g(2))$?
 (A) 17 (B) 13 (C) 10 (D) 15
- The polynomial $p(x) = x^3 + kx^2 - 5x + 6$, where k is a constant, has $x - 3$ as a factor. What is the value of k ?
- An exponential function $f(t) = a \cdot b^t$, with $b > 0$, has $f(0) = 300$ and $f(2) = 192$. What is the value of b ?
- The quadratic function $g(x) = a(x + 2)(x - 6)$ has a maximum value of 32. What is the value of a ?

Answers Advanced Math: Answers and Explanations

Solutions for every practice problem, numbered to match.

M6 Equivalent Expressions

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

PRACTICE · EQUIVALENT EXPRESSIONS

1. A. Expand: $x^2 + 8x + 16 - (x^2 - 8x + 16) = 16x$. Choice B comes from adding the constants and ignoring the x terms.
2. D. The greatest common factor is $3x^2y$. Dividing each term gives $2x$ and $3y$, so the result is $3x^2y(2x - 3y)$. Choice A drops the y from the second term.
3. C. Inside: $a^{-2-4}b^{3-1} = a^{-6}b^2$. The exponent -1 flips it: $a^6b^{-2} = \frac{a^6}{b^2}$. Choice A forgot the outer -1 .
4. A. Factor: $\frac{(x-4)(x+3)}{(x-3)(x+3)}$. Cancel the common factor $x + 3$ to get $\frac{x-4}{x-3}$. Choice B cancels terms, which is not allowed.
5. B. Common denominator $(x+2)(x-1) = x^2 + x - 2$. Numerator: $3(x-1) + 4(x+2) = 3x - 3 + 4x + 8 = 7x + 5$. Choices A and C add denominators.
6. 40. A perfect square $(4x+5)^2 = 16x^2 + 40x + 25$, so the middle term is $2(4)(5)x = 40x$.
7. 7. Compute $(2x+1)(x-2) = 2x^2 - 3x - 2$. The numerator minus this is $5 - (-2) = 7$, so $r = 7$. (Division gives the same remainder.)
8. 6. Combine over $(x-1)(x+2)$: the numerator is $a(x+2) + b(x-1) = (a+b)x + (2a-b)$. Match $5x+4$: $a+b=5$ and $2a-b=4$. Adding gives $3a=9$, so $a=3$, $b=2$, and $ab=6$.

M7 Nonlinear Equations and Systems**PRACTICE · NONLINEAR EQUATIONS AND SYSTEMS**

1. C. Factor: $(x+5)(x-3) = 0$, so $x = -5$ or $x = 3$. Choice B has the signs reversed.
2. A. Solve $2x - 5 = 9$ to get $x = 7$ and $2x - 5 = -9$ to get $x = -2$. The sum is $7 + (-2) = 5$. Choice B uses only the positive case.
3. B. Write both sides in base 2: $2^{2x-2} = 2^{3x+3}$. Then $2x - 2 = 3x + 3$, so $x = -5$. Choice A has a sign error.
4. C. Square both sides: $2x + 7 = x^2 + 4x + 4$, so $x^2 + 2x - 3 = 0$ and $x = 1$ or $x = -3$. Check $x = -3$: $\sqrt{1} = 1$ but $x + 2 = -1$, so it is extraneous. Only $x = 1$ works ($\sqrt{9} = 3$).
5. D. Complete the square: $(x^2 + 8x + 16) + (y^2 - 2y + 1) = 8 + 16 + 1 = 25$, so $(x+4)^2 + (y-1)^2 = 25$. The center is $(-4, 1)$ and $r = \sqrt{25} = 5$.
6. -2. Set equal: $x^2 + 2x + k = 4x - 3$, so $x^2 - 2x + (k+3) = 0$. One intersection means $D = 4 - 4(k+3) = 0$, so $k+3 = 1$ and $k = -2$.
7. 3. Multiply by $2x(x+3)$: $2(x+3) + 2x = x(x+3)$, so $x^2 - x - 6 = 0$ and $(x-3)(x+2) = 0$. The solutions 3 and -2 both keep the denominators nonzero. The positive one is 3.
8. 6. Substitute: $x^2 + (x+k)^2 = 18$ gives $2x^2 + 2kx + k^2 - 18 = 0$. Tangent means $D = 0$: $4k^2 - 8(k^2 - 18) = 0$, so $k^2 = 36$ and $k = 6$ (positive). Check: the distance from the origin to the line is $\frac{6}{\sqrt{2}} = \sqrt{18}$, the radius.

M8 Nonlinear Functions**PRACTICE · NONLINEAR FUNCTIONS**

1. A. Complete the square: $x^2 - 6x + 9 - 9 + 5 = (x-3)^2 - 4$. The minimum value is -4 at $x = 3$. Choice B has the wrong sign inside the square.
2. C. $f(0) = 3$ gives $c = 3$. The zeros at $x = 1$ and $x = 3$ give $f(x) = (x-1)(x-3) = x^2 - 4x + 3$. Check $f(2) = -1$. The other choices fail $f(1) = 0$.
3. B. A 3% increase multiplies by $1 + 0.03 = 1.03$ each year. Choice A would shrink the population, and choice B triples it each year.
4. D. The zeros are $-2, 0$ and 2 , so the factors are $x+2, x$ and $x-2$. The graph falls on the right, so the leading coefficient is negative. Choice A rises on the right, choice B has a double root at 2, and choice C has zeros at ± 1 .
5. B. First $g(2) = 4 + 1 = 5$. Then $f(5) = 15 - 2 = 13$. Choice A is $g(f(2)) = g(4) = 17$, from composing in the wrong order.
6. -2. By the factor theorem $p(3) = 0$: $27 + 9k - 15 + 6 = 0$, so $9k = -18$ and $k = -2$.
7. 0.8 (or $4/5$). $f(0) = a = 300$. Then $300b^2 = 192$, so $b^2 = 0.64$ and $b = 0.8$ (positive). This is a 20% decay per period.
8. -2. The zeros are -2 and 6 , so the vertex is at $x = 2$. A maximum means $a < 0$. Then $g(2) = a(4)(-4) = -16a = 32$, so $a = -2$.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



Problem-Solving and Data Analysis

7 chapters of original lessons with worked examples, then 56 practice problems with full answers and explanations.

M9 Ratios, Rates, Proportional Relationships, and Units

Learn to set up ratios and proportions, read unit rates from tables and graphs, and convert units correctly, including squared and cubed units, so that rate and scale questions become routine.

What the SAT Asks

Ratios, rates, and unit conversions are the workhorse of the Problem-Solving and Data Analysis domain. You will see stems such as "If the ratio of red to blue marbles is 3 to 5...", "A car travels at a constant speed of...", "How many square feet are in...", or "The table shows a proportional relationship between...". A typical test has about 2 to 4 of these questions, spread across both modules, and they range from one-step proportions to multi-step conversions with cubed units.

The skills are not hard individually, but the questions reward care. The most common lost points come from a unit that was never converted, a ratio read in the wrong order, or a squared unit converted as if it were a plain length. This chapter builds each skill from zero and then shows how to check your work quickly.

Core Ideas

A **ratio** compares two quantities by division. The ratio of a to b can be written $a : b$, " a to b ", or $\frac{a}{b}$. Order matters: the ratio of cats to dogs is not the ratio of dogs to cats. A ratio such as $3 : 5$ describes parts, not actual amounts. If the parts total $3 + 5 = 8$, then cats are $\frac{3}{8}$ of the whole and dogs are $\frac{5}{8}$.

Ratios and parts of a whole

If two quantities are in the ratio $a : b$, then the first is $\frac{a}{a+b}$ of the total and the second is $\frac{b}{a+b}$ of the total.

Find the **value of one part** by dividing the total by the number of parts, then multiply.

A **proportion** is an equation stating that two ratios are equal, such as $\frac{x}{12} = \frac{5}{8}$. Solve it by cross multiplying: $8x = 60$, so $x = 7.5$. A **rate** is a ratio of two quantities with different units, such as miles per hour. A **unit rate** has a denominator of 1, such as 48 miles per 1 hour. Unit rates make comparisons easy: a store charging 2.40 dollars for 8 ounces (0.30 dollars per ounce) is cheaper than one charging 3.50 dollars for 10 ounces (0.35 dollars per ounce).

Proportional relationships

Two quantities x and y are **proportional** when $y = kx$ for a constant k , called the **constant of proportionality** (also the unit rate).

Its graph is a straight line **through the origin**. In a table, $\frac{y}{x}$ is the same for every row. The equation $y = kx + b$ with $b \neq 0$ is linear but **not** proportional.

In a context, k is the amount of y for each 1 unit of x . If y is the cost in dollars and x is the number of pounds, then $k = 3.5$ means 3.5 dollars per pound.

Units and conversion

To convert units, multiply by a fraction equal to 1 that cancels the old unit. This method is called **dimensional analysis**. For example, to change 90 kilometers per hour to meters per second, write

$$90 \frac{\text{km}}{\text{h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 25 \frac{\text{m}}{\text{s}}.$$

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Squared and cubed units

If 1 yard = 3 feet, then 1 square yard = $3^2 = 9$ square feet and 1 cubic yard = $3^3 = 27$ cubic feet.
Raise the **conversion factor itself** to the power: lengths use the factor once, areas use it twice, volumes use it three times.

Quantity	Formula	Typical unit
Speed	distance ÷ time	miles per hour
Density	mass ÷ volume	grams per cubic centimeter
Work rate	job ÷ time	jobs per hour
Unit price	cost ÷ amount	dollars per ounce

Work rates add. If one worker finishes a job in 6 hours, that worker does $\frac{1}{6}$ of the job per hour. Two workers together do $\frac{1}{6} + \frac{1}{12}$ of the job per hour. A **scale drawing** is a proportion in disguise: a map scale of 1 centimeter : 2.5 kilometers means every centimeter on the map is 2.5 kilometers of real distance, and areas scale by the square of the length scale.

Worked Examples

EXAMPLE 1 (EASY, STUDENT-PRODUCED RESPONSE)

The ratio of red marbles to blue marbles in a bag is 3 : 5. There are 72 marbles in all. How many marbles are blue?

SOLUTION

The ratio has $3 + 5 = 8$ parts, and the 8 parts make up 72 marbles.
One part is $72 \div 8 = 9$ marbles.
Blue is 5 parts: $5 \times 9 = 45$.
Answer: 45. Check: red is $3 \times 9 = 27$, and $27 + 45 = 72$.

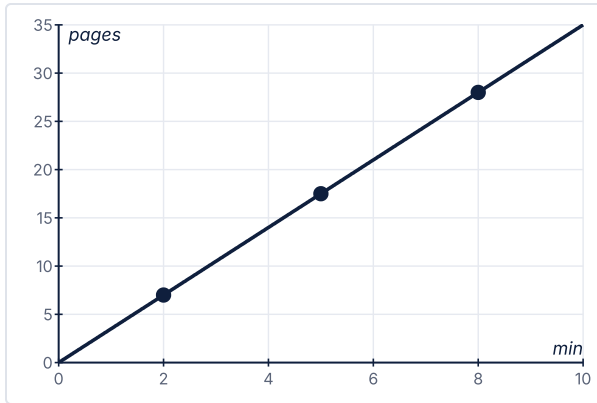
Want a plan built around **your** student?
Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 2 (MEDIUM, MULTIPLE CHOICE)

The table shows the number of pages y printed by a machine after x minutes. The relationship is proportional. Which equation represents it?

- (A) $y = x + 5$
 (B) $y = 3.5x$
 (C) $y = \frac{x}{3.5}$
 (D) $y = 7x$



All three points lie on a line through the origin.

x (minutes)	2	5	8
y (pages)	7	17.5	28

SOLUTION

For a proportional relationship, $k = \frac{y}{x}$ must be the same in every row.

Row 1: $\frac{7}{2} = 3.5$. Row 2: $\frac{17.5}{5} = 3.5$. Row 3: $\frac{28}{8} = 3.5$.

So $y = 3.5x$. **Answer: B.**

Choice A adds 5 (that is not proportional). Choice C divides when it should multiply. Choice D uses the first y -value as the rate.

EXAMPLE 3 (MEDIUM, STUDENT-PRODUCED RESPONSE)

A rectangular lawn measures 12 feet by 15 feet. Sod is sold at 8 dollars per square yard. What is the cost, in dollars, to cover the lawn?

SOLUTION

Area in square feet: $12 \times 15 = 180 \text{ ft}^2$.

Since $1 \text{ yd} = 3 \text{ ft}$, we have $1 \text{ yd}^2 = 9 \text{ ft}^2$. So $180 \div 9 = 20$ square yards.

Cost: $20 \times 8 = 160$.

Answer: 160. A common slip is dividing by 3 instead of 9, which gives 60 square yards and a wrong cost of 480.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 4 (MEDIUM TO HARD, MULTIPLE CHOICE)

On a map, 1 centimeter represents 2.5 kilometers. A straight road measures 7.2 centimeters on the map. A bus drives the whole road at a constant 60 kilometers per hour. How many minutes does the trip take?

- (A) 0.3
- (B) 2.88
- (C) 18
- (D) 432

SOLUTION

Real length: $7.2 \times 2.5 = 18$ kilometers (multiply, because the real world is bigger than the map).

Time: $\frac{18 \text{ km}}{60 \text{ km/h}} = 0.3$ hours.

Convert: $0.3 \times 60 = 18$ minutes. **Answer: C.**

Choice A forgets to convert hours to minutes. Choice B divides 7.2 by 2.5. Choice D multiplies 7.2 by 60.

EXAMPLE 5 (HARD, STUDENT-PRODUCED RESPONSE)

Ana can paint a fence alone in 6 hours. Ben can paint the same fence alone in 12 hours. Working together at their usual rates, how many hours will they take?

SOLUTION

Ana's rate is $\frac{1}{6}$ fence per hour. Ben's rate is $\frac{1}{12}$ fence per hour.

Combined rate: $\frac{1}{6} + \frac{1}{12} = \frac{2}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$ fence per hour.

Time = $\frac{1 \text{ fence}}{1/4 \text{ fence per hour}} = 4$ hours. **Answer: 4.**

Sense check: together they must be faster than the faster worker alone (6 hours), so the answer must be under 6. Averaging the times to get 9 hours fails that check.

SAT Strategy

- 1 Write the units on every number.** Carry units through the arithmetic. If the units on your answer are not the units the question asks for, you are not done.
- 2 Convert early, and convert area and volume correctly.** For a squared or cubed unit, square or cube the length factor ($12^2 = 144$ square inches per square foot, $12^3 = 1728$ cubic inches per cubic foot).
- 3 Use unit rates to compare.** Convert both options to the same unit rate, then compare. For a table, compute $\frac{y}{x}$ in each row.
- 4 Plug in the answer choices.** For a proportion or equation question, test a table row in each choice. One row usually eliminates two or three options.
- 5 Sanity check size.** Scaling up should make a number bigger, converting to a smaller unit should make the count bigger, and combined workers should finish faster than either alone.

Common traps

(1) Reading the ratio backwards: "red to blue" means red first. (2) Treating the parts of a ratio as the actual amounts, instead of dividing the total by the number of parts. (3) Converting a squared or cubed unit with the plain length factor (using 3 instead of 9 for square yards). (4) Averaging times in a work problem instead of adding rates. (5) Forgetting that a line must pass through the origin to be proportional.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Practice

PRACTICE Ratios, Rates, Proportional Relationships, and Units

Choose the best answer or enter your answer. Calculator allowed.

- A shelter has cats and dogs in the ratio 4 : 7. There are 55 animals in all. How many are dogs?
(A) 20 (B) 35 (C) 28 (D) 44
- A cyclist rides 36 kilometers in 2.5 hours at a constant speed. At this speed, how many kilometers will the cyclist ride in 4 hours?
(A) 14.4 (B) 39.6 (C) 90 (D) 57.6
- The relationship between the flour needed f , in cups, and the number of servings s is $f = 0.75s$. Which statement is true?
(A) Each cup of flour makes 0.75 serving.
(B) The recipe needs 0.75 cup of flour in total.
(C) Each serving needs 0.75 cup of flour.
(D) Every serving needs 0.75 cup more flour than the one before.
- A box has a volume of 3 cubic feet. What is its volume in cubic inches? (1 foot = 12 inches)
(A) 5,184 (B) 36 (C) 432 (D) 1,728
- A blueprint uses the scale 1 inch : 4 feet. A rectangular room measures 3.5 inches by 2 inches on the blueprint. What is the actual floor area, in square feet?
(A) 28 (B) 112 (C) 56 (D) 448
- A solid metal cube has edges of length 5 centimeters and a mass of 1 kilogram. What is its density, in grams per cubic centimeter?
- A train travels at a constant 108 kilometers per hour. How many meters does it travel in 20 seconds?
- A pool is filled by two pipes. Pipe A alone can fill it in 12 hours. Pipe B alone can fill it in 6 hours. Pipe A runs alone for the first 3 hours, and then Pipe B is also opened. How many more hours, after Pipe B opens, until the pool is full?

M10 Percentages

Master percent of, percent change, successive changes, reverse percents, and interest. You will learn to use growth factors so that discounts, tax, and compound growth become one multiplication.

What the SAT Asks

Percent questions appear in almost every test, usually 2 to 4 times across the two modules, in plain word problems, in tables, and inside other topics (statistics, exponential models, geometry). Typical stems: "What is 35% of...", "The price increased from ... to ... What was the percent increase?", "After a 20% discount and then 8% tax...", and "An account earns 4% interest compounded annually..."

Difficulty runs from one-step arithmetic to multi-step questions about successive changes, reverse percents, and compound growth. The mistakes are rarely arithmetic. They are about **what the percent is a percent of**. Always ask: percent of which amount?

Core Ideas

Percent means "per hundred", so $35\% = \frac{35}{100} = 0.35$. The basic relationship connects three numbers: the part, the whole, and the percent.

Percent of a number

$$\text{part} = \frac{\text{percent}}{100} \times \text{whole}.$$

To find the percent, use $\frac{\text{part}}{\text{whole}} \times 100$. The word "of" tells you what the whole is: in "18 is what percent of 45", the whole is 45.

Percent change

$$\text{percent change} = \frac{\text{new} - \text{old}}{\text{old}} \times 100\%.$$

The denominator is always the **original** (old) value. A positive result is an increase and a negative result is a decrease.

Growth factors turn percent changes into multiplication. Increasing a quantity by $r\%$ multiplies it by $1 + \frac{r}{100}$. Decreasing by $r\%$ multiplies it by $1 - \frac{r}{100}$. So a 15% increase is a factor of 1.15 and a 15% decrease is a factor of 0.85. This is the most useful idea in the chapter, because it makes multi-step problems into one product.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Words	Meaning	Multiply by
increase by 8%	new = old + 8% of old	1.08
decrease by 30%	new = old - 30% of old	0.70
is 150% of	new = 1.5 times old	1.5
is 50% greater than	new = old + 50% of old	1.5
is 25% less than	new = old - 25% of old	0.75

Notice that "is 150% of" and "is 50% greater than" describe the same relationship. A percent over 100 is perfectly normal: it simply means the part is larger than the whole you compare with. Likewise "a is 200% of b" means $a = 2b$, which is a 100% increase from b to a . Reading the exact words is half the battle.

Successive changes multiply

If a quantity goes through changes with growth factors f_1 and f_2 , the result is $\text{original} \times f_1 \times f_2$.

A 20% increase followed by a 20% decrease is $1.2 \times 0.8 = 0.96$, a net **4% decrease**, not zero. Percent changes do not add or cancel, because the second percent is taken of a different amount.

Reverse percent problems give you the final amount and ask for the original. Divide by the growth factor. If an item costs 84 dollars after a 25% discount, then $0.75 \times \text{original} = 84$, so the original is $84 \div 0.75 = 112$ dollars. Do not add 25% back to 84; that takes 25% of the wrong number.

Percent versus percentage points

If support for a plan rises from 40% to 46%, it rose by **6 percentage points** (the plain difference of the two percents). The **percent increase** is $\frac{6}{40} = 15\%$. The SAT likes this distinction, and it often appears in tables where you must decide which numbers to subtract and which to divide.

Interest

Simple and compound interest

Simple interest on principal P at annual rate r (as a decimal) for t years: $I = Prt$, so the balance is $P(1 + rt)$. It grows **linearly**.

Compounded annually: $A = P(1 + r)^t$. It grows **exponentially**, because each year's interest is earned on the previous balance. With n compounding periods per year, $A = P \left(1 + \frac{r}{n}\right)^{nt}$.

Percent information in a table works the same way. If a table gives the number of students in each grade and the percent of each grade that walks to school, the number who walk is a part for each row. To get the overall percent you must add the parts and divide by the total whole, not average the percents.

Worked Examples

EXAMPLE 1 (EASY, STUDENT-PRODUCED RESPONSE)

A survey asked 240 students when they prefer to study. If 35% said they prefer mornings, how many students prefer mornings?

SOLUTION

$$\text{Part} = 0.35 \times 240.$$

$$0.35 \times 240 = 84.$$

Answer: 84.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 2 (EASY TO MEDIUM, MULTIPLE CHOICE)

A bicycle's price rose from 40 dollars to 52 dollars. By what percent did the price increase?

- (A) 12%
- (B) 23%
- (C) 30%
- (D) 130%

SOLUTION

Change: $52 - 40 = 12$.

Divide by the **original**: $\frac{12}{40} = 0.30$, which is 30%. **Answer: C.**

Choice A is the raw change (12 dollars) mislabeled as a percent. Choice B divides by the new price ($\frac{12}{52} \approx 23\%$). Choice D is the new price as a percent of the old.

EXAMPLE 3 (MEDIUM, STUDENT-PRODUCED RESPONSE)

A store reduces the price of a coat by 20%. A week later it reduces the new price by another 15%. The two reductions together are equal to a single discount of what percent off the original price?

SOLUTION

Factors: 0.80 and 0.85.

Net factor: $0.80 \times 0.85 = 0.68$.

The coat now costs 68% of the original, so the total discount is $100 - 68 = 32$ percent.

Answer: 32. Adding the percents (35) is the trap.

EXAMPLE 4 (MEDIUM TO HARD, MULTIPLE CHOICE)

A jacket is on sale for 25% off its original price. A buyer pays 8% sales tax on the sale price, for a total of 97.20 dollars. What was the original price, in dollars?

- (A) 112.50
- (B) 120.00
- (C) 129.60
- (D) 117.11

SOLUTION

Work forward with factors. Sale price: $0.75 \times P$. After tax: $1.08 \times 0.75 \times P$.

So $0.81P = 97.20$ and $P = 97.20 \div 0.81 = 120$.

Answer: B.

Choice C only undoes the discount (divides by 0.75) and ignores the tax. Choice D divides by 0.83 (adds the percents $-25 + 8$), which is wrong because the tax applies to the discounted price. Choice A multiplies by 1.25 and divides by 1.08, mixing up the order of the two inverse steps.

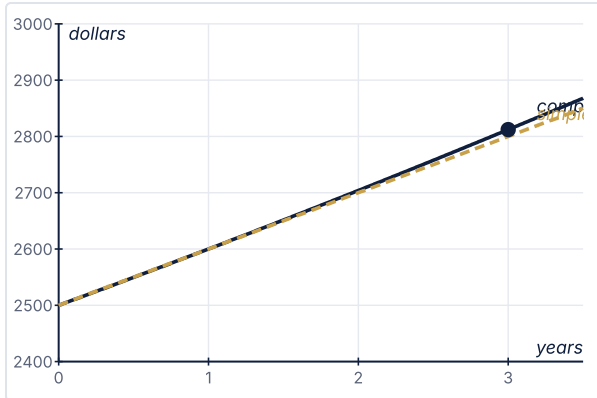
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 5 (HARD, STUDENT-PRODUCED RESPONSE)

An account holds 2,500 dollars and earns 4% interest compounded annually. To the nearest cent, how many dollars are in the account after 3 years? (Enter the number only.)



Compound growth (navy curve) pulls above simple interest (gold dashed line).

SOLUTION

Growth factor per year: 1.04.

$$A = 2500(1.04)^3 = 2500 \times 1.124864 = 2812.16.$$

Answer: 2812.16.

Compare simple interest: $2500 + 2500(0.04)(3) = 2800$. Compounding earns 12.16 dollars more because interest is earned on interest.

SAT Strategy

- 1 Find the whole first.** Underline what the percent is of. If a problem has a discount and then tax, the whole changes at each stage.
- 2 Use decimal multipliers.** Convert every percent change to a factor (1.07, 0.90, 1.5) and multiply. Reverse problems divide by the product.
- 3 Pick a number.** When no starting amount is given, choose 100 (or 1,000). A 10% rise followed by a 10% drop from 100 gives $110 \rightarrow 99$, so the net change is -1% .
- 4 Plug in the answer choices.** For reverse-percent questions, start with a choice, apply the discount and tax, and see if you land on the given total.
- 5 Use the calculator for compounding.** Type $2500 * (1.04)^3$ directly. To find when an amount reaches a target, enter the model as a function and read the table or graph.

Common traps

(1) Taking the percent change of the wrong base, usually the new value instead of the old. (2) Adding successive percents (20% off then 15% off is not 35% off). (3) Undoing a discount by adding the same percent back. (4) Confusing percentage points with percent change. (5) Averaging the percents in a table instead of weighting each by its group size.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Practice

PRACTICE Percentages

Choose the best answer or enter your answer. Calculator allowed.

1. What percent of 45 is 18?
(A) 25% (B) 2.5% (C) 40% (D) 250%
2. The population of a town fell from 12,500 to 11,000. By what percent did it decrease?
(A) 13.6% (B) 12% (C) 88% (D) 15%
3. A price is increased by 10%, and the new price is then decreased by 10%. Compared with the original price, the final price is
(A) the same. (B) 1% higher. (C) 10% lower.
(D) 1% lower.
4. In one school, the percent of students who ride the bus fell from 30% to 24%. Which statement describes this change?
(A) It fell 6 percentage points, which is a 20% decrease.
(B) It fell 6%, which is 6 percentage points.
(C) It fell 20 percentage points, which is a 6% decrease.
(D) It fell 25%.
5. The table shows the number of students in each grade at a school and the percent of those students who walk to school. Which is the percent of all students at the school who walk?

Grade	Students	Percent who walk
9	100	50%
10	300	20%
11	400	25%
12	200	40%

- (A) 33.75% (B) 35% (C) 29% (D) 290%
6. An account has 1,800 dollars at a simple interest rate of 3.5% per year. How many dollars of interest does it earn in 4 years?
7. After a 15% discount, a jacket costs 76.50 dollars. What was the original price, in dollars?
8. A number is increased by 60%. The result is then decreased by $x\%$, and the final value equals the original number. What is x ?

M11 One-Variable Data: Distributions and Measures of Center and Spread

Compute and compare mean, median, mode, range, and standard deviation, read dot plots, histograms and box plots, and predict how outliers and transformations change a data set.

What the SAT Asks

One-variable data questions ask you to describe a single list or display of numbers. You will see stems such as "What is the median of the data set?", "The mean of five values is 14. If four of them are ..., what is the fifth?", "Which data set has the greater standard deviation?", and "What happens to the mean if the value 90 is added?". Expect about 2 to 4 questions per test, with data shown as a list, a frequency table, a dot plot, a histogram, or a box plot.

The questions mostly test understanding rather than long arithmetic. You are rarely asked to calculate a standard deviation, but you must know what it measures and how it responds to changes in the data.

Core Ideas

Measures of center describe a typical value. **Measures of spread** describe how far values scatter. Learn each one precisely.

Measure	How to find it	Notes
Mean	sum of values ÷ number of values	Pulled toward extreme values
Median	middle value of the ordered list (average of the two middle values if the count is even)	Not much affected by extreme values
Mode	most frequent value	Can have none, one, or several

Want a plan built around **your** student?
Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Measure	How to find it	Notes
Range	maximum – minimum	Depends only on the two extremes
Interquartile range (IQR)	$Q_3 - Q_1$	Width of the middle half of the data

Mean and total

$$\text{mean} = \frac{\text{sum}}{\text{count}}, \quad \text{so} \quad \text{sum} = \text{mean} \times \text{count}.$$

Missing-value problems all use the second form. A group with mean 82 over 4 quizzes has a total of $82 \times 4 = 328$ points.

Standard deviation (SD) measures the typical distance of the values from the mean. Data bunched near the mean has a small SD; data scattered far from the mean has a large SD. For the list 2, 4, 6, 8, 10 (mean 6) the deviations are $-4, -2, 0, 2, 4$, so the SD is about 2.8. For the list 4, 5, 6, 7, 8 (also mean 6) the deviations are $-2, -1, 0, 1, 2$, and the SD is about 1.4. Same mean, but the second list is tighter and has the smaller SD. On the SAT you compare spread by looking, not by computing.

Outliers and transformations

Outliers: a very large or very small value moves the **mean** and the **range** and **SD** a lot, but moves the **median** only a little.

Add a constant c to every value: mean and median increase by c ; range, IQR and SD do not change. **Multiply** every value by $c > 0$: mean, median, range, IQR and SD are all multiplied by c .

Displays. A *dot plot* stacks a dot for each value above a number line; the median is the middle dot. A *histogram* shows counts in intervals, so you can find the interval containing the median but not the exact value. A *box plot* shows the five-number summary: minimum, Q_1 , median, Q_3 , maximum. The box (from Q_1 to Q_3) holds the middle half of the data, so its width is the IQR. A *frequency table* lists each value with its count; the total count is the sum of the frequencies, and the sum of the data is the sum of value $\times$ frequency.

Shape. If the long tail points right (a few large values), the mean is usually greater than the median. If the tail points left, the mean is usually less than the median. A symmetric distribution has mean and median about equal.

Worked Examples

EXAMPLE 1 (EASY, STUDENT-PRODUCED RESPONSE)

The data set is 6, 9, 9, 12, 14. What is the sum of its mean and its median?

SOLUTION

Mean: $\frac{6+9+9+12+14}{5} = \frac{50}{5} = 10$.

The list is already ordered, so the median is the middle value, 9.

Sum: $10 + 9 = 19$. **Answer: 19.**

EXAMPLE 2 (MEDIUM, MULTIPLE CHOICE)

A student's first four quiz scores have a mean of 82. What score on the fifth quiz will raise the mean of all five quizzes to exactly 85?

- (A) 85
- (B) 91
- (C) 97
- (D) 103

SOLUTION

Total needed for five quizzes: $85 \times 5 = 425$.

Total already earned: $82 \times 4 = 328$.

Fifth score: $425 - 328 = 97$. **Answer: C.**

Check the others: $328 + 85 = 413$, $328 + 91 = 419$, and $328 + 103 = 431$, none of which equals 425. Choice A is just the target mean, not the score needed.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 3 (MEDIUM, STUDENT-PRODUCED RESPONSE)

The table gives the number of children in 20 households. What is the mean number of children per household?

Children	1	2	3	4	5
Households	3	5	8	2	2

SOLUTION

Sum of the data: $1(3) + 2(5) + 3(8) + 4(2) + 5(2) = 3 + 10 + 24 + 8 + 10 = 55$.

Count: $3 + 5 + 8 + 2 + 2 = 20$.

Mean: $\frac{55}{20} = 2.75$. **Answer: 2.75.**

(The median is 3: the 10th and 11th values are both 3.)

EXAMPLE 4 (MEDIUM TO HARD, MULTIPLE CHOICE)

The data set 20, 22, 23, 25, 30 has mean 24 and median 23. The value 90 is added to the data set. Which statement is true about the new data set?

- (A) The mean and median each increase by about 11.
- (B) The mean increases by 11 and the median increases by 1.
- (C) The mean increases by 1 and the median increases by 11.
- (D) Neither the mean nor the median changes.

SOLUTION

New total: $120 + 90 = 210$ over 6 values, so the mean is 35, an increase of 11.

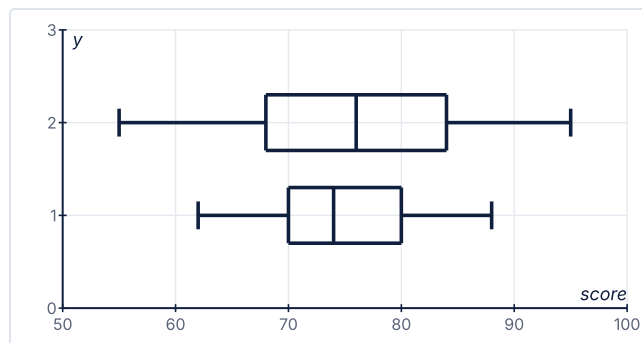
New ordered list: 20, 22, 23, 25, 30, 90. The median is $\frac{23+25}{2} = 24$, an increase of 1.

Answer: B. The outlier 90 drags the mean far upward but barely shifts the median.

EXAMPLE 5 (HARD, MULTIPLE CHOICE)

Two classes took the same test. The box plots show the scores for Class A (top) and Class B (bottom). Which statement is true?

- (A) Class A has the greater median and the smaller IQR.
- (B) Class B has the greater median and the greater range.
- (C) Class A has the greater median, the greater range, and the greater IQR.
- (D) Class B has the greater IQR and the smaller range.



Top box plot: Class A. Bottom box plot: Class B.

SOLUTION

Class A: min 55, Q_1 68, median 76, Q_3 84, max 95. Range = 40, IQR = $84 - 68 = 16$.

Class B: min 62, Q_1 70, median 74, Q_3 80, max 88. Range = 26, IQR = $80 - 70 = 10$.

Class A is larger on all three measures. **Answer: C.**

SAT Strategy

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 1 **Sort first.** Median questions need an ordered list. For an even count, average the middle two.
- 2 **Turn means into totals.** Whenever you see a mean and a count, compute the sum. New mean times new count minus old sum gives the added value.
- 3 **Think "outlier pulls the mean."** Adding or removing an extreme value changes the mean and SD a lot and the median a little.
- 4 **Compare spread visually.** For SD, ask which data set has values farther from its center. For box plots, compare box widths (IQR) and whisker ends (range).
- 5 **Use the calculator's list tool.** For long lists or frequency tables, enter the data in the on-screen calculator's table and read mean and SD from its statistics output; then check it against what the problem says.

Common traps

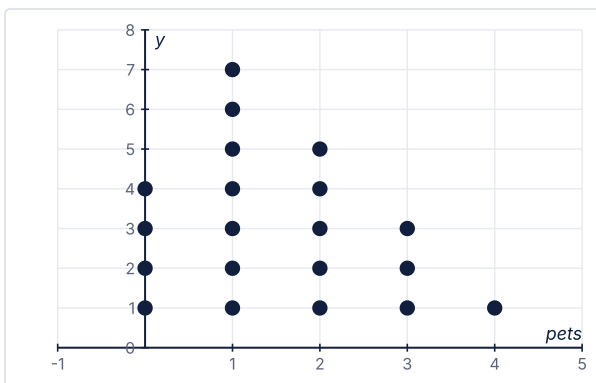
(1) Taking the middle of an unordered list as the median. (2) Using the number of distinct values instead of the total count when finding a mean from a frequency table. (3) Believing that adding a constant to every value changes the SD (it does not), or that multiplying by 2 leaves it unchanged (it doubles). (4) Reading a histogram bar's height as a data value instead of a count. (5) Assuming a larger range always means a larger SD; the range uses only two values.

Practice

PRACTICE One-Variable Data: Distributions and Measures of Center and Spread

Choose the best answer or enter your answer. Calculator allowed.

1. What is the median of the data set 14, 9, 22, 17, 9, 12?
(A) 9 (B) 12 (C) 13 (D) 13.8
2. The mean of 6 numbers is 20. When the number 32 is removed, what is the mean of the remaining 5 numbers?
(A) 16.0 (B) 17.6 (C) 18.0 (D) 20.0
3. Every value in a data set is increased by 10. Which statement about the new data set, compared with the original, is true?
(A) The mean increases by 10 and the standard deviation increases by 10.
(B) The mean stays the same and the standard deviation increases by 10.
(C) The mean increases by 10 and the standard deviation is multiplied by 10.
(D) The mean increases by 10 and the standard deviation stays the same.
4. The dot plot shows the number of pets owned by each of 20 students. Which statement is true?



Each dot is one student.

- (A) The mean is greater than the median.
- (B) The mean is less than the median.
- (C) The mean equals the median.
- (D) The mean cannot be compared with the median.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

5. Data Set A is 2, 4, 6, 8, 10. Data Set B is 4, 5, 6, 7, 8. Which statement is true?
- (A) The standard deviations are equal because the means are equal.
- (B) Set B has the greater standard deviation because it has more distinct values.
- (C) Set A has the greater standard deviation because its values are farther from the mean.
- (D) The standard deviations cannot be compared without a calculation.
6. The mean of 7 numbers is 12. An eighth number is added, and the mean of all 8 numbers is 14. What is the eighth number?
7. The table shows the scores of 20 students on a quiz. What is the mean score? (Enter a decimal.)

Score	60	70	80	90	100
Students	2	5	8	4	1

8. Five positive integers have a mean of 12 and a median of 11. The only mode of the list is 8. What is the greatest possible value in the list?

M12 Two-Variable Data: Models and Scatterplots

Read scatterplots, interpret the slope and intercept of a line of best fit, compute residuals, and choose between linear, exponential, and quadratic models, while knowing when a prediction is trustworthy.

What the SAT Asks

Two-variable data questions ask you to read a relationship between two quantities from a scatterplot or a table and then use a model. Stems include "The scatterplot shows ... The line of best fit is ... Which is the best interpretation of the slope?", "For the data point (10, 29), what is the residual?", "Which type of function best models the data in the table?", and "Based on the model, what is the predicted value when ...?". A typical test has about 2 to 4 of these questions. Difficulty ranges from reading a graph to choosing and then using an exponential or quadratic model.

Core Ideas

A **scatterplot** shows pairs (x, y) as points. The pattern tells you the **association**. If y tends to rise as x rises, the association is **positive**; if y tends to fall, it is **negative**. The **strength** depends on how tightly the points hug a pattern: points close to a line show a strong association, a loose cloud shows a weak one, and no pattern at all shows no association. (A correlation coefficient near 1 or -1 means strong; near 0 means weak.)

Line of best fit

A **line of best fit** has the form $\hat{y} = mx + b$, where $\hat{y}$ is the **predicted** value.

Slope m : the predicted change in y for each 1-unit increase in x . **Intercept** b : the predicted y when $x = 0$. Say it with units and the word "predicted" (or "estimated"). The line balances the points; it need not pass through any of them.

Residual

$$\text{residual} = \text{actual } y - \text{predicted } y.$$

A positive residual means the point lies **above** the line (the model underestimates). A negative residual means the point lies **below** the line (the model overestimates).

Choosing a model. Linear, exponential, and quadratic models leave different fingerprints.

Model	Equation form	Fingerprint in a table or graph
Linear	$y = mx + b$	Equal steps in x give equal differences in y . Straight line.
Exponential	$y = a \cdot b^x$	Equal steps in x give equal ratios (growth if $b > 1$, decay if $0 < b < 1$). A curve that bends steadily upward or flattens toward an axis.
Quadratic	$y = ax^2 + bx + c$	Rises then falls (or falls then rises) with one turning point; symmetric about it. Differences change at a steady rate.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Reading an exponential model

In $y = a \cdot b^x$, a is the starting value (at $x = 0$) and b is the **growth factor** per 1 unit of x . If $b = 1.08$, y grows 8% per unit; if $b = 0.85$, y decays 15% per unit.

Given two points of an exponential, divide their y -values: $\frac{y_2}{y_1} = b^{x_2 - x_1}$.

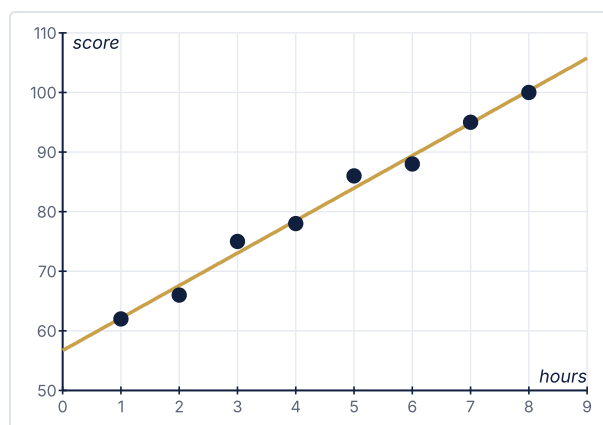
Interpolation and extrapolation. Predicting for an x inside the range of the data is **interpolation** and is usually reliable. Predicting outside that range is **extrapolation** and can be badly wrong, because the pattern may not continue. A line fitted to ages 10 to 16 should not be used to predict height at age 40.

Worked Examples

EXAMPLE 1 (EASY, MULTIPLE CHOICE)

The scatterplot shows the number of hours x that 8 students studied and their test scores y . The line of best fit is $\hat{y} = 5.45x + 56.7$. Which is the best interpretation of the number 5.45?

- (A) The predicted score of a student who did not study
- (B) The predicted increase in score for each additional hour studied
- (C) The number of hours needed to score 100
- (D) The predicted number of hours for each additional point scored



A strong positive association.

SOLUTION

In $\hat{y} = mx + b$, the number multiplying x is the slope m .

The slope is the change in predicted y for each 1-unit increase in x . Here: about 5.45 more points for each extra hour. **Answer: B.**

Choice A describes the intercept 56.7. Choice D reverses the roles of x and y .

EXAMPLE 2 (MEDIUM, STUDENT-PRODUCED RESPONSE)

Using the line of best fit from Example 1, $\hat{y} = 5.45x + 56.7$, find the residual for the student who studied 5 hours and scored 86. Enter the value.

SOLUTION

Predicted score at $x = 5$: $5.45(5) + 56.7 = 27.25 + 56.7 = 83.95$.

Residual = actual – predicted = $86 - 83.95 = 2.05$.

Answer: 2.05. The point lies slightly above the line.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 3 (MEDIUM, MULTIPLE CHOICE)

The table shows a quantity y at four values of x . Which equation best models the data?

- (A) $y = 8 + 4x$
 (B) $y = 8x^2 + 4$
 (C) $y = 8 + 1.5^x$
 (D) $y = 8(1.5)^x$

x	0	1	2	3
y	8	12	18	27

SOLUTION

Check differences: $12 - 8 = 4$, $18 - 12 = 6$, $27 - 18 = 9$. Not constant, so not linear.

Check ratios: $\frac{12}{8} = 1.5$, $\frac{18}{12} = 1.5$, $\frac{27}{18} = 1.5$. Constant, so exponential with $b = 1.5$ and $a = 8$ (the value at $x = 0$).

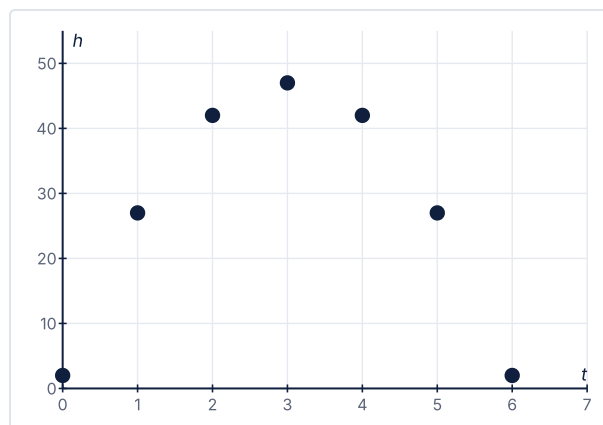
Model: $y = 8(1.5)^x$. **Answer: D.**

Test the others at $x = 2$: choice A gives 16, choice B gives 36, choice C gives 10.25. Only choice D gives 18.

EXAMPLE 4 (MEDIUM TO HARD, MULTIPLE CHOICE)

A ball is thrown upward from a platform. The scatterplot shows its height h , in feet, at time t seconds. Which equation models the data?

- (A) $h = 5(t - 3)^2 + 47$
 (B) $h = -5(t + 3)^2 + 47$
 (C) $h = -5(t - 3)^2 + 47$
 (D) $h = -5(t - 47)^2 + 3$



Height of the ball over time.

SOLUTION

The points rise then fall and are symmetric about $t = 3$, so the model is quadratic with a **maximum** at $(3, 47)$. That means vertex form $h = a(t - 3)^2 + 47$ with $a < 0$.

Use the point $(0, 2)$ to find a : $2 = a(0 - 3)^2 + 47$, so $9a = -45$ and $a = -5$.

Model: $h = -5(t - 3)^2 + 47$. **Answer: C.**

Choice A opens upward (it would have a minimum). Choice B puts the vertex at $t = -3$. Choice D swaps the coordinates of the vertex.

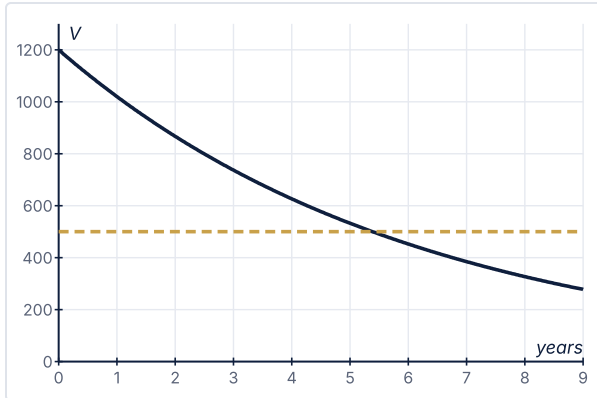
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 5 (HARD, STUDENT-PRODUCED RESPONSE)

The value V , in dollars, of a machine t years after purchase is modeled by $V = 1200(0.85)^t$. According to the model, what is the least whole number of years after purchase at which the machine's value is below 500 dollars?



The curve crosses the dashed line $y = 500$ between $t = 5$ and $t = 6$.

SOLUTION

The factor 0.85 means the value falls 15% each year. Set up $1200(0.85)^t < 500$.

Try whole years: $t = 5$ gives $1200(0.85)^5 \approx 532.4$, still above 500. $t = 6$ gives $1200(0.85)^6 \approx 452.6$, below 500.

Answer: 6. (On the graphing calculator, graph $y = 1200(0.85)^x$ and $y = 500$ and find that they cross at $x \approx 5.4$; the next whole year is 6.)

SAT Strategy

- 1 Identify the model from numbers, not looks.** Subtract neighboring y -values (linear) and divide them (exponential). A symmetric rise and fall means quadratic.
- 2 Plug a point into each choice.** One or two points from the table usually eliminate three of the four equations.
- 3 Use the graphing calculator.** Enter the data in a table, plot the points, and type the candidate equation to see whether the curve passes through them. You can also let it compute a regression and read the coefficients.
- 4 Interpret with words and units.** Slope: "for each additional x -unit, the predicted y changes by m y -units." Intercept: "the predicted y when $x = 0$ ".
- 5 Check the range.** Before trusting a prediction, ask whether the x -value lies within the data.

Common traps

(1) Computing a residual as predicted minus actual instead of actual minus predicted. (2) Interpreting the intercept as a rate, or the slope as a starting value. (3) Calling an increasing curve exponential when its differences, not its ratios, are constant. (4) Using a quadratic opening up when the data rises then falls. (5) Trusting an extrapolated prediction as if it were interpolated.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

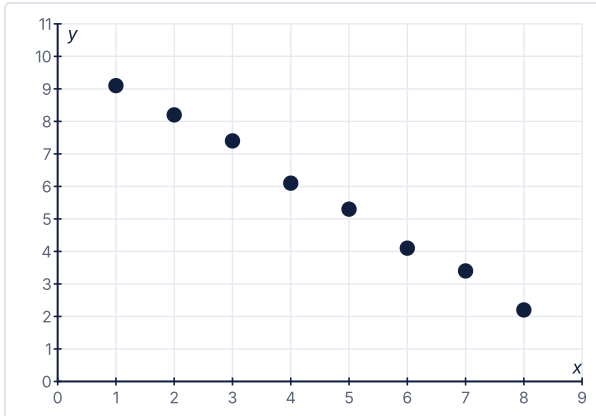
BOOK NOW →

Practice

PRACTICE Two-Variable Data: Models and Scatterplots

Choose the best answer or enter your answer. Calculator allowed.

1. The scatterplot shows 8 data points. Which of the following best describes the association between x and y ?



Eight data points.

- (A) Weak positive (B) No association (C) Strong negative
(D) Strong positive
2. A line of best fit for daily ice cream sales y (in cones) against the daily high temperature x (in degrees Celsius) is $\hat{y} = 3.2x - 18$. Which statement is the best interpretation of 3.2?
- (A) Predicted sales increase by 3.2 cones for each 1 degree increase in temperature.
(B) Predicted sales at 0 degrees are 3.2 cones.
(C) The temperature increases by 3.2 degrees for each additional cone sold.
(D) Sales are always 3.2 cones more than the day before.
3. A line of best fit models the heights of children from the data for ages 10 to 16. For which age would using the line to predict height be the least reliable?
- (A) 12 years (B) 14 years (C) 16 years (D) 40 years
4. A line of best fit is $\hat{y} = 1.5x + 12$. A data point is $(10, 29)$. What is the residual for this point?
- (A) -2 (B) 27 (C) 2 (D) 29
5. The table shows a quantity y for four values of x . Which equation best models the data?

x	0	1	2	3
y	100	80	64	51.2

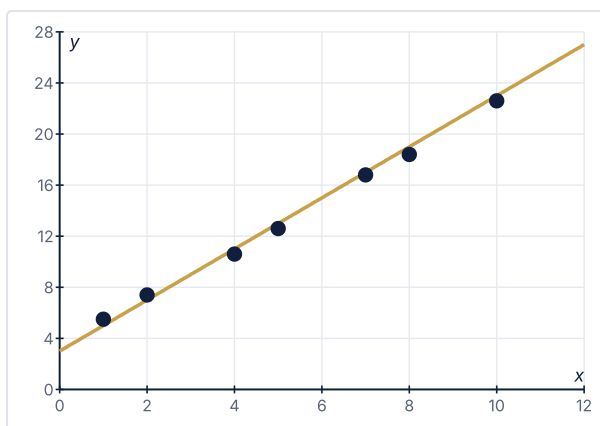
- (A) $y = 100(0.8)^x$ (B) $y = 100 - 20x$ (C) $y = 100(0.2)^x$
(D) $y = 100(1.2)^x$

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

6. The scatterplot shows data and a line of best fit. The line passes through the points (2, 7) and (10, 23). According to the line of best fit, what is the predicted value of y when $x = 14$?



Line of best fit through (2, 7) and (10, 23).

7. The height h , in feet, of a thrown object t seconds after release is modeled by $h = -16t^2 + 48t + 6$. What is the maximum height, in feet, according to the model?
8. The value V of an investment in dollars after t years is modeled by $V = ab^t$ for positive constants a and b . The value is 605 dollars at $t = 2$ and 732.05 dollars at $t = 4$. What is the initial value of the investment, in dollars?

M13 Probability and Conditional Probability

Learn to find probabilities from counts, tables and word descriptions, and to choose the correct denominator for conditional questions. These skills show up in several data questions on every test.

What the SAT Asks

Probability questions sit in the Problem-Solving and Data Analysis domain. A typical test has about 1 to 3 of them per module. The stems are short: "What is the probability that a randomly selected...", "Given that the person chose..., what is the probability...", or "What is the expected...". Most come with a two-way table, and the hardest step is reading the right row or column.

Difficulty ranges from one-step fractions (favorable over total) to conditional probabilities where you must shrink the group you are counting from. The skill is not heavy arithmetic. It is deciding **what the denominator is**. Answers are given as fractions or decimals between 0 and 1.

Core Ideas

A **sample space** is the list of all equally likely outcomes. An **event** is a collection of outcomes you care about. When outcomes are equally likely, the probability of an event is a ratio of counts.

Probability of an event

$$P(A) = \frac{\text{number of outcomes in } A}{\text{total number of outcomes}}, \quad 0 \leq P(A) \leq 1.$$

Complement rule: the chance an event does not happen is $P(\text{not } A) = 1 - P(A)$. Use it whenever "at least one" or "not" makes the direct count long.

Relative frequency uses data instead of theory. If an event happened 18 times in 400 trials, its relative frequency is $\frac{18}{400} = 0.045$, and you can use that to predict counts in a larger group: expect about $0.045 \times 12,000$ occurrences in 12,000 trials.

A **two-way table** sorts the same people by two categories. The row and column totals are your denominators. The key habit: the words after "given that" or "of those who" tell you which row or column to use as the whole.

Conditional probability from a table

$$P(A \text{ given } B) = \frac{\text{number in both } A \text{ and } B}{\text{number in } B}.$$

The group "given" becomes the denominator. $P(A \text{ given } B)$ and $P(B \text{ given } A)$ are usually different.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Combining events

Independent events (one does not change the other): $P(A \text{ and } B) = P(A) \cdot P(B)$.

General 'or' rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. You subtract the overlap because it was counted twice.

Dependent events: $P(A \text{ and } B) = P(A) \cdot P(B \text{ given } A)$.

Counting and expected value

Counting principle: if one choice can be made in m ways and another in n ways, together there are $m \cdot n$ outcomes. With repeats allowed, 3 slots from 10 digits give 10^3 ; with no repeats, $10 \cdot 9 \cdot 8$.

Expected value: $E = \sum(\text{value}) \times (\text{probability})$. It is the long-run average per trial, and it need not be a value that can actually occur.

Worked Examples

EXAMPLE 1 (EASY)

A jar holds 5 red, 7 blue and 8 green marbles. One marble is chosen at random. What is the probability that it is **not** blue?

SOLUTION

Total marbles: $5 + 7 + 8 = 20$.

$P(\text{blue}) = \frac{7}{20}$, so $P(\text{not blue}) = 1 - \frac{7}{20} = \frac{13}{20}$.

Check by counting directly: red plus green is $5 + 8 = 13$ out of 20. The answer is $\frac{13}{20}$, or 0.65.

EXAMPLE 2 (MEDIUM, MULTIPLE CHOICE)

The table shows how 200 students get to school. Of the students who ride the bus, what fraction are in grades 9 and 10? (A) $\frac{45}{200}$ (B) $\frac{45}{100}$ (C) $\frac{3}{5}$ (D) $\frac{3}{4}$

	Walk	Bus	Car	Total
Grades 9 and 10	30	45	25	100
Grades 11 and 12	20	30	50	100
Total	50	75	75	200

SOLUTION

The words "of the students who ride the bus" make the **bus column** the whole group.

Bus column total: $45 + 30 = 75$. Of those, grades 9 and 10 account for 45.

$P = \frac{45}{75} = \frac{3}{5}$. The answer is **(C)**. Choice (B) divides by the grade 9 and 10 row total (100) instead, which answers a different question, and (A) divides by everyone.

EXAMPLE 3 (MEDIUM, STUDENT-PRODUCED RESPONSE)

Events A and B are independent, with $P(A) = 0.4$ and $P(B) = 0.25$. What is $P(A \text{ or } B)$? Enter your answer as a decimal.

SOLUTION

Independent, so $P(A \text{ and } B) = 0.4 \times 0.25 = 0.10$.

$P(A \text{ or } B) = 0.4 + 0.25 - 0.10 = 0.55$.

Quick check with the complement: the chance neither happens is $0.6 \times 0.75 = 0.45$, and $1 - 0.45 = 0.55$. Answer: **0.55**.

EXAMPLE 4 (MEDIUM, STUDENT-PRODUCED RESPONSE)

A raffle sells 500 tickets at 2 dollars each. One ticket wins a 200-dollar prize and four tickets each win a 50-dollar prize. What is the expected **net** gain, in dollars, for one ticket? (A negative answer means an expected loss.)

SOLUTION

Expected prize per ticket: $\frac{1}{500}(200) + \frac{4}{500}(50) = 0.4 + 0.4 = 0.8$ dollars.

Subtract the cost of 2 dollars: $0.8 - 2 = -1.2$.

Answer: **-1.2**. On average a ticket loses 1.20 dollars.

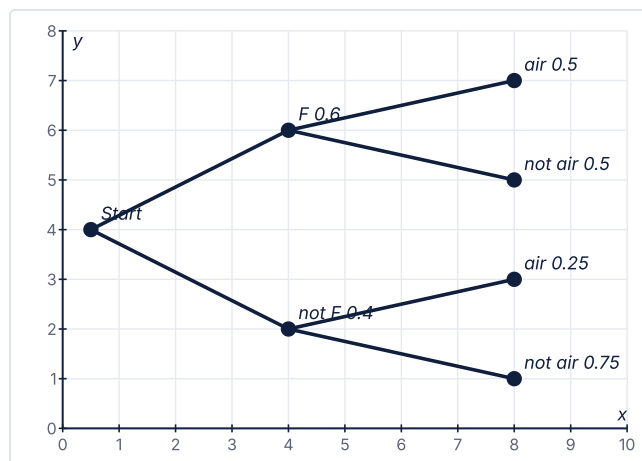
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 5 (HARD, MULTIPLE CHOICE)

Sixty percent of the packages at a depot are marked Fragile (event F). Of the fragile packages, half are shipped by air (event S). Of the packages that are not fragile, one quarter are shipped by air. A package shipped by air is chosen. What is the probability that it is fragile? (A) $\frac{3}{10}$ (B) $\frac{1}{2}$ (C) $\frac{15}{19}$ (D) $\frac{3}{4}$



Tree diagram for Example 5.

SOLUTION

Follow the branches of the tree below.

Build the tree: $P(F \text{ and } S) = 0.6 \times 0.5 = 0.30$ and $P(\text{not } F \text{ and } S) = 0.4 \times 0.25 = 0.10$.

All air packages: $0.30 + 0.10 = 0.40$. The "given" group is air packages, so it is the denominator.

$P(F \text{ given } S) = \frac{0.30}{0.40} = \frac{3}{4}$. The answer is **(D)**. Choice (A) forgets to divide by the air group, and (B) is the given fact $P(S \text{ given } F)$, which is the reverse condition.

SAT Strategy

- 1 Name the denominator first.** Before touching numbers, underline the group in the question ("of those who...", "given that..."). Circle that row or column total.
- 2 Use the complement for 'at least one' and 'not'.** It is usually one subtraction instead of many cases.
- 3 Check the answer range.** A probability must be between 0 and 1. If you get 1.4, you added when you should have subtracted the overlap.
- 4 Convert to counts when percents confuse you.** Pick a total such as 1,000 people, turn each percent into a count, then use the table method.
- 5 Use the on-screen calculator for arithmetic only.** Type fractions as division, then compare with the answer choices written as decimals.

Common traps

(1) Dividing by the grand total when the question says "given" or "of those who". (2) Reversing the condition: $P(A \text{ given } B)$ is not $P(B \text{ given } A)$. (3) Adding $P(A) + P(B)$ for "or" without subtracting the overlap. (4) Multiplying probabilities for events that are not independent, or forgetting that the second probability changes when items are removed without replacement.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Practice

PRACTICE Probability and Conditional Probability

Choose the best answer or enter your answer. Calculator allowed.

1. A jar holds 12 red, 18 yellow and 10 white beads. One bead is chosen at random. What is the probability that it is **not** yellow?
 (A) $\frac{9}{20}$ (B) $\frac{3}{10}$ (C) $\frac{11}{20}$ (D) $\frac{22}{18}$

2. The table shows the course choices of 150 students. Of the students taking French, what fraction are in grade 10?

	Spanish	French	Latin	Total
Grade 9	24	18	8	50
Grade 10	33	15	12	60
Grade 11	18	14	8	40
Total	75	47	28	150

- (A) $\frac{1}{4}$ (B) $\frac{15}{47}$ (C) $\frac{47}{150}$ (D) $\frac{15}{150}$
3. Events A and B are independent, with $P(A) = 0.5$ and $P(B) = 0.3$. What is $P(A \text{ or } B)$?
 (A) 0.80 (B) 0.15 (C) 0.20 (D) 0.65
4. A lock code is two letters chosen from $\{A, B, C, D, E\}$ (letters may repeat) followed by one digit from 0 to 9. How many different codes are possible?
 (A) 20 (B) 250 (C) 200 (D) 100
5. In a random sample of 400 light bulbs, 18 were defective. Based on this sample, about how many defective bulbs would be expected in a shipment of 12,000 bulbs?
 (A) 54 (B) 216 (C) 667 (D) 540
6. The table shows 160 adults sorted by age group and pet ownership. Of the adults who are 30 or older, what proportion own a pet? Enter your answer as a decimal.

	Pet	No pet	Total
Under 30	42	28	70
30 or older	36	54	90
Total	78	82	160

7. A spinner pays 10 dollars with probability $\frac{1}{5}$, pays 4 dollars with probability $\frac{2}{5}$, and pays nothing otherwise. It costs 3 dollars to spin. What is the expected net gain per spin, in dollars? Enter a decimal.
8. Event A has $P(A) = 0.6$. If A occurs, then B occurs with probability 0.5. If A does not occur, B occurs with probability 0.25. Given that B occurred, what is the probability that A also occurred? Enter a decimal.

M14 Inference from Sample Statistics and Margin of Error

Learn when a sample can speak for a population, how to estimate a proportion or mean, and how to read a margin of error as a range of plausible values.

What the SAT Asks

These questions ask what a sample can tell you about a larger group. Stems include "Which of the following is a reasonable conclusion?", "Based on the sample, which is the best estimate for the population?" and "The margin of error is... which value is plausible?". A typical test has about 1 to 3 of them. They mix light arithmetic (scale a proportion up to a population) with careful reading of what a conclusion may claim.

Difficulty is driven by wording, not computation. The easiest ones need a single multiplication. The hardest ask you to decide whether a stated interval supports a claim such as "a majority of voters". Train yourself to separate **what the sample shows** from **what the population might be**.

Core Ideas

The **population** is the whole group you want to know about. The **sample** is the part you actually measure. A **statistic** (sample proportion, sample mean) is computed from the sample. A **parameter** is the true value for the population, which you almost never see.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

When a sample can speak for a population

A sample supports a conclusion about a population only if it is **random** (every member has a fair chance of being chosen) and reasonably **large**.

A random sample from a population lets you generalize **to that population only**. A survey of a school's students says nothing reliable about all teenagers in the country.

Estimating. If a random sample has proportion $\hat{p}$ with a trait, estimate that the population proportion is about $\hat{p}$, so the number in the population is about $\hat{p} \times N$. For means, estimate the population mean by the sample mean.

A statistic is never exactly the parameter, because a different random sample would give a slightly different result. The **margin of error** measures how far off the estimate could reasonably be.

Margin of error and plausible values

$$\text{interval} = \text{estimate} \pm \text{margin of error.}$$

Every value inside the interval is a **plausible** value for the population parameter. Values outside it are unlikely, not impossible. The interval says nothing about individual members, only about the population parameter.

Sample size

For a random sample of size n , the margin of error shrinks as n grows, roughly in proportion to $\frac{1}{\sqrt{n}}$. To cut the margin of error in half, you need **four times** the sample size.

A larger sample reduces random error. It does **not** fix a biased way of choosing the sample.

Term	Meaning	Example
Population	Everyone you want to describe	All 1,200 students in a school
Sample	The members actually measured	100 students picked at random from the roster
Statistic	Number from the sample	42 of the 100 support a later start: 0.42
Parameter	True number for the population	The real share of all 1,200 students

Worked Examples

EXAMPLE 1 (EASY, MULTIPLE CHOICE)

A principal wants to estimate the share of her 1,200 students who support a later start time. Which plan gives the best basis for a conclusion about all students? (A) Ask the 100 members of the debate team. (B) Ask the first 100 students to arrive at school. (C) Number the roster and ask 100 students chosen by a random number generator. (D) Ask 100 students who volunteer after an announcement.

SOLUTION

Only a **random** sample lets you generalize to the whole school.

(A), (B) and (D) each select a special group (debaters, early arrivers, people with strong opinions), which creates bias.

The answer is **(C)**.

EXAMPLE 2 (EASY, STUDENT-PRODUCED RESPONSE)

A city has 48,000 adult residents. In a random sample of 250 of them, 90 favor building a new park. Based on the sample, about how many adults in the city favor the park?

SOLUTION

Sample proportion: $\frac{90}{250} = 0.36$.

Scale up: $0.36 \times 48,000 = 17,280$.

Answer: **17280**. If the margin of error were 6 percentage points, the plausible range would be 30% to 42%, which is 14,400 to 20,160 adults.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 3 (MEDIUM, MULTIPLE CHOICE)

A random sample of commuters had a mean trip time of 42.6 minutes, with a margin of error of 1.8 minutes. Which value is **not** a plausible mean trip time for all commuters? (A) 41 (B) 43 (C) 44 (D) 45

SOLUTION

Interval: $42.6 - 1.8 = 40.8$ to $42.6 + 1.8 = 44.4$ minutes.

Values 41, 43 and 44 are inside it. The value 45 is above 44.4.

The answer is **(D)**.

EXAMPLE 4 (MEDIUM, STUDENT-PRODUCED RESPONSE)

A poll of 400 randomly chosen voters has a margin of error of about 5 percentage points. A second poll uses the same method and should have a margin of error of about 2.5 percentage points. How many voters should it include?

SOLUTION

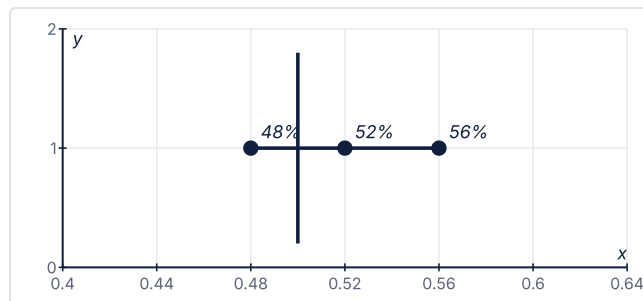
Halving the margin of error requires $2^2 = 4$ times as many voters, because the error shrinks like $\frac{1}{\sqrt{n}}$.

$4 \times 400 = 1600$.

Answer: **1600**. Check: $\frac{1}{\sqrt{1600}} = 0.025$, half of $\frac{1}{\sqrt{400}} = 0.05$.

EXAMPLE 5 (HARD, MULTIPLE CHOICE)

A random sample of voters showed that 52% support a ballot measure, with a margin of error of 4 percentage points. A reporter writes that the measure "has majority support among all voters." Which statement is best? (A) The reporter is correct because 52% is above 50%. (B) The claim is not justified, because the plausible values range from 48% to 56%, and 50% is inside that range. (C) The claim is justified because the margin of error is small. (D) The claim is wrong because exactly 48% support it.



The interval from 0.48 to 0.56 contains the 0.50 line.

SOLUTION

Interval: 52 ± 4 , which is 48% to 56%, shown on the number line below.

The value 50% is a plausible population proportion, and so are values below it. The data cannot rule out that the true share is under half.

The answer is **(B)**. Choice (A) treats the sample result as the population value, and (D) invents a certainty.

SAT Strategy

- 1 Check the sampling method first.** Ask: random? from the whole population being discussed? If not, eliminate any answer that generalizes.
- 2 Turn the margin of error into an interval.** Subtract and add it, then test each answer choice against the interval.
- 3 Match the conclusion to the interval.** If the interval contains a cutoff such as 50%, you cannot be sure which side the true value is on.
- 4 Scale carefully.** Convert the sample count to a proportion, then multiply by the population size. Use the on-screen calculator and keep the unrounded proportion.
- 5 Prefer cautious wording.** Correct choices often say "plausible", "estimate" or "about". Wrong ones say "exactly", "proves" or "all".

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Common traps

(1) Applying a result to a population different from the one sampled. (2) Thinking a larger sample removes bias; it only reduces random error. (3) Reading the margin of error as a statement about individual people ("52% plus or minus 4% of voters") instead of about the population proportion. (4) Treating the end of the interval as a hard limit: values just outside are unlikely, but this does not make them impossible.

Practice**PRACTICE Inference from Sample Statistics and Margin of Error**

Choose the best answer or enter your answer. Calculator allowed.

1. A school has 900 students. Which survey method would best support a conclusion about the opinions of all 900 students on a new lunch menu?
(A) Survey 60 students leaving the cafeteria.
(B) Survey the 60 students in the cooking club.
(C) Survey 60 students chosen at random from a full student list.
(D) Survey 60 students who respond to a posted link.
2. In a random sample of 80 riders on a city bus system, 28 said they are satisfied with the service. The system has 20,000 riders. Based on the sample, about how many riders are satisfied?
(A) 5,600 (B) 9,000 (C) 560 (D) 7,000
3. A random sample of workers had a mean weekly study time of 15.4 hours, with a margin of error of 0.9 hours. Which of the following is a plausible value for the mean of all workers?
(A) 14.2 (B) 14.8 (C) 16.5 (D) 17.0
4. Two random surveys of the same population use the same method. Survey X has 200 participants and survey Y has 800 participants. Which statement is true?
(A) Survey X has the smaller margin of error because it is easier to manage.
(B) Survey Y has the smaller margin of error because its sample is larger.
(C) Both have the same margin of error.
(D) Survey Y must have the same estimate as survey X.
5. A random sample gave an estimate of 45% support for a proposal, with a margin of error of 4 percentage points. Which conclusion is best supported?
(A) Exactly 45% of the population supports the proposal.
(B) Between 41% and 49% of the people in the sample support it.
(C) Every plausible value for the population percent is below 50%, so a majority is unlikely.
(D) Forty-five percent of every group in the population supports it.
6. In a random sample of 150 adults from a town of 3,000 adults, 27 own an electric bicycle. Based on the sample, about how many adults in the town own an electric bicycle?
7. A random sample gives an estimated support of 62% for a plan, with a margin of error of 5 percentage points. What is the largest plausible percent of support in the whole population?
8. A poll of 225 randomly chosen people has a margin of error of 12 percentage points. Assume the margin of error is proportional to $\frac{1}{\sqrt{n}}$. Using the same method with 900 people, about what would the margin of error be, in percentage points?

M15 Evaluating Statistical Claims: Observational Studies and Experiments

Learn to read a study design and decide whether it supports a claim about association, cause and effect, or a larger population. Two ideas, random sampling and random assignment, answer most of these questions.

What the SAT Asks

Here you read a short description of a study and decide which conclusion its design allows. Stems look like "Which conclusion is best supported by the study?" or "Why can the researchers not conclude that...?". A typical test has about 1 to 2 such questions. No calculation is needed, so these are points you earn by knowing two simple ideas and applying them calmly.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Difficulty depends on how many details are packed into the description. The hardest ones describe a study with random sampling but no random assignment, or the reverse, and you must keep the two ideas apart.

Core Ideas

In an **observational study** researchers record what happens without deciding who gets what. In an **experiment** researchers assign the treatment to the participants. Experiments often use a **control group** (no treatment or a placebo) so that the treatment group can be compared with something.

Two different random steps matter, and they answer different questions.

Random step	What it protects	What it allows
Random sampling (choosing who is in the study)	Participants represent the population	Generalize results to the population the sample came from
Random assignment (choosing who gets which treatment)	Groups are alike except for the treatment	A cause and effect conclusion

Which conclusion is justified

Random sample and random assignment: you may generalize to the population AND conclude cause and effect.

Random sample, no random assignment (observational): you may generalize to the population, but only say there is an **association**, not causation.

Random assignment, volunteers (not a random sample): you may conclude cause and effect for participants like those in the study, but you cannot generalize to a wider population.

Neither: only describe the participants.

Confounding and bias

A **confounding variable** is related to both the explanatory variable and the outcome, so it can create an association with no cause behind it. Random assignment spreads confounders evenly across groups.

Bias is a systematic tilt in who is chosen or what is measured (volunteer-only samples, leading questions, only surveying people who are easy to reach).

Statistical significance. A result is called statistically significant when a difference this large would be very unlikely to appear by chance alone if the treatment did nothing. On the SAT, a small p-value (for example, less than 0.05) means the difference is probably not just random variation. It does **not** say the difference is large or important, and it does not by itself say what caused the difference. The design decides that.

Worked Examples

EXAMPLE 1 (EASY, MULTIPLE CHOICE)

Researchers choose 300 adults at random from a city and ask each how many hours they sleep and how many cups of coffee they drink per day. What type of study is this? (A) An experiment, because the sample was random. (B) An observational study, because the researchers did not assign anything. (C) An experiment, because data were collected. (D) Neither, because it has no control group.

SOLUTION

Ask: did the researchers decide who drinks how much coffee? No, they only recorded it.

No treatment was assigned, so it is **observational**. A random sample does not make a study an experiment.

The answer is **(B)**.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 2 (MEDIUM, MULTIPLE CHOICE)

In the study of Example 1, the researchers found that adults who drink more coffee sleep fewer hours. Which conclusion is best supported? (A) Drinking coffee causes people to sleep less. (B) Among adults in that city, more coffee is associated with fewer hours of sleep. (C) Sleeping less causes people to drink more coffee. (D) Coffee drinkers in every country sleep less.

SOLUTION

The sample was random from the city, so it is fine to generalize to **adults in that city**.

There was no random assignment, so causation (in either direction) is not justified. That removes (A) and (C).

(D) goes beyond the city. The answer is **(B)**.

EXAMPLE 3 (MEDIUM, MULTIPLE CHOICE)

Sixty volunteers from a college are randomly assigned to two groups of 30. One group uses a new study method for a month and the other keeps its usual method. The new-method group has a higher mean test score, and the difference is statistically significant. Which conclusion is best supported? (A) The new method causes higher scores for all students in the country. (B) The new method caused the higher scores among volunteers like these, but the result may not apply to all students. (C) The two groups are associated, but causation cannot be claimed. (D) The new method has no effect.

SOLUTION

Random **assignment** is present, so a cause and effect conclusion is allowed for the participants.

The participants were **volunteers**, not a random sample of all students, so the result cannot be generalized to all students.

The answer is **(B)**. Choice (A) generalizes too far; (C) ignores random assignment.

EXAMPLE 4 (MEDIUM, MULTIPLE CHOICE)

A survey of towns finds that towns with more libraries per resident also have higher average test scores. A newspaper says that building more libraries will raise test scores. Which is the best criticism? (A) The sample was too large. (B) Towns may differ in average income, which could affect both library funding and test scores. (C) Test scores cannot be measured. (D) Libraries are unrelated to reading.

SOLUTION

The data show an association between libraries and test scores, not that libraries cause the scores.

A **confounding variable** such as town income could raise both. That is the criticism in **(B)**.

Choice (A) is not a flaw, and (D) and (C) overstate or invent problems.

EXAMPLE 5 (HARD, STUDENT-PRODUCED RESPONSE)

In a randomized experiment, 120 volunteers were split into two equal groups. In the treatment group, 45 of 60 improved. In the control group, 30 of 60 improved. By how many percentage points did the improvement rate in the treatment group exceed that of the control group?

SOLUTION

Treatment rate: $\frac{45}{60} = 75\%$. Control rate: $\frac{30}{60} = 50\%$.

Difference: $75 - 50 = 25$ percentage points.

Answer: **25**. Because of random assignment, a large difference like this (if statistically significant) supports a cause and effect conclusion for volunteers like these.

SAT Strategy

- 1 Ask two questions in order.** (1) Was the sample random? That decides generalizing. (2) Were the groups randomly assigned? That decides causation.
- 2 Underline the verbs in the answer choices.** "Causes", "leads to" and "results in" need random assignment. "Is associated with" is safe for observational data.
- 3 Check the group a choice talks about.** If the choice names all people or all students and the sample was volunteers or one school, eliminate it.
- 4 Look for an alternative explanation.** If a variable like age, income or fitness could drive both traits, the answer about confounding is probably right.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 5 Do not over-read significance.** Significant means unlikely to be chance. It does not mean large, important or proven for everyone.

Common traps

(1) Confusing random sampling with random assignment. (2) Saying "causes" about an observational study, even a huge one. (3) Generalizing an experiment on volunteers to everyone. (4) Believing a bigger sample can cure a bad selection method; it cannot.

Practice

PRACTICE Evaluating Statistical Claims: Observational Studies and Experiments

Choose the best answer or enter your answer. Calculator allowed.

1. A researcher randomly divides 40 plants into two groups. One group gets a new fertilizer and the other gets none. After a month the researcher measures heights. What type of study is this?
(A) An observational study, because plants cannot choose.
(B) A survey, because heights are measured.
(C) An experiment, because the researcher assigned the fertilizer.
(D) A sample, because 40 is less than all plants.
2. A random sample of 500 students from a large district was surveyed. Students who read for fun scored higher on a vocabulary test. Which conclusion is best supported?
(A) Reading for fun causes higher vocabulary scores for these students.
(B) In this district, reading for fun is associated with higher vocabulary scores.
(C) Higher vocabulary scores cause students to read for fun.
(D) Students in every district who read for fun score higher.
3. Eighty volunteers were randomly assigned to either take a new sleep routine or keep their usual routine. The routine group reported better mood, and the difference was statistically significant. Which is justified?
(A) The routine caused better mood among volunteers like these.
(B) The routine caused better mood for people in general.
(C) The routine is associated with mood only.
(D) The routine had no effect.
4. A study finds that people who carry lighters are more likely to develop a certain lung disease. Which is the most likely explanation for the association that does not involve lighters causing the disease?
(A) Lighters are heavy.
(B) Lighters are inexpensive.
(C) Most people own a lighter.
(D) Smoking is related to both carrying a lighter and the disease.
5. In an experiment, why do researchers include a control group?
(A) To make the sample larger.
(B) To provide a comparison group that does not receive the treatment.
(C) To make sure everyone gets the treatment.
(D) To remove the need for random assignment.
6. In a randomized experiment, 36 of 80 people in the treatment group improved and 20 of 80 in the control group improved. By how many percentage points was the improvement rate higher in the treatment group?
7. In a randomized experiment, the mean improvement for the treatment group was 14.2 points and for the control group was 9.7 points. What is the difference between the group means, in points?
8. Eighty volunteers are randomly assigned to a treatment group and a control group. The treatment group is to have 3 times as many people as the control group. How many people are in the control group?

Answers Problem-Solving and Data Analysis: Answers and Explanations

Solutions for every practice problem, numbered to match.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

M9 Ratios, Rates, Proportional Relationships, and Units**PRACTICE · RATIOS, RATES, PROPORTIONAL RELATIONSHIPS, AND UNITS**

1. B. Dogs are 7 of the $4 + 7 = 11$ parts. One part is $55 \div 11 = 5$, so dogs are $7 \times 5 = 35$. Choice A (20) is the number of cats.
2. D. Rate: $36 \div 2.5 = 14.4$ km per hour. Distance in 4 hours: $14.4 \times 4 = 57.6$. Choice A is only the unit rate; choice D multiplies 36 by 2.5.
3. C. In $f = kx$ form, $k = 0.75$ is the cups of flour per 1 serving. Choice A reverses the units.
4. A. A cubic foot is $12^3 = 1728$ cubic inches, so $3 \times 1728 = 5184$. Choice C uses 3×144 (square units), choice B uses 3×12 , and choice D is a single cubic foot.
5. B. Each dimension scales by 4, so the room is 14 ft by 8 ft and the area is $14 \times 8 = 112$. Equivalently $3.5 \times 2 = 7$ square inches and $7 \times 4^2 = 112$. Choice A (28) uses 4 instead of 4^2 on the area; choice C is 7×8 ; choice D multiplies by 8^2 .
6. 8. Volume: $5^3 = 125 \text{ cm}^3$. Mass: 1000 g. Density: $1000 \div 125 = 8 \text{ g per cm}^3$.
7. 600. Speed: $108 \times 1000 \div 3600 = 30$ meters per second. In 20 seconds: $30 \times 20 = 600$ meters.
8. 3. In 3 hours Pipe A fills $\frac{3}{12} = \frac{1}{4}$ of the pool. Remaining: $\frac{3}{4}$. Together the rate is $\frac{1}{12} + \frac{1}{6} = \frac{1}{4}$ pool per hour, so $\frac{3}{4} \div \frac{1}{4} = 3$ hours.

M10 Percentages**PRACTICE · PERCENTAGES**

1. C. Percent = $\frac{18}{45} \times 100 = 40\%$. The whole is 45, not 18. Choice D (250%) divides 45 by 18.
2. B. The decrease is $12,500 - 11,000 = 1,500$. Divide by the original: $\frac{1500}{12500} = 0.12$, or 12%. Choice A divides by the new population ($\frac{1500}{11000} \approx 13.6\%$).
3. D. Factors: $1.10 \times 0.90 = 0.99$, so the final price is 99% of the original, 1% lower. The second percent is taken of the larger price.
4. A. Difference: $30 - 24 = 6$ percentage points. Percent decrease: $\frac{6}{30} = 20\%$. Choice A confuses points with percent; choice C swaps the two numbers.
5. C. Walkers: $50 + 60 + 100 + 80 = 290$ out of 1000 students, so 29%. Choice A averages the four percents, ignoring group sizes.
6. 252. Interest = $Prt = 1800 \times 0.035 \times 4 = 252$ dollars.
7. 90. $0.85 \times \text{original} = 76.50$, so $\text{original} = 76.50 \div 0.85 = 90$.
8. 37.5. Factor after the increase is 1.6. We need $1.6(1 - \frac{x}{100}) = 1$, so $1 - \frac{x}{100} = 0.625$ and $x = 37.5$. Check: $1.6 \times 0.625 = 1$.

M11 One-Variable Data: Distributions and Measures of Center and Spread**PRACTICE · ONE-VARIABLE DATA: DISTRIBUTIONS AND MEASURES OF CENTER AND SPREAD**

1. C. Ordered: 9, 9, 12, 14, 17, 22. The median is $\frac{12+14}{2} = 13$. Choice D is the mean ($83 \div 6 \approx 13.8$).
2. B. Total = $20 \times 6 = 120$. Remove 32: 88 over 5 numbers, $88 \div 5 = 17.6$. Choice D forgets the total changes.
3. D. Adding a constant shifts the center but not the spread: mean +10, SD unchanged. SD would be multiplied only if the values were multiplied.
4. A. Counts 4, 7, 5, 3, 1 give 20 students. The 10th and 11th values are both 1, so the median is 1. Mean = $\frac{0+7+10+9+4}{20} = 1.5$. The tail points right, so the mean is greater.
5. C. Both have mean 6, but Set A deviates from 6 by up to 4 while Set B deviates by up to 2. The SD of A is about 2.8 and of B about 1.4.
6. 28. Total of 7 numbers: $7 \times 12 = 84$. Total of 8 numbers: $8 \times 14 = 112$. The eighth number is $112 - 84 = 28$.
7. 78.5. Sum = $60(2) + 70(5) + 80(8) + 90(4) + 100(1) = 120 + 350 + 640 + 360 + 100 = 1570$. Mean = $1570 \div 20 = 78.5$.
8. 21. Sum is 60. Since 8 is the only mode and 8 is below the median 11, the two smallest values are both 8. The list is 8, 8, 11, d , e with $d + e = 33$. If $d = 11$, then 11 appears twice and ties with 8, so $d \geq 12$. To make e as large as possible take $d = 12$, giving $e = 21$. Check: 8, 8, 11, 12, 21 has sum 60 and unique mode 8.

M12 Two-Variable Data: Models and Scatterplots**PRACTICE · TWO-VARIABLE DATA: MODELS AND SCATTERPLOTS**

1. C. The points trend steadily downward and lie very close to a line, so the association is strong and negative.
2. A. In $\hat{y} = mx + b$ the slope 3.2 is the predicted change in y per 1-unit increase in x . Choice B describes the intercept (which here is -18 , not 3.2).
3. D. Ages 10 to 16 are the range of the data. Predicting at 12, 14, or 16 is interpolation (or at the edge), but age 40 is far outside it, so the line probably does not apply.
4. C. Predicted: $1.5(10) + 12 = 27$. Residual = $29 - 27 = 2$. Choice A has the wrong sign (predicted minus actual).
5. A. Differences are $-20, -16, -12.8$, not constant. Ratios are $80/100 = 0.8, 64/80 = 0.8, 51.2/64 = 0.8$, so it is exponential decay with $a = 100, b = 0.8$. Choice C uses the percent lost (0.2) as the factor.
6. 31. Slope = $\frac{23-7}{10-2} = 2$. Then $7 = 2(2) + b$ gives $b = 3$, so $\hat{y} = 2x + 3$. At $x = 14$: $2(14) + 3 = 31$.
7. 42. The vertex is at $t = -\frac{48}{2(-16)} = 1.5$. Then $h(1.5) = -16(2.25) + 48(1.5) + 6 = -36 + 72 + 6 = 42$.
8. 500. Divide the values: $\frac{ab^4}{a^4b^2} = b^2 = \frac{732.05}{605} = 1.21$, so $b = 1.1$. Then $a(1.1)^2 = 605$ gives $a = \frac{605}{1.21} = 500$.

M13 Probability and Conditional Probability

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

PRACTICE · PROBABILITY AND CONDITIONAL PROBABILITY

1. C. $P(\text{yellow}) = \frac{18}{40}$, so $P(\text{not yellow}) = 1 - \frac{18}{40} = \frac{22}{40} = \frac{11}{20}$. Choice (A) is $P(\text{yellow})$.
2. B. The French column totals 47 and grade 10 has 15 in it, so $\frac{15}{47}$. Choice (A) is $\frac{15}{60}$, which divides by the grade 10 row and answers the reverse question.
3. D. $P(A \text{ and } B) = 0.5 \times 0.3 = 0.15$. Then $P(A \text{ or } B) = 0.5 + 0.3 - 0.15 = 0.65$. Choice (A) forgets to subtract the overlap.
4. B. Two letter slots with repeats: $5 \cdot 5 = 25$. Times 10 digits gives $25 \cdot 10 = 250$. Choice (A) adds instead of multiplying.
5. D. Relative frequency is $\frac{18}{400} = 0.045$, and $0.045 \times 12,000 = 540$.
6. 0.4. The 30 or older row totals 90 and 36 own a pet, so $\frac{36}{90} = 0.4$.
7. 0.6. Expected winnings: $10 \cdot \frac{1}{5} + 4 \cdot \frac{2}{5} = 2 + 1.6 = 3.6$. Net: $3.6 - 3 = 0.6$.
8. 0.75. $P(A \text{ and } B) = 0.6 \times 0.5 = 0.30$ and $P(\text{not } A \text{ and } B) = 0.4 \times 0.25 = 0.10$. So $P(B) = 0.40$ and $P(A \text{ given } B) = \frac{0.30}{0.40} = 0.75$.

M14 Inference from Sample Statistics and Margin of Error**PRACTICE · INFERENCE FROM SAMPLE STATISTICS AND MARGIN OF ERROR**

1. C. Only a random sample from the full list lets you generalize. The others pick groups with special views or only willing volunteers.
2. D. $\frac{28}{80} = 0.35$ and $0.35 \times 20,000 = 7,000$.
3. B. Interval: 15.4 ± 0.9 gives 14.5 to 16.3. Only 14.8 lies inside; 14.2 is below, and 16.5 and 17.0 are above.
4. B. A larger random sample from the same method reduces the margin of error. The estimates can differ, so the last choice is false.
5. C. The interval is 45 ± 4 , or 41% to 49%, and all of it lies below 50%. The sample percent is only an estimate (not exact), and the interval describes the population, not the sample.
6. 540. $\frac{27}{150} = 0.18$, and $0.18 \times 3000 = 540$.
7. 67. Upper end: $62 + 5 = 67$ percent.
8. 6. The sample is 4 times as large, so the margin of error is divided by $\sqrt{4} = 2$: $12 \div 2 = 6$.

M15 Evaluating Statistical Claims: Observational Studies and Experiments**PRACTICE · EVALUATING STATISTICAL CLAIMS: OBSERVATIONAL STUDIES AND EXPERIMENTS**

1. C. The researcher chose which plants got the fertilizer, so it is an experiment. Random division is the random assignment.
2. B. The sample was random, so the result applies to this district, but no one assigned reading habits, so only association is supported.
3. A. Random assignment allows cause and effect, but volunteers are not a random sample, so it applies to people like the participants.
4. D. A confounder (smoking) tied to both variables can create an association with no cause from lighters.
5. B. The control group shows what happens without the treatment, so the treatment effect can be compared.
6. 20. Treatment rate $\frac{36}{80} = 45\%$, control rate $\frac{20}{80} = 25\%$. The difference is 20 percentage points.
7. 4.5. $14.2 - 9.7 = 4.5$.
8. 20. Let the control group be x people. The treatment group is $3x$, so $x + 3x = 80$, $4x = 80$, $x = 20$.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



Geometry and Trigonometry

4 chapters of original lessons with worked examples, then 32 practice problems with full answers and explanations.

M16 Area and Volume

Master area of triangles, trapezoids, circles and sectors, volume and surface area of common solids, and the scaling rules that save time on harder questions.

What the SAT Asks

Geometry questions on area and volume appear about 1 to 3 times per test. Typical stems: "What is the area of the shaded region?", "What is the volume of the cone?", "A cylinder has volume ... and radius ...; what is its height?" and "If every dimension is doubled, by what factor does the volume change?". Many include a figure, and some ask you to convert units such as cubic centimeters to liters.

A formula box is provided on the test, but you save time if you know the common ones. Difficulty rises from direct substitution to composite figures, to questions where you solve for a dimension or reason about scaling without computing anything at all.

Core Ideas

Area is measured in square units and volume in cubic units. Every area or volume problem is the same plan: choose the formula, substitute with matching units, and watch whether the answer wants "in terms of π " or a decimal.

Shape	Area / Surface area	Notes
Triangle	$A = \frac{1}{2}bh$	h is perpendicular to b
Parallelogram	$A = bh$	Use the height, not the slanted side
Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$	Average of the bases times height
Circle	$A = \pi r^2$, $C = 2\pi r$	Radius is half the diameter
Sector (angle θ degrees)	$A = \frac{\theta}{360}\pi r^2$	Fraction of the circle; in radians $A = \frac{1}{2}r^2\theta$
Rectangular prism	$SA = 2(lw + lh + wh)$	
Cylinder	$SA = 2\pi r^2 + 2\pi rh$	Two circles plus the side
Sphere	$SA = 4\pi r^2$	

Volume formulas

Prism or cylinder: $V = Bh$ (area of base times height), so a cylinder is $V = \pi r^2 h$ and a rectangular prism is $V = lwh$.

Cone: $V = \frac{1}{3}\pi r^2 h$. Pyramid: $V = \frac{1}{3}Bh$. Sphere: $V = \frac{4}{3}\pi r^3$.

A cone or pyramid holds exactly one third of the prism or cylinder with the same base and height.

Scaling

If every length is multiplied by k , then areas are multiplied by k^2 and volumes by k^3 .

If only some dimensions change, multiply the separate factors: radius tripled and height halved gives $3^2 \cdot \frac{1}{2} = \frac{9}{2}$ times the volume of a cylinder.

Unit conversion for volume

Convert each length first, or cube the length conversion factor. $1 \text{ m} = 100 \text{ cm}$, so $1 \text{ m}^3 = 100^3 = 1,000,000 \text{ cm}^3$. Also $1 \text{ L} = 1000 \text{ cm}^3$.

Want a plan built around **your** student?

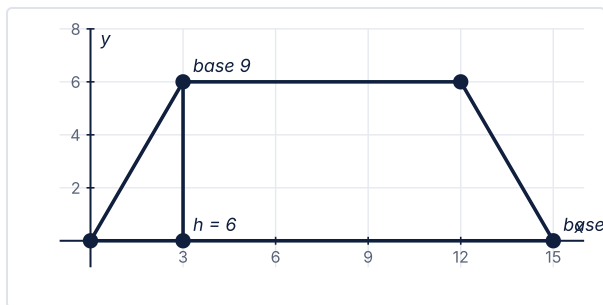
Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Worked Examples

EXAMPLE 1 (EASY, STUDENT-PRODUCED RESPONSE)

A trapezoid has parallel sides of length 15 and 9, and the distance between them is 6. What is its area?



Trapezoid with bases 15 and 9 and height 6.

SOLUTION

$$A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(15 + 9)(6).$$

$$= \frac{1}{2}(24)(6) = 72.$$

Answer: **72**. The figure shows bases 15 and 9 and the height 6.

EXAMPLE 2 (MEDIUM, MULTIPLE CHOICE)

A circle has radius 10. A sector of the circle has a central angle of 72 degrees. What is the area of the sector? (A) 2π (B) 10π (C) 20π (D) 72π

SOLUTION

The sector is the fraction $\frac{72}{360} = \frac{1}{5}$ of the circle.

Circle area: $\pi(10)^2 = 100\pi$. So the sector area is $\frac{1}{5}(100\pi) = 20\pi$.

The answer is **(C)**. Choice (D) multiplies by the angle in degrees, and (B) takes $\frac{1}{10}$ of the circle.

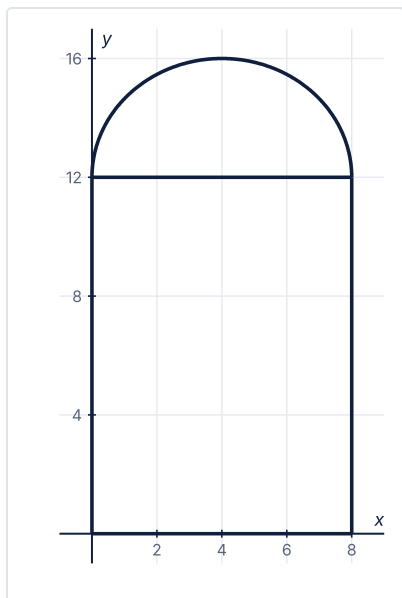
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 3 (MEDIUM, STUDENT-PRODUCED RESPONSE)

A figure is a rectangle 8 wide and 12 tall with a semicircle of diameter 8 attached to the top side. The area is $96 + k\pi$. Find k .



Rectangle 8 by 12 with a semicircle on the top side.

SOLUTION

Rectangle: $8 \times 12 = 96$.

Semicircle: radius 4, so area $\frac{1}{2}\pi(4)^2 = 8\pi$.

Total $96 + 8\pi \approx 121.13$. Answer: **8**.

EXAMPLE 4 (MEDIUM, STUDENT-PRODUCED RESPONSE)

A cone has radius 6 and volume 96π . What is its height?

SOLUTION

$V = \frac{1}{3}\pi r^2 h$, so $96\pi = \frac{1}{3}\pi(36)h = 12\pi h$.

Divide both sides by 12π : $h = \frac{96\pi}{12\pi} = 8$.

Answer: **8**.

EXAMPLE 5 (HARD, MULTIPLE CHOICE)

A spherical balloon's radius grows from 5 cm to 7.5 cm. The new volume is how many times the original volume? (A) 1.5 (B) 2.25 (C) 3.375 (D) 4.5

SOLUTION

Radius factor: $k = \frac{7.5}{5} = 1.5$.

Volume scales by $k^3 = 1.5^3 = 3.375$.

The answer is **(C)**. Choice (B) is k^2 , the factor for surface area, and (A) forgets to power the factor. No formula for the sphere is needed.

SAT Strategy

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 1 **Write the formula, then substitute.** Check that every length uses the same unit before you multiply.
- 2 **Split composite figures.** Break them into rectangles, triangles and circles; add pieces or subtract holes.
- 3 **Solve for the missing dimension by division.** Put numbers in the formula, leave π in, and cancel it on both sides.
- 4 **Use scaling instead of recomputing.** If the question asks "how many times as large", raise the factor to the 2nd or 3rd power.
- 5 **Use the on-screen calculator for decimals.** Enter "pi" for exact values and compare with the choices (some are in terms of π).

Common traps

(1) Using the diameter where the formula needs the radius. (2) Forgetting the $\frac{1}{3}$ for cones and pyramids or the $\frac{1}{2}$ for triangles. (3) Using the slanted side instead of the perpendicular height. (4) Converting units by 100 instead of 100^3 for volume, or forgetting that 1 liter is 1000 cubic centimeters (not 100).

Practice

PRACTICE Area and Volume

Choose the best answer or enter your answer. Calculator allowed.

1. A right triangle has sides of length 9, 12 and 15. What is its area?
(A) 108 (B) 90 (C) 67.5 (D) 54
2. A circle has radius 12. A sector has a central angle of $\frac{\pi}{3}$ radians. What is the area of the sector?
(A) 12π (B) 24π (C) 48π (D) 72π
3. A cylinder has diameter 10 and height 7. What is its volume?
(A) 175π (B) 70π (C) 350π (D) 700π
4. A rectangular aquarium measures 60 cm by 30 cm by 40 cm. One liter equals 1000 cubic centimeters. How many liters does the aquarium hold when full?
(A) 7.2 (B) 72 (C) 720 (D) 72,000
5. A closed cylinder has radius 3 and height 5. What is its total surface area?
(A) 45π (B) 30π (C) 96π (D) 48π
6. A square pyramid has a base with side length 6 and a volume of 96. What is its height?
7. A box has a volume of 40 cubic inches. Each of its dimensions is multiplied by 3 to make a larger box. What is the volume of the larger box, in cubic inches?
8. A solid cylinder has radius 3 and height 8. A cone with the same radius and height is removed from it. The volume that remains is $k\pi$. What is k ?

M17 Lines, Angles, and Triangles

Master the angle facts, triangle rules, and similar-triangle proportions that solve most plane geometry questions on the SAT, then practice turning diagrams into equations.

What the SAT Asks

Geometry questions on the digital SAT usually give you a small diagram or a short description and ask for an angle, a length, or a ratio. On a typical test you will see about 2 to 4 geometry and trigonometry questions per module, and a good share of them use the ideas in this chapter: angles formed by parallel lines, the angle sum of a triangle, and similar triangles. Stems sound like "What is the value of x ?", "What is the measure of angle B ?", or "What is the length of $\overline{BC}$?"

Most of these questions are easy to medium if you know the facts cold, and they turn hard only when an algebra step is hidden inside (an angle written as $4x + 15$, or a side written as $x + 2$). Learn the small list of facts below, then practice turning the picture into an equation. Diagrams are often not drawn to scale, so never measure them: always reason from the given facts.

Core Ideas

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Parallel lines and a transversal. When a line (the transversal) crosses two parallel lines, it creates eight angles at the two crossing points. Only two sizes appear: a size a and its supplement $180^\circ - a$. Angles that look "alike" (both acute or both obtuse) are equal. Angles that look "different" (one acute, one obtuse) add to 180° .

Angle pair	Where they sit	Relationship
Vertical angles	Opposite each other at one crossing	Equal
Corresponding angles	Same corner at each crossing	Equal (lines parallel)
Alternate interior angles	Between the lines, opposite sides of the transversal	Equal (lines parallel)
Same-side interior angles	Between the lines, same side of the transversal	Add to 180°
Linear pair	Side by side on a straight line	Add to 180°

Triangle Angle Facts

The three interior angles of a triangle add to 180° .

An **exterior angle** equals the sum of the two remote (non-adjacent) interior angles. If the exterior angle at C is e , then $e = A + B$.

In an isosceles triangle the angles opposite the equal sides are equal. An equilateral triangle has three 60° angles.

The triangle inequality. You cannot build a triangle from any three lengths. The two shorter sides together must be longer than the longest side. If two sides are a and b , the third side c satisfies $|a - b| < c < a + b$. For sides 7 and 12, the third side lies strictly between 5 and 19.

Similar Triangles

Two triangles are **similar** if their angles match (it is enough that two angles match, called AA). Then corresponding sides are proportional:

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = k.$$

Lengths (sides, perimeters, heights) scale by k . Areas scale by k^2 . Volumes scale by k^3 .

Spotting similar triangles. A line drawn parallel to one side of a triangle cuts off a smaller triangle similar to the whole (the angles match because of parallel lines). Two intersecting lines crossed by parallel segments form two triangles joined at a vertex, again similar. A **midsegment** joins the midpoints of two sides of a triangle; it is parallel to the third side and exactly half as long.

Congruence means same shape and same size. Triangles are congruent by SSS, SAS, ASA, AAS, or HL (right triangles). Note that AAA only gives similarity, never congruence. When two triangles are congruent, all matching parts are equal, which is how the SAT lets you "move" a length or angle from one place to another.

Angle Sums in Polygons

A convex polygon with n sides has interior angle sum

$$180(n - 2)^\circ.$$

Its exterior angles (one per vertex) always add to 360° . A regular polygon with n sides has each interior angle $\frac{180(n - 2)}{n}$ and each exterior angle $\frac{360}{n}$.

Worked Examples

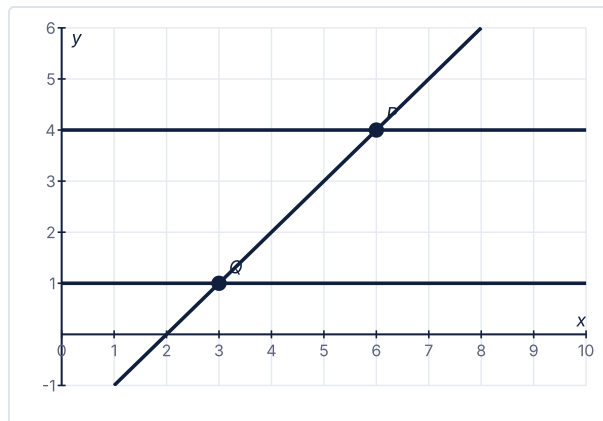
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 1: PARALLEL LINES (STUDENT-PRODUCED RESPONSE)

In the figure, line ℓ ($y = 4$) is parallel to line m ($y = 1$), and a transversal crosses both. Two same-side interior angles measure $(4x + 15)^\circ$ and $(2x + 45)^\circ$. What is the measure, in degrees, of the larger of the two angles?



Lines ℓ and m cut by a transversal at P and Q .

SOLUTION

Same-side interior angles add to 180° : $(4x + 15) + (2x + 45) = 180$.

So $6x + 60 = 180$, giving $6x = 120$ and $x = 20$.

The angles are $4(20) + 15 = 95^\circ$ and $2(20) + 45 = 85^\circ$. The check: $95 + 85 = 180$.

The larger angle is **95**.

EXAMPLE 2: EXTERIOR ANGLE (MULTIPLE CHOICE)

In triangle ABC , the exterior angle at C measures 128° . Angle A measures $(x + 20)^\circ$ and angle B measures $(2x)^\circ$. What is the measure of angle C inside the triangle?

(A) 36° (B) 52° (C) 56° (D) 72°

SOLUTION

The exterior angle equals the sum of the two remote interior angles: $(x + 20) + 2x = 128$.

So $3x = 108$ and $x = 36$. Then $A = 56^\circ$ and $B = 72^\circ$.

Interior angle $C = 180 - 128 = 52^\circ$ (a linear pair), and the check $56 + 72 + 52 = 180$ works.

The answer is **(B)**. Choice (C) is angle A and (D) is angle B ; (A) is just x .

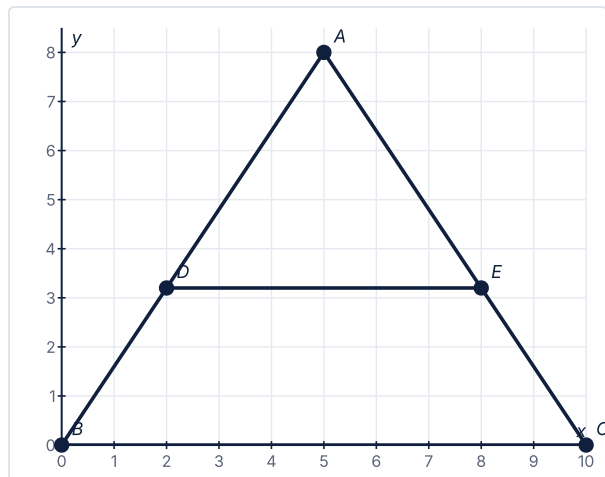
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 3: SIMILAR TRIANGLES (STUDENT-PRODUCED RESPONSE)

In triangle ABC , point D lies on $\overline{AB}$ and point E lies on $\overline{AC}$ so that $\overline{DE} \parallel \overline{BC}$. If $AD = 6$, $DB = 4$, and $DE = 9$, what is the length of $\overline{BC}$?



DE is parallel to BC , so $\triangle ADE$ is similar to $\triangle ABC$.

SOLUTION

Since $DE \parallel BC$, triangle ADE is similar to triangle ABC (AA).

The whole side is $AB = AD + DB = 10$. A common error is to compare AD with DB ; the correct ratio compares the small triangle with the whole triangle.

$$\frac{DE}{BC} = \frac{AD}{AB} \Rightarrow \frac{9}{BC} = \frac{6}{10}.$$

$$BC = \frac{9 \cdot 10}{6} = 15. \text{ The answer is } \mathbf{15}.$$

EXAMPLE 4: AREA OF SIMILAR FIGURES (MULTIPLE CHOICE)

Triangle PQR is similar to triangle XYZ . The area of PQR is 18 square units and the area of XYZ is 50 square units. If $PQ = 6$, what is the length of the corresponding side XY ?

- (A) 10 (B) $\frac{50}{3}$ (C) 12 (D) $\frac{18}{5}$

SOLUTION

Areas scale by k^2 , so $k^2 = \frac{50}{18} = \frac{25}{9}$ and $k = \frac{5}{3}$.

Lengths scale by k : $XY = 6 \cdot \frac{5}{3} = 10$.

The answer is **(A)**. The tempting wrong choice is (B): it uses the area ratio $\frac{50}{18}$ as if it were the scale factor, which forgets the square root.

EXAMPLE 5: TRIANGLE INEQUALITY WITH AN ISOSCELES TWIST (STUDENT-PRODUCED RESPONSE)

An isosceles triangle has two sides of lengths 5 and 11. What is its perimeter?

SOLUTION

An isosceles triangle has two equal sides, so the third side is either 5 or 11.

Try 5, 5, 11: the two short sides total 10, which is less than 11. This cannot form a triangle.

Try 5, 11, 11: $5 + 11 > 11$ and the other inequalities hold, so it works.

Perimeter = $5 + 11 + 11 = 27$. The answer is **27**.

SAT Strategy

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 1 **Mark the diagram.** Write every given angle and length on the figure, and add the angles you can find (vertical angles, linear pairs, parallel-line partners). Most geometry answers appear after two or three of these small moves.
- 2 **Turn words into one equation.** Same-side interior angles give $a + b = 180$. Corresponding or alternate angles give $a = b$. A triangle gives $a + b + c = 180$. Then solve; do not guess from the picture.
- 3 **Redraw similar triangles separately.** Pull the small and large triangles apart, label matching sides, and write the proportion small over large equals small over large.
- 4 **Check with the answer choices.** For angle problems, plug a choice back into the equation. For length problems, test whether the choice respects the triangle inequality or a scale factor.
- 5 **Use the calculator only for arithmetic.** The graphing calculator can solve the linear equation you build (type it as an equation and read the intersection), but the setup must come from you.

Common traps

(1) Treating same-side interior angles as equal when they are supplementary. (2) Comparing a piece to a piece (for example AD to DB) instead of a piece to the whole side (AD to AB) in similar triangles. (3) Using the side scale factor k for area instead of k^2 , or forgetting to take a square root when you start from areas. (4) Forgetting that the exterior angle equals the sum of two angles, not the third angle. (5) Ignoring the triangle inequality when a side length can be one of two values.

Practice

PRACTICE Lines, Angles, and Triangles

Choose the best answer or enter your answer. Calculator allowed.

1. Lines ℓ and m are parallel and are cut by a transversal. Two corresponding angles measure $(5x + 8)^\circ$ and $(3x + 32)^\circ$. What is the measure of the angle $(5x + 8)^\circ$?
 (A) 12° (B) 68° (C) 112° (D) 34°
2. An isosceles triangle has exactly one angle measuring 100° . What is the measure of each of the other two angles?
 (A) 30° (B) 50° (C) 80° (D) 40°
3. The interior angles of a convex polygon add to 1440° . How many sides does the polygon have?
 (A) 10 (B) 8 (C) 9 (D) 12
4. Triangle ABC is similar to triangle DEF with $AB = 8$ and $DE = 12$. The perimeter of triangle ABC is 30. What is the perimeter of triangle DEF ?
 (A) 20 (B) 36 (C) 45 (D) 34
5. In triangle ABC , segment $\overline{MN}$ joins the midpoints of $\overline{AB}$ and $\overline{AC}$. The length of $\overline{MN}$ is $3x - 1$ and the length of $\overline{BC}$ is $5x + 3$. What is the length of $\overline{BC}$?
 (A) 14 (B) 28 (C) 25 (D) 17
6. In triangle ABC , $AB = AC$. The exterior angle at B (formed by extending $\overline{CB}$ beyond B) measures 112° . What is the measure, in degrees, of angle A ?
7. Two similar triangles have areas 20 and 125. The smaller triangle has perimeter 18. What is the perimeter of the larger triangle?
8. In triangle ABC , point D is on $\overline{AB}$ and point E is on $\overline{AC}$ with $\overline{DE} \parallel \overline{BC}$. Also $AD = x$, $DB = x + 2$, $AE = 6$, $EC = 9$, and $DE = 7$. What is the length of $\overline{BC}$?

M18 Right Triangles and Trigonometry

Use the Pythagorean theorem, special triangles, SOH CAH TOA, complementary angles, and radians to solve the right-triangle problems that appear on every SAT.

What the SAT Asks

Right triangles are the workhorse of SAT geometry. Expect about 2 to 4 geometry and trigonometry questions per module, and a large share involve a right triangle: finding a missing side with the Pythagorean theorem, using the 45° - 45° - 90° or 30° - 60° - 90° shortcuts, or writing a sine, cosine or tangent ratio. Stems read "What is $\cos B$?", "What is the value of x ?", or "What is the length of the diagonal?"

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Trigonometry on this test is gentle. You will not need identities beyond the complementary-angle fact, and you will almost never need to find a sine by hand: the on-screen calculator does that (check that it is in degree mode when the angle has a degree sign, and in radian mode otherwise). What the test rewards is choosing the right ratio and noticing special triangles.

Core Ideas

The Pythagorean Theorem

In a right triangle with legs a and b and hypotenuse c (the side opposite the right angle),

$$a^2 + b^2 = c^2.$$

The hypotenuse is always the longest side. The converse is true: if $a^2 + b^2 = c^2$, the triangle is right.

Pythagorean triples are whole-number solutions that appear again and again. Learn these, and their multiples, so that you can skip the squaring: 3, 4, 5; 5, 12, 13; 8, 15, 17; 7, 24, 25. A triangle with legs 6 and 8 is just 3, 4, 5 doubled, so its hypotenuse is 10.

Special Right Triangles

45-45-90: the legs are equal and the hypotenuse is a leg times $\sqrt{2}$. Sides in the ratio 1 : 1 : $\sqrt{2}$.

30-60-90: sides in the ratio 1 : $\sqrt{3}$: 2, where 1 is opposite 30° (the short leg), $\sqrt{3}$ is opposite 60° (the long leg), and 2 is the hypotenuse.

The diagonal of a square with side s is $s\sqrt{2}$. The height of an equilateral triangle with side s is $\frac{s\sqrt{3}}{2}$, and its area is $\frac{s^2\sqrt{3}}{4}$.

Right-Triangle Trigonometry (SOH CAH TOA)

For an acute angle θ in a right triangle:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}.$$

"Opposite" and "adjacent" are measured from the angle θ , so they swap when you switch to the other acute angle.

Complementary Angles

The two acute angles of a right triangle add to 90° , and the sine of one equals the cosine of the other:

$$\sin x^\circ = \cos(90 - x)^\circ, \quad \cos x^\circ = \sin(90 - x)^\circ.$$

In radians, $\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$.

Solving triangles. To find a side, pick the ratio that links the side you want with a side you know and the given angle, write the equation, and solve (multiply or divide, then use the calculator). To find an angle, use the inverse key: if $\tan \theta = 0.3$, then $\theta = \tan^{-1}(0.3) \approx 16.7^\circ$. Similar right triangles have equal ratios, so a drawing at any size gives the same $\sin$, $\cos$, $\tan$ for the same angle.

Degrees	Radians	$\sin$	$\cos$	$\tan$
0°	0	0	1	0
30°	$\pi/6$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\pi/3$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\pi/2$	1	0	undefined

Radians and the Unit Circle

$180^\circ = \pi$ radians. To convert, multiply degrees by $\frac{\pi}{180}$, or radians by $\frac{180}{\pi}$.

On the unit circle (radius 1, center at the origin), the point at angle θ is $(\cos \theta, \sin \theta)$. So the x-coordinate is the cosine and the y-coordinate is the sine.

The table above is the unit circle in the first quadrant: at 30° the point is $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, at 45° it is $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, and at 60° it is $\left(\frac{1}{2}, \sqrt{3}\right)$. Notice that the sine column reads $0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, 1$ going up, and the cosine column is the same list in reverse. That is the complementary-angle rule in action.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Worked Examples

EXAMPLE 1: DIAGONAL OF A RECTANGLE (STUDENT-PRODUCED RESPONSE)

A rectangle is 20 centimeters wide and 21 centimeters tall. What is the length, in centimeters, of its diagonal?

SOLUTION

The diagonal is the hypotenuse of a right triangle with legs 20 and 21.

$$d^2 = 20^2 + 21^2 = 400 + 441 = 841.$$

$$d = \sqrt{841} = 29. \text{ The answer is } \mathbf{29}. \text{ (You can also recognize the triple 20, 21, 29.)}$$

EXAMPLE 2: EQUILATERAL TRIANGLE (MULTIPLE CHOICE)

An equilateral triangle has side length 12. What is its area?

(A) 36 (B) $36\sqrt{3}$ (C) $72\sqrt{3}$ (D) $18\sqrt{3}$

SOLUTION

Drop the altitude: it splits the triangle into two 30-60-90 triangles with hypotenuse 12 and short leg 6.

The long leg (the height) is $6\sqrt{3}$.

$$\text{Area} = \frac{1}{2} \cdot 12 \cdot 6\sqrt{3} = 36\sqrt{3}.$$

The answer is **(B)**. Choice (C) forgets the factor $\frac{1}{2}$.

EXAMPLE 3: READING RATIOS (MULTIPLE CHOICE)

In right triangle ABC , the right angle is at C , $AC = 15$, and $BC = 8$. What is $\sin A$?

(A) $\frac{8}{17}$ (B) $\frac{15}{17}$ (C) $\frac{8}{15}$ (D) $\frac{17}{8}$

SOLUTION

$$\text{Hypotenuse: } AB = \sqrt{15^2 + 8^2} = \sqrt{289} = 17.$$

From angle A , the opposite side is $BC = 8$.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{17}.$$

The answer is **(A)**. Choice (B) is $\cos A$, and (C) is $\tan A$.

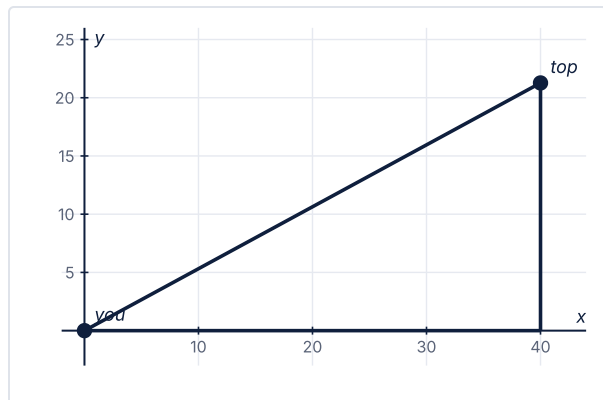
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 4: ANGLE OF ELEVATION (STUDENT-PRODUCED RESPONSE)

From a point on level ground 40 meters from the base of a tower, the angle of elevation to the top of the tower is 28° . To the nearest tenth of a meter, how tall is the tower?



The angle at the observer is 28 degrees; the horizontal side is 40 m.

SOLUTION

The side we want (the height) is opposite the angle, and the side we know (40) is adjacent, so use tangent.

$$\tan 28^\circ = \frac{h}{40}, \text{ so } h = 40 \tan 28^\circ.$$

With the calculator in degree mode, $\tan 28^\circ \approx 0.5317$, so $h \approx 21.27$.

The answer is **21.3**.

EXAMPLE 5: COMPLEMENTARY ANGLES (STUDENT-PRODUCED RESPONSE)

If $\sin(2k)^\circ = \cos(k + 9)^\circ$, where $2k$ and $k + 9$ are both acute angles, what is the value of k ?

SOLUTION

Sine of an angle equals cosine of its complement, so the two angles must add to 90° .

$$2k + (k + 9) = 90, \text{ so } 3k = 81 \text{ and } k = 27.$$

Check: $\sin 54^\circ = \cos 36^\circ$ since $54 + 36 = 90$. The answer is **27**.

SAT Strategy

- 1 Name the sides first.** Mark the hypotenuse, then label opposite and adjacent from the angle in the question. Most wrong answers come from mixing these up.
- 2 Hunt for triples and special triangles.** Sides like 5, 12, 13 or an angle of 30° , 45° , 60° mean no calculator is needed, and the answer will often contain $\sqrt{2}$ or $\sqrt{3}$.
- 3 When two angles are linked by $\sin = \cos$, add them to 90.** Equations like $\sin a^\circ = \cos b^\circ$ are solved by $a + b = 90$, not by taking inverse functions.
- 4 Check the calculator mode.** A degree sign means degree mode; a value like $\frac{\pi}{6}$ means radian mode. A wrong mode gives a plausible-looking wrong answer.
- 5 Test the answer choices.** For a ratio question, a sine or cosine of an acute angle must be between 0 and 1, and the hypotenuse must be the largest side. That eliminates choices fast.

Common traps

(1) Putting the hypotenuse in the wrong position of a ratio, so that $\sin$ comes out larger than 1. (2) Using the legs ratio $1 : \sqrt{3} : 2$ in the wrong order: the long leg is $\sqrt{3}$ times the short leg, and the hypotenuse is twice the short leg. (3) Adding the legs instead of squaring them in $a^2 + b^2 = c^2$, or forgetting the final square root. (4) Leaving the calculator in radian mode for a degree problem. (5) Writing $\sin A = \cos A$ for two different angles; the rule is $\sin A = \cos(90 - A)$.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Practice

PRACTICE Right Triangles and Trigonometry

Choose the best answer or enter your answer. Calculator allowed.

- A right triangle has legs of length 7 and 24. What is the length of its hypotenuse?
(A) 17 (B) 25 (C) 31 (D) 49
- The hypotenuse of a 45° - 45° - 90° triangle has length 6. What is the length of each leg?
(A) 3 (B) $6\sqrt{2}$ (C) 6 (D) $3\sqrt{2}$
- In a right triangle, angle θ is acute and $\sin \theta = \frac{5}{13}$. What is $\tan \theta$?
(A) $\frac{12}{13}$ (B) $\frac{13}{12}$ (C) $\frac{5}{12}$ (D) $\frac{12}{5}$
- An angle measures 150° . What is its measure in radians?
(A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) $\frac{5\pi}{3}$
- In a right triangle, the hypotenuse is 20 and an acute angle A satisfies $\sin A = 0.6$. What is the area of the triangle?
(A) 48 (B) 192 (C) 120 (D) 96
- If $\sin(5x)^\circ = \cos(2x + 6)^\circ$ and both angles are acute, what is the value of x ?
- A ladder leans against a wall and makes a 62° angle with the level ground. Its top touches the wall 18 feet above the ground. To the nearest tenth of a foot, how long is the ladder?
- A right triangle has legs 6 and 8. What is the length of the altitude drawn from the right angle to the hypotenuse?

M19 Circles

Learn circle equations, completing the square, arc length and sector area, and the angle and tangent rules that unlock every circle question on the SAT.

What the SAT Asks

Circle questions come in two flavors. The first lives in the coordinate plane: you are given an equation such as $(x - 2)^2 + (y + 4)^2 = 25$ and asked for the center, the radius, or a point on the circle, or you must rewrite an expanded equation by completing the square. The second is classical geometry: arc length, sector area, and angle relationships built from central angles, inscribed angles, chords, and tangents. Expect 1 to 3 circle questions per test, mostly at medium difficulty.

Typical stems are "Which equation represents the circle...", "What is the radius of the circle with equation...", "What is the length of arc AB ?", or "What is the measure of angle ACB ?" Radians appear regularly, so be comfortable using the arc length formula $s = r\theta$ as well as the degree version.

Core Ideas

Equation of a Circle

A circle with center (h, k) and radius r has equation

$$(x - h)^2 + (y - k)^2 = r^2.$$

Read signs carefully: $(x + 3)^2$ means $h = -3$. The right side is r^2 , not r , so $= 49$ means radius 7.

Completing the square turns an expanded equation like $x^2 + y^2 - 6x + 10y + 18 = 0$ into standard form. Group the x terms and the y terms, take half of each linear coefficient, square it, and add that amount to both sides. Half of -6 is -3 , squared is 9; half of 10 is 5, squared is 25. Then $(x^2 - 6x + 9) + (y^2 + 10y + 25) = -18 + 9 + 25$. In general, for $x^2 + y^2 + Dx + Ey + F = 0$, the center is $(-\frac{D}{2}, -\frac{E}{2})$ and $r^2 = \frac{D^2}{4} + \frac{E^2}{4} - F$.

Lines and circles. A line can miss a circle (0 points), touch it (1 point, a tangent line), or cut it (2 points, a secant line). To find intersections, substitute the line's equation into the circle's equation and solve the resulting quadratic. The discriminant tells you how many points: positive means 2, zero means 1, negative means 0.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Arc Length and Sector Area

If a central angle measures n° , the arc is the fraction $\frac{n}{360}$ of the circle:

$$\text{arc length} = \frac{n}{360} \cdot 2\pi r, \quad \text{sector area} = \frac{n}{360} \cdot \pi r^2.$$

If the central angle is θ radians:

$$s = r\theta, \quad A = \frac{1}{2}r^2\theta.$$

A full circle is $360^\circ = 2\pi$ radians.

Angles and Lines in a Circle

A **central angle** equals its intercepted arc. An **inscribed angle** (vertex on the circle) equals half its intercepted arc, so it is half the central angle on the same arc.

An angle inscribed in a semicircle is a right angle.

A **tangent** line touches the circle at one point and is perpendicular to the radius drawn to that point. Two tangent segments from the same outside point are equal in length.

A radius perpendicular to a chord bisects the chord.

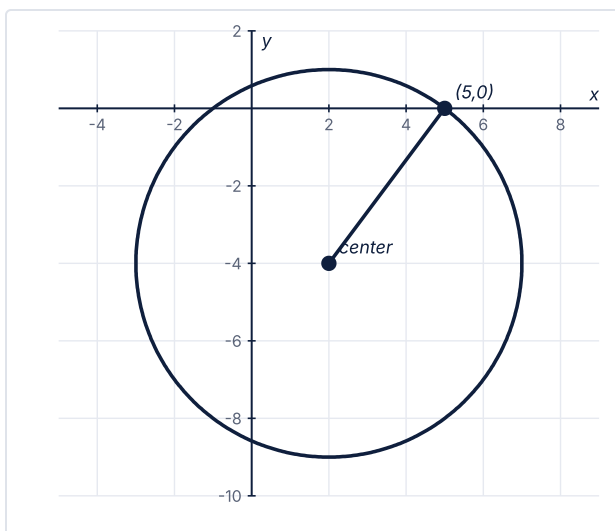
Quantity	Degrees version	Radians version
Arc length	$\frac{n}{360} \cdot 2\pi r$	$r\theta$
Sector area	$\frac{n}{360} \cdot \pi r^2$	$\frac{1}{2}r^2\theta$
Full circle	360°	2π

Worked Examples

EXAMPLE 1: WRITING AN EQUATION (MULTIPLE CHOICE)

A circle in the xy -plane has center $(2, -4)$ and passes through the point $(5, 0)$. Which equation represents the circle?

(A) $(x - 2)^2 + (y + 4)^2 = 5$ (B) $(x + 2)^2 + (y - 4)^2 = 25$ (C) $(x - 2)^2 + (y + 4)^2 = 25$ (D) $(x - 5)^2 + y^2 = 25$



Radius from $(2, -4)$ to $(5, 0)$ has length 5.

SOLUTION

The radius is the distance from the center to the given point: $r^2 = (5 - 2)^2 + (0 - (-4))^2 = 9 + 16 = 25$, so $r = 5$.

With $h = 2$ and $k = -4$: $(x - 2)^2 + (y + 4)^2 = 25$.

The answer is **(C)**. Choice (A) writes r instead of r^2 , and (B) flips the signs of the center.

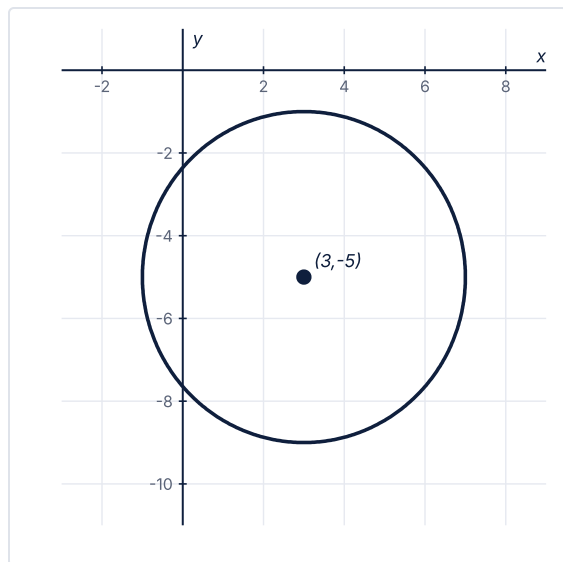
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 2: COMPLETING THE SQUARE (STUDENT-PRODUCED RESPONSE)

The equation $x^2 + y^2 - 6x + 10y + 18 = 0$ represents a circle in the xy -plane. What is the radius of the circle?



Center $(3, -5)$, radius 4.

SOLUTION

Group terms: $(x^2 - 6x) + (y^2 + 10y) = -18$.

Complete each square: add 9 and 25 to both sides. $(x - 3)^2 + (y + 5)^2 = -18 + 9 + 25 = 16$.

So the center is $(3, -5)$ and $r^2 = 16$, giving $r = 4$. The answer is **4**. A frequent slip is to answer 16, which is r^2 .

EXAMPLE 3: ARC AND SECTOR (MULTIPLE CHOICE)

A circle has radius 9. A sector of the circle has a central angle of 80° . What is the area of the sector?

(A) 4π (B) 9π (C) 18π (D) 81π

SOLUTION

The sector is the fraction $\frac{80}{360} = \frac{2}{9}$ of the whole circle.

Whole area: $\pi r^2 = 81\pi$.

Sector area = $\frac{2}{9} \cdot 81\pi = 18\pi$. The answer is **(C)**.

Choice (A) is the arc length $\frac{2}{9} \cdot 18\pi = 4\pi$, the wrong quantity. Choice (D) is the full circle.

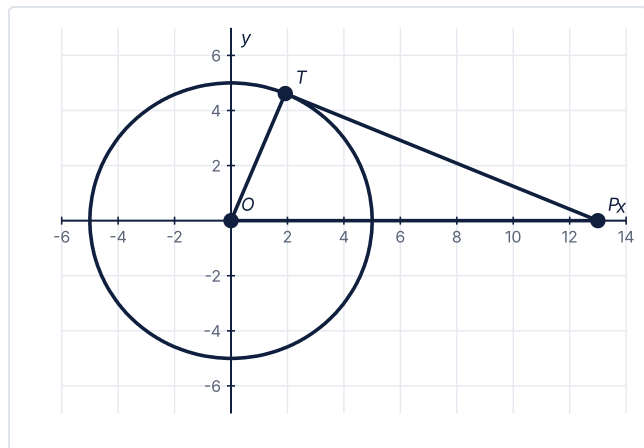
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

EXAMPLE 4: TANGENT AND RADIUS (STUDENT-PRODUCED RESPONSE)

Point P lies 13 units from the center O of a circle of radius 5. A line through P is tangent to the circle at point T . What is the length of $\overline{PT}$?



Right angle at T between radius OT and tangent PT .

SOLUTION

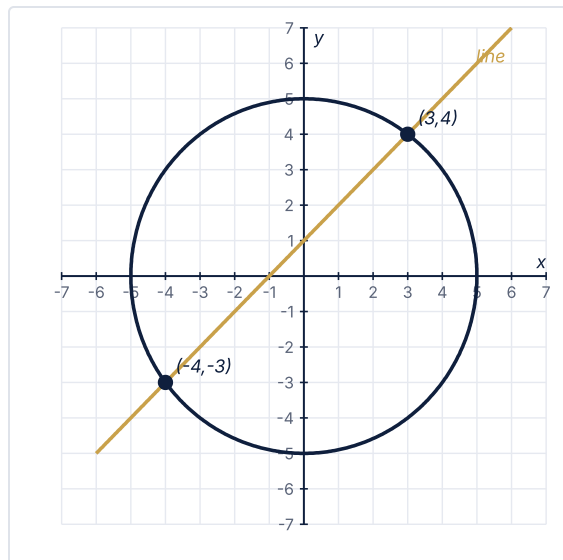
A tangent is perpendicular to the radius at the point of tangency, so triangle OTP has a right angle at T .

The hypotenuse is $OP = 13$ and one leg is $OT = 5$.

$PT = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$. The answer is **12**.

EXAMPLE 5: A LINE MEETS A CIRCLE (STUDENT-PRODUCED RESPONSE)

The line $y = x + 1$ intersects the circle $x^2 + y^2 = 25$ at two points. What is the x -coordinate of the intersection point that lies in the first quadrant?



The line crosses the circle twice.

SOLUTION

Substitute $y = x + 1$: $x^2 + (x + 1)^2 = 25$.

Expand: $2x^2 + 2x + 1 = 25$, so $2x^2 + 2x - 24 = 0$ and $x^2 + x - 12 = 0$.

Factor: $(x + 4)(x - 3) = 0$, so $x = -4$ or $x = 3$.

For $x = 3$, $y = 4$ (first quadrant). For $x = -4$, $y = -3$ (third quadrant). The answer is **3**. Check: $3^2 + 4^2 = 25$.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

SAT Strategy

- 1 Read the equation before you compute.** For $(x - h)^2 + (y - k)^2 = r^2$ you can write the center and radius at once. Flip the signs for the center and take the square root for the radius.
- 2 Complete the square, or use the shortcut.** For $x^2 + y^2 + Dx + Ey + F = 0$, the center is $(-D/2, -E/2)$ and $r^2 = D^2/4 + E^2/4 - F$. Always finish by taking the square root if the question asks for the radius.
- 3 Graph it.** Type the circle equation into the on-screen graphing calculator to see its center and where it crosses a line, then compare with the choices. For a line and a circle, the intersection points can be read directly from the graph.
- 4 Test the answer choices.** Plug a given point into each equation; the correct equation must make the left side equal the right side.
- 5 Convert arcs to fractions.** Write the central angle as a fraction of 360° (or of 2π) and multiply by the whole circumference or area. Do not mix the two formulas.

Common traps

(1) Copying the signs of the center straight from the equation: $(y + 4)^2$ means $k = -4$. (2) Reporting r^2 as the radius. (3) Using circumference for a sector area question, or area for an arc length question. (4) Forgetting that an inscribed angle is half the arc, not equal to it. (5) Plugging a degree measure into $s = r\theta$: in that formula the angle must be in radians.

Practice

PRACTICE Circles

Choose the best answer or enter your answer. Calculator allowed.

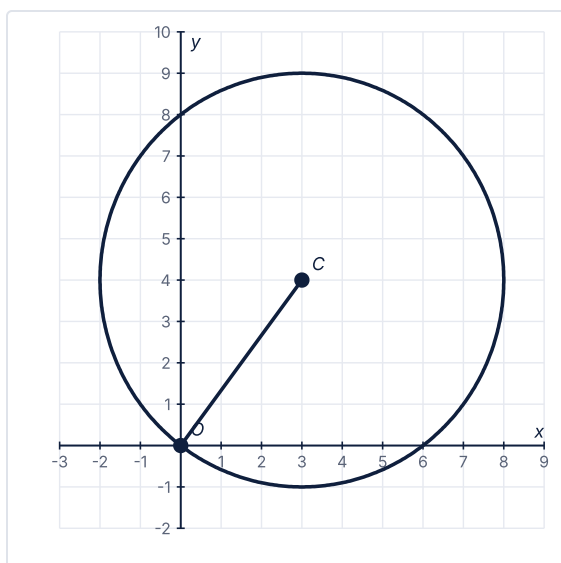
1. A circle has equation $(x + 2)^2 + (y - 7)^2 = 20$. What are the coordinates of its center?
 (A) $(2, -7)$ (B) $(-2, 7)$ (C) $(2, 7)$ (D) $(-2, -7)$
2. In a circle of radius 12, a central angle measures $\frac{\pi}{3}$ radians. What is the length of the intercepted arc?
 (A) 2π (B) 8π (C) 12π (D) 4π
3. An inscribed angle in a circle intercepts an arc of 140° . What is the measure of the inscribed angle?
 (A) 70° (B) 140° (C) 280° (D) 35°
4. The endpoints of a diameter of a circle are $(-2, 3)$ and $(6, 9)$. Which equation represents the circle?
 (A) $(x + 2)^2 + (y + 6)^2 = 25$ (B) $(x - 2)^2 + (y - 6)^2 = 5$ (C) $(x - 2)^2 + (y - 6)^2 = 25$
 (D) $(x - 2)^2 + (y - 6)^2 = 100$
5. A sector of a circle of radius 6 centimeters has area 10π square centimeters. What is the measure of the sector's central angle?
 (A) 50° (B) 100° (C) 60° (D) 120°
6. The equation $x^2 + y^2 - 10x + 6y + k = 0$ represents a circle of radius 7 in the xy -plane. What is the value of k ?
7. A circle has radius 9. From a point Q outside the circle, a tangent segment of length 12 is drawn to the circle. How far is Q from the center of the circle?

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

8. The circle shown has center $C(3, 4)$ and passes through the origin O . A line tangent to the circle at O is drawn. What is the slope of this tangent line? (Enter your answer as a decimal or fraction.)



Circle with center $(3, 4)$ through the origin.

Answers Geometry and Trigonometry: Answers and Explanations

Solutions for every practice problem, numbered to match.

M16 Area and Volume

PRACTICE · AREA AND VOLUME

1. D. The legs 9 and 12 are perpendicular, so $A = \frac{1}{2}(9)(12) = 54$. Choice (C) uses the hypotenuse 15 as a base with a leg as height, which is wrong.
2. B. $A = \frac{1}{2}r^2\theta = \frac{1}{2}(144)\frac{\pi}{3} = 24\pi$. Choice (C) forgets the $\frac{1}{2}$.
3. A. The radius is 5, so $V = \pi(5)^2(7) = 175\pi$. Choice (D) uses 10 as the radius.
4. B. $60 \times 30 \times 40 = 72,000 \text{ cm}^3$, and dividing by 1000 gives 72 liters.
5. D. $SA = 2\pi r^2 + 2\pi rh = 18\pi + 30\pi = 48\pi$. Choice (A) is the volume.
6. 8. $V = \frac{1}{3}Bh$ with $B = 36$: $96 = 12h$, so $h = 8$.
7. 1080. Volume scales by $3^3 = 27$, so $40 \times 27 = 1080$.
8. 48. Cylinder: $\pi(9)(8) = 72\pi$. Cone: $\frac{1}{3}(72\pi) = 24\pi$. Remaining: $72\pi - 24\pi = 48\pi$.

M17 Lines, Angles, and Triangles

PRACTICE · LINES, ANGLES, AND TRIANGLES

1. B. Corresponding angles are equal: $5x + 8 = 3x + 32$, so $x = 12$ and the angle is $5(12) + 8 = 68^\circ$. Choice A is only x , and C is the supplement, which would apply to same-side interior angles.
2. D. The 100° angle must be the vertex angle (two base angles of 100° would exceed 180°). The base angles split the rest: $(180 - 100)/2 = 40^\circ$.
3. A. Solve $180(n - 2) = 1440$: $n - 2 = 8$, so $n = 10$. Choice B forgets to add the 2 back.
4. C. The scale factor from ABC to DEF is $12/8 = 3/2$, and perimeters scale by the same factor: $30 \cdot \frac{3}{2} = 45$. Choice A divides by the factor instead of multiplying.
5. B. A midsegment is half the third side, so $2(3x - 1) = 5x + 3$, which gives $x = 5$. Then $BC = 5(5) + 3 = 28$. Choice A is the midsegment length $3(5) - 1 = 14$.
6. 44. The interior angle at B is $180 - 112 = 68^\circ$. Since $AB = AC$, angle C is also 68° . So $A = 180 - 68 - 68 = 44^\circ$. Check: the exterior angle $112 = A + C = 44 + 68$.
7. 45. Area ratio $\frac{125}{20} = \frac{25}{4}$, so the scale factor is $\frac{5}{2}$. Perimeter $= 18 \cdot \frac{5}{2} = 45$.
8. 17.5 (or $35/2$). Parallel gives $\frac{AD}{AB} = \frac{AE}{AC} = \frac{x}{2x+2} = \frac{6}{15}$, so $15x = 12x + 12$ and $x = 4$. Then $AB = 10$, and $\frac{DE}{BC} = \frac{AD}{AB}$ gives $BC = \frac{7 \cdot 10}{4} = 17.5$.

M18 Right Triangles and Trigonometry

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

PRACTICE · RIGHT TRIANGLES AND TRIGONOMETRY

1. B. $7^2 + 24^2 = 49 + 576 = 625$, and $\sqrt{625} = 25$. Choice A is the difference $24 - 7$ and C is the sum, neither of which the theorem uses.
2. D. The hypotenuse is a leg times $\sqrt{2}$, so the leg is $\frac{6}{\sqrt{2}} = 3\sqrt{2}$. Choice A halves the hypotenuse as if the triangle were 30-60-90.
3. C. Take opposite = 5 and hypotenuse = 13. The adjacent side is $\sqrt{169 - 25} = 12$, so $\tan \theta = \frac{5}{12}$. Choice A is $\cos \theta$ and D is $\frac{1}{\tan \theta}$.
4. C. Multiply by $\frac{\pi}{180}$: $150 \cdot \frac{\pi}{180} = \frac{5\pi}{6}$. Choice A is 120° and B is 135° .
5. D. The side opposite A is $0.6 \cdot 20 = 12$, so the other leg is $\sqrt{400 - 144} = 16$ (a 3-4-5 multiple). Area = $\frac{1}{2} \cdot 12 \cdot 16 = 96$. Choice B forgets the factor $\frac{1}{2}$.
6. 12. The angles are complementary: $5x + 2x + 6 = 90$, so $7x = 84$ and $x = 12$. Check: $\sin 60^\circ = \cos 30^\circ$.
7. 20.4. The height 18 is opposite the 62° angle and the ladder is the hypotenuse, so $\sin 62^\circ = \frac{18}{L}$. Then $L = \frac{18}{\sin 62^\circ} \approx \frac{18}{0.8829} \approx 20.4$.
8. 4.8 (or $24/5$). The hypotenuse is 10. The area is $\frac{1}{2} \cdot 6 \cdot 8 = 24$, and also $\frac{1}{2} \cdot 10 \cdot h$. So $5h = 24$ and $h = 4.8$.

M19 Circles

PRACTICE · CIRCLES

1. B. Write $x + 2 = x - (-2)$ and $y - 7$, so $h = -2$ and $k = 7$. Choice A flips both signs.
2. D. Use $s = r\theta = 12 \cdot \frac{\pi}{3} = 4\pi$. Choice C is the radius times π , which ignores the fraction.
3. A. An inscribed angle is half its intercepted arc: $\frac{140}{2} = 70^\circ$. Choice B treats it as a central angle.
4. C. The center is the midpoint, $(\frac{-2+6}{2}, \frac{3+9}{2}) = (2, 6)$. The diameter is $\sqrt{8^2 + 6^2} = 10$, so $r = 5$ and $r^2 = 25$. Choice D uses the diameter squared, and B uses r instead of r^2 .
5. B. The whole circle has area 36π , so the sector is $\frac{10\pi}{36\pi} = \frac{5}{18}$ of it. Then the angle is $\frac{5}{18} \cdot 360 = 100^\circ$.
6. -15. Complete the square: $(x - 5)^2 + (y + 3)^2 = 25 + 9 - k = 34 - k$. Setting $34 - k = 7^2 = 49$ gives $k = -15$.
7. 15. The tangent is perpendicular to the radius at the point of tangency, so the distance from Q to the center is the hypotenuse of a right triangle with legs 9 and 12: $\sqrt{81 + 144} = 15$.
8. -0.75 (or $-3/4$). The radius OC has slope $\frac{4}{3}$. The tangent at O is perpendicular to it, so its slope is the negative reciprocal, $-\frac{3}{4}$.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



Practice Exam 1

A full-length digital SAT in four modules: Reading and Writing (2 × 27 questions) and Math (2 × 22 questions), followed by complete answers and explanations.

Exam 1 Reading and Writing: Module 1

Reading and Writing · Module 1

27 questions · 32 minutes · Read each passage and question carefully, then choose the best answer.

- 1** Despite the generalized insights about urban mobility they have offered, many traffic engineering models have drawn from unrepresentative data sets: traffic flow patterns recorded solely during peak holiday weekends. To _____ this analytical deficiency, engineers must consistently gather baseline traffic data during standard workweeks.
Which choice completes the text with the most logical and precise word or phrase?
- A. warrant
B. alleviate
C. validate
D. speculate
- 2** The scientist's hypothesis regarding the extinction event of the late Devonian period is _____, as it leaves out significant geochemical data gathered from recent deep-sea drilling projects. To gain widespread acceptance, her model must reconcile these new findings of sudden oceanic anoxia.
Which choice completes the text with the most logical and precise word or phrase?
- A. flawless
B. complex
C. unsubstantiated
D. tedious
- 3** As Ghana's founding president following independence from British rule, Kwame Nkrumah became one of the most _____ politicians in modern African history: among the many lasting legacies of his tenure, Nkrumah championed Pan-African unity and funded massive infrastructure projects like the Akosombo Dam.
Which choice completes the text with the most logical and precise word or phrase?
- A. erratic
B. prominent
C. reserved
D. ordinary
- 4** Epidemiologists and public health officials reiterate that bacterial outbreaks due to water contamination are _____. For example, local communities can take basic steps like boiling water drawn from open wells to eliminate dangerous pathogens before consumption.
Which choice completes the text with the most logical and precise word or phrase?
- A. treatable
B. preventable
C. widespread
D. surprising

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 5 The following text is adapted from Zora Neale Hurston's 1921 short story "John Redding Goes to Sea." John is a imaginative child who lives in a town in the Florida woods. The St. John River flowed a scarce three hundred feet from his back door. On its banks grow numerous palms, luxuriant magnolias and bay trees. On the bosom of the stream float millions of delicately colored hyacinths. John loved to wander down to the water's edge, and, casting in dry twigs, watch them sail away down stream to Jacksonville, the sea, and the wide world.

Which choice best describes the function of the underlined sentence in the text as a whole?

- A. It provides an extended botanical description of a location that John likes to visit.
- B. It reveals that some residents of John's town are confused by his artistic behavior.
- C. It illustrates a daily chore that John must complete near his home.
- D. It suggests that John longs to experience a larger life outside the Florida woods.

- 6 As stage actor Lin-Manuel Miranda acts, it looks like his lines are being delivered on the spot. He is remarkably proficient craft-wise, but owing to how smoothly he speaks and his courage to improvise, he appears calm and unstudied. His visible fluid style, nonetheless, stems from an immense quantity of rehearsing. Regardless of Miranda's background and mastery, he never presumes that he understands a theatrical script. Rather, he treats the script as though discovering it on a brand-new occasion and attempts to analyze it completely fresh.

Which choice best states the main purpose of the text?

- A. To argue that Miranda prefers improvising lines over memorizing traditional theatrical scripts
- B. To explain why Miranda struggles to remember lines during live theatrical shows
- C. To claim that Miranda's acting appears natural owing to his extensive rehearsal process
- D. To compare Miranda's acting techniques with those used by other famous stage performers

7 Text 1

Linguistic anthropologists have long argued that endangered indigenous languages inevitably lose descriptive precision regarding local flora and fauna as younger generations shift toward dominant regional languages. This perspective assumes that generational language shift causes an immediate erasure of specialized ecological vocabulary.

Text 2

In her 2020 linguistic fieldwork, researcher Julia Kensington observes that bilingual youth in indigenous communities often retain complex ecological taxonomies even while predominantly speaking regional trade languages. Kensington notes that traditional plant and animal names are frequently preserved within domestic communication domains and cultural storytelling rituals.

Based on the texts, how would Kensington (Text 2) most likely respond to the conventional linguistic perspective in Text 1?

- A. By disputing the idea that shifting to a dominant language results in the immediate and total loss of specialized ecological knowledge
- B. By acknowledging that regional trade languages possess superior vocabularies for identifying local flora compared to indigenous tongues
- C. By validating the claim that younger generations possess no interest in preserving traditional cultural stories or botanical terms
- D. By asserting that endangered languages naturally evolve to discard ecological terminology in favor of modern technological words

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 8** A student in a political science course is writing a paper on Aristotle's *The Politics*, in which Aristotle offers his opinion on political instability and gives advice on how constitutions can be preserved. Aristotle observes that different forms of government can fall in different ways—for example, oligarchies might grant power to military leaders during wartime who refuse to relinquish that power during peacetime—but some methods of preserving order apply across all forms of government. The student claims that in particular Aristotle asserts that in a healthy state obedience to law must be as close to absolute as possible and that even minor infractions should not be ignored.

Which quotation from a philosopher's analysis of *The Politics* would best support the student's claim?

- A.** When constructing his argument regarding the characteristics of a well-functioning government, Aristotle asserts that 'Transgression creeps in unperceived and at last ruins the state,' illustrating this idea with a comparison to frequent small expenditures slowly and almost imperceptibly chipping away at a fortune until it is ultimately depleted.
- B.** When Aristotle writes on the necessity of avoiding corruption in government, he proposes that 'every state should be so administered and so regulated by law that its magistrates cannot possibly make money.' In particular, he thinks oligarchies are particularly susceptible to corruption through bribery.
- C.** When Aristotle considers the health of constitutions, he states that 'Constitutions are preserved when their destroyers are at a distance, and sometimes also because they are near, for the fear of them makes the government keep in hand the constitution.' He holds that rulers who wish to see constitutions preserved must continually remind the populace of the dangers that would result from a constitutional collapse.
- D.** When contrasting different forms of government, Aristotle holds that 'oligarchies may last, not from any inherent stability in such forms of government, but because the rulers are on good terms both with the unenfranchised and with the governing classes.' That is, oligarchic leaders who wish to hold on to power will introduce members of disenfranchised classes into government in a participatory role.

- 9** As corporate workplaces have adopted remote-first communication tools and asynchronous messaging platforms, traditional office oversight has eroded in favor of an administrative practice known as “continuous digital monitoring” (CDM), in which employers passively track keystrokes, webcam activity, and login durations to gauge worker productivity. Sociologists Naomi Brooks and Thomas Hughes reviewed telemetry data from a large sample of multinational service firms to determine employees' perceived stress levels, organizational trust, and daily output consistency. Although no major labor strike occurred concurrently with the research for Brooks and Hughes to establish absolute causality between CDM and employee turnover, they did postulate that CDM may reduce long-term staff retention through an indirect erosion of psychological safety.

Which finding, if true, would most directly support the premise advanced by Brooks and Hughes?

- A.** CDM implementation tends to increase in strictness during quarterly financial audits, and office workers are significantly more likely to meet project deadlines when working remotely than in traditional offices.
- B.** CDM implementation has a strong negative effect on perceived organizational trust, and there is known to be a strong positive correlation between low organizational trust and voluntary employee resignation rates.
- C.** Daily output consistency is known to have a strong positive effect on promotion likelihood but shows only a weak correlation with worker stress, and CDM implementation shows little association with either output consistency or promotion rates.
- D.** The frequency of software-based tracking increases as corporate management teams expand, and the relationship between digital monitoring and employee anxiety tends to strengthen as team sizes grow larger.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 10** An archaeological team quantified artifact densities across three subterranean layers at two historic settlements, Site Alpha and Site Beta, to deduce shifts in relative population concentrations over time.

Artifact Density (Items per Cubic Meter)		
Excavation Layer	Site Alpha	Site Beta
Stratum I (Top)	14	8
Stratum II	32	19
Stratum III (Base)	45	22

Artifact Density (Items per Cubic Meter)

Based on the table, by approximately how much did the artifact density of Stratum III exceed that of Stratum II at Site Alpha?

- A. By 23 items per cubic meter
- B. By 13 items per cubic meter
- C. By 3 items per cubic meter
- D. By 33 items per cubic meter

- 11** Compiled in the mid-1600s largely through the efforts of Jesuit botanists, the Andean bark compendium is the most important collection of pharmacology in the Spanish Viceroyalty, detailing the primary medicinal plants of South America. The manuscripts portray indigenous healing before the botanical rationalization ordered by the crown, and marginal notes in the compendium indicate that much of the volume's content predates the scientific overhaul. Nonetheless, some of the folios contain inarguable references to chemical extraction methods established years after the decree. Thus, some historians have concluded that _____

Which choice most logically completes the text?

- A. while its content largely predates the rationalization, the Andean bark compendium also contains entries added after the scientific overhaul.
- B. although those who compiled the compendium were fluent in Latin, they had limited knowledge of botanical taxonomy.
- C. before the scientific overhaul by the crown, Andean herbalists borrowed heavily from the medical traditions of foreign empires.
- D. the references to chemical extraction methods in the decree should be attributed to a coincidental resemblance between early and late drug models.

- 12** Sogdian merchants, the mercantile elite from which various Central Asian trading diasporas descended, flourished as early as the 4th century C.E. along the trans-Eurasian trade routes of Transoxiana and established fortified caravansary networks equipped with specialized sericulture infrastructure. Recent biochemical analysis comparing silk moth cocoon residues excavated at Samarkand, a primary Sogdian trade hub in modern Uzbekistan, to samples from early medieval textile workshops in Balkan trading outposts determined that the western fibers descended in part from cocoons processed in Central Asian facilities, with shared protein signatures appearing only after 600 C.E. Thus, historians concluded that _____

Which choice most logically completes the text?

- A. Balkan textile artisans operated independently of any Eurasian trade networks during the early medieval period.
- B. climate conditions across the Balkans and Transoxiana were identical during the sixth century C.E.
- C. some Sogdian merchant families relocated westward to Balkan trading outposts by the seventh century and introduced sericulture methods.
- D. Central Asian trading networks relied exclusively on raw silk imported directly from East Asian imperial dynastic centers.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 13** The following text is adapted from: Botanical Society of America, Carnivorous Plants

Photosynthesis is not the only way plants gain energy; some plants have evolved to capture nutrients in other ways. Carnivorous plants, for example, grow in soil that is thin or lacks nutrients, particularly nitrogen. To compensate, they have evolved to trap and digest insects or other small animals. These plants use a variety of clever traps, such as the snap-trap of the Venus flytrap or the sticky surfaces of the sundew. While they still perform photosynthesis to get carbon, the prey provides the essential nitrogen and phosphorus needed for protein synthesis. This adaptation allows them to thrive in environments where other plants might perish due to nutrient deficiency.

Why do carnivorous plants trap insects?

- A. To get carbon for photosynthesis.
- B. To protect themselves from predators.
- C. To obtain nitrogen and phosphorus.
- D. Because they do not have leaves.

- 14** Paleontologist Liam Thorne investigated the diet of prehistoric mammals using tooth fossil analysis. He determined dietary composition by measuring microscopic wear patterns on fossilized molars from forest-dwelling and open-plains herbivores. Forest-dwelling teeth displayed smooth surface polishes typical of soft foliage, whereas open-plains teeth exhibited deep pits and scratches caused by abrasive grasses.

According to the text, how did Thorne determine what the prehistoric mammals ate?

- A. He analyzed chemical residues left inside fossilized stomach cavities.
- B. He measured microscopic wear patterns on fossilized molars.
- C. He compared jawbone size between forest and plains mammals.
- D. He calculated the age of the sediment layers where fossils were found.

- 15** Paleontologists examining fossilized amber inclusions have discovered why certain Cretaceous insect specimens in Baltic deposits display specialized respiratory spiracles when processing _____ in Dominican deposits, the resin viscosity preserved delicate internal tracheal tubes, whereas in Baltic amber, rapid polymerization preserved only outer cuticular structures.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. oxygen:
- B. oxygen
- C. oxygen while
- D. oxygen,

- 16** To geomorphologists, it does not appear that meandering rivers carve their sweeping curves through uniform soil _____ the flowing water seems to erode outer banks aggressively while depositing sediment on inner bends. Researcher Oliver Vance of the Geological Survey has explained that this asymmetric erosion may in fact be driven by secondary helical flow patterns.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. composition, rather,
- B. composition rather,
- C. composition, rather;
- D. composition; rather,

- 17** Italian novelist Italo Calvino's celebrated literary oeuvre includes *Invisible Cities*, a poetic meditation on urban imagination and memory written in 1970s _____ a collection of critical essays on semiotics and literary theory; and *If on a Winter's Night a Traveler*, his innovative metafictional masterpiece.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. Italy, *The Castle of Crossed Destinies*,
- B. Italy; *The Castle of Crossed Destinies*,
- C. Italy, *The Castle of Crossed Destinies*:
- D. Italy; *The Castle of Crossed Destinies*

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 18** The first digital seismic monitoring network, Charles Richter's automated seismograph array, revolutionized geophysics not only by transmitting ground motion data instantly but also by automatically computing earthquake magnitudes that were dependent on these _____ to the array's global expansion, tracking seismic waves required seismologists to manually measure ink squiggles on rolling paper drums in dark basement vaults, a time-consuming manual calculation.
Which choice completes the text so that it conforms to the conventions of Standard English?
- A. waveforms, prior
B. waveforms and prior
C. waveforms prior
D. waveforms. Prior
-
- 19** In the historical biography detailing nineteenth-century industrial development, the writer chronicles the rise of textile manufacturing. Such intricate machinery, operated at breakneck speeds by young workers who toiled long hours in soot-filled factories, _____ the grueling conditions of early factory labor.
Which choice completes the text so that it conforms to the conventions of Standard English?
- A. typify
B. typifies
C. has typified
D. was typifying
-
- 20** Naomi Henderson published the Upper Paleolithic rock shelter pigment analysis atlas in September 2019, establishing _____ the first time archaeologists would utilize portable X-ray fluorescence spectrometry to date prehistoric cave paintings.
Which choice completes the text so that it conforms to the conventions of Standard English?
- A. those
B. some
C. this
D. these
-
- 21** Although the botanical garden prefers cultivating tropical orchids, dropping autumn temperatures drastically reduce greenhouse humidity levels, _____ the horticulture staff to relocate delicate specimens indoors.
Which choice completes the text so that it conforms to the conventions of Standard English?
- A. pressures
B. pressured
C. to pressure
D. pressuring
-
- 22** When Pakistani education activist Malala Yousafzai won the Nobel Peace Prize in 2014, she attained global recognition at a remarkably young age. _____ only adult statesmen and veteran diplomats had traditionally been selected for the honor, making Yousafzai the youngest laureate in history.
Which choice completes the text with the most logical transition?
- A. Furthermore,
B. Previously,
C. Instead,
D. Similarly,
-
- 23** When Brazilian architect Oscar Niemeyer was awarded the Pritzker Architecture Prize in 1988 for his visionary modernist designs, he achieved international acclaim. _____ no other Latin American designer had received the prestigious prize, making Niemeyer a pioneer of modern architecture in South America.
Which choice completes the text with the most logical transition?
- A. Similarly,
B. Previously,
C. Therefore,
D. Alternatively,
-
- 24** Hernán Cortés's 1519 colonial expedition chronicles detail the Spanish military campaign entering the Aztec capital of Tenochtitlan. Conquistador troops utilize steel weaponry and indigenous alliances to overrun regional fortifications. _____ Aztec emperor Montezuma II attempts diplomatic negotiations and ritual gift-giving, failing to anticipate total military conquest. Their disastrous clash destroys the empire.
Which choice completes the text with the most logical transition?
- A. Conversely,
B. Meanwhile,
C. Nevertheless,
D. Furthermore,

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 25**
- Early atomic models attempted to describe the distribution of subatomic particles.
 - 1904: J.J. Thomson proposed the "plum pudding" model, where electrons were embedded in a diffuse sphere of positive charge.
 - Thomson's model assumed the positive charge was spread uniformly throughout the atom.
 - 1911: Ernest Rutherford conducted the gold foil experiment and proposed the nuclear model.
 - The nuclear model posited that the atom's positive charge and most of its mass are concentrated in a tiny, dense central nucleus.

The student wants to emphasize a difference in the distribution of positive charge between Thomson's and Rutherford's models. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A.** J.J. Thomson's 1904 plum pudding model described an atom with electrons embedded in a sphere of positive charge, whereas Rutherford's 1911 model was based on the gold foil experiment.
- B.** In 1911, Ernest Rutherford discovered that the positive charge of an atom is concentrated in a tiny nucleus, challenging Thomson's 1904 model.
- C.** Unlike Thomson's plum pudding model, which assumed positive charge was spread uniformly throughout the atom, Rutherford's nuclear model posited that the positive charge was concentrated in a dense central nucleus.
- D.** While Thomson and Rutherford both agreed that atoms contain positive charge and electrons, Rutherford was the first to propose that the positive charge and mass are concentrated in a nucleus.

- 26**
- Symphonic metal is a subgenre of heavy metal music.
 - It combines heavy metal's aggressive drums and distorted electric guitars with classical orchestral arrangements and operatic vocals.
 - The genre emerged in Northern Europe during the mid-1990s.
 - The band Nightwish used a full 52-piece orchestra and choir alongside heavy guitar riffs on their album *Once*.
 - The band Therion incorporated operatic choir vocals and classical woodwinds on their album *Theli*.

The student wants to provide an explanation and example of "symphonic metal." Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A.** Originating in Northern Europe during the mid-1990s, symphonic metal has grown into a globally recognized music subgenre featuring operatic vocals and expansive classical instrumentation.
- B.** Symphonic metal is a music subgenre that blends heavy metal guitars and drums with classical orchestral arrangements, such as Nightwish incorporating a 52-piece orchestra alongside heavy guitar riffs.
- C.** The band Nightwish featured a 52-piece orchestra on their album *Once*, while the band Therion integrated operatic choir vocals and woodwinds into heavy metal on *Theli*.
- D.** Nightwish's album *Once* exemplifies symphonic metal, a musical style created in the 1990s when Northern European metal musicians began collaborating with classical operatic vocalists.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 27** • Medieval monastic scriptoria served as vital intellectual hubs for preserving sacred texts through meticulous hand-copied manuscript production.
- Monks labored in specialized writing rooms utilizing imported vellum and crushed mineral inks to craft illuminated codices.
 - The scriptorium at Kells Abbey in Ireland, active around 800 CE, housed dedicated teams of scribes who produced intricately illustrated gospel manuscripts.
 - Kells Abbey's master scribes utilized up to 150 calfskins to manufacture the vellum required for a single comprehensive illuminated volume.
 - The scriptorium at Lindisfarne Priory in Northumbria, England, active during the early 8th century, supported specialized monks dedicated to manuscript illumination.
 - Lindisfarne's monastic artisans utilized similar quantities of animal skin vellum to craft richly colored liturgical texts for regional evangelization.

The student wants to emphasize a similarity between the intensive vellum material requirements utilized by both the Kells and Lindisfarne monastic scriptoria. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A.** While Kells Abbey flourished in Ireland around 800 CE, the Lindisfarne Priory scriptorium operated in Northumbria, England, during the early portion of the 8th century.
- B.** Kells Abbey utilized specialized scribes to produce intricately illustrated gospel manuscripts, whereas Lindisfarne Priory focused on crafting richly colored liturgical texts for evangelization.
- C.** Medieval scriptoria functioned as vital intellectual hubs where monks labored in specialized writing rooms to preserve sacred texts through meticulous hand-copying.
- D.** Both medieval scriptoria required immense animal resource commitments for their literary output, with Kells Abbey utilizing up to 150 calfskins and Lindisfarne employing comparable vellum quantities per volume.

Exam 1 Reading and Writing: Module 2

Reading and Writing · Module 2

27 questions · 32 minutes · Read each passage and question carefully, then choose the best answer.

- 1** While recent evaluations have weakened the claim that the architectural blueprints of Renaissance master Andrea Palladio were _____ other northern Italian designers of his generation, it is undisputed that his prolonged focus on rural villas limited his immediate engagement with urban palace design.
- Which choice completes the text with the most logical and precise word or phrase?
- A.** antagonistic toward
 - B.** peripheral to
 - C.** simulated by
 - D.** polarizing among
- 2** Whether abstract expressionist Joan Mitchell was painting small canvas panels or executing sweeping multi-panel triptychs, she was profoundly _____ visual recollections, translating impressions of French fields or the shifting light over the Seine river into raw color fields.
- Which choice completes the text with the most logical and precise word or phrase?
- A.** checked by
 - B.** detached from
 - C.** unconcerned with
 - D.** stirred by
- 3** Marine biologist Dr. Kenji Tanaka published a 2022 paper documenting an octopus utilizing discarded coconut shells as portable shelters. Critics initially posited that the octopus's proximity to the shells was entirely _____, but Tanaka's video evidence confirmed that the cephalopod actively gathered and transported the items for defense.
- Which choice completes the text with the most logical and precise word or phrase?
- A.** inadvertent
 - B.** conspicuous
 - C.** profound
 - D.** deliberate

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 4 Many historical illuminated manuscripts are preserved behind specialized UV-blocking glass. This is because the organic pigments used in medieval inks are the most _____ component of these texts. They are delicate and chemically unstable when exposed to ambient light, making them exceptionally prone to fading.

Which choice completes the text with the most logical and precise word or phrase?

- A. identifiable
B. friable
C. ubiquitous
D. intricate

- 5 The following text is from Charlotte Brontë's 1847 novel *Jane Eyre*. Jane is sitting on a stone wall overlooking the fields surrounding Thornfield Hall. Jane had no formal training in landscape painting, but she possessed an intense passion for the dramatic and could be keenly sensitive to a vista which served as the vivid background of her own internal thoughts. The rugged moorland stretching below her seemed a grand reflection of her current mood, and she found an echo of herself in its wildness, its breadth, its long windswept paths. On the nearer ridges, the ancient oaks twisted like petrified lightning; lower down was a massing of dark woods, and here and there the silver flash of a small stream.

Which choice best describes the function of the underlined sentence in the text as a whole?

- A. It establishes that a character is experiencing a bitter internal conflict with her family.
B. It creates a purely physical description of the setting without any psychological context.
C. It illustrates a concept that was introduced in the immediately preceding sentence.
D. It makes a theoretical assertion that the subsequent sentence goes on to disprove.

- 6 The following text is adapted from a 2022 journal article on deep-sea organismal physiology.

For decades, marine biologists operating under the thermal adaptation framework posited that bathynomid gigantism—the tendency for deep-sea crustaceans to reach immense physical proportions—was driven primarily by cold-induced metabolic deceleration, analogous to polar gigantism. Seeking to empirically validate this model, a team of physiologists subjected specimens of *Bathynomus giganteus* to hyperbaric chamber trials, measuring oxygen consumption rates across a controlled thermal gradient replicating abyssal conditions. The trials revealed that metabolic downregulation in *B. giganteus* remained negligible across temperature shifts, and that cellular density showed no thermal optimization compared to shallow-water relatives. Instead, the organism's hyper-extended lifespan and massive lipid storage were governed by starvation-endurance pathways rather than low-temperature metabolic efficiency. This empirical deficit led the lead researchers to assert that thermal adaptation is unlikely to be the primary driver of deep-sea gigantism in these abyssal scavengers.

Which choice best states the main purpose of the text?

- A. To present an empirical investigation whose outcome challenges a prevalent biological model regarding deep-sea gigantism.
B. To analyze the technical limitations of using hyperbaric chambers to measure metabolic rates in abyssal marine life.
C. To compare two competing physiological models that explain how deep-sea crustaceans store lipid reserves during periods of starvation.
D. To argue that polar gigantism and bathynomid gigantism are governed by identical evolutionary mechanisms.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

7 Text 1

Early astronomers claimed that Martian valley networks were carved exclusively by catastrophic floods. Sharon Blackwell argued that massive subterranean aquifer breaches unleashed sudden torrents. Furthermore, these colossal deluges supposedly stripped away surface minerals in a matter of days.

Text 2

Spectroscopic tracking conducted by Cynthia Weatherford indicates that slow glacial sublimation and creeping ice sheets shaped these valleys over long epochs. Mineralogical maps reveal consistent weathering signatures that align with continuous, low-energy thermal erosion rather than brief floods. Moreover, cryogenic models confirm that water remained frozen beneath surface dust for millions of years.

How would the author of Text 2 respond to “catastrophic floods” in Text 1?

- A. By contending that slow glacial processes rather than sudden floods carved the valleys
- B. By proving that Mars never possessed liquid water in any form whatsoever
- C. By claiming that subterranean aquifers are completely non-existent on Mars
- D. By showing that volcanic eruptions formed all the surface features on the planet

- 8** Early Earth surface geology is widely theorized to have operated under an immobile-lid tectonic mode, inside which the top crustal stratum remained strictly stationary with zero interaction occurring between the stratum and the lower mantle. Investigators explored the exact timing of the structural pivot from an immobile-lid mode to a shifting-plate mode, inside which the top crustal stratum breaks into mobile segments that ultimately enable stratum and mantle materials to intermingle. Parsing spectroscopic data from stratum-associated and mantle-drawn formations ranging from 250 million to 3.9 billion years old, the investigators placed the structural pivot at 3.38 billion years ago.

Which finding, if true, would most directly support the investigators’ placed pivot?

- A. Mantle-drawn formations older than 3.38 billion years demonstrate significantly greater structural complexity than stratum-associated formations older than 3.38 billion years do.
- B. There is a positive mathematical correlation between the age of stratum-associated formations and their chemical equivalence to mantle-drawn formations, which correlation surges significantly in strength at approximately 3.38 billion years old.
- C. Mantle-drawn formations younger than 3.38 billion years contain specific elemental constituents absent from older mantle-drawn formations but present in older and contemporaneous stratum-associated formations.
- D. Among formations verified as predating 3.38 billion years, significantly more are mantle-drawn than stratum-associated, but the direct opposite pattern governs formations postdating 3.38 billion years.

- 9** Almost all classical theatrical promptbooks from the Elizabethan era contain stage directions regarding scene transitions, incorporating specific time-of-day indicators. In a 2022 dramatic literature study, Christopher Marlowe, Eleanor Vance, and Thomas Croft argue that an observable frequency trend in these theatrical time cues reveals a shift in audience performance conventions prompted by the construction of roofed indoor playhouses in the early seventeenth century. The researchers arrived at this conclusion after reviewing more than 45,000 promptbooks spanning multiple London acting companies, employing text-analysis software to isolate and tally both explicit chronological markers—that is, exact hour phrases, such as two o’clock or midnight—and implicit atmospheric cues, such as mentions of torchlight or candle-snuffing routines typically associated with specific dramatic hours.

Which finding from the study, if true, would most directly support the researchers’ argument?

- A. Promptbooks written after the year 1600 include the exact hour phrase two o’clock less often than promptbooks written before the year 1600 do.
- B. The exact hour phrases of 12 p.m. and 12 a.m. appear with roughly the same frequency across all analyzed theatrical texts.
- C. Among promptbooks originating in the seventeenth century, implicit atmospheric cues become steadily more common than explicit chronological markers as publication dates approach 1700.
- D. Promptbooks written after 1610 contain significantly more stage directions requiring artificial torch illumination and late-night action than do promptbooks from earlier decades.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 10** Public works scientists in Minneapolis tested four products used to melt ice on roads. Samples of ice were treated with equal amounts of each product, and the mass of ice melted after one hour was recorded at two temperatures. A newer product, a blend containing beet juice, is promoted as an alternative to rock salt. The scientists hypothesized that the beet-juice blend outperforms rock salt at the milder temperature but not at the severe temperature.

Table 5: Ice melted by four de-icing products in one hour at two temperatures

De-icing product	Ice melted at -5°C (g)	Ice melted at -15°C (g)
Rock salt	95	38
Beet-juice blend	118	36
Calcium chloride	130	74
Sand-salt mix	60	20

Each product was applied at the same rate to identical ice samples.

Table 5: Ice melted by four de-icing products in one hour at two temperatures

Which choice best uses data from the table to support the scientists' hypothesis?

- A.** Rock salt melted 95 g of ice at -5°C and 38 g of ice at -15°C, so its performance fell by more than half as the temperature dropped.
- B.** The beet-juice blend melted 118 g of ice at -5°C compared with 95 g for rock salt, but at -15°C it melted 36 g, slightly less than the 38 g melted by rock salt.
- C.** The beet-juice blend melted 36 g of ice at -15°C, which was less than the 118 g it melted at -5°C and more than the sand-salt mix's 20 g.
- D.** Calcium chloride melted more ice than any other product at both temperatures, melting 130 g at -5°C and 74 g at -15°C in the one-hour tests.

- 11** To address the susceptibility of materials used in components of high-performance automotive turbocharger turbine housings, such as exhaust gas inlet flanges, to high-temperature thermal fatigue (cyclic oxidation and microstructural cracking that is induced by rapid engine startup and shutdown cycles), materials researchers have developed silicon-molybdenum ductile cast irons for producing turbocharger casings. Testing the thermal fatigue parameters of several commercially available housing castings, Daniela Montoro et al. found that in comparison with two standard austenitic ductile iron castings, a niobium-alloyed ductile iron casting exhibited a lower minimum crack initiation rate, a measure of the rate at which a cyclically heated metal develops structural flaws at a constant thermal amplitude and mechanical load. The finding suggests that _____

Which choice most logically completes the text?

- A.** housing castings based on the two standard austenitic ductile iron castings have metallurgical properties that may improve the thermal and mechanical stability of engine equipment to a greater extent than do castings based on the niobium-alloyed ductile iron casting.
- B.** the two standard austenitic ductile iron castings likely hold similar potential for reducing the thermal resilience of materials exposed to rapid engine cycling, thus extending the operating life span of turbochargers.
- C.** exhaust gas inlet flanges containing the niobium-alloyed ductile iron casting may have the ability to withstand thermal cycles for a shorter period of time than can flanges containing either of the two standard austenitic ductile iron castings.
- D.** unlike the two standard austenitic ductile iron castings, the niobium-alloyed ductile iron casting can substantially inhibit high-temperature thermal fatigue, provided that thermal amplitudes and mechanical loads are consistent.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 12** River damming (constructing barriers across waterways to produce electricity) is commonly regarded as incompatible with aquatic conservation (the effort to protect freshwater ecosystems). However, a large-scale freshwater hydrology study examining 800,000 hectares of the Danube River basin has revealed evidence that complicates this assumption. The researchers observed that river sections managed through modulated pooling (the regulated storage of specific water volumes) demonstrated greater ecological stability than comparable sections with no artificial water management. The findings suggest that _____

Which choice most logically completes the text?

- A. natural waterways cannot maintain healthy conditions unless humans introduce physical barriers.
- B. electricity production from rivers should always take priority over environmental considerations.
- C. thoughtfully managed adjustments to water flow can sometimes support the stability of river environments.
- D. artificial barriers produce no significant changes in the movement patterns of freshwater species.

- 13** The following text is adapted from George Barr McCutcheon's 1902 novel *Graustark*.

Grenfall Lorry's constant motion was a symptom of a deeper restlessness that no destination could satisfy. He viewed the world as a challenge to be conquered rather than a place to be understood. While others might seek stability, Lorry sought the feeling of being on the brink of something new, always fearing that if he stopped, he would lose his sense of purpose.

What drives Grenfall Lorry's behavior?

- A. A desire to map the world for future travelers.
- B. A fear of stagnation and a need for constant stimulation.
- C. A need to find a place where he truly belongs.
- D. A goal of becoming a famous explorer.

- 14** Environmental scientist Lucas Meyer investigated microplastic pollution in freshwater lakes. He determined pollution concentration by filtering water samples collected at varying depths and counting microplastic particles per liter. Surface water samples showed moderate microplastic counts, but deep benthic sediment-line water samples contained significantly higher particle concentrations.

According to the text, how did Meyer determine the level of microplastic pollution in the lakes?

- A. He measured toxin accumulation levels in local freshwater fish species.
- B. He surveyed nearby industrial factories to estimate waste runoff.
- C. He filtered water samples and counted microplastic particles per liter.
- D. He calculated chemical oxygen demand in water samples from the lake surface.

- 15** Initially attributed by nineteenth-century art dealers to a minor follower of Rembrandt, the 1654 oil painting *Bathsheba at Bathsheba's Bath* underwent intensive pigment analysis when conservationists proved that the work—which shows a biblical figure holding a letter—was executed by the Dutch Golden Age master _____ Rembrandt van Rijn (1606–1669).

Which choice completes the text so that it conforms to the conventions of Standard English?

- | | |
|-------------|-------------|
| A. painter, | B. painter: |
| C. painter | D. painter— |

- 16** After a severe ligament tear during a regional trial, Clara Danes was told she might never stick a landing on the balance beam again. Defying all odds, Danes didn't just leap, she _____ the 2022 National Invitational in Chicago, she executed a flawless series of aerials and won the title, becoming the highest-scoring gymnast of the weekend.

Which choice completes the text so that it conforms to the conventions of Standard English?

- | | |
|---------------------------|---------------------------|
| A. stepped—surely. During | B. stepped—surely—during |
| C. stepped—surely during | D. stepped—surely, during |

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 17** Choreographers have dubbed Nadine Cross the “pioneer of kinetic dance,” a title circulated in performance arts gazettes since the 1980s. Cross herself repudiated the label _____ arguing that it masked her mathematical spatial formulations compared to “movement architect,” the designation she preferred.

Which choice completes the text so that it conforms to the conventions of Standard English?

A. title, however,
C. title; however,

B. title, however;
D. title, however

18 The pioneering automated magnetic resonance scanner, Raymond Damadian’s medical imaging prototype, advanced clinical diagnostics not only by mapping internal soft tissues without ionizing radiation but also by digitally calibrating gradient frequencies that were dependent on these _____ to the prototype’s clinical clearance, identifying internal tumors necessitated exploratory surgery or indirect biochemical assays, an invasive diagnostic pathway fraught with patient risk.

Which choice completes the text so that it conforms to the conventions of Standard English?

A. frequencies prior
C. frequencies and prior

B. frequencies, prior
D. frequencies. Prior

19 Like other predatory birds, the peregrine falcon (*Falco peregrinus*) is unable to capture agile flying prey through horizontal chases alone, so during periods of high-altitude hunting, it _____ by executing a high-speed aerodynamic stoop, a gravitational dive that helps strike targets with immense momentum.

Which choice completes the text so that it conforms to the conventions of Standard English?

A. dives
C. dived

B. had dived
D. would dive

20 Horological restorer Beatrice Vance repaired an 18th-century pendulum clock mechanism by cleaning brass gear trains and, using steel pins, tightening the pallet jewels snugly against the _____. When the escapement was reassembled, the resulting tick-tock cadence maintained accurate timekeeping throughout the week.

Which choice completes the text so that it conforms to the conventions of Standard English?

A. wheel’s teeth.
C. wheel’s tooth’s.

B. wheels’ teeth.
D. wheels teeth’.

21 Chartered in 1492 to regulate nautical cartography within the Iberian maritime empire, the Seville hydrographic office was originally made up of four divisions: coastal mapping, star cataloging, instrument calibration, and pilot examination. By the end of the sixteenth century, the crown _____ its initial administrative purview.

Which choice completes the text so that it conforms to the conventions of Standard English?

A. has expanded
C. expands

B. had expanded
D. will expand

22 Global climate monsoons—large-scale seasonal shifts in regional wind and precipitation patterns—result, broadly speaking, from thermal property differences. _____ vast landmasses heat up much faster during summer months than adjacent oceans, creating a persistent low-pressure thermal draw over the continent. This continental pressure differential then pulls moisture-laden maritime air inland to drench the region.

Which choice completes the text with the most logical transition?

A. Hence,
C. Specifically,

B. Nevertheless,
D. Similarly,

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

23 In nineteenth-century Russian literary history, realist prose narratives composed during the radical populist era are expected to prioritize socio-political critique over stylized aesthetic formalism, whereas decadent symbolist works would embrace pure artifice. Isaiah Berlin and colleagues, _____ analyzed an obscure provincial novel combining stark peasant poverty themes with intricate avant-garde narrative frame structures, that has bridged utilitarian ideology and formalist experimentation.

Which choice completes the text with the most logical transition?

- A. likewise,
B. though,
C. for example,
D. characteristically,

24 Meteorological researcher Dr. Valerie Vance notes that climatologist Marcus Thorne “re-examined historical ice-core samples,” constantly tweaking paleoclimatic temperature curves long after initial journal dissemination. Yet, the variations between the 1995 preliminary isotopic analysis and the 2018 final Greenland ice-core dataset are extreme, even by paleoclimate correction standards; _____ atmospheric scientists view the adjusted proxies as indicative of an entirely revised timeline for prehistoric warming.

Which choice completes the text with the most logical transition?

- A. by contrast,
B. in fact,
C. nonetheless,
D. similarly,

- 25**

 - The first successful vaccine was developed by Edward Jenner in 1796.
 - Jenner created a vaccine to protect against smallpox.
 - Smallpox was once a widespread disease that caused many deaths.
 - Vaccination eventually helped eliminate smallpox worldwide.

The student wants to emphasize the significance of Jenner's vaccine. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A. Jenner developed a vaccine against smallpox in 1796.
- B. Smallpox was once a widespread disease that caused many deaths.
- C. Jenner's vaccine helped establish vaccination as a medical practice and contributed to the eventual worldwide elimination of smallpox.
- D. Vaccines have been used to protect people from diseases.

- 26
- Some museums have developed augmented reality exhibits that allow visitors to interact with digital representations of historical objects.
 - Traditional museum displays usually require visitors to observe physical artifacts behind protective barriers.
 - Augmented reality technology allows visitors to view additional information, animations, and reconstructions through digital devices.
 - Museums use these exhibits to provide visitors with more detailed explanations of historical objects.
 - Researchers have found that interactive exhibits can increase visitor engagement.

The student wants to explain an advantage of augmented reality exhibits. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A.** Augmented reality exhibits provide visitors with additional information and interactive experiences that can increase engagement with historical objects.
- B.** Museums use digital devices to display animations and reconstructions.
- C.** Traditional museum displays often place artifacts behind protective barriers.
- D.** Some museums have developed augmented reality exhibits that allow visitors to interact with digital representations.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 27** • Hygromorphism refers to structural adaptations in plants that enable survival in continuously humid or waterlogged habitats.
- Features include thin cuticles, large thin leaves, permanently open stomata, and prominent hydathodes for releasing excess water.
 - These adaptations facilitate transpiration in environments saturated with atmospheric moisture.
 - The water lettuce (*Pistia stratiotes*) possesses thin, porous leaf structures covered in water-repellent micro-hairs that prevent leaf submergence in stagnant water.
 - Giant water lilies (*Victoria amazonica*) feature broad floating leaves with upturned rims and drainage notches to shed rainwater.

The student wants to provide an explanation and example of "hygromorphism." Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A.** Hygromorphism is a suite of structural plant adaptations for high-humidity environments, such as water lettuce featuring porous leaves with water-repellent micro-hairs to avoid submergence.
- B.** Designed to facilitate transpiration in water-saturated environments, hygromorphic adaptations include thin leaf cuticles, broad surface areas, and specialized water-excreting hydathodes.
- C.** Water lettuce features micro-hair-covered leaves to prevent sinking, while giant water lilies possess broad floating leaves with upturned rims to shed accumulated surface rainwater.
- D.** Water lettuce using water-repellent micro-hairs on its porous leaves illustrates hygromorphism, a structural modification analyzed by botanists investigating freshwater wetland ecology.

Exam 1 Math: Module 1

Math · Module 1

22 questions · 35 minutes · A calculator is allowed on all questions. For student-produced responses, enter your answer as a number or fraction.

1 $(5x - 2)/3 = (x + 4)/2 + 9$

What is the value of x ?

- A.** $25/7$
- B.** $43/7$
- C.** $46/7$
- D.** 10

2 The linear function f satisfies $f(1) = 9$ and $f(4) = 21$. What is the value of $f(8)$?

- A.** 37
- B.** 41
- C.** 45
- D.** 53

3 The function f is defined by $f(x) = 3x + 2$. The graph of the function g is the graph of $y = f(x)$ translated 4 units to the left in the xy -plane. Which equation defines g ?

- A.** $g(x) = 3x - 10$
- B.** $g(x) = 3x + 6$
- C.** $g(x) = 7x + 2$
- D.** $g(x) = 3x + 14$

4 The point $(t, t - 6)$ lies on the graph of $4x - 3y = 31$ in the xy -plane. What is the value of t ?

Student-produced response: enter your answer. _____

5 $4x + 3y = 29$

$3x - 2y = 9$

The solution to the given system of equations is (x, y) . What is the value of xy ?

Student-produced response: enter your answer. _____

6 A dairy shipped 20 crates of cheese. Each small crate holds 10 blocks of cheese and each large crate holds 16 blocks. The 20 crates held 266 blocks in total. How many small crates were shipped?

- A.** 9
- B.** 10
- C.** 11
- D.** 20

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

7 How many integer values of x satisfy the compound inequality $-7 \leq 2x - 3 \leq 15$?

- A. 9
B. 10
C. 11
D. 12

8 What is the value of $2(x - 3)^2 + 5$ when $x = 7$?

Student-produced response: enter your answer. _____

9 The value, in dollars, of a laundromat's industrial dryer is modeled by $V(t) = 1800(0.85)^t$, where t is the number of years since the dryer was purchased. Which of the following is the best interpretation of $1800(0.85)$ in this context?

- A. The value of the dryer, in dollars, 1 year after it was purchased
B. The decrease in the value of the dryer, in dollars, during the first year
C. The purchase price of the dryer, in dollars, after a 15% discount
D. The value of the dryer, in dollars, 0.85 year after it was purchased

10 In the equation $x^2 + (k + 2)x + 9 = 0$, k is a positive constant. If the equation has exactly one real solution, what is the value of k ?

Student-produced response: enter your answer. _____

11 One solution of the equation $x^2 - 9x + k = 0$ is 4. What is the value of k ?

Student-produced response: enter your answer. _____

12 The page numbers on two facing pages of an open library book are consecutive positive integers whose product is 132. What is the smaller of the two page numbers?

- A. 11
B. 12
C. 66
D. 131

13

Values of the quadratic function f

x	$f(x)$
-1	5
0	-1
1	-3
2	-1
3	5

Values of the quadratic function f

The table shows five values of x and their corresponding values of the quadratic function f . Which of the following equations defines f ?

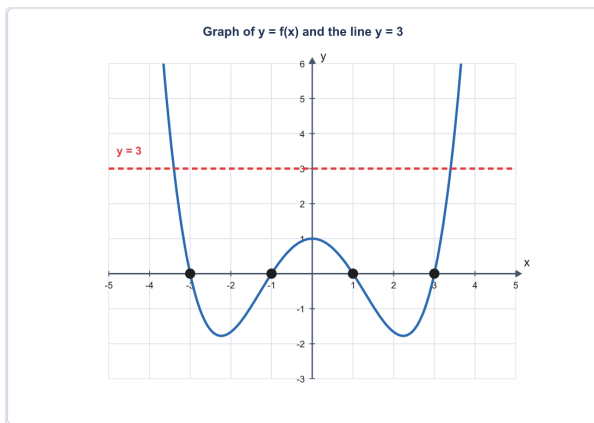
- A. $f(x) = 2(x - 1)^2 - 3$
B. $f(x) = 2(x + 1)^2 - 3$
C. $f(x) = (x - 1)^2 - 3$
D. $f(x) = 2(x - 1)^2 + 3$

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

14

Graph of $y = f(x)$ and the line $y = 3$

The graph of $y = f(x)$ and the line $y = 3$ are shown in the xy -plane. For how many distinct values of x does $f(x) = 3$?

- A. 0
B. 2
C. 3
D. 4

15

A rectangular patio at a community center measures 18 feet by 12 feet. Sealant for the patio costs \$4 per square yard. What is the total cost of the sealant needed to cover the entire patio? (1 yard = 3 feet)

- A. \$24
B. \$96
C. \$288
D. \$864

16

A water park sells a day pass for \$60. This week, the pass is on sale for 10% off. A 5% sales tax is then added to the sale price. What is the total cost of the pass?

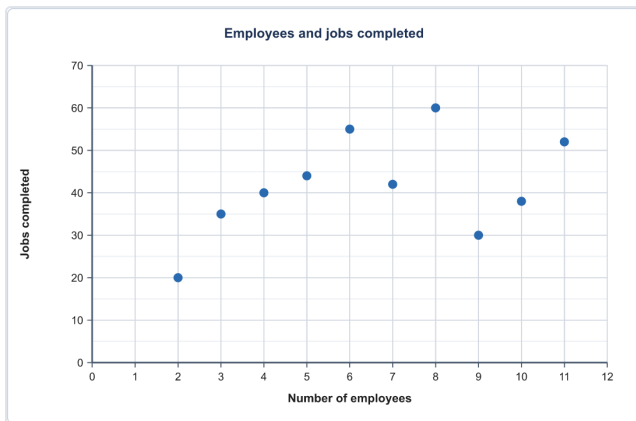
- A. \$51.30
B. \$54.00
C. \$56.70
D. \$57.00

17

At a robotics competition, 30 teams had a mean score of 78 points. The 6 lowest-scoring teams, which had a mean score of 60 points, were eliminated. What was the mean score, in points, of the remaining 24 teams?

- A. 66
B. 69
C. 78
D. 82.5

18



Employees and jobs completed

The scatterplot shows the number of employees x and the number of jobs completed y in one week for each of 10 print shops. How many of the print shops completed more than 40 jobs that week?

- A. 3
B. 4
C. 5
D. 6

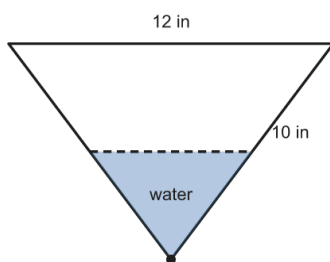
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

19

Note: Figure not drawn to scale.



Note: Figure not drawn to scale.

A grain funnel is shaped like a cone with its point at the bottom, as shown. The top of the funnel has a diameter of 12 inches, and the slant height of the funnel is 10 inches. The funnel is filled with water to exactly half of its height. What is the volume, in cubic inches, of the water in the funnel?

- A. 12π
B. 24π
C. 48π
D. 60π

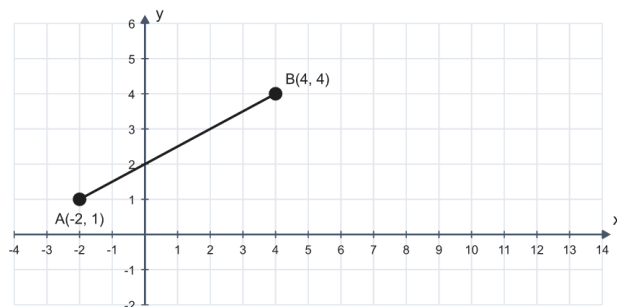
20

A bakery's cylindrical cake pan has a radius of r inches and a height of h inches. A new pan has a radius that is 50% greater and a height that is 20% less than the original pan's. The volume of the new pan is what percent greater than the volume of the original pan?

- A. 80% B. 125%
- C. 180% D. 225%

21

Points A and B in the xy-plane



Points A and B in the xy-plane

Points A(-2, 1) and B(4, 4) are shown in the xy-plane. Point C lies on the x-axis, and triangle ABC has a right angle at B. What are the coordinates of point C?

- A. (2, 0) B. (6, 0)
C. (8, 0) D. (12, 0)

22

What is the value of $4 \cos 30^\circ$?

- A. 2
B. $2\sqrt{2}$
C. 3
D. $2\sqrt{3}$

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Exam 1 Math: Module 2**Math · Module 2**

22 questions · 35 minutes · A calculator is allowed on all questions. For student-produced responses, enter your answer as a number or fraction.

- 1 The equation $w = (3/5)(p + 10) - 4$ relates the variables w and p . Which of the following expresses p in terms of w ?
 A. $p = (5w - 10)/3$ B. $p = (5w + 20)/3$
 C. $p = (5w + 50)/3$ D. $p = (3w - 38)/5$
- 2 A food truck sold 36 burritos in the first 3 hours of Monday and 50 burritos in the first 5 hours of Tuesday. If the truck sold burritos at a constant rate each day, how many more burritos per hour did it sell on Monday than on Tuesday?
 A. 2 B. 10
 C. 12 D. 14
- 3 The functions f and g are defined by $f(x) = 2x + 1$ and $g(x) = 3x + 8$. If $f(g(x)) = 41$, what is the value of x ?
 A. -4 B. 2
 C. 4 D. 5
- 4 In the xy -plane, line ℓ has slope $3/4$ and passes through the point $(4, 6)$. Line ℓ crosses the x -axis at $(a, 0)$ and the y -axis at $(0, b)$. What is the value of ab ?
 Student-produced response: enter your answer. _____
- 5 $y = kx + 8$
 $9x - 6y = -48$
 In the given system of equations, k is a constant. If the system has infinitely many solutions, what is the value of k ?
 Student-produced response: enter your answer. _____
- 6 $4x + 4y = 28$
 $6x - 6y = 18$
 The solution to the given system of equations is (x, y) . What is the value of $x^2 - y^2$?
 A. 10 B. 21
 C. 28 D. 49
- 7 Which of the following is the solution set of $-(2/3)(6x - 9) \geq 5 - (x - 4)/2$?
 A. $x \geq -2/7$ B. $x \leq -2/7$
 C. $x \leq 6/7$ D. $x \leq -26/7$
- 8 For what positive value of k is the expression $4x^2 + (k + 2)x + 9$ a perfect square trinomial?
 Student-produced response: enter your answer. _____
- 9 For positive values of x and y , which of the following is equivalent to $(x^3y^{-2})^2 / (x^{-1}y)^3$?
 A. x^3/y^7 B. x^9/y^7
 C. x^9/y D. $x^{9y}/7$
- 10 In the equation $3x^2 - kx + 24 = 0$, k is a positive constant. One of the solutions to the equation is twice the other solution. What is the value of k ?
 Student-produced response: enter your answer. _____
- 11 What is the product of all real solutions to the equation $(x^2 - 3)^2 - 4(x^2 - 3) - 5 = 0$?
 Student-produced response: enter your answer. _____
- 12 If $3^{2x} = 7$, what is the value of 3^{4x+1} ?
 A. 49 B. 147
 C. 343 D. 441

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 13 The function f is defined by $f(x) = 2x^2 - 8x + 3$. The graph of $y = g(x)$ is obtained by reflecting the graph of $y = f(x)$ over the x -axis and then translating it 4 units to the left. Which of the following defines g ?

A. $g(x) = -2x^2 - 8x + 3$ B. $g(x) = -2x^2 + 24x - 67$
 C. $g(x) = 2x^2 + 8x + 3$ D. $g(x) = -2x^2 - 8x - 3$

14

Values of the function f

x	$f(x)$
0	1
1	0
3	0
5	2

Values of the function f

The table shows four values of the function f . Which of the following equations could define f ?

A. $f(x) = (x - 1)(x - 3)/3$ B. $f(x) = |x - 2| - 1$
 C. $f(x) = |x - 1|$ D. $f(x) = |2x - 3| - 2$

- 15 A printing machine at a print shop produces 5 posters every 3 minutes. At this rate, how many minutes will it take the machine to produce 90 posters?

A. 18 B. 30
 C. 54 D. 150

- 16 An observatory gift shop buys a star projector at a cost and marks up the cost by 40% to set the marked price. During a sale, the marked price is reduced by 25%, and 8% sales tax is then added to the sale price. A customer pays a total of \$226.80. What was the shop's cost for the star projector?

A. \$200.00 B. \$210.00
 C. \$280.00 D. \$302.40

- 17 In a bakery, 40% of the loaves baked each day are sourdough and the rest are other types. The probability that a randomly selected sourdough loaf is unsold at closing is twice the probability that a randomly selected loaf of another type is unsold. Overall, 14% of the loaves baked are unsold at closing. What is the probability that a randomly selected sourdough loaf is unsold?

A. 0.10 B. 0.20
 C. 0.28 D. 0.40

- 18 A library estimated the mean wait time to check out at two branches using independent random samples. The 95% confidence interval for branch 1 was 12.4 minutes to 15.6 minutes. The 95% confidence interval for branch 2 was 9.1 minutes to 11.7 minutes. Which of the following statements is best supported by these results?

A. Every patron at branch 1 waits longer than every patron at branch 2.
 B. The mean wait time for all patrons at branch 1 is likely greater than at branch 2.
 C. The mean wait time is the same at both branches because both are intervals.
 D. The mean wait time for all patrons at branch 2 is exactly 10.4 minutes.

- 19 A carpentry shop is building a rectangular tabletop with a perimeter of 40 feet. The length of the tabletop is 5 feet more than twice its width. What is the area, in square feet, of the tabletop?

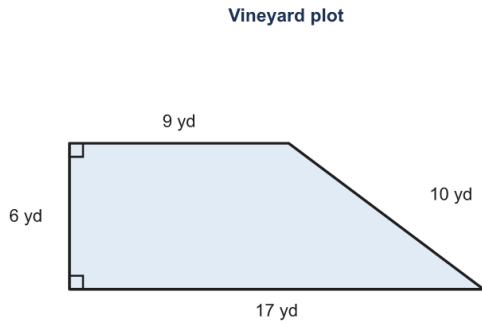
A. 25 B. 75
 C. 100 D. 225

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

20

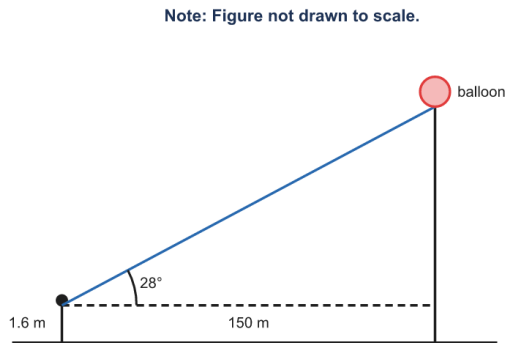


Vineyard plot

The figure shows a vineyard plot in the shape of a trapezoid. The two parallel sides are 9 yards and 17 yards long, and one of the other sides is perpendicular to both. The perpendicular side is 6 yards long and the slanted side is 10 yards long. What is the area, in square yards, of the plot?

- A. 78
B. 102
C. 130
D. 156

21



Note: Figure not drawn to scale.

A technician at a weather station stands 150 meters from the point on the ground directly below a weather balloon. The technician's eyes are 1.6 meters above the ground, and the angle of elevation from the technician's eyes to the balloon is 28° . Which expression gives the height, in meters, of the balloon above the ground?

- A. $1.6 + 150 \tan 28^\circ$
B. $150 \tan 28^\circ - 1.6$
C. $1.6 + 150/\tan 28^\circ$
D. $1.6 + 150 \sin 28^\circ$

22

If $\sin 28^\circ = \cos x^\circ$, where $0 < x < 90$, what is the value of x ?

- A. 28
B. 56
C. 62
D. 152

Key Practice Exam 1: Answers and Explanations

Check each module, then use the skill map to find what to review. Every explanation shows why the right answer works and why the tempting wrong answers fail.

READING AND WRITING · MODULE 1 · ANSWER KEY

1 B 2 C 3 B 4 B 5 D 6 C 7 A 8 A 9 B 10 B 11 A 12 C 13 C 14 B 15 A 16 D 17 B 18 D 19 B 20 C 21 D 22 B 23 B 24 A
25 C 26 B 27 D

READING AND WRITING · MODULE 2 · ANSWER KEY

1 B 2 D 3 A 4 B 5 C 6 A 7 A 8 C 9 D 10 B 11 D 12 C 13 B 14 C 15 C 16 A 17 A 18 D 19 A 20 A 21 B 22 C 23 B 24 B
25 C 26 A 27 A

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

MATH · MODULE 1 · ANSWER KEY

1 D 2 A 3 D 4 13 5 15 6 A 7 D 8 37 9 A 10 4 11 20 12 A 13 A 14 B 15 B 16 C 17 D 18 C 19 A 20 A 21 B 22 D

MATH · MODULE 2 · ANSWER KEY

1 A 2 A 3 C 4 -12 5 3/2 6 B 7 B 8 10 9 B 10 18 11 16 12 B 13 D 14 B 15 C 16 A 17 B 18 B 19 B 20 A 21 A 22 C

SKILL MAP: WHICH QUESTIONS TEST WHICH SKILL

Module	Skill	Questions
R&W M1	Words in Context	1, 2, 3, 4
R&W M1	Text Structure	5
R&W M1	Author's Purpose	6
R&W M1	Cross-Text Connections	7
R&W M1	Textual Evidence	8, 9
R&W M1	Quantitative Evidence	10
R&W M1	Inferences	11, 12
R&W M1	Central Ideas and Details	13, 14
R&W M1	Boundaries	15, 16, 17, 18
R&W M1	Form, Structure, and Sense	19, 20, 21
R&W M1	Transitions	22, 23, 24
R&W M1	Rhetorical Synthesis	25, 26, 27
R&W M2	Words in Context	1, 2, 3, 4
R&W M2	Text Structure	5
R&W M2	Author's Purpose	6
R&W M2	Cross-Text Connections	7
R&W M2	Textual Evidence	8, 9
R&W M2	Quantitative Evidence	10
R&W M2	Inferences	11, 12
R&W M2	Central Ideas and Details	13, 14
R&W M2	Boundaries	15, 16, 17, 18
R&W M2	Form, Structure, and Sense	19, 20, 21
R&W M2	Transitions	22, 23, 24
R&W M2	Rhetorical Synthesis	25, 26, 27
Math M1	Linear Equations (1 Var)	1
Math M1	Linear Functions	2, 3
Math M1	Linear Equations (2 Var)	4
Math M1	Systems of Equations	5, 6
Math M1	Linear Inequalities	7
Math M1	Equivalent Expressions	8, 9
Math M1	Nonlinear Equations	10, 11, 12
Math M1	Nonlinear Functions	13, 14
Math M1	Ratios and Rates	15
Math M1	Percentages	16
Math M1	One-Variable Data	17

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Module	Skill	Questions
Math M1	Two-Variable Data	18
Math M1	Area and Volume	19, 20
Math M1	Lines, Angles, Triangles	21
Math M1	Right Triangles and Trig	22
Math M2	Linear Equations (1 Var)	1
Math M2	Linear Functions	2, 3
Math M2	Linear Equations (2 Var)	4
Math M2	Systems of Equations	5, 6
Math M2	Linear Inequalities	7
Math M2	Equivalent Expressions	8, 9
Math M2	Nonlinear Equations	10, 11, 12
Math M2	Nonlinear Functions	13, 14
Math M2	Ratios and Rates	15
Math M2	Percentages	16
Math M2	Probability	17
Math M2	Sample Statistics	18
Math M2	Area and Volume	19, 20
Math M2	Right Triangles and Trig	21, 22

R&W 1 Reading and Writing, Module 1: Explanations

1. Answer: B · WORDS IN CONTEXT · MEDIUM

Correct answer: B. 'Alleviate' means to make less severe. Gathering baseline workweek data eases the deficiency of using holiday-only traffic data.

Why the other choices are wrong:

- A) 'Warrant' means to justify, which contradicts the aim.
- C) 'Validate' means to confirm, which contradicts the aim, so it lacks the sense of 'alleviate' that the blank needs.
- D) 'Speculate' means to guess without firm evidence, which does not fix a deficiency.

2. Answer: C · WORDS IN CONTEXT · MEDIUM

Correct answer: C. 'Unsubstantiated' means unsupported by evidence. The hypothesis leaves out significant data, so it is weak.

Why the other choices are wrong:

- A) 'Flawless' means perfect, which contradicts leaving out data.
- B) 'Complex' means complicated, which does not explain why the hypothesis needs to reconcile new data.
- D) 'Tedious' means boring, which does not describe a flaw in argument.

3. Answer: B · WORDS IN CONTEXT · MEDIUM

Correct answer: B. 'Prominent' means important and widely known. Nkrumah was Ghana's founding president who championed Pan-African unity and major projects, so he was a prominent politician.

Why the other choices are wrong:

- A) 'Erratic' means unpredictable, which does not follow.
- C) 'Reserved' means quiet, contradicting a public leader with lasting legacies.
- D) 'Ordinary' means usual and not special, which contradicts 'one of the most'.

4. Answer: B · WORDS IN CONTEXT · EASY

Correct answer: B. 'Preventable' means able to be stopped. Communities can stop bacterial outbreaks by boiling well water, so such outbreaks are preventable.

Why the other choices are wrong:

- A) 'Treatable' refers to treating illness after it occurs, but the steps described prevent outbreaks.
- C) 'Widespread' means found over a large area, which does not contrast with the simple steps.
- D) 'Surprising' means unexpected, which does not fit the sentence, where the blank needs the sense of 'preventable'.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

5. Answer: D · TEXT STRUCTURE · EASY

Correct answer: D. The underlined sentence describes John casting twigs into the water and watching them float toward "the sea" and "the wide world," explicitly stating that he "wanted to follow them." This acts as a literary metaphor indicating his desire to explore beyond his current confines.

Why the other choices are wrong:

- A) While the paragraph contains botanical descriptions (palms, magnolias), this underlined sentence specifically focuses on John's actions and desires, not flora.
- B) While other parts of the original story mention the townsfolk being puzzled by John, nothing in this specific text block or sentence states that residents are confused.
- C) The text mentions "dry twigs" and "water's edge," but frames this as a playful, imaginative pastime rather than a structured chore or obligation.

6. Answer: C · AUTHOR'S PURPOSE · EASY

Correct answer: C. This choice explains Miranda's natural acting stems from extensive rehearsal.

Why the other choices are wrong:

- A) The passage says Miranda appears to improvise but his ease stems from rehearsal; it never says he prefers improvising over memorizing, so this reverses the point.
- B) The text does not suggest he struggles with lines; it says he looks unstudied while being thoroughly rehearsed, so this claims a difficulty the passage never mentions.
- D) No other performers appear in the passage; it focuses solely on Miranda's own approach, so a comparison with other stage performers has no support.

7. Answer: A · CROSS-TEXT CONNECTIONS · EASY

Correct answer: A. Kensington observes that youth often retain complex ecological taxonomies, directly disputing the idea of immediate and total vocabulary loss.

Why the other choices are wrong:

- B) Kensington does not claim regional trade languages possess superior vocabularies for identifying flora.
- C) Kensington demonstrates youth retain traditional names, contradicting the claim that they lack interest.
- D) Kensington shows endangered languages preserve terminology in domestic domains rather than discarding it.

8. Answer: A · TEXTUAL EVIDENCE · HARD

Correct answer: A. It accurately matches Aristotle's argument regarding how small, unperceived transgressions ruin the state, supporting the student's claim about minor infractions.

Why the other choices are wrong:

- B) It focuses on magistrate corruption and bribery rather than the strict obedience to law regarding minor infractions.
- C) It discusses the fear of distant or near destroyers preserving constitutions rather than the danger of minor infractions.
- D) It focuses on oligarchies maintaining power through good terms with disenfranchised classes rather than strict law obedience.

9. Answer: B · TEXTUAL EVIDENCE · HARD

Correct answer: B. It demonstrates the hypothesized indirect path: CDM harms organizational trust, which strongly drives voluntary employee resignations.

Why the other choices are wrong:

- A) It ties CDM increases to financial audits and remote deadlines rather than staff retention.
- C) It claims CDM shows little association with output consistency or promotion rates, undermining the causal claim.
- D) It links tracking frequency to management size and anxiety rather than staff retention mechanisms.

10. Answer: B · QUANTITATIVE EVIDENCE · MEDIUM

Correct answer: B. At Site Alpha, Stratum III has 45 items per cubic meter and Stratum II has 32, so Stratum III exceeds Stratum II by $45 - 32 = 13$ items per cubic meter.

Why the other choices are wrong:

- A) A difference of 23 does not match the Site Alpha values; subtracting 32 from 45 gives 13. It appears to come from an arithmetic slip rather than from reading the table.
- C) A gap of 3 is the Site Beta difference ($22 - 19$), not Site Alpha's. The question asks about Alpha, where the values are 45 and 32.
- D) A difference of 33 does not result from $45 - 32 = 13$ and matches no pair of Alpha values. It is an arithmetic error unrelated to the table.

11. Answer: A · INFERENCES · MEDIUM

Correct answer: A. Chemical extraction references post-dating the decree indicate the compendium contains entries added after the scientific overhaul.

Why the other choices are wrong:

- B) Latin fluency does not account for the chronological addition of post-decree extraction methods.
- C) Foreign medical borrowing is unsupported by evidence of post-overhaul extraction notes.
- D) Extraction method references reflect actual historical changes rather than coincidental resemblances.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

12. Answer: C • INFERENCES • MEDIUM

Correct answer: C. Protein signatures linking Balkan medieval textiles to Sogdian trade hubs logically lead to the conclusion that Sogdian merchant families relocated westward and introduced sericulture methods.

Why the other choices are wrong:

- A) Shared protein signatures linking Balkan textiles to Samarkand workshops refute the assumption that Balkan artisans operated entirely independently of Eurasian trade networks.
- B) Protein residue continuity reflects human trade and sericulture movement rather than identical regional climate conditions.
- D) Tracing western fibers directly to Central Asian processing facilities indicates regional Sogdian production rather than reliance solely on raw East Asian imports.

13. Answer: C • CENTRAL IDEAS AND DETAILS • MEDIUM

Correct answer: C. The text says carnivorous plants trap insects because the prey provides the nitrogen and phosphorus they lack in thin soil.

Why the other choices are wrong:

- A) The text says they still get carbon through photosynthesis, so they do not trap insects for carbon.
- B) The text does not mention protection from predators as a reason.
- D) The text says nothing about the plants lacking leaves.

14. Answer: B • CENTRAL IDEAS AND DETAILS • MEDIUM

Correct answer: B. The text explicitly notes Thorne measured "microscopic wear patterns on fossilized molars."

Why the other choices are wrong:

- A) The passage never mentions this method (stomach residue); it names a different one.
- C) The passage never mentions this method (jaw size); it names a different one.
- D) The passage never mentions this method (sediment age); it names a different one.

15. Answer: A • BOUNDARIES • HARD

Correct answer: A. A colon properly links two independent clauses where the second explains the first.

Why the other choices are wrong:

- B) A run-on sentence is created by omitting punctuation between clauses.
- C) While incorrectly joins two independent clauses.
- D) A comma results in a comma splice.

16. Answer: D • BOUNDARIES • EASY

Correct answer: D. A semicolon before the transitional adverb "rather" and a comma after it correctly join the two independent clauses.

Why the other choices are wrong:

- A) Surrounding "rather" with commas joins two independent clauses with only a comma, which is a comma splice.
- B) With no punctuation before "rather" the two independent clauses run together as a run-on sentence.
- C) The semicolon is placed after the transitional adverb instead of before it, and a comma before "rather" leaves the clauses incorrectly joined.

17. Answer: B • BOUNDARIES • HARD

Correct answer: B. The series items already contain commas, so a semicolon is needed to end the first item after "Italy", and a comma after "The Castle of Crossed Destinies" then sets off its description, matching the structure of the other items.

Why the other choices are wrong:

- A) A comma after "Italy" does not separate the items, so the first and second items run together as a comma splice in a series whose items contain commas.
- C) The comma after "Italy" fails to separate the items, and the colon after "The Castle of Crossed Destinies" wrongly interrupts the sentence before its description.
- D) The semicolon is right, but omitting the comma after "The Castle of Crossed Destinies" leaves its description attached without the punctuation that sets it off.

18. Answer: D • BOUNDARIES • MEDIUM

Correct answer: D. A period correctly separates the two independent clauses, and "Prior to ..." begins a new complete sentence.

Why the other choices are wrong:

- A) A comma alone cannot join two independent clauses; this creates a comma splice.
- B) Joining the clauses with "and" and no comma-plus-conjunction structure leaves the second independent clause unseparated and the sentence garbled.
- C) With no punctuation at all, the two independent clauses are fused into a run-on sentence.

19. Answer: B • FORM, STRUCTURE, AND SENSE • EASY

Correct answer: B. The subject "machinery" is a singular mass noun, so it takes the singular simple present verb "typifies", which fits the present-tense context.

Why the other choices are wrong:

- A) "typify" is a plural verb form, but the subject "machinery" is a singular mass noun and needs a singular verb.
- C) "has typified" agrees in number with "machinery", but the present perfect is less direct than the simple present that the descriptive context calls for.
- D) "was typifying" agrees in number but is past progressive, which clashes with the present-tense context of the passage.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

20. Answer: C · FORM, STRUCTURE, AND SENSE · MEDIUM

Correct answer: C. The blank is the object of "making/representing/establishing" and points back to the entire preceding clause as one event, so the singular demonstrative "this" is needed.

Why the other choices are wrong:

- A) The plural demonstrative "those" does not agree with the single event described in the preceding clause, which requires the singular "this".
- B) The indefinite quantifier "some" cannot work as a demonstrative that points back to the whole preceding clause as a single event, so it does not fit here.
- D) The plural demonstrative "these" does not agree with the single event described in the preceding clause, which requires the singular "this".

21. Answer: D · FORM, STRUCTURE, AND SENSE · EASY

Correct answer: D. "pressuring" is a present participle that begins a modifying phrase showing the result of the main clause, so the sentence is grammatical after the comma.

Why the other choices are wrong:

- A) "pressures" is a finite verb; placed after a comma with no conjunction it joins two independent clauses and creates a comma splice.
- B) "pressured" is a past participle with a passive sense, which does not fit an active modifier describing the result of the main clause.
- C) The infinitive "to pressure" cannot introduce this modifier and leaves the final phrase grammatically unconnected to the main clause.

22. Answer: B · TRANSITIONS · MEDIUM

Correct answer: B. "Previously" is correct because it properly highlights the prior historical norm that only adult statesmen had received the prize before.

Why the other choices are wrong:

- A) "Furthermore" is incorrect because it adds supporting information rather than establishing chronological background.
- C) "Instead" is incorrect because it suggests an alternative rather than a temporal precedent.
- D) "Similarly" is incorrect because it signals a parallel comparison rather than a historical timeline.

23. Answer: B · TRANSITIONS · MEDIUM

Correct answer: B. "Previously" is correct because it properly introduces the historical timeline indicating that no Latin American designer had won prior to this event.

Why the other choices are wrong:

- A) "Similarly" is incorrect because it suggests a parallel comparison rather than chronological precedent.
- C) "Therefore" is incorrect because it indicates a logical deduction rather than a historical timeline marker.
- D) "Alternatively" is incorrect because it offers a choice or alternative rather than background historical context.

24. Answer: A · TRANSITIONS · EASY

Correct answer: A. "Conversely" is correct because it establishes an opposing contrast between Cortés's military conquest tactics and Montezuma II's diplomatic ritual gift-giving.

Why the other choices are wrong:

- B) "Meanwhile" is incorrect because it denotes concurrent timing rather than a tactical opposition.
- C) "Nevertheless" is incorrect because of clausal preamble constraints.
- D) "Furthermore" is incorrect because it adds additive details rather than an opposition.

25. Answer: C · RHETORICAL SYNTHESIS · HARD

Correct answer: C. It contrasts the distribution of positive charge: spread uniformly in Thomson's model, concentrated in a dense central nucleus in Rutherford's.

Why the other choices are wrong:

- A) It mentions the dates and Rutherford's experiment, not how positive charge is distributed.
- B) It describes Rutherford's act of discovery and doesn't state what Thomson's model said about charge.
- D) It says both models include positive charge and electrons and mentions the nucleus only for Rutherford, so the contrast in distribution is not stated.

26. Answer: B · RHETORICAL SYNTHESIS · MEDIUM

Correct answer: B. Provide explanation ("music subgenre that blends heavy metal guitars and drums with classical orchestral arrangements") AND example ("Nightwish incorporating a 52-piece orchestra alongside heavy guitar riffs").

Why the other choices are wrong:

- A) Explains the genre's characteristics generally, but omits a specific band or album example.
- C) Lists two band/album examples (Nightwish, Therion) without defining the genre itself.
- D) Mentions Nightwish's album Once as an example, but provides historical context rather than the genre's definition.

27. Answer: D · RHETORICAL SYNTHESIS · HARD

Correct answer: D. Choice D highlights the immense animal resource commitment required by both scriptoria, noting Kells Abbey's use of up to 150 calfskins and Lindisfarne's comparable vellum quantities.

Why the other choices are wrong:

- A) It contrasts locations and operating centuries.
- B) It contrasts the specific types of illuminated texts produced.
- C) It states general background information on scriptoria rather than a specific similarity between the two.

R&W 2 Reading and Writing, Module 2: Explanations

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

1. Answer: B · WORDS IN CONTEXT · HARD

Correct answer: B. 'Peripheral to' means marginal to. The claim that Palladio's blueprints were marginal to other designers is weakened, though his focus on rural villas limited his urban work.

Why the other choices are wrong:

- A) 'Antagonistic toward' means actively hostile to, which does not fit the sentence, where the blank needs the sense of 'peripheral to'.
- C) 'Simulated by' means imitated by, which does not fit the sentence, where the blank needs the sense of 'peripheral to'.
- D) 'Polarizing among' means dividing opinion among, which does not fit the sentence, where the blank needs the sense of 'peripheral to'.

2. Answer: D · WORDS IN CONTEXT · MEDIUM

Correct answer: D. 'Stirred by' means moved or stimulated by. Mitchell translated impressions of French fields and river light into color fields.

Why the other choices are wrong:

- A) 'Checked by' means restrained, which contradicts translating.
- B) 'Detached from' means separated from, which contradicts translating impressions.
- C) 'Unconcerned with' means not interested in, which contradicts translating impressions.

3. Answer: A · WORDS IN CONTEXT · MEDIUM

Correct answer: A. 'Inadvertent' means unintentional. Critics thought proximity was unintentional, but the octopus gathered shells deliberately.

Why the other choices are wrong:

- B) 'Conspicuous' means noticeable, which does not contrast with deliberate action.
- C) 'Profound' means deep, which does not contrast.
- D) 'Deliberate' means done on purpose, which would agree with the evidence, not the critics.

4. Answer: B · WORDS IN CONTEXT · HARD

Correct answer: B. 'Friable' means easily crumbled. Organic medieval pigments are chemically unstable and fade easily in light, so they are the most fragile part.

Why the other choices are wrong:

- A) 'Identifiable' means able to be recognized, which does not explain protective glass.
- C) 'Ubiquitous' means everywhere, which does not explain the need for protection.
- D) 'Intricate' means complex, which does not explain fragility.

5. Answer: C · TEXT STRUCTURE · MEDIUM

Correct answer: C. The underlined sentence directly illustrates the idea from the previous sentence: her thoughts mapping onto the setting.

Why the other choices are wrong:

- A) The passage shows her connection to the landscape, with no mention of conflict with her family; a bitter conflict is unsupported.
- B) The sentence offers imagery connected to her mood, since the previous sentence says the landscape reflects her mood, so it is not purely physical.
- D) The next sentence does not disprove anything; it continues to describe the scene in line with the preceding claim.

6. Answer: A · AUTHOR'S PURPOSE · HARD

Correct answer: A. The text presents hyperbaric chamber trials on Bathynomus giganteus whose metabolic data fail to support the thermal adaptation hypothesis for deep-sea gigantism.

Why the other choices are wrong:

- B) The passage does not analyze limits of hyperbaric chambers; the chamber is the experimental tool, not the subject, so this misses the main purpose.
- C) The text does not compare competing models of lipid storage; it tests the thermal adaptation model and notes starvation endurance as the alternative factor, so this misstates the focus.
- D) The researchers argue thermal adaptation is unlikely to drive bathynomid gigantism, which undercuts rather than supports identical mechanisms with polar gigantism.

7. Answer: A · CROSS-TEXT CONNECTIONS · EASY

Correct answer: A. Text 2 presents glacial sublimation and creeping ice sheets shaping the valleys over long epochs, directly countering Text 1's catastrophic flood theory.

Why the other choices are wrong:

- B) Text 2 does not claim Mars never had liquid water; it discusses ice and sublimation.
- C) Text 2 does not discuss the existence or absence of subterranean aquifers.
- D) Text 2 attributes valley shaping to ice sheets and sublimation, not volcanic eruptions.

8. Answer: C · TEXTUAL EVIDENCE · HARD

Correct answer: C. This option directly supports structural pivoting by revealing elemental constituents common to both mantle-drawn and stratum-associated formations.

Why the other choices are wrong:

- A) It compares variability within older samples from the two sources; greater diversity in one source before the date says nothing about material moving between the reservoirs after the transition.
- B) A change in the strength of an age-based statistical correlation near the date does not identify any specific material that passed between the two reservoirs, so it does not directly show the mixing that defines the transition.
- D) It only compares how many samples come from each source before and after the date; counting sample types does not show any material shared between the two reservoirs, which is what mixing after the transition would require.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

9. Answer: D · TEXTUAL EVIDENCE · MEDIUM

Correct answer: D. This option validates an increase in stage directions requiring torch illumination and late-night action in indoor playhouses.

Why the other choices are wrong:

- A) It reports only that one specific explicit time marker became rarer; a change in wording of the records does not show that people were actually working or gathering at new hours because of the development named in the passage.
- B) Two explicit markers appearing at roughly the same frequency across the whole collection shows no change over time, so it cannot link a shift in behavior to the development named in the passage.
- C) A steady rise of indirect indicators relative to explicit time markers within one century describes how records were written, not a change in actual routines tied to the date of the development named in the passage.

10. Answer: B · QUANTITATIVE EVIDENCE · HARD

Correct answer: B. At -5°C the blend (118 g) beats rock salt (95 g), but at -15°C the blend (36 g) falls just below rock salt (38 g). That matches the hypothesized outperformance at the milder temperature only.

Why the other choices are wrong:

- A) This describes only rock salt's own decline and never compares it with the beet-juice blend, so it does not address the hypothesis.
- C) This compares the blend only with itself and with sand-salt, not with rock salt, so it cannot show where the blend outperforms rock salt.
- D) This is true but concerns calcium chloride, not the blend versus rock salt, so it is irrelevant to the hypothesis.

11. Answer: D · INFERENCES · HARD

Correct answer: D. The new material showed a lower minimum crack initiation rate than the two comparison materials under constant conditions, which supports the claim that it can substantially inhibit the damage, provided those conditions stay consistent.

Why the other choices are wrong:

- A) This reverses the finding: the new material, not the comparison materials, showed the lower minimum crack initiation rate, so it is the better performer and the comparison materials cannot be the more protective choice.
- B) The study found a difference between the new material and the two comparison materials, so it does not support treating them as equivalent; the option is also incoherent, since reducing stability would not extend service life.
- C) A lower minimum crack initiation rate indicates greater resistance to damage, so the new material should last longer, not shorter, which makes this the reverse of what the finding suggests.

12. Answer: C · INFERENCES · EASY

Correct answer: C. River sections with regulated pooling were more ecologically stable than unmanaged ones, so carefully managed water flow can sometimes support river stability.

Why the other choices are wrong:

- A) Claims natural waterways cannot stay healthy without barriers, an extreme reading the study does not support.
- B) Prioritizes electricity over environmental concerns, a value judgment the ecological findings do not address.
- D) Says barriers cause no meaningful changes to species movement, which the passage does not address and the stability finding does not imply.

13. Answer: B · CENTRAL IDEAS AND DETAILS · MEDIUM

Correct answer: B. Lorry's constant motion comes from a fear that if he stopped he would lose his sense of purpose, so he fears stagnation and needs constant stimulation.

Why the other choices are wrong:

- A) The text says he sees the world as a challenge to conquer, not as something to map for others.
- C) He seeks the brink of something new, so he does not want to settle.
- D) The passage does not say he wants fame as an explorer.

14. Answer: C · CENTRAL IDEAS AND DETAILS · MEDIUM

Correct answer: C. The text states Meyer determined pollution by "filtering water samples collected at varying depths and counting microplastic particles per liter."

Why the other choices are wrong:

- A) The passage never mentions this method (fish toxins); it names a different one.
- B) The passage never mentions this method (factory runoff); it names a different one.
- D) The passage never mentions this method (chemical oxygen demand); it names a different one.

15. Answer: C · BOUNDARIES · MEDIUM

Correct answer: C. Zero punctuation correctly merges the restrictive title and proper noun. Rule: Essential identifying appositives must not be separated from proper nouns by punctuation marks.

Why the other choices are wrong:

- A) The option "painter," is incorrect because the comma between the title and proper noun is grammatically incorrect.
- B) The option "painter:" is incorrect because a colon violates appositive rules.
- D) The option "painter—" is incorrect because an em dash incorrectly separates the title from the proper name.

16. Answer: A · BOUNDARIES · MEDIUM

Correct answer: A. The period correctly halts the sentence before the next independent clause.

Why the other choices are wrong:

- B) Incorrect because em-dashes cannot replace terminal punctuation between clauses.
- C) Incorrect because fusing clauses without punctuation creates a run-on.
- D) Incorrect because a comma creates an ungrammatical comma splice.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

17. Answer: A · BOUNDARIES · MEDIUM

Correct answer: A. Commas before and after "however" correctly set it off as a parenthetical between the main clause and the participial phrase that follows.

Why the other choices are wrong:

- B) The semicolon after "however" is incorrect because it cannot link the main clause to the participial phrase that follows.
- C) A semicolon before "however" is incorrect because the words after it are a participial phrase, not an independent clause.
- D) It omits the comma after "however," leaving the parenthetical interrupter unclosed.

18. Answer: D · BOUNDARIES · HARD

Correct answer: D. A period correctly separates the two independent clauses, and "Prior to ..." begins a new complete sentence.

Why the other choices are wrong:

- A) With no punctuation at all, the two independent clauses are fused into a run-on sentence.
- B) A comma alone cannot join two independent clauses; this creates a comma splice.
- C) Joining the clauses with "and" and no comma-plus-conjunction structure leaves the second independent clause unseparated and the sentence garbled.

19. Answer: A · FORM, STRUCTURE, AND SENSE · MEDIUM

Correct answer: A. The simple present tense correctly indicates a regular, instinctive predatory action.

Why the other choices are wrong:

- B) Incorrect because past perfect tense places the action in an inappropriate past timeframe.
- C) Incorrect because simple past tense implies a single isolated event rather than a natural law.
- D) Incorrect because conditional phrasing weakens the factual nature of the behavior.

20. Answer: A · FORM, STRUCTURE, AND SENSE · HARD

Correct answer: A. Singular possessive noun "wheel's" correctly modifies the plural noun "teeth".

Why the other choices are wrong:

- B) This choice is wrong because plural possessive "wheels'" mismatches the single escape wheel context.
- C) This choice is wrong because double possession is ungrammatical.
- D) This choice is wrong because the apostrophe placement is invalid.

21. Answer: B · FORM, STRUCTURE, AND SENSE · MEDIUM

Correct answer: B. Past perfect "had expanded" correctly shows an action completed before the past reference point "By the end of the sixteenth century."

Why the other choices are wrong:

- A) Present perfect "has expanded" links the action to the present, which conflicts with the closed past time frame ending in the sixteenth century.
- C) Simple present "expands" is inconsistent with the past-tense narrative about a development completed by the end of the sixteenth century.
- D) Future "will expand" contradicts a completed historical development placed at the end of the sixteenth century.

22. Answer: C · TRANSITIONS · HARD

Correct answer: C. "Specifically" is correct because it accurately defines the precise thermal property differences that cause seasonal monsoon shifts.

Why the other choices are wrong:

- A) "Hence" is incorrect because it signals a concluding result rather than detailing the specific meteorological heating process.
- B) "Nevertheless" is incorrect because it introduces a concessive contrast, which conflicts with the direct explanatory passage.
- D) "Similarly" is incorrect because it proposes a parallel comparison rather than a detailed breakdown of a weather system.

23. Answer: B · TRANSITIONS · HARD

Correct answer: B. "though" is correct because it properly contrasts utilitarian populist prose norms with an obscure novel combining peasant themes and avant-garde structures.

Why the other choices are wrong:

- A) "likewise" is incorrect because it denotes similarity rather than a formalist literary exception.
- C) "for example" is incorrect because the novel defies standard nineteenth-century populist realism expectations.
- D) "characteristically" is incorrect because it implies expected sociological critique rather than formalist experimentation.

24. Answer: B · TRANSITIONS · HARD

Correct answer: B. "in fact" is correct because it intensifies the extremity of the ice-core data shift, supporting the scientists' revised timeline.

Why the other choices are wrong:

- A) "by contrast" is incorrect because it introduces a comparison of opposing items rather than emphasizing the magnitude of scientific data correction.
- C) "nonetheless" is incorrect because it concedes an obstacle, which fails to amplify the degree of climatic data variation.
- D) "similarly" is incorrect because it draws a parallel comparison rather than intensifying the scale of the dataset discrepancies.

25. Answer: C · RHETORICAL SYNTHESIS · EASY

Correct answer: C. This choice explains the significance by connecting Jenner's vaccine to the development of vaccination and elimination of smallpox.

Why the other choices are wrong:

- A) This choice explains what Jenner did but does not explain why the vaccine was historically important.
- B) This choice describes the problem smallpox caused but does not explain the significance of Jenner's contribution.
- D) This choice gives a general statement about vaccines without explaining Jenner's specific impact.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

26. Answer: A · RHETORICAL SYNTHESIS · HARD

Correct answer: A. This choice explains the advantage by showing that augmented reality provides interactive experiences that improve visitor engagement.

Why the other choices are wrong:

- B) This choice describes a feature of augmented reality but does not explain the benefit.
- C) This choice describes traditional museum displays rather than the advantage of augmented reality.
- D) This choice introduces the technology but does not explain why it is useful.

27. Answer: A · RHETORICAL SYNTHESIS · HARD

Correct answer: A. Defines hygromorphism (structural adaptations for high-humidity habitats) AND provides the example (water lettuce micro-hairs preventing submergence). B/C/D miss either the explanation or the example.

Why the other choices are wrong:

- B) It explains the features and purpose of hygromorphic adaptations but gives no specific plant example, so the example is missing.
- C) It describes two plants with wet-habitat features but never defines hygromorphism, so the explanation half is missing.
- D) It supplies the water lettuce example but does not clearly define hygromorphism, and adds an unsupported claim about botanists investigating wetland ecology.

Math 1 Math, Module 1: Explanations**1. Answer: D · LINEAR EQUATIONS (1 VAR) · HARD**

Multiply both sides by 6: $2(5x - 2) = 3(x + 4) + 54$. This gives $10x - 4 = 3x + 66$. Subtract $3x$ and add 4 to get $7x = 70$, so $x = 10$.

2. Answer: A · LINEAR FUNCTIONS · EASY

The slope is $(21 - 9)/(4 - 1) = 4$. Then $f(x) = 4x + b$, and $f(1) = 9$ gives $b = 5$, so $f(x) = 4x + 5$. Therefore $f(8) = 4(8) + 5 = 37$.

3. Answer: D · LINEAR FUNCTIONS · MEDIUM

Translating the graph 4 units to the left means $g(x) = f(x + 4)$. Then $g(x) = 3(x + 4) + 2 = 3x + 12 + 2 = 3x + 14$. The slope stays 3.

4. Answer: 13 · LINEAR EQUATIONS (2 VAR) · MEDIUM

Substitute $x = t$ and $y = t - 6$: $4t - 3(t - 6) = 31$. Distributing gives $4t - 3t + 18 = 31$, so $t + 18 = 31$ and $t = 13$.

Common mistakes:

- 31 comes from dropping the -6 and solving $4t - 3t = 31$.
- 37 comes from distributing incorrectly: $4t - 3t - 6 = 31$ gives $t = 37$.
- 49 comes from a sign error, $4t - 3t - 18 = 31$, giving $t = 49$.

5. Answer: 15 · SYSTEMS OF EQUATIONS · MEDIUM

Multiply the first equation by 2 and the second by 3: $8x + 6y = 58$ and $9x - 6y = 27$. Adding gives $17x = 85$, so $x = 5$. Then $3y = 29 - 20 = 9$, so $y = 3$. Therefore $xy = 5 \times 3 = 15$.

Common mistakes:

- 8 is $x + y$, the sum rather than the product.
- 2 is $x - y$, the difference rather than the product.

6. Answer: A · SYSTEMS OF EQUATIONS · MEDIUM

Let s be the number of small crates and l the number of large crates. Then $s + l = 20$ and $10s + 16l = 266$. Substituting $l = 20 - s$ gives $10s + 320 - 16s = 266$, so $-6s = -54$ and $s = 9$.

7. Answer: D · LINEAR INEQUALITIES · MEDIUM

Add 3 to all three parts: $-4 \leq 2x \leq 18$. Divide by 2: $-2 \leq x \leq 9$. Both endpoints are included, so the integers are $-2, -1, 0, 1, \dots, 9$. Counting from -2 to 9 inclusive gives $9 - (-2) + 1 = 12$ integers.

8. Answer: 37 · EQUIVALENT EXPRESSIONS · EASY

Substitute $x = 7$: $2(7 - 3)^2 + 5$. The parentheses give 4, and $4^2 = 16$. Then $2(16) = 32$, and $32 + 5 = 37$.

Common mistakes:

- 69 comes from squaring $2(7 - 3) = 8$ to get 64 and then adding 5. The exponent applies only to the parentheses.
- 13 comes from skipping the exponent, computing $2(4) + 5$.
- 21 comes from evaluating $(7 - 3)^2$ as $4 \cdot 2 = 8$ instead of 16, giving $2(8) + 5$.

9. Answer: A · EQUIVALENT EXPRESSIONS · MEDIUM

Substituting $t = 1$ gives $V(1) = 1800(0.85)^1 = 1800(0.85) = 1530$. So $1800(0.85)$ is the value of the dryer, in dollars, 1 year after purchase. The factor 0.85 means the dryer keeps 85% of its value each year.

10. Answer: 4 · NONLINEAR EQUATIONS · MEDIUM

The equation has exactly one real solution when its discriminant is 0: $(k + 2)^2 - 4(1)(9) = 0$. So $(k + 2)^2 = 36$ and $k + 2 = 6$ or $k + 2 = -6$, giving $k = 4$ or $k = -8$. Since k is positive, $k = 4$.

Common mistakes:

- -8 is the other value that makes the discriminant 0, but k is stated to be positive.
- 6 comes from stopping at $k + 2 = 6$ without subtracting the 2.
- 36 comes from stopping at $(k + 2)^2 = 36$ and not taking the square root.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

11. Answer: 20 · NONLINEAR EQUATIONS · EASY

Substitute $x = 4$ into the equation: $4^2 - 9(4) + k = 0$. This gives $16 - 36 + k = 0$, so $k = 20$.

Common mistakes:

- 20 comes from a sign error when moving $16 - 36$ to the other side of the equation.
- 4 is the given solution, not the constant k .

12. Answer: A · NONLINEAR EQUATIONS · EASY

Let n be the smaller page number, so the other is $n + 1$. Then $n(n + 1) = 132$, which gives $n^2 + n - 132 = 0$, or $(n + 12)(n - 11) = 0$. Since n is positive, $n = 11$, and the pages are 11 and 12.

13. Answer: A · NONLINEAR FUNCTIONS · MEDIUM

The values are symmetric about $x = 1$ ($f(0) = f(2)$ and $f(-1) = f(3)$), and the minimum value is -3 at $x = 1$, so the vertex is $(1, -3)$ and $f(x) = a(x - 1)^2 - 3$. Using $f(0) = -1$ gives $a(1) - 3 = -1$, so $a = 2$. Therefore $f(x) = 2(x - 1)^2 - 3$.

14. Answer: B · NONLINEAR FUNCTIONS · MEDIUM

Solutions of $f(x) = 3$ are the x -coordinates of the points where the graph meets the horizontal line $y = 3$. The middle part of the graph never rises above $y = 1$, so it stays below the line. Only the left and right ends rise above the line, so the graph meets the line exactly 2 times.

15. Answer: B · RATIOS AND RATES · MEDIUM

The patio's area is $18 \times 12 = 216$ square feet. Because 1 square yard = $3 \times 3 = 9$ square feet, the area is $216 \div 9 = 24$ square yards. At \$4 per square yard, the cost is $24 \times \$4 = \$96$.

16. Answer: C · PERCENTAGES · EASY

The sale price is $60 \times 0.90 = \$54.00$. The 5% tax is added to the sale price, so the total is $54 \times 1.05 = \$56.70$.

17. Answer: D · ONE-VARIABLE DATA · HARD

The 30 teams scored a total of $30 \times 78 = 2,340$ points. The 6 eliminated teams scored $6 \times 60 = 360$ points. The remaining 24 teams scored $2,340 - 360 = 1,980$ points, so their mean is $1,980/24 = 82.5$.

18. Answer: C · TWO-VARIABLE DATA · EASY

A shop with more than 40 jobs has a point above the 40 line. The points $(5, 44)$, $(6, 55)$, $(7, 42)$, $(8, 60)$, and $(11, 52)$ are above it, so 5 print shops completed more than 40 jobs.

19. Answer: A · AREA AND VOLUME · HARD

The radius of the top is 6 and the slant height is 10, so the height is $\sqrt{10^2 - 6^2} = 8$. Water filling half the height forms a similar cone with height 4 and radius 3. Its volume is $(1/3)\pi(3^2)(4) = 12\pi$ cubic inches.

20. Answer: A · AREA AND VOLUME · HARD

The original volume is $\pi r^2 h$. The new pan has radius $1.5r$ and height $0.8h$, so its volume is $\pi(1.5r)^2(0.8h) = \pi(2.25r^2)(0.8h) = 1.8\pi r^2 h$. The new volume is 1.8 times the original volume, so it is 80% greater.

21. Answer: B · LINES, ANGLES, TRIANGLES · HARD

The slope of AB is $(4 - 1)/(4 - (-2)) = 1/2$, so a line perpendicular to AB at B has slope -2 . Its equation is $y - 4 = -2(x - 4)$. Setting $y = 0$ gives $-4 = -2(x - 4)$, so $x = 6$. Point C is $(6, 0)$.

22. Answer: D · RIGHT TRIANGLES AND TRIG · EASY

In a 30-60-90 triangle the side lengths are in the ratio $1 : \sqrt{3} : 2$, so $\cos 30^\circ = (\sqrt{3})/2$. Therefore $4 \cos 30^\circ = 4 \times \sqrt{3}/2 = 2\sqrt{3}$.

Math 2 Math, Module 2: Explanations**1. Answer: A · LINEAR EQUATIONS (1 VAR) · MEDIUM**

Add 4 to both sides to get $w + 4 = (3/5)(p + 10)$. Multiply by $5/3$ to get $p + 10 = 5(w + 4)/3$, then subtract 10: $p = (5w + 20)/3 - 10 = (5w - 10)/3$.

2. Answer: A · LINEAR FUNCTIONS · EASY

Monday's rate is $36/3 = 12$ burritos per hour. Tuesday's rate is $50/5 = 10$ burritos per hour. The truck sold $12 - 10 = 2$ more burritos per hour on Monday.

3. Answer: C · LINEAR FUNCTIONS · MEDIUM

Substitute $g(x)$ into f : $f(g(x)) = 2(3x + 8) + 1 = 6x + 17$. Setting $6x + 17 = 41$ gives $6x = 24$, so $x = 4$.

4. Answer: -12 · LINEAR EQUATIONS (2 VAR) · HARD

Point-slope form gives $y - 6 = (3/4)(x - 4)$, so $y = (3/4)x + 3$. The y -intercept is $b = 3$. Setting $y = 0$ gives $(3/4)x = -3$, so $x = -4$ and $a = -4$. Therefore $ab = (-4)(3) = -12$.

Common mistakes:

- 12 results from losing the sign of a , taking the x -intercept as 4, the x -coordinate of the given point.
- -1 is the sum $a + b = -4 + 3$, not the product ab .
- -9 comes from using $b = 6 - 3 = 3$ but $a = -3$ (solving $(3/4)x = -3$ as $x = -3$).

5. Answer: 3/2 or 1.5 · SYSTEMS OF EQUATIONS · HARD

A system of two linear equations has infinitely many solutions when both equations describe the same line. Solving $9x - 6y = -48$ for y gives $y = (3/2)x + 8$. This matches $y = kx + 8$ exactly when $k = 3/2$. Equivalently, substituting gives $(9 - 6k)x = 0$ for every x , so $9 - 6k = 0$.

Common mistakes:

- $2/3$ comes from dividing the coefficients in reverse order ($6/9$ instead of $9/6$).
- $-3/2$ comes from a sign error when isolating y , writing $y = -(3/2)x - 8$.
- 8 is the y -intercept of the first line, not its slope.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

6. Answer: B · SYSTEMS OF EQUATIONS · HARD

Divide the first equation by 4 to get $x + y = 7$, and divide the second by 6 to get $x - y = 3$. Since $x^2 - y^2 = (x + y)(x - y)$, the value is $7 \times 3 = 21$. (Indeed $x = 5$ and $y = 2$, and $25 - 4 = 21$.)

7. Answer: B · LINEAR INEQUALITIES · HARD

Distribute on the left: $-4x + 6$. On the right, $5 - (x - 4)/2 = 5 - x/2 + 2 = 7 - x/2$. So $-4x + 6 \geq 7 - x/2$, which gives $-(7/2)x \geq 1$. Dividing by $-7/2$ reverses the symbol: $x \leq -2/7$.

8. Answer: 10 · EQUIVALENT EXPRESSIONS · HARD

A perfect square trinomial has the form $(2x \pm 3)^2 = 4x^2 \pm 12x + 9$. So the coefficient of x must be 12 or -12. Setting $k + 2 = 12$ gives $k = 10$, while $k + 2 = -12$ gives $k = -14$, which is not positive. The positive value is 10.

Common mistakes:

- 12 comes from setting k equal to the middle coefficient and forgetting the +2 in $k + 2$.
- 14 comes from adding 2 to 12 instead of subtracting it.
- 6 comes from using $(2x)(3) = 6x$ as the middle term and forgetting to double it.

9. Answer: B · EQUIVALENT EXPRESSIONS · MEDIUM

The numerator is x^{6y-4} and the denominator is $x^{-3}y^3$. Subtracting exponents gives $x^{6-(-3)} = x^9$ and $y^{-4-3} = y^{-7}$, so the expression equals x^9/y^7 .

10. Answer: 18 · NONLINEAR EQUATIONS · HARD

Let the solutions be s and $2s$. Their product is $24/3 = 8$, so $2s^2 = 8$ and $s = \pm 2$. Their sum is $3s = k/3$, so $k = 9s$. Since k is positive, $s = 2$, the solutions are 2 and 4, and $k = 18$. Check: $3x^2 - 18x + 24 = 3(x - 2)(x - 4)$.

Common mistakes:

- 18 comes from $s = -2$, with solutions -2 and -4, but k is stated to be positive.
- 9 is the value of $9s$ with $s = 1$, which does not satisfy $2s^2 = 8$.
- 6 is the sum of the solutions 2 and 4, not the value of k ; the sum equals $k/3$.

11. Answer: 16 or 16.0 · NONLINEAR EQUATIONS · HARD

Let $u = x^2 - 3$. Then $u^2 - 4u - 5 = 0$, so $(u - 5)(u + 1) = 0$ and $u = 5$ or $u = -1$. This gives $x^2 = 8$ or $x^2 = 2$, so the real solutions are $\pm 2\sqrt{2}$ and $\pm \sqrt{2}$. Their product is $(2\sqrt{2})(-2\sqrt{2})(\sqrt{2})(-\sqrt{2}) = (-8)(-2) = 16$.

Common mistakes:

- -5 is the product of the two values of u , not of the solutions for x . The values of u must be converted back with $x^2 = u + 3$.
- 0 is the sum of the real solutions, which cancel in pairs. The question asks for their product.

12. Answer: B · NONLINEAR EQUATIONS · HARD

Write $3^{4x+1} = 3^{4x} \times 3^1 = (3^{2x})^2 \times 3$. Since $3^{2x} = 7$, this equals $7^2 \times 3 = 49 \times 3 = 147$.

13. Answer: D · NONLINEAR FUNCTIONS · HARD

Reflecting over the x -axis gives $-f(x)$, and translating 4 units left replaces x with $x + 4$. So $g(x) = -f(x + 4) = -[2(x + 4)^2 - 8(x + 4) + 3] = -[2x^2 + 8x + 3] = -2x^2 - 8x - 3$.

14. Answer: B · NONLINEAR FUNCTIONS · HARD

Test each equation against all four rows. For $f(x) = |x - 2| - 1$: $f(0) = 2 - 1 = 1$, $f(1) = 1 - 1 = 0$, $f(3) = 1 - 1 = 0$, and $f(5) = 3 - 1 = 2$. All four values match the table, so this equation could define f .

15. Answer: C · RATIOS AND RATES · MEDIUM

The machine produces 5 posters every 3 minutes, so 90 posters make up $90/5 = 18$ groups of 5 posters. Each group takes 3 minutes, so the time is $18 \times 3 = 54$ minutes.

16. Answer: A · PERCENTAGES · HARD

Let c be the shop's cost. The marked price is $1.40c$, the sale price is $0.75(1.40c) = 1.05c$, and the total with tax is $1.08(1.05c) = 1.134c$. Setting $1.134c = 226.80$ gives $c = 200$. The shop's cost was \$200.

17. Answer: B · PROBABILITY · HARD

Let p be the probability that a loaf of another type is unsold, so a sourdough loaf is unsold with probability $2p$. Then $0.40(2p) + 0.60(p) = 0.14$, so $1.4p = 0.14$ and $p = 0.10$. The sourdough probability is $2(0.10) = 0.20$.

18. Answer: B · SAMPLE STATISTICS · MEDIUM

The interval for branch 1, 12.4 to 15.6, and the interval for branch 2, 9.1 to 11.7, share no values. This suggests the mean wait time at branch 1 is greater than at branch 2.

19. Answer: B · AREA AND VOLUME · MEDIUM

Let the width be w . Then the length is $2w + 5$, and the perimeter is $2(w + 2w + 5) = 6w + 10 = 40$, so $w = 5$. The length is $2(5) + 5 = 15$, and the area is $5 \times 15 = 75$ square feet.

20. Answer: A · AREA AND VOLUME · MEDIUM

The area of a trapezoid is $(1/2)(b_1 + b_2)h$. The parallel sides are 9 and 17, and the perpendicular distance between them is 6, so the area is $(1/2)(26)(6) = 78$ square yards. The 10-yard slanted side is not needed.

21. Answer: A · RIGHT TRIANGLES AND TRIG · MEDIUM

The horizontal distance 150 m is the side adjacent to the 28° angle, and the balloon's height above eye level is the opposite side. Since $\tan 28^\circ = \text{opposite}/150$, the opposite side is $150 \tan 28^\circ$. Adding the 1.6 m from the ground to the technician's eyes gives $1.6 + 150 \tan 28^\circ$.

22. Answer: C · RIGHT TRIANGLES AND TRIG · EASY

For acute angles, $\sin \theta = \cos(90^\circ - \theta)$, so the sine of an angle equals the cosine of its complement. Therefore $x = 90 - 28 = 62$.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



Practice Exam 2

A full-length digital SAT in four modules: Reading and Writing (2 × 27 questions) and Math (2 × 22 questions), followed by complete answers and explanations.

Exam 2 Reading and Writing: Module 1

Reading and Writing · Module 1

27 questions · 32 minutes · Read each passage and question carefully, then choose the best answer.

- 1** To show that the composition of sealed historical pottery jars can be analyzed without breaking the wax seals, conservator Clara Hughes and colleagues _____ the tendency of specialized gamma rays to scatter differently depending on the density of internal materials: the team verified that these patterns can be mapped from outside.
Which choice completes the text with the most logical and precise word or phrase?
- A. posited
B. dismissed
C. overstretched
D. leveraged
- 2** Like other indigenous autonomous polities, the Navajo Nation operates a sovereign judicial system; its Legislative Council produces comprehensive statutory laws governing civil and commercial affairs, while the executive branch and cabinet delegates _____ those mandates by articulating explicit administrative protocols and delivering essential public services in alignment with them.
Which choice completes the text with the most logical and precise word or phrase?
- A. execute
B. presume
C. improvise
D. mimic
- 3** For a 2020 outdoor installation, environmental artist and botanist Dr. Elena Rostova _____ an array of architectural living walls based on sketches of hanging gardens made by nineteenth-century naturalists. Rostova engineered specialized hydroponic channels and arranged native mosses over reclaimed steel frameworks, creating what she termed "living industrial historical facades."
Which choice completes the text with the most logical and precise word or phrase?
- A. rubber-stamped
B. constructed
C. opposed
D. salvaged
- 4** For a 2021 virtual reality exhibit, software engineer and visual artist Dr. David Kim _____ a dynamic digital landscape modeled on a collection of landscape woodblock prints from the Edo period. Kim mapped the traditional ink gradients into three-dimensional polygons and animated the environmental assets, producing a fully navigable historical art space.
Which choice completes the text with the most logical and precise word or phrase?
- A. validated
B. produced
C. disputed
D. restored

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 5 The subglacial lakes of Antarctica are among the most chemically extreme aquatic environments on Earth. Dr. Ravi Lawson and other astrobiologists isolate extremophilic archaea from these dark, high-pressure depths because the subglacial water tables closely replicate the conditions hypothesized to exist in the subsurface oceans of Europa. The metabolic pathways utilized by microbes in these Antarctic lakes may yield vital insights into extraterrestrial biochemistry. By examining how these specialized organisms process nutrients in the complete absence of sunlight, Lawson's team aims to determine whether similar life-forms could survive on Jupiter's icy moons and to refine the spectrometry sensors needed to detect them during future flyby missions.

Which choice best describes the function of the underlined sentence in the text as a whole?

- A. To identify the primary reason why astrobiologists look to Antarctic subglacial lakes as an analog for extraterrestrial environments.
- B. To contrast the biochemical composition of Antarctic lakes with the oceanic chemistry of Europa.
- C. To enumerate the specific experimental failures Lawson's team encountered when simulating alien environments.
- D. To argue that microbial life on Earth originated in subglacial lakes before migrating to the oceans.

- 6 The following text is adapted from a 1918 account of life on a dairy farm. The narrator is describing the selection process for a local livestock competition.

[The fair is] selecting the county milking team and there's just a chance that I shall make it. I'm small of course, but terribly swift and precise and rhythmic. While the heavy farmhands are tiring out their hands, I can empty the buckets and tend the cattle.

Which choice best states the main purpose of the text?

- A. To explain why the narrator thinks she might make the milking team
- B. To compare cattle raising with sheep farming
- C. To provide a detailed guide on how to process raw milk
- D. To state how judges award ribbons at the fair

7 Text 1

Long-distance obsidian exchange networks in the early Levantine Neolithic period are frequently analyzed through provenance studies. Some researchers contend that maritime cabotage along the Mediterranean coastline dominated material distribution, noting that sea routes provided the most efficient transport vector for heavy mineral cargo.

Text 2

Trace-element geochemical characterization using laser ablation inductively coupled plasma mass spectrometry offers new distributional insights. Researcher Shubhranshu Roy analyzed artifact distribution frequencies across inland and coastal settlements and, based on the compositional data, compellingly argues that overland pack-animal trails accounted for the primary commercial flow of obsidian.

Based on the texts, how would the author of Text 2 most likely respond to what "some researchers contend" as described in Text 1?

- A. By confirming that maritime cabotage was the sole method utilized by Neolithic obsidian traders
- B. By showing that Mediterranean coastal waters were impassable for ancient seafaring canoes
- C. By indicating that overland pack-animal trails accounted for the primary commercial distribution of obsidian
- D. By establishing that obsidian sources were located directly inside every Neolithic dwelling

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 8** As higher education institutions have expanded online micro-credentialing and modular certificate programs, traditional multi-year degree completion has softened in favor of a career strategy known as “just-in-time upskilling” (JITU), in which learners passively stack short-term digital badges based on real-time corporate software demands. Economists Beatrice Palmer and Jonathan Wright analyzed longitudinal employment records for recent college graduates to measure participants’ JITU adoption rates, salary growth, and career stability. Although no nationwide economic boom occurred during the study for Palmer and Wright to establish definitive causality between JITU and long-term occupational security, they did conjecture that JITU may lower comprehensive domain mastery through an indirect fragmentation of foundational knowledge.

Which finding, if true, would most directly support the assertion advanced by Palmer and Wright?

- A. JITU adoption rates tend to increase during economic downturns, and young professionals are significantly more likely to change employers within two years of graduation than in previous decades.
- B. JITU adoption has a strong negative effect on comprehensive theoretical comprehension, and there is known to be a strong positive correlation between deep theoretical comprehension and long-term career adaptability across technological shifts.
- C. Starting salaries are known to have a strong positive correlation with technical certificate counts but show only a weak relationship with job tenure, and JITU adoption shows minimal correlation with starting salary or tenure.
- D. The availability of online micro-credentials increases as software subscription costs drop, and the relationship between digital badge acquisition and short-term hiring rates tends to strengthen as industry skill requirements evolve.

- 9** Sociologists Julian Moretti and Irina Belovska have proposed that observing vast desert horizons—a landscape feature evoking immense spatial magnitude—can induce a stabilizing sense of humility and thereby motivate people to offer voluntary assistance. Moretti, in collaboration with Takumi Endara, Selene Vardoux, and associates, claims to have found confirmation for this behavioral pattern in a trial where participants were directed to either view sweeping photographic projections of an expansive desert or examine images of a cluttered garage. Moments later, a facilitator purposely spilled a box of colored beads.

Which finding from the researchers’ trial, if true, would most strongly support their claim?

- A. Participants who had been viewing the desert projections retrieved significantly more spilled beads for the facilitator than did participants who had been examining the cluttered garage.
- B. Participants who did not retrieve the spilled beads were significantly more likely to report experiencing visual stillness, regardless of whether they viewed the desert or the garage.
- C. Participants who retrieved the spilled beads used a greater frequency of expansive spatial terms when describing their visual focus during a follow-up discussion than did participants who did not retrieve them.
- D. Participants who had been examining the cluttered garage were significantly more likely to notice that the facilitator had spilled the colored beads than were participants who had been viewing the desert projections.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 10** A housing economist compared six metropolitan areas to see how the share of a typical household's income that goes to rent relates to the area's median household income. She suspected that lower-income areas devote a larger share of income to rent than higher-income areas do, even if the rents themselves are lower.

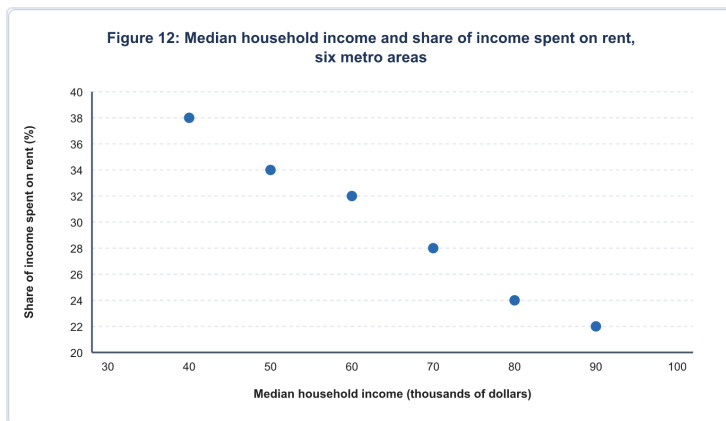


Figure 12: Median household income and share of income spent on rent, six metro areas

Which choice best describes data from the graph?

- A. The area with a median income of \$40,000 had a rent share of 38%, the highest of the six areas, and the area at \$90,000 had the lowest rent share, 22%.
- B. The areas with median incomes of \$60,000 and \$70,000 had the same rent share of 32%.
- C. Rent share generally rose as median income rose, from 22% at \$40,000 to 38% at \$90,000.
- D. The area with a median income of \$50,000 had a rent share above 38%.

- 11** The connection between water channeling (the redirection of spring water for agricultural purposes) and desert habitat preservation has remained a subject of debate among hydrologists. A study covering 190,000 hectares in the Sahara Desert provided evidence that complicated earlier assumptions about this relationship. The investigation demonstrated that spring systems receiving regulated ducting (the organized movement of specific water quantities) maintained greater long-term stability than similar systems without such management. The findings suggest that _____

Which choice most logically completes the text?

- A. redirecting natural resources produces harmful consequences for fragile environments.
- B. deliberate adjustments to how essential materials are distributed can improve the functioning of vulnerable regions.
- C. untouched desert areas are generally unable to survive without assistance from outside sources.
- D. large-scale industrial projects are necessary to maintain desert environments over time.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 12** To address the susceptibility of materials used in components of municipal water chlorination disinfection systems, such as residual hypochlorite dosing pumps, to oxidative stress cracking (polymeric embrittlement that is induced by active chlorine exposure and that often occurs under continuous mechanical pumping stress), materials researchers have developed fluorocarbon elastomer compounds for producing pump diaphragm seals. Testing the chemical degradation parameters of several commercially available elastomer membranes, Nicolás Paredes et al. found that in comparison with two nitrile rubber membranes, a perfluoroelastomer membrane exhibited a lower minimum elongation loss rate, a measure of the rate at which an elastomer loses elasticity against a chlorine solution at a constant fluid pressure and operating temperature. The finding suggests that _____

Which choice most logically completes the text?

- A. unlike the two nitrile rubber membranes, the perfluoroelastomer membrane can substantially inhibit oxidative stress cracking, provided that fluid pressures and operating temperatures are consistent.
- B. pump diaphragm seals containing the perfluoroelastomer membrane may have the ability to withstand pumping stress for a shorter period of time than can seals containing either of the two nitrile rubber membranes.
- C. the two nitrile rubber membranes likely hold similar potential for reducing the chemical stability of materials exposed to chlorine, thus extending the life span of municipal water systems.
- D. membranes based on the two nitrile rubber membranes have chemical properties that may improve the mechanical and chemical performance of disinfection equipment to a greater extent than do membranes based on the perfluoroelastomer membrane.

- 13** In 2013, astronomers searching for distant planets monitored the brightness of thousands of stars over several years. When a planet passed directly between its star and Earth, the star's brightness briefly decreased by a very small amount. Computers examined enormous collections of observations to identify these repeating patterns, allowing researchers to detect planets too distant to observe directly with telescopes. By confirming that the same dimming occurred at regular intervals, scientists distinguished genuine planets from random changes in a star's brightness caused by measurement errors or natural stellar activity.

According to the text, how did astronomers identify distant planets?

- A. By photographing the planets with powerful telescopes.
- B. By measuring changes in the planets' temperatures.
- C. By collecting samples from the planets' atmospheres.
- D. By detecting repeated decreases in a star's brightness.

- 14** The following text is adapted from Carlos Ramos's 2019 memoir *Along the Railway*. The narrator is taking a long-distance train ride with his grandfather, Mateo.

Mateo worked on crossword puzzles printed in a daily newspaper, while I counted the wooden telegraph poles flashing past our passenger window. We were used to short ten-minute bus rides to the city market, but I wondered what life was like in the peaceful agricultural communities spread across these plains.

©2019 by Carlos Ramos

According to the text, what did the narrator do during the train trip?

- A. He solved crossword puzzles in a daily newspaper.
- B. He read about agricultural communities in a book.
- C. He counted wooden telegraph poles past the window.
- D. He bought snacks from the passenger car train vendor.

- 15** Moisture retention capacities differ significantly among high-altitude flora. For instance, the cushion moss (*Raoulia eximia*) can survive months of complete desiccation _____ the alpine buttercup (*Ranunculus glacialis*) withers rapidly without a continuous water supply.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. desiccation. While
- B. desiccation: while
- C. desiccation; while
- D. desiccation, while

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 16** Volcanic degassing injects massive quantities of sulfur dioxide and carbon dioxide into the stratosphere, influencing global climate patterns. In a detailed 2014 volcanology treatise, geochemist _____ established that high-altitude sulfate aerosols reflect incoming solar radiation, resulting in temporary hemispheric surface cooling.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. geochemist, Alistair Vance
- B. geochemist Alistair Vance,
- C. geochemist, Alistair Vance,
- D. geochemist Alistair Vance

- 17** After severe altitude sickness cut short her initial expedition, Julian Thorne was told he might never scale high peaks again. Defying all odds, Thorne didn't just climb, she _____ the 2018 Himalayan Ascent in Nepal, she reached the treacherous summit ahead of schedule and planted her team flag, becoming the lead pioneer of the route.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. climbed—steadily—during
- B. climbed—steadily during
- C. climbed—steadily, during
- D. climbed—steadily. During

- 18** In 1859, during a period of fierce biological debate, naturalist Charles Darwin published his foundational treatise on natural selection to explain evolutionary divergence. Darwin's subsequent research would involve more than just _____ as an obsessive investigator of barnacle anatomy, Darwin would help transform modern biology.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. selection. Though,
- B. selection though
- C. selection, though:
- D. selection, though,

- 19** Aquatic biologists studying river restoration projects highlight the ecological benefits of removing obsolete concrete dams from migrating fish routes. The most direct approach to restoring natural sediment transport is to breach barrier walls. Demolishing one of these abandoned concrete barriers _____ river ecosystems recover their natural flow dynamics and aquatic biodiversity.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. are helping
- B. help
- C. helps
- D. have helped

- 20** Energy-conservation programs designed to reduce residential electricity consumption can be successful, says urban researcher Julian Voss of Utrecht University in the Netherlands, when these campaigns give homeowners clear feedback: for example, Rotterdam achieved a 30 percent reduction in peak-hour grid load after neighborhood tenants were sent monthly statements indicating how much _____ used.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. one
- B. they
- C. it
- D. you

- 21** Gideon Finch inaugurated the nationwide invasive species acoustic barrier network in September 2023, establishing _____ the first time state wildlife agencies would deploy real-time sonic deterrents in major river basins.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. those
- B. some
- C. this
- D. these

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

22 Atmospheric ozone depletion—thinning layers of protective atmospheric gas above polar zones—result, broadly speaking, from industrial chemical pollutants. _____ synthetic chlorofluorocarbon molecules drift upward into the stratosphere, where intense ultraviolet radiation breaks them apart to release free chlorine atoms. These reactive chlorine atoms then catalytically destroy thousands of fragile ozone molecules before neutralizing.

Which choice completes the text with the most logical transition?

- A. Similarly,
C. Hence,
- B. Specifically,
D. Nevertheless,

23 In classical physics governing macroscopic objects, gravity and motion operate with absolute, predictable mathematical certainty. Quantum mechanical subatomic particles, _____ behave probabilistically, existing in multiple states simultaneously until observed. This fundamental duality bridges classical mechanics and modern physics.

Which choice completes the text with the most logical transition?

- A. for example,
C. accordingly,
- B. by contrast,
D. furthermore,

24 William Golding's 1954 allegorical novel *Lord of the Flies* chronicles the descent into savagery of a group of British schoolboys stranded on an uninhabited tropical island. Jack Merridew abandons civilized rules entirely, establishing a brutal hunting tribe governed by fear and violence. _____ Ralph clings desperately to democratic order and rescue signals, attempting to maintain civic responsibility among the fracturing children. Their bitter rivalry leads to tragic bloodshed.

Which choice completes the text with the most logical transition?

- A. Conversely, B. Meanwhile,
C. Nevertheless, D. Furthermore.

- 25**

 - Sociological theories provide frameworks for understanding social order and stability.
 - 1893: Émile Durkheim published *The Division of Labour in Society*, detailing functionalism.
 - Functionalism views society as a system of interdependent parts working together to maintain social cohesion (organic solidarity).
 - 1848: Karl Marx and Friedrich Engels published *The Communist Manifesto*, outlining conflict theory.
 - Conflict theory views social order as the result of power struggles between competing classes (the bourgeoisie and the proletariat).

The student wants to emphasize a difference in the view of social order between functionalism and conflict theory. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A.** Whereas functionalism views social order as the maintenance of cohesion through interdependent parts, conflict theory interprets it as the product of power struggles between competing social classes.
- B.** Functionalism and conflict theory, proposed in 1893 and 1848 respectively, both attempt to explain how social stability is maintained in modern societies.
- C.** While Durkheim's functionalism focuses on organic solidarity, Marx's conflict theory focuses on the class struggle between the bourgeoisie and the proletariat.
- D.** Conflict theory suggests that social order is maintained through power struggles, a view that differs from functionalism's focus on the division of labour and social cohesion.

- 26** • The Solar Panel Efficiency Research Project tested new materials for solar energy technology.
- Engineers compared traditional solar panels with panels using a newly developed coating.
 - Panels with the coating converted more sunlight into electricity.
 - Researchers concluded that the coating improved solar panel efficiency.

The student wants to present the Solar Panel Efficiency Research Project and its conclusion. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A.** Engineers compared solar panels using traditional materials with panels using a newly developed coating.
- B.** The Solar Panel Efficiency Research Project found that a new coating increased solar panel efficiency by improving electricity production from sunlight.
- C.** Researchers tested new materials related to solar energy technology.
- D.** Solar panels can produce electricity from sunlight using different materials.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 27** • A psychrolute is an organism adapted to thrive in extremely cold, high-pressure, or deep-sea environmental conditions.
- Morphological adaptations often include reduced bone ossification, jelly-like tissues, and low-density muscular structures to withstand crushing hydrostatic pressure.
 - The blobhead sculpin (*Psychrolutes phrictus*) lives at abyssal ocean depths up to 2,800 meters.
 - Its body consists primarily of gelatinous tissue with minimal bone mass, allowing it to drift above the sea floor without expending energy on swimming.
 - Psychrolutes maintain neutral buoyancy without needing a gas-filled swim bladder, which would collapse under abyssal pressure.

The student wants to provide an explanation and example of a "psychrolute." Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A. Deep-sea organisms living at extreme abyssal depths lack gas-filled swim bladders because extreme hydrostatic pressure would cause traditional air-filled organs to collapse immediately.
- B. The blobhead sculpin lives at ocean depths reaching 2,800 meters, utilizing low-density gelatinous tissue to float above the ocean floor without expending muscular energy.
- C. A psychrolute is an organism adapted to deep-sea conditions through soft tissue and minimal bone structure, such as the blobhead sculpin using gelatinous tissue to survive ocean pressure.
- D. The blobhead sculpin's gelatinous body structure exemplifies a psychrolute, illustrating how deep-sea animals maintain neutral buoyancy without relying on gas-filled swim bladders.

Exam 2 Reading and Writing: Module 2

Reading and Writing · Module 2

27 questions · 32 minutes · Read each passage and question carefully, then choose the best answer.

- 1** Without the discovery of hidden royal diaries, biographers cannot provide an entirely _____ assessment of Queen Ranavalona I's domestic policy, since her political rivals systematically expunged her official decrees from the national archives.

Which choice completes the text with the most logical and precise word or phrase?

- A. meticulous
- B. flawed
- C. deceptive
- D. skeletal

- 2** In the mid-nineteenth century, the inventor Louis Braille perfected a tactile reading system for the visually impaired. Because the raised-dot cell configuration corresponded to alphabetical characters so intuitively, his code was simple to master and thus quickly gained _____ utility: by 1860, the vast majority of specialized institutions across Europe utilized it.

Which choice completes the text with the most logical and precise word or phrase?

- A. universal
- B. meticulous
- C. unexpected
- D. sparse

- 3** In the mid-1800s, Japanese inventor Tanaka Hisashige designed several mechanical devices that simplified tasks previously requiring specialized training. Because ordinary craftspeople could master the devices with relative ease, their use quickly became _____ throughout workshops in major urban centers.

Which choice completes the text with the most logical and precise word or phrase?

- A. intermittent
- B. widespread
- C. unintended
- D. meticulous

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 4 The academic practice of formalizing macroeconomic models is frequently criticized as _____ because the frameworks rely heavily on highly idealized, unrealistic assumptions that lack immediate real-world utility. However, econometrician Marcus Vance has formulated a practical methodology that anchors abstract equations to everyday market data, rendering dense economic theory actionable.

Which choice completes the text with the most logical and precise word or phrase?

- A. abstruse
- B. commonplace
- C. superficial
- D. unyielding

- 5 Paleoclimatologists are confident that the ancient Eemian interglacial period experienced an abrupt sea-level rise due to ice sheet decay. They are much less confident, however, about which ice sheet contributed the most meltwater, since that depends on localized subglacial erosion rates that are largely untraceable. Geochemist Marcus Vance and colleagues recently investigated whether strontium isotope ratios in marine sediments could be used to determine internal ice-sheet states but concluded that this method could not sufficiently reveal ancient internal characteristics to allow the primary meltwater source to be firmly fixed.

Which choice best describes the function of the second sentence in the overall structure of the text?

- A. It identifies the historical question that Vance and colleagues set out to solve.
- B. It details a limitation of using strontium isotope ratios in geological sediment.
- C. It summarizes the overarching conclusion reached by Vance's research team.
- D. It explains how Vance's study reshaped modern paleoclimate consensus models.

- 6 The following text is adapted from Edith Wharton's 1920 novel *The Age of Innocence*. Newland Archer is struggling with his situation.

Archer looked at May, sitting there with her serene, untroubled beauty, and felt a sudden longing for a life that was less predictable. He loved her—or at least, he loved the idea of her—but he could not shake the feeling that he was trapped in a future that had already been decided for him by the very people who claimed to be protecting his happiness.

Which choice best states the main purpose of the text?

- A. To explain how Newland met May
- B. To describe May's beauty in great detail
- C. To argue that arranged marriages are always unhappy
- D. To reveal Newland's internal conflict between social duty and personal desire

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

7 Text 1

Standard cellular membranes maintain fluidity via a balanced distribution of saturated fatty acids, but psychrophilic Antarctic bacteria survive freezing sub-zero temperatures by synthesizing specialized polyunsaturated lipids that prevent intracellular crystallization. Microbiologists are studying this cold-adaptation strategy partly to improve the preservation longevity of human donor tissues, and they are recording remarkable experimental milestones. A particularly promising biochemical discovery is that both Antarctic bacteria and standard mammalian liver cells possess a desaturase enzyme gene linked to membrane fluidity regulation.

Text 2

When Ilona Petrescu and his laboratory team reported that standard mammalian liver cells, like Antarctic bacteria, possess a desaturase enzyme gene, it generated considerable enthusiasm among molecular biologists. Nevertheless, as the team pointed out, the gene likely controls lipid modification through vastly different regulatory pathways in warm-blooded mammals than it does in psychrophilic microbes. Petrescu has likened the enzyme to a molecular wedge that keeps cell walls pliable in freezing waters, but its functional expression in mammalian cellular environments remains unmapped.

Based on the texts, what would the author of Text 2 most likely say about Text 1's characterization of the finding involving the desaturase enzyme gene?

- A. It is reasonable given that Petrescu and her laboratory team have isolated how the desaturase gene functions in both cold-adapted and mammalian cells.
- B. It is unexpected given that Petrescu and her laboratory team's discoveries were initially met with immediate public skepticism.
- C. It is unfairly dismissive given the pioneering genetic milestones that Petrescu and her laboratory team have formally reported.
- D. It is overly optimistic given additional observations from Petrescu and her laboratory team regarding enzymatic activation thresholds.

- 8** Within deep-sea hydrothermal ecosystems, toxic hydrogen sulfide concentrations alongside fluctuating oxygen levels threaten respiratory integrity in giant tube worms; yet these plume-bearing invertebrates thrive without mouths or guts. Directed by Dr. Tessa Frost, marine ecologists noted a positive correlation between coelomic hemoglobin subunit multiplicity and ambient hydrogen sulfide toxicity, leading them to hypothesize that specialized dual-binding hemoglobins transport oxygen and sulfide simultaneously without autoinhibition.

Which finding, if true, would most directly support the researchers' hypothesis?

- A. Plume surface area measurements vary linearly with total body length in adult specimens.
- B. Bacteriocyte symbiont population densities fluctuate according to regional volcanic venting activity.
- C. Binding affinity assays confirm that vascular hemoglobin molecules attach oxygen and sulfide at distinct allosteric sites without competitive interference.
- D. Growth rate estimates derived from chitin tube increments remain consistent across deep-sea vent sites.

- 9** Born in 1876 to a farming family in rural Jutland, Denmark, Astrid Lindgren is remembered as an innovative agricultural geneticist. In a research paper for a botany seminar, a student claims that Lindgren's selective breeding programs successfully increased frost resistance in cereal crops—in her experiments, Lindgren cross-bred local rye strains with hardy northern grasses, yielding significantly higher survival rates during harsh winters.

Which finding, if true, would most directly support the student's claim?

- A. Lindgren's hybrid rye variants successfully survived temperatures dropping to minus twenty degrees Celsius, outyielding traditional crops by forty percent during the severe winter of 1912.
- B. Lindgren published a popular cookbook featuring traditional Danish pastries, breads, and seasonal vegetable soups.
- C. Lindgren served for three decades as the treasurer of the local agricultural cooperative board in her native county.
- D. Lindgren traveled extensively throughout central Europe to study classical architecture and medieval cathedral design.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 10** Microbiologist Dr. Ethan Hunt examined how the frequency of structural sound wave micro-vibrations influences the rate of cellular membrane permeability in cultured *E. coli* cells. Hunt suggested that membrane porousness does not respond linearly to auditory pitch. Instead, he proposed that structural resonance happens exclusively within a highly restricted frequency window; within this specific band, permeability spikes dramatically, whereas frequencies falling either below or above this narrow activation range leave cellular membranes completely unperturbed.

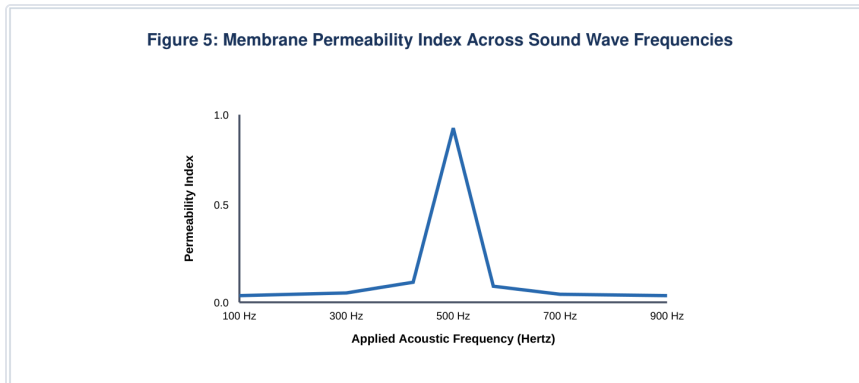


Figure 5: Membrane Permeability Index Across Sound Wave Frequencies

Which choice best uses data from the line graph to support Hunt's acoustic resonance proposal?

- A.** The cell membrane permeability index stays uniformly high across all acoustic frequencies that exceed 500 Hertz.
- B.** The cellular membrane permeability index remains near-baseline (under 0.1) at low frequencies and high frequencies, but exhibits an acute, isolated spike to nearly 0.9 precisely at the 500 Hertz threshold.
- C.** Acoustic frequencies between 100 Hz and 300 Hz provoke a gradual, predictable linear descent in cellular membrane structural integrity.
- D.** The structural resonance threshold is reached at 300 Hz, after which the cell membrane remains completely porous across the rest of the spectrum.

- 11** The anomalous isotopic fractionation observed in Ediacaran-period carbonate rocks has challenged traditional models of ancient oceanic oxygenation and carbon cycling. Examining drill cores from shallow-water shelf deposits, Eirik Solheim found that localized organic matter remineralization decoupled local carbon signals from global atmospheric trends, demonstrating that _____

Which choice most logically completes the text?

- A.** global oceanic chemistry during the Ediacaran was completely uniform and devoid of regional biogeochemical variation.
- B.** localized diagenetic processes can create misleading isotopic signatures that complicate global paleoclimate interpretations.
- C.** Precambrian carbonate formations contain zero organic carbon compounds suitable for stable isotope mass spectrometry.
- D.** atmospheric oxygen levels peaked during the Ediacaran period before experiencing a catastrophic collapse.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 12** To address the susceptibility of materials used in components of high-pressure synthetic gas compressors, such as reciprocating piston rod packings, to hydrogen gas embrittlement (metal fatigue cracking that is induced by high-pressure molecular hydrogen penetration and that often occurs under rapid gas pressure cycling), materials researchers have developed precipitation-hardening stainless steel alloys for producing packing rings. Testing the fatigue crack growth parameters of several commercially available alloy rings, Emmanuel Kone et al. found that in comparison with two conventional carbon steel rings, a cobalt-alloyed maraging steel ring exhibited a lower minimum crack propagation rate, a measure of the rate at which a hydrogen-exposed metal develops fractures at a constant gas pressure and cycling frequency. The finding suggests that _____

Which choice most logically completes the text?

- A.** the two conventional carbon steel rings likely hold similar potential for reducing the fracture resistance of materials exposed to high-pressure hydrogen, thus extending the life span of industrial compressors.
- B.** packing rings based on the two conventional carbon steel rings have metallurgical properties that may improve the mechanical and chemical stability of compressor equipment to a greater extent than do rings based on the cobalt-alloyed maraging steel ring.
- C.** packing rings containing the cobalt-alloyed maraging steel ring may have the ability to withstand gas pressure for a shorter period of time than can packing rings containing either of the two conventional carbon steel rings.
- D.** unlike the two conventional carbon steel rings, the cobalt-alloyed maraging steel ring can substantially inhibit hydrogen gas embrittlement, provided that gas pressures and cycling frequencies are consistent.

- 13** The following text is adapted from Theodore Dreiser's 1900 novel *Sister Carrie*.

Carrie Meeber's view of Chicago was filtered through a lens of naive romanticism. She saw the city as a grand stage where she was destined to play a leading role, ignoring the grim reality that a young woman alone in the city was often a victim of its indifference. Her arrival was not the beginning of a triumph, but the entry into a struggle she was ill-equipped to win.

What is the irony in Carrie's perception of the city?

- A.** She thinks it is dangerous when it is actually quite safe.
- B.** She anticipates success while being vulnerable to failure.
- C.** She expects to be welcomed, but the city is actually hostile.
- D.** She wants to be a performer but the city only offers office jobs.

- 14** The Greenland ice sheet contains a large amount of Earth's freshwater stored as ice. Scientists monitor the ice sheet because rising temperatures have contributed to increased melting in recent decades. Researchers study these changes to understand how they may affect global sea levels and climate patterns.

According to the text, why do scientists monitor the Greenland ice sheet?

- A.** It contains fewer glaciers than scientists expected.
- B.** It is becoming impossible to measure accurately.
- C.** It is producing new freshwater supplies too quickly.
- D.** Increased melting could affect global sea levels.

- 15** Russian matchstick-craft specialist Svetlana Petrov is renowned for his wooden locomotives _____ so intricate that they take months to construct, Petrov's replicas capture historic railway engines.

Which choice completes the text so that it conforms to the conventions of Standard English?

- | | |
|-----------------------------|--------------------------------|
| A. models, creations | B. models creations |
| C. models. Creations | D. models and creations |

- 16** Typically, stained parchment, broken wax seals, and smeared ink on historical charters lower a document's _____ when the issuer is unverified, such flaws invite scrutiny regarding authenticity.

Which choice completes the text so that it conforms to the conventions of Standard English?

- | | |
|-----------------------------|----------------------------|
| A. authenticity, yet | B. authenticity |
| C. authenticity, | D. authenticity yet |

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 17 Glass artisans have dubbed Loretta Finch the “mistress of molten silica,” an appellation printed in craft reviews since the 1970s. Finch herself rejected the term _____ judging that it overlooked her thermodynamic annealing methods alongside “glass theorist,” the moniker she preferred.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. term, however;
B. term; however,
C. term, however,
D. term, however

- 18 Long misidentified in historical municipal ledgers as a standard architectural drafting exercise, the 1923 blueprints for the Lovell Health House underwent comprehensive structural analysis by architectural historians who proved that the radical steel-frame residence—which integrated modern medical hygiene principles—was designed by pioneering modernist architect _____ Richard Neutra (1892–1970).

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. architect,
B. architect:
C. architect—
D. architect

- 19 Organized in 1485 to oversee sustainable timber harvesting in Franconia, the Nuremberg Timber Board was originally made up of four committees: woodland surveying, felling licensing, transport regulation, and reforestation oversight. By the end of the fifteenth century, the city council _____ its initial protected acreage.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. has enlarged
B. had enlarged
C. enlarged
D. will enlarge

- 20 Exercise physiologists Percival Vance and Julian Vance investigated how _____. In a clinical performance study conducted in 2024, they established that high-intensity interval workouts maximized cardiovascular efficiency within compressed timeframes.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. interval training improves cardiovascular output?
B. interval training improves cardiovascular output.
C. do interval training improve cardiovascular output?
D. do interval training improve cardiovascular output.

- 21 Danish Lysander King was a cartographer whose work helped navigators chart uncharted polar waters. Applauded in 1998 as the most accurate navigational survey of the Arctic, _____ led to safer maritime shipping routes.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. King's 1994 mapping of the seabed topography
B. King mapped the seabed topography in 1994; it
C. in 1994 King mapped the seabed topography, which
D. the seabed topography was mapped by King in 1994 and

- 22 A sponge spicule appears external to the soft tissue, stiffening its body like a scaffold. _____ the spicule is often incorrectly assumed to be a synthetic crystal, an abiotic mineral grain like that of beach sand or quartz gravel, when in fact it is a biogenic structure, a part of the sponge's skeletal support framework, more akin to a microscopic needle or a structural anchor.

Which choice completes the text with the most logical transition?

- A. However,
B. For instance,
C. Hence,
D. That being said,

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 23** Quantum computers utilize qubits capable of existing in multiple states simultaneously through the physical phenomenon of superposition. This fundamental computing advantage, _____ allows algorithms to evaluate complex combinatorial problems exponentially faster than classical binary architectures.

Which choice completes the text with the most logical transition?

- A.** for example,
C. instead,
- B.** in turn,
D. similarly,

- 24** In classical numismatics, silver tetradrachms minted by Hellenistic Seleucid monarchs during periods of military defeat are expected to exhibit weight standard reductions and debased metal alloys, whereas stable imperial issues would maintain high purity. Arthur Houghton and colleagues, _____ cataloged an intact hoard of heavy silver coins minted during a catastrophic dynastic civil war, that has preserved pristine silver fineness despite severe political collapse.

Which choice completes the text with the most logical transition?

- A. instead,
B. therefore,
C. moreover,
D. likewise,

- 25**

 - Chemotropism is the growth or directional movement of a plant or fungus in response to a chemical gradient.
 - Positive chemotropism involves growth toward a chemical stimulus, whereas negative chemotropism involves growth away from harmful chemicals.
 - It is controlled by localized cellular growth responses along concentration gradients.
 - Pollen tubes of flowering plants display positive chemotropism, growing toward calcium chemical signals released by the plant's ovule to enable fertilization.
 - Fungal hyphae grow toward peptone gradients released by decomposing organic matter.

The student wants to provide an explanation and example of "chemotropism." Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A. Controlled by localized cellular growth mechanisms, chemotropism allows plant roots and fungal structures to navigate surrounding soil environments by sensing nearby chemical concentration shifts.
- B. Pollen tubes grow toward calcium signals emitted by ovules during fertilization, while fungal hyphae extend toward peptone gradients released by decaying organic matter in soil.
- C. Chemotropism is directional plant or fungal growth in response to chemical gradients, such as pollen tubes growing toward calcium signals released by ovules during plant fertilization.
- D. Pollen tubes extending toward calcium chemical signals inside plant ovules demonstrate chemotropism, a cellular orientation mechanism studied by plant reproductive biologists.

- 26**

 - Conventional libraries require visitors to travel to physical locations to borrow materials.
 - Online databases provide digital access to academic articles, books, and research papers.
 - Students and researchers can search online databases using keywords and filters.
 - Online databases often allow users to locate relevant sources more quickly than searching through physical collections.
 - Universities frequently subscribe to online academic databases.

The student wants to explain an advantage of online academic databases. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A.** Universities frequently subscribe to online academic databases because researchers need access to scholarly materials.
- B.** Online academic databases provide digital access to research materials and allow users to quickly locate relevant sources without searching through physical collections.
- C.** Conventional libraries require visitors to travel to physical locations to borrow materials.
- D.** Online databases contain academic articles, books, and research papers that students and researchers can use.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 27** • Linear A and Linear B are ancient, undeciphered or partially deciphered scripts used by Bronze Age Aegean civilizations.
 • Linear A was used by the Minoan civilization on Crete from roughly 1800 to 1450 BCE and remains undeciphered by modern linguists.
 • Linear B was used by the Mycenaean civilization and was successfully deciphered in 1952 by Michael Ventris as an archaic form of written Greek.

The student wants to compare the decipherment status of Linear A and Linear B. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A.** Whereas Linear A remains completely undeciphered by modern linguists, Linear B was successfully deciphered in 1952 as an archaic form of Greek.
B. Linear A was developed by the Minoans on Crete, while Linear B was utilized by the Mycenaean civilization.
C. Both Linear A and Linear B are ancient writing scripts created during the Bronze Age by Aegean civilizations.
D. Linear B was deciphered in 1952 by Michael Ventris, shedding light on ancient Mycenaean record-keeping practices.

Exam 2 Math: Module 1

Math · Module 1

22 questions · 35 minutes · A calculator is allowed on all questions. For student-produced responses, enter your answer as a number or fraction.

- 1** A candle maker earns \$18 for each hour worked plus a bonus of \$4 for each candle she makes. She makes 12 candles each hour. The equation $18h + 4(12h) = 330$ represents the situation in which she earned a total of \$330 for working h hours. Which of the following is the best interpretation of $12h$ in this context?
- A.** Her hourly pay, in dollars
B. The total bonus, in dollars, she earned
C. The total number of candles she made in h hours
D. The number of hours she worked
- 2** If $f(x) = 3x - 8$ and $f(2k + 1) = k + 10$, what is the value of k ?
- A.** 1 **B.** 2
C. 3 **D.** 7
- 3** The function f is defined by $f(x) = 3x - 9$. For what value of x is $f(x) = 27$?
- A.** 0 **B.** 6
C. 12 **D.** 36
- 4** In the xy -plane, the lines $2x - 5y = 14$ and $kx + 3y = 21$ intersect at a point on the x -axis. What is the value of k ?
- Student-produced response: enter your answer. _____
- 5** If $5x + 3y = 17$ and $2x + 5y = 10$, what is the value of $8x + y$?
- Student-produced response: enter your answer. _____
- 6** A science fair quiz has 30 questions. Some are worth 2 points each and the rest are worth 4 points each, and the quiz is worth 96 points in all. The system below represents this situation, where x is the number of 2-point questions and y is the number of 4-point questions.
- $$x + y = 30$$
- $$2x + 4y = 96$$
- The solution to the system is $(12, 18)$. Which of the following is the best interpretation of this solution?
- A.** The quiz has 12 four-point questions and 18 two-point questions.
B. The 2-point questions account for 12 points and the 4-point questions account for 18 points.
C. The quiz has 12 two-point questions and 18 four-point questions.
D. The quiz has 12 more two-point questions than four-point questions.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

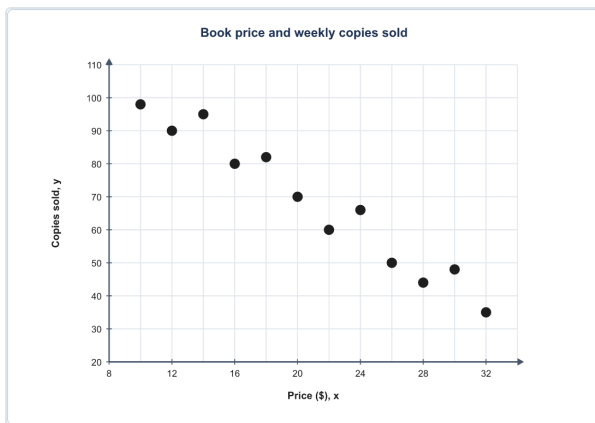
- 7 Which of the following is the solution set of the inequality $3(2 - x) - 4x < 5(x - 6)$?
A. $x < -3$ B. $x < 3$
C. $x > -3$ D. $x > 3$
- 8 When the expression $(x + 4)(2x^2 + kx + 3)$ is expanded, the coefficient of x^2 is 17. What is the value of k ?
Student-produced response: enter your answer. _____
- 9 The function f is defined by $f(x) = 2x^2 - 5x + 4$. What is the value of $f(-3)$?
A. 1 B. 7
C. 37 D. 55
- 10 The graphs of $xy = 20$ and $y = 5x$ intersect at two points in the xy -plane. What is the y -coordinate of the point of intersection with a positive x -coordinate?
Student-produced response: enter your answer. _____
- 11 In the equation $x^2 - 9x + c = 0$, c is a constant. The two solutions to the equation differ by 3. What is the value of c ?
Student-produced response: enter your answer. _____
- 12 For what value of k does the equation $(x - 3)(x + 5) = k$ have exactly one real solution?
A. -64 B. -16
C. -15 D. 16
- 13 A ball is thrown upward from ground level. The ball's height above the ground, in feet, is a quadratic function of the time t , in seconds, since it was thrown. The ball is 96 feet above the ground at $t = 1$ and again at $t = 6$. At what time t does the ball return to the ground?
A. 3.5 B. 7
C. 9.5 D. 14
- 14 What is the minimum value of the function g defined by $g(x) = (x - 5)^2 - 8$?
A. -8 B. -5
C. 5 D. 8
- 15 A potter mixes 2 pounds of clay with 5 cups of water to make slip. At this rate, how many cups of water should be mixed with 8 pounds of clay?
A. 3.2 B. 11
C. 13 D. 20
- 16 The number of daily orders at a food truck increased by 10% in June and then by 20% in July. In August, the number of orders decreased by 30%. Compared with the number of daily orders before June, the number in August is
A. a decrease of 7.6% B. no change
C. an increase of 2% D. an increase of 32%
- 17 A rowing club has 2,400 members. The club emailed a satisfaction survey about its new boathouse to every member. Of the 2,400 members, 310 responded, and 74% of the respondents said they were satisfied. Which of the following conclusions is best supported by the survey results?
A. About 74% of all 2,400 members of the club are satisfied with the new boathouse.
B. The new boathouse caused 74% of the members of the club to become satisfied with it.
C. 74% of the 310 respondents were satisfied, but this may not describe all 2,400 members because the respondents chose to respond.
D. Most of the 2,090 members who did not respond to the survey are dissatisfied with the boathouse.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

18



Book price and weekly copies sold

The scatterplot shows the price, x , in dollars, and the number of copies sold in one week, y , for each of 12 paperback books at a bookstore. Which of the following statements is supported by the scatterplot?

- A. Every increase in price was followed by a decrease in the number of copies sold.
- B. Books with higher prices tended to sell fewer copies.
- C. Books with higher prices tended to sell more copies.
- D. There is no relationship between the price of a book and the number of copies sold.

19

The floor of a rectangular bakery kitchen measures 12 feet by 10 feet. The owner will cover the entire floor with square tiles that are 2 feet on each side. The tiles will not be cut or overlap. How many tiles are needed?

- A. 30
- B. 44
- C. 60
- D. 120

20

A bakery packs slices in two triangular boxes whose faces are similar triangles ABC and DEF, where AB corresponds to DE. $AB = 3$ centimeters and $DE = 6$ centimeters. The area of triangle DEF is 48 square centimeters. What is the area, in square centimeters, of triangle ABC?

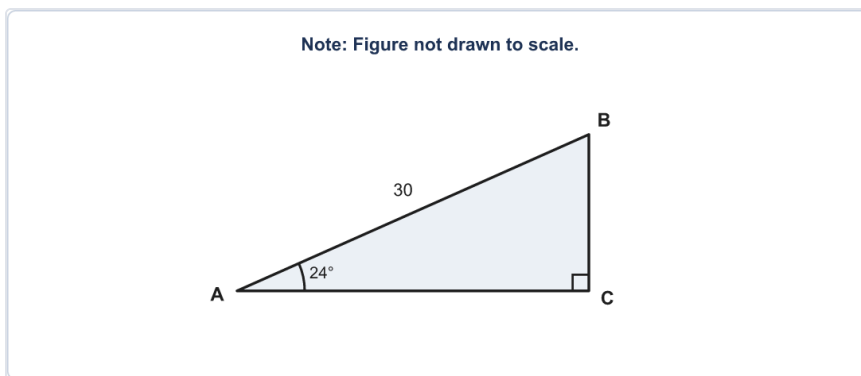
- A. 12
- B. 24
- C. 96
- D. 192

21

In triangle PQR, angle Q is a right angle and $\tan P = 8/15$. What is the value of $\sin P$?

- A. $8/17$
- B. $8/15$
- C. $15/17$
- D. $17/8$

22



Note: Figure not drawn to scale.

In the figure, triangle ABC has a right angle at C and $AB = 30$. If $\sin 24^\circ = 0.4$, what is the length of BC?

- A. 12
- B. 18
- C. 30
- D. 75

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Exam 2 Math: Module 2**Math · Module 2**

22 questions · 35 minutes · A calculator is allowed on all questions. For student-produced responses, enter your answer as a number or fraction.

- 1** Three seats in the same row of a theater have consecutive odd seat numbers, and the sum of the three seat numbers is 117. What is the least of the three seat numbers?
- A. 37
B. 38
C. 39
D. 41
- 2** The graph of the linear function f in the xy -plane has a slope of 3 and passes through the point $(2, 5)$. The graph of the function g is obtained by translating the graph of f 4 units to the left and 1 unit down. Which of the following defines $g(x)$?
- A. $g(x) = 3x - 14$
B. $g(x) = 3x + 10$
C. $g(x) = 3x + 12$
D. $g(x) = 3x - 2$
- 3** In the xy -plane, the graphs of $f(x) = (k + 1)x + 6$ and $g(x) = 3kx + 4$, where k is a constant, intersect at the point with x -coordinate 2. What is the value of k ?
- A. 1
B. 2
C. 3
D. 4
- 4** The lines $2x + y = 11$ and $x - y = 1$ intersect at point P in the xy -plane. The line $kx + 3y = 20$, where k is a constant, also passes through point P . What is the value of k ?
- Student-produced response: enter your answer. _____
- 5** A greenhouse mixes two nutrient solutions. Solution A is 20% fertilizer by volume and solution B is 50% fertilizer by volume. The greenhouse needs 60 liters of a mixture that is 38% fertilizer by volume. How many liters of solution A are needed?
- Student-produced response: enter your answer. _____
- 6** $(2/3)x + (1/4)y = 6$
 $(5/6)x - (1/8)y = 4$
The solution to the given system of equations is (x, y) . What is the value of $x + y$?
- A. 8
B. 14
C. 20
D. 48
- 7** Which of the following inequalities is equivalent to $-(x - 4)/3 \geq (x + 5)/2$?
- A. $x \geq -7/5$
B. $x \leq -23/5$
C. $x \leq -7/5$
D. $x \geq -23/5$
- 8** If $x^{3/2} = 216$ and $x > 0$, what is the value of x ?
- Student-produced response: enter your answer. _____
- 9** For $x > 0$, which of the following expressions is equivalent to $2/x + 3/(x + 1)$?
- A. $(5x + 2)/(x(x + 1))$
B. $(3x + 2)/(x(x + 1))$
C. $5/(2x + 1)$
D. $(5x + 1)/(x(x + 1))$
- 10** What is the negative solution to the equation $2x^2 + x - 21 = 0$?
- Student-produced response: enter your answer. _____
- 11** $x^2 + y^2 = 20$ and $y = 2x + k$. In the given system, k is a positive constant and the system has exactly one solution (x, y) . What is the value of k ?
- Student-produced response: enter your answer. _____

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 12 What is the solution to the equation $\sqrt{x + 7} = x - 5$?

A. 2
B. 9
C. 11
D. 18

- 13 A coffee roaster's sales S , in thousands of dollars, t years after 2020 are modeled by $S(t) = 500(1.4)^{t/3}$. Which of the following functions gives the same sales in terms of m , the number of months after 2020?

A. $S(m) = 500(1.4)^{4m}$
B. $S(m) = 500(1.4)^{m/12}$
C. $S(m) = 500(1.4)^{m/36}$
D. $S(m) = 500(1.4)^{36m}$

14

Values of the function f

x	0	1	2	3
$f(x)$	5	10	20	40

Values of the function f

The table shows values of the function f . Which of the following equations defines f ?

A. $f(x) = 5x + 5$
B. $f(x) = 5x^2 + 5$
C. $f(x) = 10(2)^x$
D. $f(x) = 5(2)^x$

- 15 A campground store sells firewood bundles in several package sizes. Which of the following packages has the lowest price per bundle?

A. 4 bundles for \$15.00
B. 6 bundles for \$21.60
C. 9 bundles for \$31.50
D. 12 bundles for \$43.20

- 16 At a language school, 60% of the students are adults. Of the adult students, 45% take evening classes, and of the students who are not adults, 20% take evening classes. What percent of all students at the school take evening classes?

A. 27%
B. 32.5%
C. 35%
D. 65%

17

Loads of laundry washed by 40 customers

Number of loads	Number of customers
1	8
2	12
3	10
4	6
5	4

Loads of laundry washed by 40 customers

The table shows the number of loads of laundry washed by each of 40 customers at a laundromat. Additional customers then arrive, and each washes 5 loads. What is the least number of additional customers needed so that the mean number of loads washed for all customers is at least 3?

A. 3
B. 5
C. 6
D. 7

- 18 A delivery service surveyed 200 customers. Of these, 140 placed their order using the app and 60 placed their order by phone. Of the customers who used the app, 91 chose same-day delivery, and of the customers who used the phone, 24 chose same-day delivery. If a customer who chose same-day delivery is selected at random, what is the probability that the customer placed the order by phone?

A. $24/115$
B. $24/60$
C. $115/200$
D. $91/115$

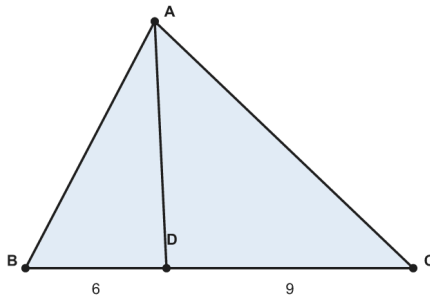
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

19

Note: Figure not drawn to scale.



Note: Figure not drawn to scale.

In the figure, point D lies on side BC of triangle ABC, with $BD = 6$ and $DC = 9$. The area of triangle ADC is 63. What is the area of triangle ABC?

- A. 42
B. 63
C. 84
D. 105

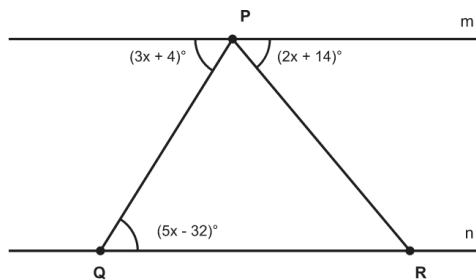
20

A print shop cuts posters that each measure 60 centimeters by 90 centimeters from a roll of paper with a total area of 270 square meters. If no paper is wasted, how many posters can be cut from the roll? (1 meter = 100 centimeters)

- A. 5
B. 50
C. 500
D. 5,000

21

Note: Figure not drawn to scale.



Note: Figure not drawn to scale.

In the figure, line m is parallel to line n . Points Q and R lie on line n , and point P lies on line m . What is the measure, in degrees, of angle QPR ?

- A. 50°
B. 58°
C. 72°
D. 108°

22

For acute angles x° and y° , $\sin x^\circ = \cos y^\circ$. If $x = 2y - 15$, what is the value of y ?

- A. 15
B. 25
C. 35
D. 55

Key Practice Exam 2: Answers and Explanations

Check each module, then use the skill map to find what to review. Every explanation shows why the right answer works and why the tempting wrong answers fail.

READING AND WRITING · MODULE 1 · ANSWER KEY

1 D 2 A 3 B 4 B 5 A 6 A 7 C 8 B 9 A 10 A 11 B 12 A 13 D 14 C 15 D 16 D 17 D 18 A 19 C 20 B 21 C 22 B 23 B 24 A
25 A 26 B 27 C

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

READING AND WRITING · MODULE 2 · ANSWER KEY

1 A 2 A 3 B 4 A 5 A 6 D 7 D 8 C 9 A 10 B 11 B 12 D 13 B 14 D 15 C 16 A 17 C 18 D 19 C 20 B 21 A 22 A 23 B 24 A
25 C 26 B 27 A

MATH · MODULE 1 · ANSWER KEY

1 C 2 C 3 C 4 3 5 24 6 C 7 D 8 9 9 C 10 10 11 18 12 B 13 B 14 A 15 D 16 A 17 C 18 B 19 A 20 A 21 A 22 A

MATH · MODULE 2 · ANSWER KEY

1 A 2 B 3 A 4 $11/4$ 5 24 6 B 7 C 8 36 9 A 10 $-7/2$ 11 10 12 B 13 C 14 D 15 C 16 C 17 D 18 A 19 D 20 C 21 C 22 C

SKILL MAP: WHICH QUESTIONS TEST WHICH SKILL

Module	Skill	Questions
R&W M1	Words in Context	1, 2, 3, 4
R&W M1	Text Structure	5
R&W M1	Author's Purpose	6
R&W M1	Cross-Text Connections	7
R&W M1	Textual Evidence	8, 9
R&W M1	Quantitative Evidence	10
R&W M1	Inferences	11, 12
R&W M1	Central Ideas and Details	13, 14
R&W M1	Boundaries	15, 16, 17, 18
R&W M1	Form, Structure, and Sense	19, 20, 21
R&W M1	Transitions	22, 23, 24
R&W M1	Rhetorical Synthesis	25, 26, 27
R&W M2	Words in Context	1, 2, 3, 4
R&W M2	Text Structure	5
R&W M2	Author's Purpose	6
R&W M2	Cross-Text Connections	7
R&W M2	Textual Evidence	8, 9
R&W M2	Quantitative Evidence	10
R&W M2	Inferences	11, 12
R&W M2	Central Ideas and Details	13, 14
R&W M2	Boundaries	15, 16, 17, 18
R&W M2	Form, Structure, and Sense	19, 20, 21
R&W M2	Transitions	22, 23, 24
R&W M2	Rhetorical Synthesis	25, 26, 27
Math M1	Linear Equations (1 Var)	1
Math M1	Linear Functions	2, 3
Math M1	Linear Equations (2 Var)	4
Math M1	Systems of Equations	5, 6
Math M1	Linear Inequalities	7
Math M1	Equivalent Expressions	8, 9
Math M1	Nonlinear Equations	10, 11, 12
Math M1	Nonlinear Functions	13, 14
Math M1	Ratios and Rates	15

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Module	Skill	Questions
Math M1	Percentages	16
Math M1	Statistical Claims	17
Math M1	Two-Variable Data	18
Math M1	Area and Volume	19
Math M1	Lines, Angles, Triangles	20
Math M1	Right Triangles and Trig	21, 22
Math M2	Linear Equations (1 Var)	1
Math M2	Linear Functions	2, 3
Math M2	Linear Equations (2 Var)	4
Math M2	Systems of Equations	5, 6
Math M2	Linear Inequalities	7
Math M2	Equivalent Expressions	8, 9
Math M2	Nonlinear Equations	10, 11, 12
Math M2	Nonlinear Functions	13, 14
Math M2	Ratios and Rates	15
Math M2	Percentages	16
Math M2	One-Variable Data	17
Math M2	Probability	18
Math M2	Area and Volume	19, 20
Math M2	Lines, Angles, Triangles	21
Math M2	Right Triangles and Trig	22

R&W 1 Reading and Writing, Module 1: Explanations

1. Answer: D · WORDS IN CONTEXT · HARD

Correct answer: D. 'Leveraged' means used to advantage. The team used gamma ray scattering differences to analyze sealed jars.

Why the other choices are wrong:

- A) 'Posited' means proposed, which does not fit.
- B) 'Dismissed' means treated as unworthy of consideration, which contradicts the demonstration, so it lacks the sense of 'leveraged' that the blank needs.
- C) 'Overstretched' means extended beyond a reasonable limit, which does not fit the sentence, where the blank needs the sense of 'leveraged'.

2. Answer: A · WORDS IN CONTEXT · MEDIUM

Correct answer: A. 'Execute' means to carry out. The executive branch carries out the legislature's laws by writing protocols and delivering services.

Why the other choices are wrong:

- B) 'Presume' means to suppose without proof, which does not fit the sentence, where the blank needs the sense of 'execute'.
- C) 'Improvise' means to create without preparation, which contradicts the sentence, where the blank needs the sense of 'execute'.
- D) 'Mimic' means to imitate, which does not fit the sentence, where the blank needs the sense of 'execute'.

3. Answer: B · WORDS IN CONTEXT · MEDIUM

Correct answer: B. 'Constructed' means built. Rostova engineered hydroponic channels and arranged mosses over steel frames to build living walls.

Why the other choices are wrong:

- A) 'Rubber-stamped' means approved without real examination, which does not fit the sentence, where the blank needs the sense of 'constructed'.
- C) 'Opposed' means was against, which does not fit the sentence, where the blank needs the sense of 'constructed'.
- D) 'Salvaged' means rescued from loss or damage, which does not fit the sentence, where the blank needs the sense of 'constructed'.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

4. Answer: B · WORDS IN CONTEXT · EASY

Correct answer: B. 'Produced' means made. Kim mapped woodblock gradients into polygons to make a digital landscape.

Why the other choices are wrong:

- A) 'Validated' means confirmed, which does not fit the sentence, where the blank needs the sense of 'produced'.
- C) 'Disputed' means argued over, which does not fit the sentence, where the blank needs the sense of 'produced'.
- D) 'Restored' means returned to former condition, which does not fit the sentence, where the blank needs the sense of 'produced'.

5. Answer: A · TEXT STRUCTURE · MEDIUM

Correct answer: A. It explains the scientific rationale/justification for using this specific Earth location for alien simulations.

Why the other choices are wrong:

- B) It claims the sentence contrasts the biochemical makeup of the lakes with Europa's oceans, but it instead says the lakes resemble Europa's conditions.
- C) It claims a list of experimental failures, but the text mentions no failures and presents the research as promising.
- D) It claims an argument that life on Earth originated in subglacial lakes, which the text never makes; the sentence concerns analog conditions for Europa.

6. Answer: A · AUTHOR'S PURPOSE · EASY

Correct answer: A. The narrator lists her strengths ("swift," "precise," "rhythmic") to argue why she has a chance to make the milking team.

Why the other choices are wrong:

- B) Sheep farming is not mentioned.
- C) No step-by-step guide on processing milk is provided.
- D) Award criteria are not discussed.

7. Answer: C · CROSS-TEXT CONNECTIONS · MEDIUM

Correct answer: C. Text 2 indicates that overland pack-animal trails accounted for the primary commercial distribution of obsidian.

Why the other choices are wrong:

- A) Text 2 argues against maritime cabotage being the sole method.
- B) Text 2 does not state coastal waters were impassable.
- D) Text 2 notes obsidian was distributed from specific geologic sources to settlements.

8. Answer: B · TEXTUAL EVIDENCE · HARD

Correct answer: B. It directly supports the authors' claim that JITU harms comprehensive theoretical understanding, which is necessary for long-term career adaptability.

Why the other choices are wrong:

- A) It correlates JITU adoption with economic downturns and job switching speed rather than domain mastery.
- C) It states JITU shows minimal correlation with starting salaries or job tenure.
- D) It links micro-credentials to software costs and hiring rates rather than foundational knowledge fragmentation.

9. Answer: A · TEXTUAL EVIDENCE · EASY

Correct answer: A. It is correct because increased bead retrieval by the desert-viewing group directly confirms the spatial awe hypothesis.

Why the other choices are wrong:

- B) This option is incorrect because uniform stillness does not measure helping.
- C) This option is incorrect because spatial vocabulary is unrelated to altruistic acts.
- D) This option is incorrect because baseline noticing does not test prosocial behavior.

10. Answer: A · QUANTITATIVE EVIDENCE · MEDIUM

Correct answer: A. The \$40,000 area sits at 38%, the top of the six points, and the \$90,000 area is lowest at 22%. Rent share generally falls as income rises, consistent with the economist's suspicion.

Why the other choices are wrong:

- B) The \$60,000 area had a 32% rent share but the \$70,000 area had 28%, so the shares are different.
- C) The values are reversed: rent share was 38% at \$40,000 and 22% at \$90,000, so rent share fell as income rose.
- D) The \$50,000 area had a rent share of 34%, which is below 38%, not above it.

11. Answer: B · INFERENCES · EASY

Correct answer: B. Spring systems with regulated ducting were more stable, so deliberate adjustments to how resources are distributed can improve the functioning of vulnerable regions.

Why the other choices are wrong:

- A) Says redirecting resources is harmful to fragile environments, contradicted by the greater stability of managed springs.
- C) Claims untouched desert areas cannot survive without help, an extreme claim the comparison does not support.
- D) Says large industrial projects are necessary, an overreach beyond the small-scale regulated ducting studied.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

12. Answer: A • INFERENCES • HARD

Correct answer: A. The new material showed a lower minimum elongation loss rate than the two comparison materials under constant conditions, which supports the claim that it can substantially inhibit the damage, provided those conditions stay consistent.

Why the other choices are wrong:

B) A lower minimum elongation loss rate indicates greater resistance to damage, so the new material should last longer, not shorter, which makes this the reverse of what the finding suggests.

C) The study found a difference between the new material and the two comparison materials, so it does not support treating them as equivalent; the option is also incoherent, since reducing stability would not extend service life.

D) This reverses the finding: the new material, not the comparison materials, showed the lower minimum elongation loss rate, so it is the better performer and the comparison materials cannot be the more protective choice.

13. Answer: D • CENTRAL IDEAS AND DETAILS • MEDIUM

Correct answer: D. The passage explains that astronomers identified planets by observing regular, repeating dips in a star's brightness.

Why the other choices are wrong:

A) The planets were too distant to observe directly with telescopes, so photographing them was not how they were identified.

B) Planet temperatures are not measured; the evidence was a star's brightness dimming at regular intervals.

C) No samples of planetary atmospheres are collected, and the method relied on starlight observations from far away.

14. Answer: C • CENTRAL IDEAS AND DETAILS • EASY

Correct answer: C. The text says the narrator counted the wooden telegraph poles flashing past the window.

Why the other choices are wrong:

A) Mateo, not the narrator, worked on crossword puzzles.

B) The narrator only wondered about life in the agricultural communities; reading about them is not described.

D) The text does not say the narrator bought snacks.

15. Answer: D • BOUNDARIES • MEDIUM

Correct answer: D. A comma correctly joins the main clause to the dependent "while" clause that expresses the contrast; no stronger punctuation is needed.

Why the other choices are wrong:

A) A period leaves "While ..." as a sentence fragment, because it begins a dependent clause with no main clause of its own.

B) A colon signals that what follows explains or elaborates the first clause, which does not fit a contrast introduced by "while".

C) A semicolon must join two independent clauses, but "while ..." begins a dependent clause, so the semicolon is incorrect here.

16. Answer: D • BOUNDARIES • HARD

Correct answer: D. The title and name form one essential, integrated noun phrase before the verb, so no commas are needed anywhere.

Why the other choices are wrong:

A) A comma between the title and the name splits one essential noun phrase for no reason.

B) A comma after the name separates the subject from its verb, which is not allowed.

C) Commas on both sides wrongly treat the name as a nonessential interrupter, although it is needed to identify the person.

17. Answer: D • BOUNDARIES • MEDIUM

Correct answer: D. A period correctly establishes a full sentence boundary before the subsequent clause.

Why the other choices are wrong:

A) Incorrect because em-dashes fail to resolve the clause boundary error.

B) Incorrect because fusing the clauses without punctuation produces a run-on structure.

C) Incorrect because a comma creates an illegal comma splice between independent clauses.

18. Answer: A • BOUNDARIES • MEDIUM

Correct answer: A. A period ends the first independent clause, and "Though, as a ..., [name] would help ..." then forms a complete second sentence with "though" as a transitional opener.

Why the other choices are wrong:

B) With no punctuation at all, two independent clauses run together as a run-on sentence.

C) A colon after "though" is misplaced and still leaves two independent clauses improperly joined.

D) Commas around "though" leave two independent clauses joined only by a comma, which is a comma splice.

19. Answer: C • FORM, STRUCTURE, AND SENSE • MEDIUM

Correct answer: C. The subject is the gerund phrase beginning "Demolishing", which is grammatically singular despite the plural nouns inside it, so it takes the singular present-tense verb "helps".

Why the other choices are wrong:

A) The plural present progressive "are helping" does not agree with the singular gerund-phrase subject beginning "Demolishing", so it creates a subject-verb agreement error.

B) The plural verb form "help" does not agree with the singular gerund-phrase subject beginning "Demolishing"; the plural nouns inside the phrase do not change its number.

D) The plural present perfect "have helped" does not agree with the singular gerund-phrase subject beginning "Demolishing", so it creates a subject-verb agreement error.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

20. Answer: B • FORM, STRUCTURE, AND SENSE • EASY

Correct answer: B. The plural pronoun "they" agrees in number with its plural antecedent "tenants".

Why the other choices are wrong:

- A) The singular pronoun "one" does not agree in number with the plural "tenants".
- C) The singular pronoun "it" does not agree in number with the plural "tenants".
- D) The second-person pronoun "you" shifts away from the third-person plural "tenants".

21. Answer: C • FORM, STRUCTURE, AND SENSE • MEDIUM

Correct answer: C. "This" is the singular demonstrative that refers back to the entire preceding action as one event, matching "the first time".

Why the other choices are wrong:

- A) The plural demonstrative "those" has no plural antecedent; the preceding clause is a single event that requires the singular "this".
- B) The indefinite quantifier "some" cannot serve as a demonstrative pointing back to the preceding event and does not fit before "the first time".
- D) The plural demonstrative "these" has no plural antecedent; the preceding clause is a single event that requires the singular "this".

22. Answer: B • TRANSITIONS • MEDIUM

Correct answer: B. "Specifically" is correct because it properly elaborates on ozone depletion by explaining the precise chemical breakdown caused by chlorofluorocarbons.

Why the other choices are wrong:

- A) "Similarly" is incorrect because it suggests a resemblance between separate phenomena rather than an elaboration of a chemical process.
- C) "Hence" is incorrect because it indicates a final result rather than detailing the specific catalytic destruction mechanism.
- D) "Nevertheless" is incorrect because it introduces a contrastive turn, which conflicts with the direct explanatory tone.

23. Answer: B • TRANSITIONS • EASY

Correct answer: B. "by contrast" is correct because quantum subatomic particles behave probabilistically, opposing macroscopic classical physics objects.

Why the other choices are wrong:

- A) "for example" is incorrect because it introduces an illustration rather than a physical contrast.
- C) "accordingly" is incorrect because it denotes a result rather than a physical contrast.
- D) "furthermore" is incorrect because it adds supplementary information rather than showing a contrast.

24. Answer: A • TRANSITIONS • EASY

Correct answer: A. "Conversely" is correct because it establishes an opposing contrast between Jack's descent into savagery and Ralph's adherence to democratic order.

Why the other choices are wrong:

- B) "Meanwhile" is incorrect because it denotes concurrent timing rather than a philosophical opposition.
- C) "Nevertheless" is incorrect because of clausal preamble constraints.
- D) "Furthermore" is incorrect because it adds additive details rather than an opposition.

25. Answer: A • RHETORICAL SYNTHESIS • HARD

Correct answer: A. It contrasts the two views of social order: functionalism sees order as cohesion among interdependent parts, while conflict theory sees it as the product of power struggles between social classes.

Why the other choices are wrong:

- B) It gives the dates of the two theories and says they share a goal, so it contrasts nothing about social order.
- C) It focuses on specific concepts (organic solidarity, bourgeoisie and proletariat) rather than the two views of social order.
- D) It describes the views but frames the contrast as power struggles versus division of labour and cohesion, which is less precise about how each theory views social order.

26. Answer: B • RHETORICAL SYNTHESIS • EASY

Correct answer: B. This option identifies the project and explains the finding that the coating improved solar panel efficiency.

Why the other choices are wrong:

- A) This option describes the experimental comparison but does not explain the result.
- C) This choice only describes what researchers tested without presenting their conclusion.
- D) This is a general statement about solar panels and does not summarize the specific research.

27. Answer: C • RHETORICAL SYNTHESIS • HARD

Correct answer: C. Defines psychrolute (adapted to deep sea via soft tissue/minimal bone) AND provides the example (blobhead sculpin). A/B/D miss either the explanation or the example.

Why the other choices are wrong:

- A) It explains the lack of swim bladders in deep-sea organisms but never defines a psychrolute or names an example such as the blobhead sculpin.
- B) It describes the sculpin's depth and gelatinous body as an example but never defines what a psychrolute is, so the explanation is missing.
- D) It offers the sculpin as an example and discusses buoyancy, but its definition is weak because it focuses on swim bladders rather than adaptation to deep-sea conditions.

R&W 2 Reading and Writing, Module 2: Explanations

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

1. Answer: A · WORDS IN CONTEXT · MEDIUM

Correct answer: A. 'Meticulous' means extremely thorough and careful. Without the diaries, biographers cannot give an entirely thorough assessment because decrees were expunged.

Why the other choices are wrong:

B) 'Flawed' describes an assessment with defects; 'cannot provide an entirely flawed assessment' would make no sense.

C) 'Deceptive' means misleading, which does not fit the idea of missing information.

D) 'Skeletal' means bare-bones; the missing records make a bare-bones assessment the likely result, not something they cannot provide.

2. Answer: A · WORDS IN CONTEXT · EASY

Correct answer: A. 'Universal' means used by all or nearly all. By 1860 the vast majority of institutions across Europe used Braille, so its utility was universal.

Why the other choices are wrong:

B) 'Meticulous' means careful, which does not describe how common it was.

C) 'Unexpected' means not anticipated, which does not match the explained reason.

D) 'Sparse' means thinly scattered, contradicting the vast majority.

3. Answer: B · WORDS IN CONTEXT · EASY

Correct answer: B. 'Widespread' means found over a large area. Ordinary craftspeople mastered the devices easily, so use spread through workshops in major urban centers.

Why the other choices are wrong:

A) 'Intermittent' means occasional, which contradicts quick use throughout workshops.

C) 'Unintended' means not planned, which does not describe how common the use was.

D) 'Meticulous' means very careful, which describes the way work is done, not how common the use was.

4. Answer: A · WORDS IN CONTEXT · HARD

Correct answer: A. 'Abstruse' means hard to understand. Idealized assumptions make macroeconomic models seem hard to understand.

Why the other choices are wrong:

B) 'Commonplace' means ordinary, which contradicts criticism of being unrealistic.

C) 'Superficial' means shallow, which does not match 'dense theory'.

D) 'Unyielding' means refusing to bend, which does not describe academic practice.

5. Answer: A · TEXT STRUCTURE · MEDIUM

Correct answer: A. It identifies the historical paleoclimate question that Vance and colleagues set out to solve.

Why the other choices are wrong:

B) The sentence says nothing about strontium isotope ratios; their limitation is addressed in the final sentence, while this one states the uncertainty over which ice sheet supplied the most meltwater.

C) Vance's team reached only a negative conclusion, which appears in the third sentence; the second sentence states an open question rather than summarizing any research result.

D) The passage does not claim Vance's study reshaped paleoclimate models; the sentence merely states uncertainty about the main meltwater source, and the study failed to settle it.

6. Answer: D · AUTHOR'S PURPOSE · MEDIUM

Correct answer: D. Archer looks at May and longs for a less predictable life, feeling trapped in a future decided by others, revealing his conflict between social duty and personal desire.

Why the other choices are wrong:

A) The passage gives no account of how the two met; it presents Archer's present thoughts as he looks at May.

B) May's beauty is mentioned only in passing as serene and untroubled; the passage focuses on Archer's feelings, not on describing her.

C) The passage shows one man's feelings of being trapped and makes no sweeping claim that arranged marriages are always unhappy; this overstates.

7. Answer: D · CROSS-TEXT CONNECTIONS · MEDIUM

Correct answer: D. Text 2 notes vastly different regulatory pathways, making Text 1's translation overly optimistic.

Why the other choices are wrong:

A) Text 2 states functional expression remains unmapped.

B) Text 2 notes enthusiasm.

C) Text 1 is positive, not dismissive.

8. Answer: C · TEXTUAL EVIDENCE · MEDIUM

Correct answer: C. It confirms that vascular hemoglobin molecules attach oxygen and sulfide at distinct allosteric sites without interference.

Why the other choices are wrong:

A) Plume surface area relative to length concerns general morphology rather than gas binding.

B) Symbiont densities reflect volcanic activity rather than hemoglobin biochemistry.

D) Chitin tube growth rates reflect structural secretion rather than internal respiratory transport.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

9. Answer: A • TEXTUAL EVIDENCE • HARD

Correct answer: A. This choice is correct because hybrid rye variants surviving extreme frost and outyielding traditional crops directly supports increased crop hardiness.

Why the other choices are wrong:

B) This option describes a cookbook featuring Danish pastries and vegetable soups. It therefore would not directly support the claim being made.

C) This option notes serving as a cooperative board treasurer. It therefore would not directly support the claim being made.

D) This option discusses traveling to study classical architecture. It therefore would not directly support the claim being made.

10. Answer: B • QUANTITATIVE EVIDENCE • HARD

Correct answer: B. Permeability stays below about 0.1 at low and high frequencies, then spikes to roughly 0.9 at 500 Hz alone. This isolated peak inside a narrow window, with unperturbed membranes on either side, is the resonance pattern Hunt proposes.

Why the other choices are wrong:

A) Above 500 Hz permeability falls back to roughly 0.03 to 0.08, so it does not stay uniformly high, which contradicts the data.

C) Between 100 Hz and 300 Hz the line is nearly flat at about 0.03 to 0.05 rather than descending, and the figure measures permeability, not structural integrity.

D) Permeability at 300 Hz is only about 0.05, and after the 500 Hz peak it drops back near baseline, so the membrane does not stay porous at higher frequencies.

11. Answer: B • INFERENCES • HARD

Correct answer: B. Localized organic matter remineralization decoupling local signals from global trends demonstrates that diagenetic processes can create misleading isotopic interpretations.

Why the other choices are wrong:

A) Localized organic remineralization decoupled local signals, proving oceanic chemistry was not completely uniform.

C) The premise relies on analyzing organic matter in carbonate rocks, refuting the claim that they contain zero organic carbon.

D) The text addresses Ediacaran carbonate isotopic fractionation and diagenesis, not a catastrophic collapse of atmospheric oxygen.

12. Answer: D • INFERENCES • HARD

Correct answer: D. The new material showed a lower minimum crack propagation rate than the two comparison materials under constant conditions, which supports the claim that it can substantially inhibit the damage, provided those conditions stay consistent.

Why the other choices are wrong:

A) The study found a difference between the new material and the two comparison materials, so it does not support treating them as equivalent; the option is also incoherent, since reducing stability would not extend service life.

B) This reverses the finding: the new material, not the comparison materials, showed the lower minimum crack propagation rate, so it is the better performer and the comparison materials cannot be the more protective choice.

C) A lower minimum crack propagation rate indicates greater resistance to damage, so the new material should last longer, not shorter, which makes this the reverse of what the finding suggests.

13. Answer: B • CENTRAL IDEAS AND DETAILS • MEDIUM

Correct answer: B. Carrie sees the city as a stage where she will play a leading role, but she is ill-equipped to win the struggle ahead, so she anticipates success while being vulnerable to failure.

Why the other choices are wrong:

A) The text says the city is indifferent and hard, so she is not wrong about danger the way the choice claims.

C) She expects triumph, not simply a welcome, and the irony lies in her vulnerability rather than a hostile reception.

D) The passage does not say she wants to be a performer or that the city offers only office jobs.

14. Answer: D • CENTRAL IDEAS AND DETAILS • EASY

Correct answer: D. The text connects melting with sea-level effects.

Why the other choices are wrong:

A) The number of glaciers is not the concern.

B) Scientists are successfully studying it.

C) Freshwater production is not discussed.

15. Answer: C • BOUNDARIES • HARD

Correct answer: C. The words before the blank already form a complete sentence once the noun is supplied, so a period is needed. The next sentence then opens with the noun phrase "Creations so ..." followed by a comma and its main clause.

Why the other choices are wrong:

A) The comma joins a complete clause to a second independent clause (after the later comma), which creates a comma splice.

B) With no punctuation, the nouns run together ungrammatically and the sentence runs on into the next clause.

D) "and" merges the noun phrase into the first clause, which leaves the later comma splicing two independent clauses.

16. Answer: A • BOUNDARIES • HARD

Correct answer: A. A comma plus the coordinating conjunction "yet" correctly joins the two independent clauses, with the intervening "when" clause then set off by its own comma.

Why the other choices are wrong:

B) With no punctuation or conjunction, the two independent clauses run together as a fused sentence.

C) A comma alone cannot join two independent clauses; it creates a comma splice.

D) The conjunction "yet" joining two independent clauses needs a comma before it; without the comma the clauses are improperly joined.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

17. Answer: C • BOUNDARIES • HARD

Correct answer: C. Commas before and after "however" correctly set it off as a parenthetical between the main clause and the participial phrase that follows.

Why the other choices are wrong:

- A) The semicolon after "however" is incorrect because it cannot link the main clause to the participial phrase that follows.
- B) A semicolon before "however" is incorrect because the words after it are a participial phrase, not an independent clause.
- D) It omits the comma after "however," leaving the parenthetical interrupter unclosed.

18. Answer: D • BOUNDARIES • HARD

Correct answer: D. Omitting all punctuation correctly unifies the title and proper name. Rule: Restrictive appositives require continuous, unpunctuated integration between title and name.

Why the other choices are wrong:

- A) The option "architect," is incorrect because an internal comma incorrectly interrupts the restrictive noun phrase.
- B) The option "architect:" is incorrect because a colon creates an ungrammatical pause.
- C) The option "architect—" is incorrect because an em dash violates restrictive appositive standards.

19. Answer: C • FORM, STRUCTURE, AND SENSE • MEDIUM

Correct answer: C. Simple past "enlarged" correctly reports an action completed by the end of the fifteenth century, matching the past-tense framing of the passage.

Why the other choices are wrong:

- A) Present perfect "has enlarged" links the action to the present, which conflicts with the closed past time frame ending in the fifteenth century.
- B) Past perfect "had enlarged" is also acceptable here, so this item has two defensible answers; the keyed answer is the simple past.
- D) Future "will enlarge" contradicts a completed historical development placed at the end of the fifteenth century.

20. Answer: B • FORM, STRUCTURE, AND SENSE • HARD

Correct answer: B. The clause after "how" is an embedded (indirect) question, so it takes normal subject-verb word order and ends with a period: "interval training improves cardiovascular output."

Why the other choices are wrong:

- A) A question mark is wrong because the sentence is a statement containing an indirect question, which ends with a period.
- C) Inverted "do" word order and a question mark are wrong because the question is embedded inside a declarative sentence. The plural "do" also does not agree with the singular subject.
- D) Inverted "do" word order is wrong in an embedded question, which needs normal subject-verb order. The plural "do" also does not agree with the singular subject.

21. Answer: A • FORM, STRUCTURE, AND SENSE • MEDIUM

Correct answer: A. The introductory participial phrase must be followed by the noun it describes. The possessive noun phrase names the work that was recognized and that led to the later result, so the modifier is not left dangling.

Why the other choices are wrong:

- B) Opening with the person's name makes the introductory participial phrase modify a person instead of the work that was recognized, creating a dangling modifier.
- C) The introductory participial phrase ends up modifying the person named right after the comma rather than the work that was recognized, leaving a dangling modifier.
- D) The passive construction makes the introductory phrase describe the object that was acted on rather than the work that was recognized, and the compound verb is illogical.

22. Answer: A • TRANSITIONS • MEDIUM

Correct answer: A. "However" is correct because it introduces a direct contrast and correction regarding the misconception of the sponge spicule.

Why the other choices are wrong:

- B) "For instance" is incorrect because it supplies an example instead of a corrective contrast.
- C) "Hence" is incorrect because it presents a causal outcome instead of a structural correction.
- D) "That being said" is incorrect because the sentence structure uses a single-word contrast transition before a comma.

23. Answer: B • TRANSITIONS • HARD

Correct answer: B. "in turn" is correct because it indicates the operational outcome or computational capability enabled consecutively by the physical phenomenon of qubit superposition.

Why the other choices are wrong:

- A) "for example" is incorrect because it gives a specific illustration of a general claim instead of connecting a cause to its direct result.
- C) "instead" is incorrect because it offers an alternative choice that replaces the prior action, which contradicts the additive technical benefit.
- D) "similarly" is incorrect because it compares two similar processes rather than linking a computing advantage to its direct systemic output.

24. Answer: A • TRANSITIONS • HARD

Correct answer: A. "instead" is correct because it accurately highlights the surprising preservation of pristine silver fineness despite political collapse.

Why the other choices are wrong:

- B) "therefore" is incorrect because it signals a mathematical or logical deduction.
- C) "moreover" is incorrect because it introduces an additive point.
- D) "likewise" is incorrect because it shows similarity.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

25. Answer: C · RHETORICAL SYNTHESIS · HARD

Correct answer: C. Defines chemotropism (directional growth in response to chemical gradients) AND provides the example (pollen tube growth toward calcium). A/B/D miss either the explanation or the example.

Why the other choices are wrong:

- A) It explains the control mechanism and mentions plant roots, which the notes do not support, and gives no specific example such as pollen tubes.
- B) It supplies two examples of growth toward chemical signals but never defines chemotropism, so the explanation half is missing.
- D) It gives the pollen tube example but does not define chemotropism as growth along chemical gradients, and adds an unsupported claim about plant reproductive biologists.

26. Answer: B · RHETORICAL SYNTHESIS · MEDIUM

Correct answer: B. This choice explains the advantage by showing that online databases increase accessibility and make finding sources faster.

Why the other choices are wrong:

- A) This choice explains why universities subscribe to databases but does not describe the advantage of the databases themselves.
- C) This choice describes a limitation of traditional libraries but does not explain the benefit of online databases.
- D) This choice lists the contents of databases but does not explain why digital access is advantageous.

27. Answer: A · RHETORICAL SYNTHESIS · MEDIUM

Correct answer: A. Compare the decipherment status of Linear A (undeciphered) vs. Linear B (deciphered in 1952 as early Greek). A explicitly contrasts their decipherment statuses.

Why the other choices are wrong:

- B) It compares the historical civilizations that used the scripts, not their decipherment status.
- C) It highlights similarities (both ancient Bronze Age scripts), failing to make a comparison.
- D) It only provides information about Linear B, omitting Linear A entirely.

Math 1 Math, Module 1: Explanations**1. Answer: C · LINEAR EQUATIONS (1 VAR) · HARD**

She makes 12 candles each hour, so in h hours she makes $12h$ candles. Multiplying by the \$4 bonus per candle gives the bonus term $4(12h)$. Therefore $12h$ is the total number of candles made.

2. Answer: C · LINEAR FUNCTIONS · HARD

Substitute $2k + 1$ into f : $3(2k + 1) - 8 = 6k - 5$. Setting $6k - 5 = k + 10$ gives $5k = 15$, so $k = 3$.

3. Answer: C · LINEAR FUNCTIONS · EASY

Set $3x - 9 = 27$. Adding 9 to both sides gives $3x = 36$. Dividing both sides by 3 gives $x = 12$. Check: $f(12) = 3(12) - 9 = 27$.

4. Answer: 3 · LINEAR EQUATIONS (2 VAR) · MEDIUM

A point on the x -axis has $y = 0$. In $2x - 5y = 14$, this gives $2x = 14$, so $x = 7$. The point $(7, 0)$ must also be on $kx + 3y = 21$, so $7k = 21$ and $k = 3$.

Common mistakes:

- 7 is the x -coordinate of the intersection point, not the value of k .
- -3 would come from a sign error when solving $7k = 21$.

5. Answer: 24 · SYSTEMS OF EQUATIONS · HARD

Multiply the first equation by 2 to get $10x + 6y = 34$. Subtract the second equation: $(10x + 6y) - (2x + 5y) = 34 - 10$. This gives $8x + y = 24$, which is the expression asked for.

Common mistakes:

- 34 is the right side of $10x + 6y = 34$ before subtracting the second equation.
- 27 comes from adding the right sides, $17 + 10$, instead of combining the equations to match $8x + y$.
- 7 comes from subtracting the right sides, $17 - 10$, without doubling the first equation.

6. Answer: C · SYSTEMS OF EQUATIONS · MEDIUM

A solution (x, y) is a pair of values satisfying both equations, with x the number of 2-point questions and y the number of 4-point questions. Since $x = 12$ and $y = 18$, the quiz has 12 two-point questions and 18 four-point questions, which total 30 questions and $24 + 72 = 96$ points.

7. Answer: D · LINEAR INEQUALITIES · MEDIUM

Expand both sides: $6 - 3x - 4x < 5x - 30$, so $6 - 7x < 5x - 30$. Add $7x$ to both sides and add 30: $36 < 12x$. Divide by 12: $3 < x$, which is $x > 3$.

8. Answer: 9 · EQUIVALENT EXPRESSIONS · MEDIUM

The x^2 terms come from $x \cdot kx = kx^2$ and $4 \cdot 2x^2 = 8x^2$. Their sum is $(k + 8)x^2$. Setting $k + 8 = 17$ gives $k = 9$.

Common mistakes:

- 25 results from adding 8 to 17 instead of subtracting it.
- 17 results from setting k equal to the coefficient without including the $8x^2$ term.
- 4.25 results from dividing 17 by 4.

9. Answer: C · EQUIVALENT EXPRESSIONS · EASY

Substitute $x = -3$: $f(-3) = 2(-3)^2 - 5(-3) + 4 = 2(9) + 15 + 4 = 18 + 15 + 4 = 37$.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

10. Answer: 10 · NONLINEAR EQUATIONS · EASY

Substitute $y = 5x$ into $xy = 20$ to get $x(5x) = 20$, so $5x^2 = 20$ and $x^2 = 4$. The positive solution is $x = 2$, so $y = 5(2) = 10$. The point is $(2, 10)$.

Common mistakes:

- Answer 2 is the x-coordinate of the intersection point, not the y-coordinate.
- Answer -10 is the y-coordinate of the other intersection point, $(-2, -10)$, which has a negative x-coordinate.
- Answer 20 is the product xy given in the first equation, not the y-coordinate.

11. Answer: 18 · NONLINEAR EQUATIONS · MEDIUM

Let the solutions be r and $r + 3$. Their sum is 9, so $2r + 3 = 9$ and $r = 3$. The solutions are 3 and 6, and the product of the solutions equals c , so $c = 3 \times 6 = 18$. Check: $x^2 - 9x + 18 = (x - 3)(x - 6)$.

Common mistakes:

- Answer 9 uses the sum of the solutions as c , instead of their product.
- Answer 3 is the smaller solution, not the product c .
- Answer 20.25 comes from $c = (9/2)^2$, which gives one repeated solution rather than two solutions that differ by 3.

12. Answer: B · NONLINEAR EQUATIONS · HARD

Expand to get $x^2 + 2x - 15 = k$, so $x^2 + 2x - 15 - k = 0$. One real solution requires a zero discriminant: $2^2 - 4(1)(-15 - k) = 64 + 4k = 0$, so $k = -16$. Equivalently, the parabola's lowest point is at its vertex $(-1, -16)$.

13. Answer: B · NONLINEAR FUNCTIONS · HARD

The graph of a quadratic is symmetric about its vertex, which is halfway between the two times with equal height: $t = (1 + 6)/2 = 3.5$. The ball is on the ground at $t = 0$, and the symmetric time is $3.5 + 3.5 = 7$. So the ball returns to the ground at $t = 7$ seconds.

14. Answer: A · NONLINEAR FUNCTIONS · EASY

The function is in vertex form, $g(x) = (x - 5)^2 - 8$, so its vertex is $(5, -8)$. The squared term is never negative, so the smallest value occurs when $x = 5$, and it equals -8 .

15. Answer: D · RATIOS AND RATES · EASY

The ratio of water to clay is 5 cups to 2 pounds. Since 8 pounds is 4 times 2 pounds, the water needed is $4 \times 5 = 20$ cups.

16. Answer: A · PERCENTAGES · MEDIUM

The overall multiplier is $1.10 \times 1.20 \times 0.70 = 0.924$. Since 0.924 is 7.6% less than 1, the August number is 7.6% lower than the number before June.

17. Answer: C · STATISTICAL CLAIMS · MEDIUM

The 310 respondents were self-selected, so they may feel differently from members who did not respond. The 74% describes the respondents but cannot be reliably extended to all 2,400 members.

18. Answer: B · TWO-VARIABLE DATA · MEDIUM

The points trend downward from left to right: books priced near \$10 sold about 95 copies, while books priced near \$32 sold about 35. So higher-priced books tended to sell fewer copies.

19. Answer: A · AREA AND VOLUME · EASY

The floor has area $12 \times 10 = 120$ square feet. Each square tile has area $2 \times 2 = 4$ square feet. The number of tiles needed is $120 \div 4 = 30$.

20. Answer: A · LINES, ANGLES, TRIANGLES · EASY

The ratio of corresponding sides is $3/6 = 1/2$. The ratio of the areas of similar triangles is the square of that, $(1/2)^2 = 1/4$. So the area of ABC is $48 \times 1/4 = 12$ square centimeters.

21. Answer: A · RIGHT TRIANGLES AND TRIG · MEDIUM

Because $\tan P = QR/PQ = 8/15$, the legs can be written as $QR = 8k$ and $PQ = 15k$. Then $PR = \sqrt{(8k)^2 + (15k)^2} = 17k$. Therefore $\sin P = QR/PR = 8k/17k = 8/17$.

22. Answer: A · RIGHT TRIANGLES AND TRIG · MEDIUM

Side BC is opposite the 24° angle and AB = 30 is the hypotenuse. Since $\sin 24^\circ = BC/AB$, $BC = 30 \times 0.4 = 12$.

Math 2 Math, Module 2: Explanations**1. Answer: A · LINEAR EQUATIONS (1 VAR) · MEDIUM**

Let the seat numbers be n , $n + 2$, and $n + 4$. Then $3n + 6 = 117$, so $3n = 111$ and $n = 37$. The seats are 37, 39, and 41, so the least is 37.

2. Answer: B · LINEAR FUNCTIONS · HARD

From slope 3 and $f(2) = 5$, we get $f(x) = 3x - 1$. Translating 4 units left replaces x with $x + 4$, and 1 unit down subtracts 1. So $g(x) = f(x + 4) - 1 = 3(x + 4) - 1 - 1 = 3x + 10$.

3. Answer: A · LINEAR FUNCTIONS · HARD

At $x = 2$, $f(2) = 2(k + 1) + 6 = 2k + 8$ and $g(2) = 6k + 4$. Setting $2k + 8 = 6k + 4$ gives $4 = 4k$, so $k = 1$.

4. Answer: 11/4 or 2.75 · LINEAR EQUATIONS (2 VAR) · MEDIUM

Adding $2x + y = 11$ and $x - y = 1$ gives $3x = 12$, so $x = 4$ and $y = 3$, and P is $(4, 3)$. Substituting into $kx + 3y = 20$ gives $4k + 9 = 20$. Then $4k = 11$, so $k = 11/4$, or 2.75.

Common mistakes:

- Using P as $(3, 4)$ instead of $(4, 3)$ gives $3k + 12 = 20$, so $k = 8/3$ rather than $11/4$.
- Forgetting to multiply y by 3 gives $4k + 3 = 20$, so $k = 17/4$, which is incorrect.
- A value of 5 comes from solving $4k = 20$ without subtracting the $3y$ term.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

5. Answer: 24 · SYSTEMS OF EQUATIONS · HARD

Let a and b be the liters of A and B. Then $a + b = 60$ and $0.20a + 0.50b = 0.38(60) = 22.8$. Substituting $b = 60 - a$ gives $0.20a + 30 - 0.50a = 22.8$, so $-0.30a = -7.2$ and $a = 24$ liters.

Common mistakes:

- 36 is the number of liters of solution B ($60 - 24$), not solution A, which the question asks for.
- 30 results from assuming equal amounts of the two solutions; that mixture would be 35% fertilizer, not 38%.
- 22.8 is the number of liters of pure fertilizer in the mixture, not the volume of solution A.

6. Answer: B · SYSTEMS OF EQUATIONS · HARD

Clear fractions: multiply the first equation by 12 to get $8x + 3y = 72$, and the second by 24 to get $20x - 3y = 96$. Adding gives $28x = 168$, so $x = 6$. Then $3y = 72 - 48 = 24$, so $y = 8$. Therefore $x + y = 14$.

7. Answer: C · LINEAR INEQUALITIES · HARD

Multiply both sides by 6: $-2(x - 4) \geq 3(x + 5)$, so $-2x + 8 \geq 3x + 15$. Subtract $3x$ and 8 from both sides: $-5x \geq 7$. Dividing by -5 reverses the symbol, so $x \leq -7/5$.

8. Answer: 36 · EQUIVALENT EXPRESSIONS · MEDIUM

Raise both sides to the power $2/3$ to undo the exponent $3/2$. Then $x = 216^{2/3} = (\sqrt[3]{216})^2 = 6^2 = 36$. Check: $\sqrt{36} = 6$ and $6^3 = 216$.

Common mistakes:

- Giving 6 takes only the cube root of 216 and skips the squaring.
- Giving 324 multiplies 216 by $3/2$, but an exponent cannot be undone by multiplying.
- Giving 144 comes from $216 \cdot 2/3$, but undoing an exponent requires raising to the reciprocal power.

9. Answer: A · EQUIVALENT EXPRESSIONS · MEDIUM

Use the common denominator $x(x + 1)$. Then $2/x = 2(x + 1)/(x(x + 1))$ and $3/(x + 1) = 3x/(x(x + 1))$. Adding gives $(2x + 2 + 3x)/(x(x + 1)) = (5x + 2)/(x(x + 1))$.

10. Answer: -7/2 or -3.5 · NONLINEAR EQUATIONS · MEDIUM

Factor the quadratic as $(2x + 7)(x - 3) = 0$. Then $2x + 7 = 0$ gives $x = -7/2$, and $x - 3 = 0$ gives $x = 3$. The negative solution is $-7/2$, which can be entered as $-7/2$ or -3.5 .

Common mistakes:

- 3 is the positive solution, not the negative one.
- 7 comes from solving $2x + 7 = 0$ without dividing by the coefficient 2.

11. Answer: 10 or 10.0 · NONLINEAR EQUATIONS · HARD

Substitute $y = 2x + k$: $x^2 + (2x + k)^2 = 20$, so $5x^2 + 4kx + k^2 - 20 = 0$. One solution requires a zero discriminant: $16k^2 - 20(k^2 - 20) = 0$, so $-4k^2 + 400 = 0$ and $k^2 = 100$. Since k is positive, $k = 10$.

Common mistakes:

- 10 also gives exactly one solution, but k is positive.
- 100 is k^2 , not k . The square root must still be taken.
- 20 is the value of $x^2 + y^2$ on the circle, not the constant in the line.

12. Answer: B · NONLINEAR EQUATIONS · EASY

Square both sides: $x + 7 = (x - 5)^2 = x^2 - 10x + 25$, so $x^2 - 11x + 18 = 0$ and $(x - 2)(x - 9) = 0$. Check $x = 2$: $\sqrt{9} = 3$, but $2 - 5 = -3$, so $x = 2$ is extraneous. Check $x = 9$: $\sqrt{16} = 4$ and $9 - 5 = 4$. The solution is 9.

13. Answer: C · NONLINEAR FUNCTIONS · HARD

The number of years is $t = m/12$. Substituting into the exponent gives $t/3 = (m/12)/3 = m/36$, so $S(m) = 500(1.4)^{m/36}$. Check: at $m = 36$ months (3 years), sales are $500(1.4) = 700$.

14. Answer: D · NONLINEAR FUNCTIONS · EASY

The values double each time x increases by 1, so f is exponential with growth factor 2. Since $f(0) = 5$, the initial value is 5, and $f(x) = 5(2)^x$. Check: $f(3) = 5 \times 8 = 40$.

15. Answer: C · RATIOS AND RATES · MEDIUM

Find the price per bundle for each package. $15 \div 4 = \$3.75$, $21.60 \div 6 = \$3.60$, $31.50 \div 9 = \$3.50$, and $43.20 \div 12 = \$3.60$. The lowest price per bundle is \$3.50, which is the package of 9 bundles for \$31.50.

16. Answer: C · PERCENTAGES · HARD

Adults taking evening classes are $0.60(0.45) = 27\%$ of all students. Non-adults taking evening classes are $0.40(0.20) = 8\%$. The total is $27\% + 8\% = 35\%$.

17. Answer: D · ONE-VARIABLE DATA · HARD

The 40 customers washed $1(8) + 2(12) + 3(10) + 4(6) + 5(4) = 106$ loads. With x more customers each washing 5 loads, the mean is $(106 + 5x)/(40 + x)$. Requiring this to be at least 3 gives $106 + 5x \geq 120 + 3x$, so $x \geq 7$. The least number is 7.

18. Answer: A · PROBABILITY · MEDIUM

Only the $91 + 24 = 115$ customers who chose same-day delivery are possible, and 24 of them ordered by phone. The probability is $24/115$.

19. Answer: D · AREA AND VOLUME · HARD

Triangles ABD, ADC, and ABC share the same height from A to line BC. Since area ADC = $(1/2)(9)h = 63$, $h = 14$. Triangle ABC has base $6 + 9 = 15$, so its area is $(1/2)(15)(14) = 105$.

20. Answer: C · AREA AND VOLUME · MEDIUM

One square meter is $100 \text{ cm} \times 100 \text{ cm} = 10,000$ square centimeters, so the roll has $270 \times 10,000 = 2,700,000$ square centimeters. Each poster has area $60 \times 90 = 5,400$ square centimeters. The number of posters is $2,700,000 \div 5,400 = 500$.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

21. Answer: C · LINES, ANGLES, TRIANGLES · HARD

Alternate interior angles are equal, so $3x + 4 = 5x - 32$, which gives $x = 18$. The angles on line m at P sum to 180° : $(3x + 4) + \text{angle } QPR + (2x + 14) = 180$. With $x = 18$, that is $58 + \text{angle } QPR + 50 = 180$, so $\text{angle } QPR = 72^\circ$.

22. Answer: C · RIGHT TRIANGLES AND TRIG · HARD

For acute angles, $\sin x^\circ = \cos y^\circ$ only when the angles are complementary, so $x + y = 90$. Substituting $x = 2y - 15$ gives $3y - 15 = 90$, so $y = 35$. Then $x = 55$, and both angles are acute.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →



Practice Exam 3

A full-length digital SAT in four modules: Reading and Writing (2 × 27 questions) and Math (2 × 22 questions), followed by complete answers and explanations.

Exam 3 Reading and Writing: Module 1

Reading and Writing · Module 1

27 questions · 32 minutes · Read each passage and question carefully, then choose the best answer.

- 1 Measures to subsidize fossil fuel extraction are often criticized for their environmental consequences, but fiscal experts note that maintaining these artificial market incentives could _____ the commercial expansion of renewable energy sectors by keeping the consumer cost of traditional utilities artificially low.
- Which choice completes the text with the most logical and precise word or phrase?
- A. hinder
B. multiply
C. mock
D. obscure
- 2 The following text is adapted from Carlos Fuentes's 1992 analytical essay collection *The Buried Mirror*. While revising the final chapter of a complex historical treatise concerning Iberian influence in Mesoamerica, the author contemplates his exhaustion. "I stepped onto the balcony to watch the twilight traffic, hoping the mundane scene would break my focus, but the intricate structural outlines and dense bibliographic citations held fast in my thoughts. I yearned for total amnesia to banish them, but it remained out of reach. I assumed my normal daily routines would resume only after the final packet was delivered to the publisher, and I eagerly anticipated terminating the endeavor."
- As used in the text, what does the word "terminating" most nearly mean?
- A. Demolishing
B. Finishing
C. Evaluating
D. Disseminating
- 3 The predatory sundew plant produces sticky, glistening sap at the precise time of day when local gnats are most active. Botanists proved that the plant manages this predatory _____ with insect patterns by altering its metabolic rate based on early morning light exposure.
- Which choice completes the text with the most logical and precise word or phrase?
- A. divide
B. matching
C. clash
D. gap
- 4 Entomologist Dr. Sylvia Earle directed a 2023 research project on a species of ant that arranged small pebbles around their nest entrances to block wind. Skeptical colleagues maintained that the pebble placement was merely _____, but Earle's manipulation of wind patterns demonstrated that the ants dynamically relocated the barriers as wind direction shifted.
- Which choice completes the text with the most logical and precise word or phrase?
- A. fortuitous
B. explicit
C. intriguing
D. strategic

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 5 Some wasp species do not construct autonomous physical enclosures for their brood. Instead, adult females surreptitiously deposit their larvae inside the constructed subterranean nests of mud-dauber wasps, directly adjacent to the host's stored spider provisions. Female cuckoo-wasps have been observed rapidly slipping into the mud chambers when the resident wasps are temporarily absent on foraging flights. After the parasitic egg hatches, the non-native larva will typically consume both the host egg and the gathered arachnid prey as if it were its natural inheritance, even if the intruder lacks any morphological similarity to the host species.

Which choice best describes the function of the underlined sentence in the text as a whole?

- A. It presents a detailed description of the structural composition of mud-dauber nests.
- B. It details the specific actions through which female cuckoo-wasps execute the brood-parasitic behavior described in the passage.
- C. It explains the behavioral modifications that host mud-dauber wasps undergo to defend their nests.
- D. It identifies an anatomical adaptation that enables cuckoo-wasps to survive high thermal stress.

- 6 The following text is adapted from Oscar Wilde's 1890 novel *The Picture of Dorian Gray*. Lord Henry is speaking to Dorian.

DORIAN: I wish I could stay young forever, like this portrait.

LORD HENRY: Youth is the only thing worth having, Dorian. When you lose it, you lose everything.

DORIAN: But how can I ensure I never grow old?

LORD HENRY: You cannot. But you can live as if you will never grow old. That is the true secret of life.

Which choice best states the main purpose of the text?

- A. To explain how to paint a perfect portrait
- B. To show Lord Henry's cynical philosophy regarding youth and pleasure
- C. To argue that Dorian should become a painter
- D. To describe Dorian's physical appearance in detail

- 7 Text 1

The magnetic recording fidelity of annual varves in proglacial lake sediment cores is central to paleomagnetic studies. Some researchers contend that post-depositional bioturbation by benthic organisms scrambles the detrital remanent magnetization, noting that biological mixing blurs annual magnetic field variations.

Text 2

High-resolution cryogenic magnetometer scanning and computed tomography core imaging reveal pristine magnetic alignment preservation in specific basins. Researcher Krishnam Reddy examined sub-millimeter magnetic grain fabrics and low organic carbon accumulation rates and, based on the paleomagnetic data, compellingly argues that rapid sediment compaction locks detrital grains before biological mixing can occur.

Based on the texts, how would the author of Text 2 most likely respond to what "some researchers contend" as described in Text 1?

- A. By asserting that rapid sediment compaction locks detrital grains before biological mixing can occur
- B. By proving that lake sediment varves possess zero magnetic mineral content
- C. By claiming that benthic organisms actively enhance magnetic field recording fidelity
- D. By demonstrating that cryogenic magnetometers cannot detect remnant magnetization in core samples

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 8** LADRILLO is a regional brickmaking cooperative fuel management network in Paraguay that provides artisanal brick makers with high-efficiency kiln burners and clay-mixing machines on credit. Makers use their brick sales revenue to repay equipment loans in monthly increments and incur no penalty for missed payments other than temporary suspension of shared fuel delivery services. Industrial engineers Faustino Churata and Melchor Quispeña analyzed LADRILLO data and found that independent brickmakers miss loan repayments more often than cooperative-affiliated makers do, partly because they fire and market their clay bricks less frequently than fuel supply schedules permit. Churata and Quispeña claim that this behavior reflects strategic decisions that enable independent brickmakers to maximize their net capital return per fired masonry unit.

Which finding, if true, would most directly support Churata and Quispeña's claim?

- A. Independent brickmakers generally incur lower raw clay excavation expenses per cubic meter than industrial brick manufacturing plants do.
- B. Construction contractors pay significantly lower per-unit prices for bricks unless suppliers deliver massive, multi-pallet batches required for large-scale foundation framing.
- C. Independent brickmakers require smaller initial equipment loans for mixing machinery than large-scale concrete producers do.
- D. Fuel distributors charge brickmakers a flat delivery fee for each transport trip regardless of whether the truck carries one pallet or five pallets of finished bricks.

- 9** A student in an environmental economics course is writing a paper on Rachel Carson's *Silent Spring*, in which Carson examines the widespread ecological disruption caused by synthetic chemical pesticides and argues for a precautionary approach to industrial expansion. Carson observes that modern technological interventions often generate unforeseen ecological feedback loops—for example, intensive pesticide applications can eliminate beneficial predatory insect populations alongside targeted pests, triggering exponential resurgences of secondary agricultural vectors—but certain regulatory safeguards can mitigate systemic environmental degradation. The student claims that in particular Carson asserts that chemical manufacturers must be held publicly accountable for long-term ecotoxicological accumulation and that synthetic pesticide dissemination should be heavily restricted.

Which quotation from an environmental historian's analysis of *Silent Spring* would best support the student's claim?

- A. "When evaluating the historical trajectory of post-war agricultural modernization, Carson notes that 'The current system of insect control is based on a philosophy of aggressive warfare,' arguing that such methods inevitably provoke biological resistance rather than achieving permanent environmental mastery."
- B. "When analyzing the complex web of life within forest ecosystems, Carson emphasizes that 'Every ecosystem is bound by delicate internal feedback loops,' suggesting that chemical interventions must always be preceded by multi-year environmental impact assessments conducted by independent academic panels."
- C. "When documenting the catastrophic ecological ramifications of indiscriminate pesticide use, Carson maintains that 'The most alarming of all man's assaults upon the environment is the contamination of air, earth, rivers, and sea with dangerous and lethal materials,' warning that these synthetic poisons persist indefinitely within living tissue and biomagnify across trophic levels."
- D. "When outlining the economic incentives driving chemical manufacturing corporations, Carson contends that 'The chemical industry operates under a cloak of financial secrecy,' observing that corporate balance sheets consistently conceal the true public health costs associated with widespread pesticide exposure."

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 10** A literacy team led by Dr. Tomas Ekwueme studied whether a daily paired-reading routine, in which students take turns reading aloud to a partner, speeds up reading fluency in third grade. Teachers in 20 classrooms measured oral reading rate every two months. Students in the paired-reading classrooms and those in the comparison classrooms started the year at nearly the same rate, but the gap between the groups widened over time. By May, the students in paired-reading classrooms

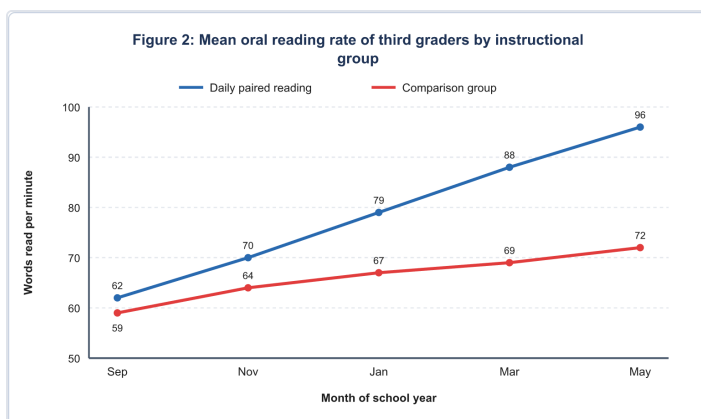


Figure 2: Mean oral reading rate of third graders by instructional group

Which choice most effectively uses data from the graph to complete the text?

- A.** read 96 words per minute on average, which was 24 words per minute faster than the 72 words per minute of the comparison group.
- B.** read 88 words per minute on average, which was 24 words per minute faster than the 64 words per minute of the comparison group.
- C.** read 96 words per minute on average, which was 34 words per minute faster than the 62 words per minute of the comparison group.
- D.** gained 18 words per minute since September, while the comparison group gained 22 words per minute over the same period.
- 11** Chemical spraying (the use of substances to remove unwanted vegetation) has long been viewed as a possible threat to botanical conservation (the protection of native plant communities). Yet a comparative agronomy investigation involving 120,000 hectares of Argentina's Pampas region has produced findings that challenge this assumption. Researchers examining the region discovered that grassland areas receiving targeted application (the placement of herbicide in carefully selected locations) developed stronger ecological conditions than similar areas that did not receive the treatment. The findings indicate that _____

Which choice most logically completes the text?

- A.** eliminating agricultural chemical practices is necessary to protect native plant populations in every environment.
- B.** untreated landscapes generally provide more successful conditions for plant communities than managed landscapes.
- C.** certain agricultural techniques can support ecological objectives when their use is appropriately managed.
- D.** chemical methods for vegetation control have little effect on relationships among plant species.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 12** Net casting (the use of commercial fishing equipment in shallow marine environments) is often assumed to conflict with reef preservation (the protection of marine biodiversity). However, a marine biology study conducted across 500,000 hectares of the Great Barrier Reef has produced evidence that challenges this assumption. Investigators found that reef areas experiencing regulated netting (the controlled removal of monitored quantities of fish) displayed greater ecological activity than comparable reef areas that had never been subjected to fishing. The results of this study suggest that _____

Which choice most logically completes the text?

- A. carefully supervised commercial harvesting can be compatible with efforts to maintain healthy reef ecosystems.
- B. the increased ecological activity found in regulated reef areas was probably caused by natural differences between reef locations rather than by fishing practices.
- C. protecting reef environments is generally more dependent on reducing total fishing levels than on improving how fishing methods are managed.
- D. successful reef conservation requires preventing commercial fishing in sensitive marine regions, even when fishing practices are strictly regulated.

- 13** In 2019, agricultural scientists tested a new irrigation system designed to reduce water use on vegetable farms. Traditional sprinklers sprayed water across entire fields, including areas without crops, resulting in substantial evaporation and runoff. The new system delivered water directly to the roots of each plant through narrow tubes placed along every row. During two growing seasons, farms using the system produced harvests comparable to those of farms using conventional irrigation while consuming significantly less water. Researchers concluded that directing water precisely where plants needed it improved irrigation efficiency without reducing crop production.

According to the text, why did the new irrigation system use less water?

- A. Because it delivered water directly to plant roots instead of spraying entire fields.
- B. Because the farms received more rainfall during the study.
- C. Because farmers planted fewer vegetables than in previous years.
- D. Because the system stored rainwater in underground reservoirs.

- 14** The following text is adapted from: World Wildlife Fund (WWF), Amazon Rainforest

The Amazon Rainforest is a moist broadleaf tropical rainforest in the Amazon biome that covers most of the Amazon basin of South America. This region includes territory belonging to nine nations. The majority of the forest is contained within Brazil, with 60% of the rainforest, followed by Peru with 13%, Colombia with 10%, and lesser amounts in other countries. The Amazon represents over half of the planet's remaining rainforests and comprises the largest and most biodiverse tract of tropical rainforest in the world. It plays a critical role in global climate regulation by acting as a "carbon sink," absorbing carbon dioxide from the atmosphere.

Which country contains the majority of the Amazon Rainforest?

- A. Peru
- B. Colombia
- C. Brazil
- D. Venezuela

- 15** A study published by Stanford University volcanology specialist Nora Finch in 2020 offers a new explanation for the origin of Earth's _____ structures called calderas, giant circular depressions that form when a shallow magma chamber empties during an eruption, triggers catastrophic roof collapse, and then leaves behind a massive sunken basin.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. magmatic. Seismic
- B. magmatic: seismic
- C. magmatic seismic
- D. magmatic; seismic

- 16** In ancient Greece, a Stoic was a follower of Zeno of Citium, a philosopher whose beliefs revolved around emotional resilience. Zeno defined virtue as "living in agreement with nature and mastering one's _____ that all human suffering derived from false judgment.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. passions," asserting
- B. passions": asserting
- C. passions"; asserting
- D. passions." Asserting

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 17** The Copenhagen interpretation, developed during the late 1920s by physicists Niels Bohr and Werner Heisenberg, maintained that quantum systems exist in superpositions of multiple states until physical measurement forces wave-function collapse _____ it glossed over the foundational philosophical paradox regarding how macroscopic observation instigates this instantaneous physical transition.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A.** out, **B.** out but
C. out **D.** out, but

- 18 From a limnological sampling dock, limnologist Liam Gallagher's phytoplankton concentration charts look like green watercolor pools, their density gradients and chlorophyll distributions so smooth that it seems only a fluid wash could have stained them. When viewed through a phase-contrast microscope, however, the water samples reveal themselves to be _____ microscopic diatoms and flagellated algae barely countable within a single drop.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. cells, the
C. cells; the
B. cells, and the
D. cells. The

- 19 Arthur Pendelton and Ines Ramos, celebrated speleologists of subterranean limestone systems, compiled the definitive survey of crystal caves, an inventory that _____ the mineral formations discovered in the deepest chambers.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. have cataloged
B. catalogs
C. were cataloging
D. catalog

- 20** During his 1936 theatrical staging of *Macbeth* in Harlem, director Orson Welles reimagined Shakespearean drama within a Haitian setting. Today, Welles _____ viewed as an innovative reinterpreter of classical theater.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. had been
B. is
C. was
D. will be

- 21** In Everglades National Park's Ten Thousand Islands, a vast mangrove estuary, are twisting tidal channels—some lined with dense red mangroves but others opening into wide bays—that connect freshwater rivers to the Gulf of Mexico. Mangrove roots in the _____ tidal flow.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. islands tide mark the channel's
B. island's tide mark the channels
C. island's tide mark the channels'
D. islands' tide mark the channels'

- 22** In classical epigraphy, fragmentary Linear B administrative tablets unearthed from Late Bronze Age palace archives are expected to preserve incomplete syllabic inventories lacking contextual verbs, whereas intact monumental inscriptions would feature comprehensive grammatical structures. Michael Ventris and colleagues, _____ deciphered a heavily damaged administrative receipt listing commodity allocations from Knossos, that has supplied complete lexical paradigms despite severe surface erosion.

Which choice completes the text with the most logical transition?

- A. nonetheless,
C. additionally,
- B. consequently,
D. similarly,

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 23** Admired for its rotating bronze golden-spire dome and neoclassical colonnades, Belfast City Hall anchors Donegall Square in Northern Ireland. However, routine electrical safety inspections carried out last spring uncovered outdated wiring hazards running through the main ceiling cavities. _____ municipal authorities closed selected public exhibition rooms while certified electricians upgraded the entire power infrastructure.

Which choice completes the text with the most logical transition?

- A. Therefore,
B. Conversely,
C. For example,
D. Likewise,

- 24** According to Michels's iron law of oligarchy, large bureaucratic organizations across modern industrial societies tend to concentrate power into the hands of a small elite leadership group, wherein institutional directors consistently control the entire hierarchy. _____ systems with direct-democracy governing frameworks tend to support egalitarianism, under which authority gets distributed among all active citizens.

Which choice completes the text with the most logical transition?

- A. Subsequently,
B. Conversely,
C. For instance,
D. In other words,

- 25**

 - Scientists studying bird migration have long investigated how birds navigate thousands of miles without visible landmarks.
 - Researchers at Lund University examined whether migratory birds use Earth's magnetic field to determine direction.
 - The scientists exposed European robins to controlled magnetic conditions inside a laboratory environment.
 - The researchers altered magnetic signals around the birds and observed changes in their orientation behavior.
 - The study provided evidence that some birds possess a magnetic sense that helps them navigate during migration.

The student wants to present the study and its methodology. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A.** To investigate whether European robins use Earth's magnetic field for navigation, researchers at Lund University exposed the birds to controlled magnetic conditions and observed changes in their orientation behavior.
- B.** Researchers discovered that migratory birds possess a magnetic sense that allows them to navigate thousands of miles without visible landmarks. This discovery helped explain how birds complete long-distance migrations.
- C.** European robins were studied because scientists wanted to understand how birds navigate during migration. The research showed that some birds are able to detect changes in Earth's magnetic field.
- D.** Scientists studying bird migration used laboratory experiments to examine the behavior of European robins. The experiments demonstrated that magnetic signals influence how birds determine direction.

- 26**

 - Wood-decay fungi decompose fallen timber in forest ecosystems using distinct enzymatic breakdown strategies.
 - Brown rot fungi (e.g., *Fomitopsis pinicola*) selectively break down cellulose and hemicellulose, leaving structural lignin largely intact as brownish, cubic fragments.
 - White rot fungi (e.g., *Trametes versicolor*) synthesize specialized enzymes that break down tough lignin alongside cellulose, leaving behind soft, bleached wood residue.

The student wants to compare the wood decomposition strategies of brown rot fungi and white rot fungi. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A. Brown rot fungi degrade cellulose while leaving lignin intact, which causes decaying wood to break into brownish cubes.
- B. While brown rot fungi leave structural lignin largely intact, white rot fungi are commonly found on decaying timber in forest ecosystems.
- C. Brown rot and white rot fungi use distinct enzymatic pathways to break down fallen trees in forest ecosystems.
- D. Brown rot fungi selectively decompose cellulose while leaving lignin intact, whereas white rot fungi break down both lignin and cellulose.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 27** • Dmitri Mendeleev formulated the periodic law and arranged the known chemical elements into a systematic table based on atomic weight.
- Mendeleev famously left deliberate blank spaces in his grid to predict the existence and properties of undiscovered elements like gallium and germanium.
 - Subsequent chemical discoveries confirmed his precise predictions, validating the robust predictive power of his structural periodic framework.
 - Modern versions of the periodic table organize elements by atomic number rather than atomic weight following discoveries about subatomic proton structures.
 - The table remains the foundational cornerstone of modern chemical education and research worldwide.

The student wants to highlight Mendeleev's innovative method for anticipating undiscovered elements. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A. Russian chemist Dmitri Mendeleev formulated the periodic law in 1869 by arranging known chemical elements into a systematic table based on atomic weight.
- B. Mendeleev famously left deliberate blank spaces in his grid to predict the existence and properties of undiscovered elements like gallium and germanium.
- C. Modern versions of the periodic table organize elements by atomic number rather than atomic weight following discoveries about subatomic proton structures.
- D. The table remains the foundational cornerstone of modern chemical education and research across scientific institutions worldwide.

Exam 3 Reading and Writing: Module 2

Reading and Writing · Module 2

27 questions · 32 minutes · Read each passage and question carefully, then choose the best answer.

- 1** Lateralization, the preferential use of one side of the brain for specific cognitive tasks, is typically easy to document in songbirds. Because this neurological trait is present but less _____ in many migratory waterfowl, avian researchers often utilize complex auditory tasks specifically designed to reveal subtle lateral preferences in flock orientation.
- Which choice completes the text with the most logical and precise word or phrase?
- A. discernable
 - B. captivating
 - C. substantial
 - D. advantageous
- 2** Elma York, the central character in Mary Robinette Kowal's 2018 novel *The Calculating Stars*, is a mathematician and pilot, professional roles which, in Kowal's alternate history narrative, are far from _____.: dozens of women populate the ranks of the aerospace program as computer specialists and aviators.
- Which choice completes the text with the most logical and precise word or phrase?
- A. flawless
 - B. idiosyncratic
 - C. commercial
 - D. tedious
- 3** Although occupying widely separate branches of the evolutionary tree, the sugar glider of Australia shares fundamental _____ the flying squirrel of North America, possessing thin patagial membranes, large nocturnal eyes, and nesting behaviors tailored to high-canopy habitats.
- Which choice completes the text with the most logical and precise word or phrase?
- A. congruencies with
 - B. anxieties about
 - C. representations of
 - D. arguments for

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 4 Rin, the central figure in R.F. Kuang's 2018 epic fantasy novel *The Poppy War*, is a military strategist and shaman, a vocation which, in Kuang's historical-inspired lore, is by no means _____: a diverse pantheon of mystical warriors and tactical adepts populate the imperial academies and border armies.

Which choice completes the text with the most logical and precise word or phrase?

- A. errorless
- B. singular
- C. high-paying
- D. laborious

- 5 In his 1887 memoir of forestry in the pine barrens of Maine, woodsman Silas Thorne recalled how early loggers extracted the volatile oleoresin from mature turpentine trees. Now the harvesters began to notch the bark—pacing rhythmically through the groves, scoring iron in hand, and striking an isolated downward blow to see if the volatile oil would bleed. The pines, much like human companions, exhibited their peculiar vitalities; some were quick to surrender their sticky wealth, whereas others were decidedly unyielding. Now a zinc receptacle was suspended under each V-shaped groove, and a curved iron gutter driven beneath the incision which the iron had sliced. From the vertex of this gutter—initially drop by drop, then in a steady trickling—the raw turpentine descended into the tin.

Which choice best describes the function of the underlined sentence in the text as a whole?

- A. It functions as an anthropomorphic elaboration on the intrinsic physical differences in the trees that the workers are monitoring.
- B. It demonstrates that human intervention invariably standardizes the behavioral responses of natural organisms.
- C. It documents the physical exhaustion and emotional alienation experienced by the loggers during the harvesting season.
- D. It serves to contrast the delicate sensibilities of the trees with the violent, disruptive habits of the industrial logging crew.

- 6 The following text is from Robert Frost's 1914 poem "Mending Wall."

Something there is that doesn't love a wall,
That sends the frozen-ground-swell under it,
And spills the upper boulders in the sun;
And makes gaps even two can pass abreast.
The work of hunters is another thing:
I have come after them and made repair
Where they have left not one stone on a stone,
But they would have the rabbit out of hiding,
To please the yelping dogs.

Which choice best states the main purpose of the text?

- A. To describe the specific geological causes of stone wall collapse.
- B. To highlight the tension between nature/human activity and the maintenance of boundaries.
- C. To criticize hunters for destroying private property during their pursuits.
- D. To argue that walls are essential for maintaining good neighborly relations.

- 7 Text 1

Graptolite taxonomy and paleoecology models traditionally assume that planktonic colonial rhabdosomes maintained a rigid vertical orientation in the water column, suspended passively by a primary nema. Invertebrate paleontologists operating under this framework assert that hydrodynamic drag forces dictated colony stability without active behavioral intervention. They maintain that colonial drift was governed exclusively by ambient ocean currents.

Text 2

Hydrodynamic flume experiments utilizing physical resin models of diverse diplograptid morphologies challenge this passive vertical suspension hypothesis. Researcher Mirek Ostrowski analyzed vortex shedding frequencies and fluid-structure interactions, persuasively arguing that flexible helical coiling and active zooid orientation allowed colonies to exploit micro-turbulent boundary layers. Consequently, behavioral posture regulation replaced passive vertical drifting as a survival strategy.

Based on the texts, how would Mirek Ostrowski most likely respond to the paleontologists' assumption described in Text 1?

- A. By verifying that ocean currents were entirely nonexistent in Ordovician epicontinental seas
- B. By proving that the primary nema served no structural purpose in graptolite colonies
- C. By establishing that diplograptids were benthic organisms incapable of swimming
- D. By demonstrating that flexible helical coiling and zooid orientation allowed active exploitation of boundary layers rather than passive vertical suspension

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 8 Sports scientists who favor the traditional view of athletic recovery claim that incorporating cold-water immersion protocols does not change a runner's subsequent maximal aerobic sprint capacity. Focusing on collegiate track and field programs from 2012 to 2022, Nathan Parker, Olive Reed, and Peter Smith tested this claim by distinguishing between athletes assigned to post-training cold immersion sessions (around half of whom completed all scheduled protocols) and otherwise similar athletes assigned to passive room-temperature recovery (and thus none of whom completed cold immersion). Parker, Reed, and Smith compared the maximal aerobic sprint capacity measurements of the groups of athletes forty-eight hours after intense exertion.

Which finding from Parker, Reed, and Smith's study, if true, would most directly weaken the claim made by sports scientists who favor the traditional view of athletic recovery?

- A. Athletes' maximal aerobic sprint capacity measurements forty-eight hours after intense exertion were strongly predicted by athletes' baseline training volume, regardless of whether athletes completed post-training cold immersion sessions.
- B. Athletes assigned to passive room-temperature recovery in a given collegiate track program recorded significantly lower maximal aerobic sprint capacity measurements forty-eight hours later than they recorded at baseline.
- C. Forty-eight hours after intense exertion, athletes who completed cold immersion sessions and athletes in passive recovery were significantly more likely to experience lower-limb muscle fatigue than cardiovascular exhaustion.
- D. Athletes who completed post-training cold immersion sessions recorded significantly higher maximal aerobic sprint capacity measurements forty-eight hours later than did athletes assigned to passive room-temperature recovery.

- 9 The musicologist Plácido Gandarillas was the first director of the Archive of Traditional Music in Lima, from 1971 to 1986. A scholar asserts that one of Gandarillas's main objectives during his directorship was to record indigenous folk songs in remote Andean valleys.

Which finding, if true, would most strongly support the scholar's claim?

- A. Gandarillas spent his academic career analyzing the symphonic works of nineteenth-century German composers.
- B. Field expeditions organized by Gandarillas produced hundreds of recorded audio reels capturing traditional harvest songs and festival melodies performed by rural musicians.
- C. The music archive focused its acquisitions primarily on classical European piano sheet music.
- D. Academic publications edited by Gandarillas concentrated on urban jazz club history.

- 10 Neurophysiologists measuring synaptic fatigue patterns within mammalian auditory pathways tracked the amplitude degradation frequencies of post-synaptic potentials across extended high-frequency stimulus bursts. Prevailing neurological models state that neurotransmitter depletion occurs uniformly across all neural pathways, making specialized receptor densities irrelevant. The investigative team counter-hypothesizes that auditory relay stations rich in ionotropic glutamate receptors maintain signal clarity past the critical 200-millisecond inflection window, confirming that _____

Chart 3: Mean Post-Synaptic Potential Amplitude Retained (mV) Across Stimulus Eras

Auditory Relay Target Pathway	0 ms Baseline	200 ms Inflection	500 ms Terminal
Glutamate-Rich Core Pathway	12.4 mV	11.8 mV	11.5 mV
Glutamate-Deficient Belt Zone	11.9 mV	6.2 mV	2.1 mV
Standard Somatosensory Control	9.5 mV	4.1 mV	1.2 mV

Chart 3: Mean Post-Synaptic Potential Amplitude Retained (mV) Across Stimulus Eras

Which choice most effectively uses data from the table to complete the sentence?

- A. the glutamate-deficient belt zone degraded rapidly from 11.9 mV to 6.2 mV at the 200 ms mark, and fell further to 2.1 mV by the terminal checkpoint.
- B. the glutamate-rich core pathway maintained amplitude stability throughout the stimulus, staying at 11.8 mV at the inflection window and 11.5 mV at the terminal mark, outperforming the belt zone's rapid collapse.
- C. the standard somatosensory control generated the absolute lowest signal retention across every distinct phase evaluated by the team.
- D. amplitude metrics across all three mammalian pathways declined to absolute zero when stimulation extended past 500 milliseconds.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 11** Fire management practices involving brush burning (the use of fire to remove dense vegetation) have generated debate among ecologists studying Mediterranean shrub landscapes. A fire ecology study examining 370,000 hectares in southern California produced evidence that challenges the belief that all forms of burning produce negative outcomes. The collected evidence indicated that scrub regions exposed to prescribed scorching (the controlled burning of selected vegetation areas) demonstrated stronger recovery patterns than similar regions without this treatment. The results suggest that _____

Which choice most logically completes the text?

- A. fires are the primary cause of environmental decline in dry climates around the world.
- B. landscapes without exposure to fire are always better able to recover from ecological pressures.
- C. carefully designed interventions can sometimes increase the ability of natural areas to withstand disruption.
- D. fire has less meaningful connection to the development of Mediterranean shrub regions than previously assumed.

- 12** To address the susceptibility of materials used in components of heavy-duty emission control systems, such as diesel oxidation catalysts, to thermal sintering (active surface area loss that is induced by noble metal particle agglomeration and that often occurs during high-temperature exhaust regeneration), materials researchers have developed ceria-zirconia washcoats for producing durable catalytic monoliths. Testing the thermal aging parameters of several commercially available washcoats, Tanvi Desai et al. found that in comparison with two alumina washcoats, a lanthanum-stabilized ceria-zirconia washcoat exhibited a lower minimum surface area loss rate, a measure of the rate at which a catalytic material deactivates at a constant exhaust temperature and gas flow rate. The finding suggests that _____

Which choice most logically completes the text?

- A. washcoats based on the two alumina washcoats have chemical properties that may improve the catalytic and thermal stability of emission equipment to a greater extent than do washcoats based on the lanthanum-stabilized ceria-zirconia washcoat.
- B. the two alumina washcoats likely hold similar potential for reducing the catalytic efficiency of materials exposed to high exhaust temperatures, thus extending the life span of diesel engines.
- C. catalytic monoliths containing the lanthanum-stabilized ceria-zirconia washcoat may have the ability to withstand exhaust heat for a shorter period of time than can catalytic monoliths containing either of the two alumina washcoats.
- D. unlike the two alumina washcoats, the lanthanum-stabilized ceria-zirconia washcoat can substantially inhibit thermal sintering, provided that exhaust temperatures and gas flow rates are consistent.

- 13** In 2020, computer scientists designed an artificial intelligence system to help doctors identify certain eye diseases. Instead of replacing physicians, the software highlighted areas within medical images that appeared unusual, allowing specialists to examine those regions more carefully before making a diagnosis. During clinical trials, doctors using the system detected more early-stage diseases than doctors reviewing the same images without technological assistance. Researchers concluded that the program improved accuracy because it directed physicians' attention toward details they might otherwise overlook while still leaving the final decision to trained medical professionals.

According to the text, why did the AI system improve doctors' accuracy?

- A. Because it automatically treated patients after diagnosis.
- B. Because it replaced doctors during examinations.
- C. Because it created higher-resolution medical images.
- D. Because it highlighted unusual areas for doctors to inspect.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 14** In 2014, geographers studied satellite images showing the expansion of mangrove forests along several tropical coastlines. By comparing images collected over many years, researchers discovered that newly established mangroves gradually trapped sediment carried by rivers and ocean tides. As more sediment accumulated, the coastline slowly expanded outward in certain locations. Scientists concluded that mangrove forests not only provided habitat for wildlife but also helped reshape coastal environments through the steady buildup of deposited material over long periods of time under favorable environmental conditions.

According to the text, how did mangrove forests contribute to coastline expansion?

- A. By producing new volcanic rock beneath the ocean.
- B. By trapping sediment that gradually accumulated over time.
- C. By preventing rivers from reaching the sea.
- D. By increasing ocean wave heights.

- 15** In 1905, amidst groundbreaking theoretical advancements, physicist Albert Einstein formulated his special theory of relativity to redefine space and time. Einstein's subsequent academic career would involve more than just _____ as a visionary thinker in theoretical physics, Einstein would help revolutionize our understanding of the cosmos.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. time. Though,
- B. time, though:
- C. time though
- D. time, though,

- 16** Entomologist Gideon Thorne contributes to the biodiversity preservation group responsible for documenting newly discovered arthropods, which includes the Goliath beetle and the Amazonian orchid bee. Thorne doesn't register such species _____ all taxonomic classifications must be endorsed by a collective of seven zoologists from the fields of morphology, phylogenetics, ecology, and entomology.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. individually, furthermore;
- B. individually; furthermore,
- C. individually, furthermore,
- D. individually furthermore

- 17** The extreme pressures found deep underground can alter the molecular structure of enzymes in subterranean microbes, distorting their normal shape. The chemical trehalose counters this stress, ensuring that cellular proteins retain their original _____ is produced in high concentrations by extremophile bacteria during dry periods.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. configurations. Trehalose
- B. configurations Trehalose
- C. configurations, Trehalose
- D. configurations and trehalose

- 18** South African novelist Nadine Gordimer's celebrated literary catalog includes *The Burger's Daughter*, a novel examining anti-apartheid activism written in 1970s _____ a collection of critical essays on censorship in southern Africa; and *July's People*, her searing exploration of racial upheaval.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. England, *Burger's Daughter*,
- B. England; *Burger's Daughter*,
- C. England, *Burger's Daughter*:
- D. England; *Burger's Daughter*

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 19** Although the conservatory orchestra traditionally performs classical symphonies, dwindling seasonal ticket sales routinely prompt board interventions, _____ the artistic director to program contemporary film scores.
Which choice completes the text so that it conforms to the conventions of Standard English?
- A. obliged
B. obliges
C. obliging
D. to oblige
-
- 20** In the early days of aviation development, an aeronautical designer named Percival Wright was testing glider wing configurations when his aerodynamic consultant Bernadette Thorne recommended altering the camber curve to improve lift-to-drag ratios. To verify the adjustment, Thorne _____ recording wind tunnel pressure readings.
Which choice completes the text so that it conforms to the conventions of Standard English?
- A. analyzed
B. analyzes
C. had analyzed
D. was analyzing
-
- 21** Australian Coraline Pendelton was a marine pharmacologist whose work helped medicine combat cellular diseases. Applauded in 2014 as the most promising pharmaceutical breakthrough from marine organisms, _____ led to the development of a novel anti-tumor drug.
Which choice completes the text so that it conforms to the conventions of Standard English?
- A. Pendelton isolated the peptide compound in 2011; it
B. in 2011 Pendelton isolated the peptide compound, which
C. Pendelton isolated the peptide compound in 2011 and
D. Pendelton's 2011 isolation of the peptide compound
-
- 22** The microfluidic channels that direct chemical reagents in portable diagnostic kits could eventually be etched into modified paper substrates. Bioengineers in Singapore have formulated a procedure to apply laser ablation and wax stamping to seal porous cellulose sheets. _____ these hydrophobic barriers become precise enough that liquid samples can be routed automatically through specific reaction zones.
Which choice completes the text with the most logical transition?
- A. Subsequently,
B. As a result,
C. Conversely,
D. Thus,
-
- 23** The Ottoman Empire, the vast transcontinental state that controlled Southeast Europe and Western Asia, was governed by the House of Osman for over six centuries. _____ administrative titles and architectural styles, such as minaret-studded civic skylines, became widespread across the Balkan provinces.
Which choice completes the text with the most logical transition?
- A. Alternatively,
B. Consequently,
C. For example,
D. Specifically,
-
- 24** Aksum, the ancient trading empire located in the Horn of Africa, commanded vital maritime trade routes linking the Roman Empire with India. _____ indigenous script innovations, such as the vocalization of Ge'ez consonant characters, established enduring epigraphic traditions in East Africa.
Which choice completes the text with the most logical transition?
- A. Conversely,
B. Consequently,
C. For example,
D. Likewise,
-

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 25**
- 16th- and 17th-century European botanical gardens were established as academic institutions for cultivating medicinal plants and conducting scientific research.
 - These specialized grounds featured systematic planting beds, herbaria, and early glasshouses for exotic species.
 - The Hortus Botanicus of Leiden, established in the Netherlands in 1590 CE, became a premier center for botanical research and Asian flora cultivation.
 - Leiden's founding collection expanded rapidly as Dutch merchant ships returned from overseas expeditions with novel biological specimens.
 - The Jardin des Plantes in Paris, France, established officially in 1635 CE, served as a royal medicinal garden and public botanical teaching center.
 - Paris's sprawling facility cultivated thousands of cataloged plant species imported from global colonies for pharmaceutical study.

The student wants to emphasize a similarity between the founding centuries of both the Leiden and Paris botanical gardens. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A.** Both major botanical gardens were established during the same historical period, with the Hortus Botanicus of Leiden founded in 1590 CE and the Jardin des Plantes established in 1635 CE.
- B.** While the Leiden botanical garden served as a center for Asian flora cultivation, the Paris facility functioned primarily as a royal medicinal teaching garden.
- C.** European botanical gardens were established as academic institutions featuring systematic planting beds and early glasshouses to cultivate exotic species for scientific study.
- D.** Leiden's collection expanded rapidly as Dutch merchant ships returned with novel specimens, whereas the Paris garden cultivated thousands of cataloged plants from global colonies.

- 26**
- The Gutenberg Bible was among the first major books printed using movable metal type in Europe.
 - It was produced by Johannes Gutenberg around 1455.
 - The invention of movable type allowed books to be produced more efficiently.
 - The printing process contributed to the spread of knowledge throughout Europe.
 - Only a limited number of original copies survive today.

The student wants to introduce the Gutenberg Bible. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A.** Johannes Gutenberg developed movable metal type printing technology in Europe during the fifteenth century.
- B.** The Gutenberg Bible, produced around 1455 by Johannes Gutenberg, was among the first major books printed with movable metal type and helped expand access to written knowledge.
- C.** Only a limited number of original Gutenberg Bible copies survive today.
- D.** Printing technology allowed knowledge to spread throughout Europe.

- 27**
- Tardigrades (*Ramazzottius varieornatus*) are microscopic animals famous for surviving severe dehydration (desiccation).
 - In a 2019 study, biologist Takekazu Kunieda analyzed *R. varieornatus* cells undergoing extreme drying.
 - He investigated how the organism prevents its cellular membrane from collapsing without water.
 - He discovered that tardigrade-disordered proteins (TDPs) form a protective, glass-like matrix inside the cells during drying.
 - He concluded that this vitrification process preserves cellular structures until rehydration occurs.

The student wants to present the study and its findings. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A.** In a 2019 study of *Ramazzottius varieornatus*, Kunieda determined that tardigrade-disordered proteins form a protective glass-like matrix inside cells during drying, preserving cellular structures.
- B.** *Ramazzottius varieornatus* is a species of microscopic tardigrade capable of surviving severe dehydration through cellular protection.
- C.** In 2019, Kunieda studied how tardigrades prevent their cellular membranes from collapsing during severe desiccation.
- D.** Although tardigrades survive extreme drying, Kunieda was initially unsure how their cellular membranes remained intact without water.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Exam 3 Math: Module 1**Math · Module 1**

22 questions · 35 minutes · A calculator is allowed on all questions. For student-produced responses, enter your answer as a number or fraction.

- 1** A chess club has \$240 in its account and deposits \$12 into the account each week from member dues. The club needs \$600 for a tournament trip. After how many weeks will the account contain exactly \$600?
- A. 30
B. 50
C. 70
D. 360
- 2** The functions f and g are defined by $f(x) = 4x - 3$ and $g(x) = 3x + 2$. If $f(g(a)) = 41$, what is the value of a ?
- A. 3
B. 4
C. 11
D. 41
- 3** The function f is defined by $f(x) = (1/2)x - 3$. The graph of the function g is obtained by stretching the graph of $y = f(x)$ vertically by a factor of 4 and then shifting the result 6 units to the left. Which of the following defines g ?
- A. $g(x) = 2x - 12$
B. $g(x) = 2x$
C. $g(x) = 2x + 12$
D. $g(x) = 2x - 24$
- 4** The equation $3x + 6y = 24$ can be written in the form $y = mx + b$ in the xy -plane, where m and b are constants. What is the value of $m + b$?
- Student-produced response: enter your answer. _____
- 5** $x/3 + y/4 = 4$
 $0.5x - 0.25y = 1$
The solution to the given system of equations is (x, y) . What is the value of $x + y$?
- Student-produced response: enter your answer. _____
- 6** $3(x + 2) - 2(y - 1) = 6$
 $2(x - 1) + 4(y + 3) = 30$
If (x, y) is the solution to the system of equations above, what is the value of y ?
- A. 2
B. 4
C. 6
D. 8
- 7** Which ordered pair (x, y) is a solution to the system of inequalities $y \geq x + 1$ and $x + y \leq 9$?
- A. (2, 3)
B. (3, 2)
C. (4, 6)
D. (5, 5)
- 8** For $x > 0$, $\sqrt[3]{(54x^7)} = a \cdot x^2 \cdot \sqrt[3]{(2x)}$, where a is a constant. What is the value of a ?
- Student-produced response: enter your answer. _____
- 9** For $x > 0$, which of the following is equivalent to $(32x^{10})^{3/5} / x^{1/2}$?
- A. $8x^{11/2}$
B. $8x^{13/2}$
C. $6x^{11/2}$
D. $8x^3$
- 10** In the equation $x^2 + kx + (k + 3) = 0$, k is a positive constant. If the equation has exactly one real solution, what is the value of k ?
- Student-produced response: enter your answer. _____
- 11** What is the solution to the equation $1/(x - 3) + 2/(x + 3) = 1/(x^2 - 9)$?
- Student-produced response: enter your answer. _____
- 12** What is the positive solution to the equation $2x^2 + 5x = 3x + 24$?
- A. -4
B. 3
C. 4
D. 12

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

13

Values of x and y

x	y
-1	5
0	2
1	1
2	2

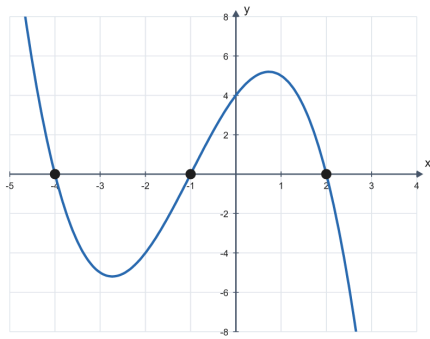
Values of x and y

The table shows four pairs of values of x and y. Which of the following equations could represent the relationship between x and y?

- A. $y = x^2 + 2x + 2$ B. $y = x^2 - 2x + 2$
 C. $y = 2x^2 - 2x + 2$ D. $y = x^2 - 2x - 2$

14

Graph of $y = f(x)$



Graph of $y = f(x)$

The graph of the polynomial function f is shown in the xy -plane. Which of the following is equal to 0?

- A. $f(-3)$ B. $f(-2)$
 C. $f(-1)$ D. $f(0)$

15

A syrup recipe at a bakery uses 5 cups of water for every 2 cups of sugar. At this ratio, how many cups of water are needed for 8 cups of sugar?

- A. 3.2 B. 11
 C. 20 D. 40

16

An outdoor-gear shop buys a tent for a cost c . The shop marks the cost up by 50% to set the marked price and then sells the tent for 20% off the marked price. If the sale price is \$72, what was the shop's cost c ?

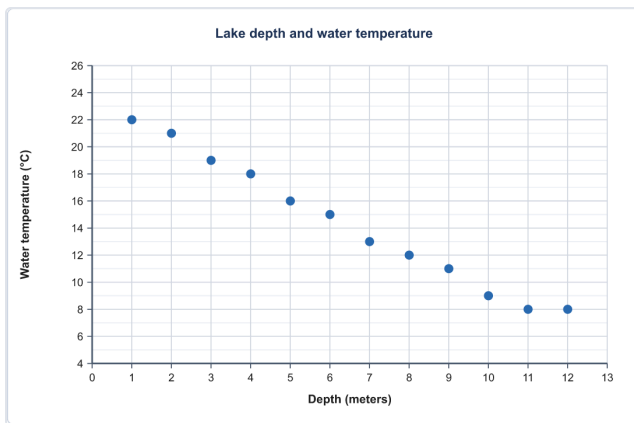
- A. \$48 B. \$60
 C. \$90 D. \$108

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

17



Lake depth and water temperature

The scatterplot shows the depth below the surface and the water temperature at 12 locations in a lake. Which of the following statements best describes the relationship between depth and temperature?

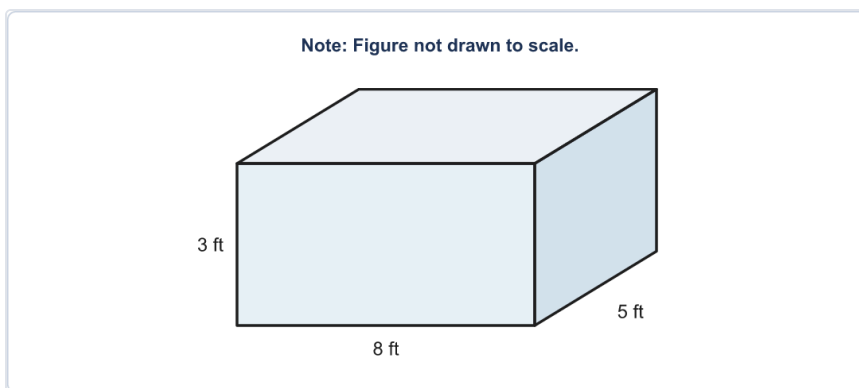
- A. As depth increases, temperature tends to increase.
- B. As depth increases, temperature tends to decrease.
- C. Depth and temperature appear to be unrelated.
- D. As temperature increases, depth tends to stay the same.

18

A city library director selected a random sample of 200 library card holders from the list of all card holders who live in the city's East District. Each selected card holder was asked how many books they borrowed last year. Which of the following is the largest population to which the results can be reasonably generalized?

- A. All library card holders who live in the East District
- B. All library card holders in the city
- C. All residents of the East District
- D. The 200 card holders who were selected

19



Note: Figure not drawn to scale.

The figure shows a closed rectangular crate that a zoo uses to ship supplies. The crate is 8 feet long, 5 feet wide, and 3 feet high. What is the total outside surface area, in square feet, of the crate?

- A. 79
- B. 120
- C. 158
- D. 240

20

A rectangular banner at a museum has a length that is 6 feet greater than its width. The area of the banner is 91 square feet. What is the perimeter, in feet, of the banner?

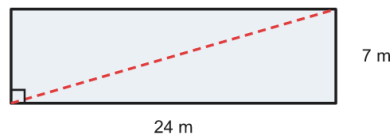
- A. 20
- B. 26
- C. 40
- D. 91

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

21

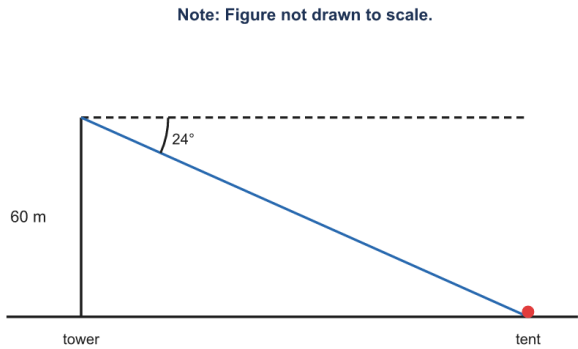


Figure

A rectangular enclosure at a zoo measures 24 meters by 7 meters. A walkway will run along a diagonal of the enclosure, as shown by the dashed segment. What is the length, in meters, of the walkway?

- A. 17
C. 25
- B. 24
D. 31

22



Note: Figure not drawn to scale.

From the top of a 60-meter observation tower at a campground, the angle of depression to a tent on level ground is 24° . Which expression gives the horizontal distance, in meters, from the base of the tower to the tent?

- A. $60/\tan 24^\circ$
B. $60 \tan 24^\circ$
C. $60/\sin 24^\circ$
D. $60 \cos 24^\circ$

Exam 3 Math: Module 2

Math · Module 2

22 questions · 35 minutes · A calculator is allowed on all questions. For student-produced responses, enter your answer as a number or fraction.

- 1 If $x = 6$ is a solution to the equation $5x + k = 38$, what is the value of k ?

A. -8
B. 2
C. 8
D. 68

2 The function f is defined by $f(x) = 2x - 3$. For what value of x does $f(x + 3) = 3f(x)$?

A. 0
B. 3
C. 6
D. 12

3 The linear function f satisfies $f(2) = 5$ and $f(f(2)) = 11$. What is the value of $f(0)$?

A. -1
B. 1
C. 3
D. 5

4 The equation $4x - 5y = 10$ can be written in the form $y = mx + b$, where m and b are constants. What is the value of $m \times b$?

Student-produced response: enter your answer.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 5 $(3/5)x + (1/2)y = 5.1$
 $0.25x - (1/3)y = 0.5$
In the system of equations above, what is the value of $x + y$?
Student-produced response: enter your answer. _____
- 6 A school library bought hardcover books at x dollars each and paperback books at y dollars each. The library spent \$70 on 5 hardcover books and 3 paperback books, and it spent \$52 on 2 hardcover books and 6 paperback books. These purchases are represented by the system $5x + 3y = 70$ and $2x + 6y = 52$. Which of the following statements is true?
A. A paperback book costs \$11 and a hardcover book costs \$5.
B. A hardcover book costs \$3 more than a paperback book.
C. A hardcover book costs \$6 more than a paperback book.
D. One hardcover book and one paperback book together cost \$21.
- 7 A wind turbine produced 840 kilowatt-hours (kWh) of energy in March. In April, its output was 5% higher than in March. For May, the operator wants the output, m kWh, to be at most 12% less than the April output. Which inequality represents this requirement?
A. $m \leq 0.88(1.05)(840)$
B. $m \geq 0.88(1.05)(840)$
C. $m \geq 0.88(840)$
D. $m \geq 0.93(840)$
- 8 For $x > 0$, the expression $\sqrt[4]{(x^3)} \cdot \sqrt[3]{(x^2)} / \sqrt{x}$ is equivalent to x^k , where k is a constant. What is the value of k ?
Student-produced response: enter your answer. _____
- 9 The total resistance R of two resistors connected in parallel is given by $1/R = 1/R_1 + 1/R_2$. Which of the following equations gives R_1 in terms of R and R_2 ?
A. $R_1 = (R \cdot R_2)/(R_2 - R)$
B. $R_1 = R_2 - R$
C. $R_1 = (R \cdot R_2)/(R_2 + R)$
D. $R_1 = (R_2 - R)/(R \cdot R_2)$
- 10 What is the solution to the equation $|2x - 5| = 3x - 1$?
Student-produced response: enter your answer. _____
- 11 If x is a real number such that $9^x - 3^{x+1} = 54$, what is the value of x ?
Student-produced response: enter your answer. _____
- 12 For which value of k does the equation $|x + 3| = |2x - k|$ have exactly one real solution?
A. -6
B. -3
C. 3
D. 6
- 13 A flare is launched upward from the balcony of a lighthouse. The height of the flare above the ground, in feet, t seconds after it is launched is given by $h(t) = -16t^2 + 48t + 160$. After how many seconds does the flare hit the ground?
A. 2
B. 5
C. 10
D. 48
- 14 The function f is defined by $f(x) = 7/(x^2 - 2x - 15)$. For which of the following values of x is $f(x)$ undefined?
A. -5
B. 3
C. 5
D. 15
- 15 At an orchard, a juice press fills 36 bottles in 45 seconds. Assuming the press keeps this rate, how many bottles does it fill in 2 hours?
A. 96
B. 160
C. 4,320
D. 5,760
- 16 A chemistry teacher mixes 12 liters of a 20% acid solution with 8 liters of a 45% acid solution. What is the percent of acid in the resulting mixture?
A. 20%
B. 30%
C. 32.5%
D. 35%

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

- 17 A recycling plant has two sorting lines. Line X processes 2,400 items per day, and 15% of these are contaminated. Line Y processes 1,600 items per day, and 20% of these are contaminated. If one of the 4,000 items processed in a day is selected at random, what is the probability that it is contaminated?

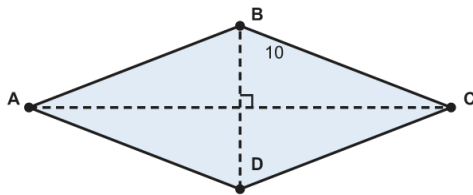
A. 0.09
B. 0.15
C. 0.17
D. 0.175

- 18 At a company, employees who requested standing desks reported less back pain than employees who did not request standing desks. A manager claims that standing desks reduce back pain. Which of the following, if true, provides the most serious alternative explanation for the difference in reported back pain?

A. Standing desks cost more than traditional desks.
B. Employees with severe back pain tended not to request standing desks because standing for long periods was painful for them.
C. The company sells standing desks in several colors.
D. Back pain scores were collected on the same day for all employees.

19

Note: Figure not drawn to scale.



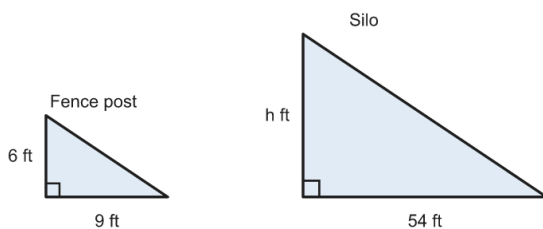
Note: Figure not drawn to scale.

In rhombus ABCD, the diagonals intersect at right angles and $BD = 10$ inches. The area of the rhombus is 120 square inches. What is the perimeter, in inches, of the rhombus?

A. 34
B. 52
C. 68
D. 120

20

Note: Figure not drawn to scale.



Note: Figure not drawn to scale.

A vertical fence post 6 feet tall casts a shadow 9 feet long. At the same time, a vertical silo casts a shadow 54 feet long. The figure shows the right triangles formed by each object, its shadow, and the line from the top of the object to the end of its shadow. What is the height, in feet, of the silo?

A. 6
B. 36
C. 51
D. 81

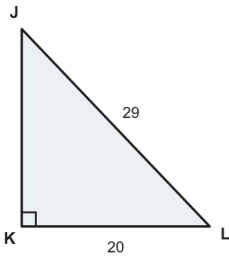
Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

21

Note: Figure not drawn to scale.



Note: Figure not drawn to scale.

In the figure, triangle JKL has a right angle at K. What is the value of $\cos J$?

- A. 20/29

B. 21/29

C. 21/20

D. 29/21

22 For acute angles measuring $(7x + 2)^\circ$ and $(2x + 7)^\circ$, $\sin (7x + 2)^\circ = \cos (2x + 7)^\circ$. What is the value of x ?

- A. 1

B. 9

C. 11

D. 19

Key

Practice Exam 3: Answers and Explanations

Check each module, then use the skill map to find what to review. Every explanation shows why the right answer works and why the tempting wrong answers fail.

READING AND WRITING · MODULE 1 · ANSWER KEY

1 A 2 B 3 B 4 A 5 B 6 B 7 A 8 B 9 C 10 A 11 C 12 A 13 A 14 C 15 C 16 A 17 D 18 A 19 B 20 B 21 D 22 A 23 A 24 B 25 A 26 D 27 B

READING AND WRITING · MODULE 2 · ANSWER KEY

1 A 2 B 3 A 4 B 5 A 6 B 7 D 8 D 9 B 10 B 11 C 12 D 13 D 14 B 15 B 16 B 17 A 18 B 19 C 20 A 21 D 22 B 23 B 24 B 25 A 26 B 27 A

MATH · MODULE 1 · ANSWER KEY

1 A 2 A 3 B 4 7/2 5 14 6 B 7 A 8 3 9 A 10 6 11 4/3 12 B 13 B 14 C 15 C 16 B 17 B 18 A 19 C 20 C 21 C 22 A

MATH · MODULE 2 · ANSWER KEY

1 C 2 B 3 B 4 -8/5 5 9 6 C 7 B 8 11/12 9 A 10 6/5 11 2 12 A 13 B 14 C 15 D 16 B 17 C 18 B 19 B 20 B 21 B 22 B

SKILL MAP: WHICH QUESTIONS TEST WHICH SKILL

Module	Skill	Questions
R&W M1	Words in Context	1, 2, 3, 4
R&W M1	Text Structure	5
R&W M1	Author's Purpose	6
R&W M1	Cross-Text Connections	7
R&W M1	Textual Evidence	8, 9
R&W M1	Quantitative Evidence	10
R&W M1	Inferences	11, 12
R&W M1	Central Ideas and Details	13, 14
R&W M1	Boundaries	15, 16, 17, 18
R&W M1	Form, Structure, and Sense	19, 20, 21
R&W M1	Transitions	22, 23, 24

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Module	Skill	Questions
R&W M1	Rhetorical Synthesis	25, 26, 27
R&W M2	Words in Context	1, 2, 3, 4
R&W M2	Text Structure	5
R&W M2	Author's Purpose	6
R&W M2	Cross-Text Connections	7
R&W M2	Textual Evidence	8, 9
R&W M2	Quantitative Evidence	10
R&W M2	Inferences	11, 12
R&W M2	Central Ideas and Details	13, 14
R&W M2	Boundaries	15, 16, 17, 18
R&W M2	Form, Structure, and Sense	19, 20, 21
R&W M2	Transitions	22, 23, 24
R&W M2	Rhetorical Synthesis	25, 26, 27
Math M1	Linear Equations (1 Var)	1
Math M1	Linear Functions	2, 3
Math M1	Linear Equations (2 Var)	4
Math M1	Systems of Equations	5, 6
Math M1	Linear Inequalities	7
Math M1	Equivalent Expressions	8, 9
Math M1	Nonlinear Equations	10, 11, 12
Math M1	Nonlinear Functions	13, 14
Math M1	Ratios and Rates	15
Math M1	Percentages	16
Math M1	Two-Variable Data	17
Math M1	Sample Statistics	18
Math M1	Area and Volume	19, 20
Math M1	Right Triangles and Trig	21, 22
Math M2	Linear Equations (1 Var)	1
Math M2	Linear Functions	2, 3
Math M2	Linear Equations (2 Var)	4
Math M2	Systems of Equations	5, 6
Math M2	Linear Inequalities	7
Math M2	Equivalent Expressions	8, 9
Math M2	Nonlinear Equations	10, 11, 12
Math M2	Nonlinear Functions	13, 14
Math M2	Ratios and Rates	15
Math M2	Percentages	16
Math M2	Probability	17
Math M2	Statistical Claims	18

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

Module	Skill	Questions
Math M2	Area and Volume	19
Math M2	Lines, Angles, Triangles	20
Math M2	Right Triangles and Trig	21, 22

R&W 1 Reading and Writing, Module 1: Explanations

1. Answer: A · WORDS IN CONTEXT · MEDIUM

Correct answer: A. 'Hinder' means to hold back. Artificially low prices for traditional utilities would hold back renewables.

Why the other choices are wrong:

- B) 'Multiply' means to increase, the opposite.
- C) 'Mock' means to ridicule, which does not describe a market effect.
- D) 'Obscure' means to hide, which does not describe hindering expansion.

2. Answer: B · WORDS IN CONTEXT · HARD

Correct answer: B. 'Finishing' means completing. The author eagerly anticipates completing the project once the packet is delivered.

Why the other choices are wrong:

- A) 'Demolishing' means destroying, which does not describe finishing a treatise.
- C) 'Evaluating' means judging, which does not fit 'terminating'.
- D) 'Disseminating' means spreading, which does not mean ending the work.

3. Answer: B · WORDS IN CONTEXT · EASY

Correct answer: B. 'Matching' means correspondence. The sundew achieves a predatory matching with the insects' activity.

Why the other choices are wrong:

- A) 'Divide' means a split, which contradicts a timed match.
- C) 'Clash' means a conflict, which contradicts the successful timing.
- D) 'Gap' means a space between, which contradicts a close match.

4. Answer: A · WORDS IN CONTEXT · HARD

Correct answer: A. 'Fortuitous' means by chance. Colleagues claimed it was chance, but the ants relocated barriers.

Why the other choices are wrong:

- B) 'Explicit' means clearly and openly stated, which does not contrast.
- C) 'Intriguing' means arousing curiosity, which does not contrast.
- D) 'Strategic' means carefully planned to achieve a goal, which would agree with the findings, not the colleagues.

5. Answer: B · TEXT STRUCTURE · HARD

Correct answer: B. The text discusses the brood-parasitic behavior of cuckoo-wasps. The underlined sentence explains the detailed mechanics of how this behavior is carried out (rapidly slipping into mud chambers when the resident is absent).

Why the other choices are wrong:

- A) The sentence describes how cuckoo-wasps slip into chambers while hosts are away, not the structural composition of the mud-dauber nests themselves.
- C) The host wasps' defensive behavior is not discussed; they are simply absent when the parasites slip in.
- D) No anatomical or thermal adaptation is mentioned; the sentence describes the wasps' behavior of entering chambers.

6. Answer: B · AUTHOR'S PURPOSE · MEDIUM

Correct answer: B. Lord Henry encourages Dorian to pursue youth and sensation above all, which is the core of his philosophy.

Why the other choices are wrong:

- A) Painting is not discussed; the portrait is only a symbol of youth in Dorian's wish.
- C) Nobody suggests Dorian paint; he is the subject of the portrait, so this is unsupported.
- D) Dorian's appearance is not described; the passage focuses on ideas about youth.

7. Answer: A · CROSS-TEXT CONNECTIONS · MEDIUM

Correct answer: A. Text 2 asserts that rapid sediment compaction locks detrital grains before biological mixing can occur.

Why the other choices are wrong:

- B) Text 2 analyzes magnetic mineral content in proglacial lake varves.
- C) Text 2 shows rapid compaction prevents bioturbation from blurring records.
- D) Text 2 successfully employs cryogenic magnetometer scanning.

8. Answer: B · TEXTUAL EVIDENCE · MEDIUM

Correct answer: B. This option demonstrates that contractors pay lower per-unit prices unless suppliers deliver multi-pallet batches, directly supporting the claim that batching production maximizes capital returns per unit.

Why the other choices are wrong:

- A) Lower clay excavation expenses do not explain brick firing schedules.
- C) Initial equipment loan amounts are irrelevant to brick firing timing strategies.
- D) A delivery fee per trip that stays the same regardless of volume gives only an indirect cost-saving reason to consolidate; it does not show the higher return per unit that the claim describes, so it is weaker support than the keyed option.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

9. Answer: C · TEXTUAL EVIDENCE · MEDIUM

Correct answer: C. This explicitly connects Carson's documentation of dangerous chemical dissemination and persistent ecotoxicological accumulation (biomagnifying across trophic levels) to the core of the student's claim.

Why the other choices are wrong:

- A) While it discusses Carson's view on aggressive insect control, it emphasizes biological resistance rather than public accountability or pesticide dissemination restrictions.
- B) This focuses on ecological feedback loops and impact assessments rather than manufacturer liability and dissemination restrictions.
- D) This focuses on financial secrecy and corporate balance sheets rather than long-term ecotoxicological accumulation.

10. Answer: A · QUANTITATIVE EVIDENCE · MEDIUM

Correct answer: A. In May the paired-reading group averaged 96 words per minute and the comparison group 72, a difference of 24. This completes the sentence accurately and shows the widened gap.

Why the other choices are wrong:

- B) The figures are misaligned: 88 and 64 are values from different months, not May, and in May the paired-reading mean was 96 versus 72 for the comparison group.
- C) The 62 value is the paired-reading group's September rate, not the comparison group's May rate; the actual May gap is 24 (96 versus 72).
- D) The paired-reading group gained 34 words per minute (62 to 96) and the comparison group gained only 13 (59 to 72), so both gains and their order are wrong.

11. Answer: C · INFERENCES · EASY

Correct answer: C. Grassland areas with targeted herbicide application had stronger ecological conditions, so certain agricultural techniques can support ecological goals when well managed.

Why the other choices are wrong:

- A) Says agricultural chemicals must be eliminated everywhere, contradicted by the better conditions under targeted use.
- B) Claims untreated landscapes generally do better, the reverse of the finding.
- D) Says chemical control has little effect on plant relationships, though treated areas differed measurably.

12. Answer: A · INFERENCES · EASY

Correct answer: A. Reef areas with regulated netting showed greater ecological activity than unfished ones, so supervised commercial harvesting can be compatible with healthy reefs.

Why the other choices are wrong:

- B) Attributes the result to natural differences between reef sites, explaining away the findings instead of reflecting them.
- C) Claims reducing total fishing matters more than improving methods, a comparison the study did not make.
- D) Says conservation requires preventing all commercial fishing even when regulated, contradicted by the benefit seen with regulated netting.

13. Answer: A · CENTRAL IDEAS AND DETAILS · MEDIUM

Correct answer: A. The passage states that water was delivered directly to plant roots, reducing waste.

Why the other choices are wrong:

- B) Rainfall is never mentioned.
- C) The amount of crops planted is not discussed.
- D) Underground reservoirs are not mentioned.

14. Answer: C · CENTRAL IDEAS AND DETAILS · MEDIUM

Correct answer: C. The text says Brazil contains 60 percent of the Amazon Rainforest.

Why the other choices are wrong:

- A) The text says Peru has 13 percent, so it is not the majority.
- B) The text says Colombia has 10 percent, so it is not the majority.
- D) Venezuela is not named in the text and falls under lesser amounts.

15. Answer: C · BOUNDARIES · EASY

Correct answer: C. "magmatic" and "seismic" are modifiers that work together to describe "structures," so no punctuation should separate them; "magmatic seismic structures" reads as one smooth noun phrase.

Why the other choices are wrong:

- A) The period ends the sentence after "magmatic," leaving a fragment and cutting the modifiers off from "structures" and the rest of the sentence.
- B) A colon must follow a complete independent clause, but "magmatic" is only a modifier within the noun phrase, so the colon wrongly interrupts "magmatic seismic structures."
- D) A semicolon must join two independent clauses, but "magmatic" does not end a complete clause, so the semicolon wrongly splits the modifiers from the noun they describe.

16. Answer: A · BOUNDARIES · EASY

Correct answer: A. The comma goes inside the closing quotation mark and attaches the participial phrase beginning "asserting" to the sentence, with no other punctuation needed before "that".

Why the other choices are wrong:

- B) A colon must follow a complete independent clause, and "asserting" begins a participial phrase, so a colon cannot introduce it.
- C) A semicolon must join two independent clauses, but "asserting..." is a participial phrase, so the semicolon leaves a fragment.
- D) A period ends the sentence and leaves the phrase beginning "asserting" as a sentence fragment.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

17. Answer: D • BOUNDARIES • HARD

Correct answer: D. A comma followed by the coordinating conjunction "but" correctly joins the two independent clauses.

Why the other choices are wrong:

- A) A comma alone between two independent clauses creates a comma splice.
- B) The conjunction "but" joining two independent clauses needs a comma before it; omitting the comma is a punctuation error.
- C) With no punctuation or conjunction, the two independent clauses are fused into a run-on sentence.

18. Answer: A • BOUNDARIES • EASY

Correct answer: A. The comma properly separates the main clause from the supplementary noun phrase describing the microscopic organisms.

Why the other choices are wrong:

- B) Adding "and" creates an ungrammatical conjunction error.
- C) A semicolon disrupts the modifier flow.
- D) A period results in a sentence fragment.

19. Answer: B • FORM, STRUCTURE, AND SENSE • HARD

Correct answer: B. The singular present tense verb "catalogs" properly matches the singular noun antecedent "inventory."

Why the other choices are wrong:

- A) The present perfect plural verb "have cataloged" violates subject-verb number agreement with the singular noun "inventory."
- C) The past continuous plural verb "were cataloging" creates an ungrammatical number mismatch.
- D) The plural base verb "catalog" fails to agree with the singular noun antecedent "inventory."

20. Answer: B • FORM, STRUCTURE, AND SENSE • MEDIUM

Correct answer: B. The time marker "Today" signals a current, ongoing status, so the simple present tense "is" is required.

Why the other choices are wrong:

- A) The past perfect "had been" places the state before an earlier past moment and does not fit the present-time marker "Today".
- C) The simple past tense "was" confines the recognition to the past, which contradicts the present-time marker "Today".
- D) The future tense "will be" projects the status into the future, but "Today" signals a status that already holds now.

21. Answer: D • FORM, STRUCTURE, AND SENSE • EASY

Correct answer: D. "islands' tide mark the channels'" supplies the possessive forms the sentence needs: "islands'" shows possession of the tide, and "channels'" must be possessive to modify "tidal flow".

Why the other choices are wrong:

- A) "islands tide" has no apostrophe, so it does not show possession, and the ending "channel's" does not match the plural noun the passage describes.
- B) "channels" has no possessive apostrophe, but it must be possessive to modify "tidal flow" after the blank.
- C) The singular possessive "island's" implies a single island, but the passage's place name is plural in form, so the plural possessive "islands'" fits.

22. Answer: A • TRANSITIONS • HARD

Correct answer: A. "nonetheless" is correct because it properly contrasts incomplete syllabic inventory expectations with complete lexical paradigms recovered from a damaged tablet.

Why the other choices are wrong:

- B) "consequently" is incorrect because it denotes causation.
- C) "additionally" is incorrect because it adds supplemental data.
- D) "similarly" is incorrect because it shows resemblance.

23. Answer: A • TRANSITIONS • EASY

Correct answer: A. "Therefore" is correct because it successfully demonstrates the closure decision executed by municipal authorities after discovering wiring hazards.

Why the other choices are wrong:

- B) "Conversely" is incorrect because it introduces an inverted contrast, which contradicts the direct response to safety hazards.
- C) "For example" is incorrect because it supplies an example rather than showing a direct administrative consequence of an inspection.
- D) "Likewise" is incorrect because it draws a parallel comparison rather than tracking a hazard to its repair solution.

24. Answer: B • TRANSITIONS • EASY

Correct answer: B. "Conversely" is correct because the second sentence contrasts the direct-democracy systems with the concentration of power in large organizations described in the first sentence.

Why the other choices are wrong:

- A) "Subsequently" signals a time order or a step in a sequence, but the second sentence contrasts with the first.
- C) "For instance" introduces an example, but the second sentence contrasts with the first.
- D) "In other words" restates the previous idea, but the second sentence contrasts with the first.

25. Answer: A • RHETORICAL SYNTHESIS • MEDIUM

Correct answer: A. This choice presents both the purpose of the study (investigating whether robins use Earth's magnetic field) and the methodology (controlled magnetic conditions and observation of orientation behavior).

Why the other choices are wrong:

- B) This choice focuses mainly on the discovery that birds possess a magnetic sense but does not explain how researchers conducted the study.
- C) This choice summarizes the topic and finding but does not clearly present the research method used by the scientists.
- D) This choice includes the methodology and finding but does not explain the specific research question the study was designed to investigate.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

26. Answer: D · RHETORICAL SYNTHESIS · MEDIUM

Correct answer: D. Compare the wood decomposition strategies of brown rot (decomposes cellulose, leaves lignin intact) vs. white rot (decomposes lignin and cellulose). D contrasts the target chemical components broken down by each.

Why the other choices are wrong:

- A) It only describes brown rot fungi, omitting white rot fungi.
- B) It contrasts brown rot chemistry with white rot habitat.
- C) It vaguely states that they use "distinct enzymatic pathways" without detailing or comparing the actual strategies.

27. Answer: B · RHETORICAL SYNTHESIS · MEDIUM

Correct answer: B. Option B states Mendeleev's innovative method: leaving deliberate blank spaces in his table to predict undiscovered elements. Option A only describes how he arranged the known elements.

Why the other choices are wrong:

- A) It describes how Mendeleev arranged the known elements by atomic weight, not his method of leaving gaps to predict undiscovered elements.
- C) It describes how modern tables are organized by atomic number, which is a later revision and not Mendeleev's method of anticipating new elements.
- D) It describes the table's present-day importance in education and research, not the innovative method Mendeleev used to predict undiscovered elements.

R&W 2 Reading and Writing, Module 2: Explanations**1. Answer: A · WORDS IN CONTEXT · HARD**

Correct answer: A. 'Discernable' means able to be recognized. Lateralization is easy to see in songbirds but less recognizable in waterfowl.

Why the other choices are wrong:

- B) 'Captivating' means charming and holding attention, which does not fit the sentence, where the blank needs the sense of 'discernable'.
- C) 'Substantial' means large in amount or importance, which does not fit the sentence, where the blank needs the sense of 'discernable'.
- D) 'Advantageous' means beneficial, which does not fit the sentence, where the blank needs the sense of 'discernable'.

2. Answer: B · WORDS IN CONTEXT · HARD

Correct answer: B. 'Idiosyncratic' means peculiar to an individual. Many women are pilots and computers in Kowal's world, so these jobs are far from rare.

Why the other choices are wrong:

- A) 'Flawless' means perfect, which does not fit the sentence, where the blank needs the sense of 'idiosyncratic'.
- C) 'Commercial' means relating to business and profit, which does not fit the sentence, where the blank needs the sense of 'idiosyncratic'.
- D) 'Tedious' means boring because of length or repetition, which does not fit the sentence, where the blank needs the sense of 'idiosyncratic'.

3. Answer: A · WORDS IN CONTEXT · HARD

Correct answer: A. 'Congruencies with' means points of agreement. The sugar glider has features matching those of the flying squirrel.

Why the other choices are wrong:

- B) 'Anxieties about' means worries concerning, which does not describe shared traits.
- C) 'Representations of' means depictions, which does not describe shared features.
- D) 'Arguments for' means reasons given in support of, which does not describe shared features.

4. Answer: B · WORDS IN CONTEXT · MEDIUM

Correct answer: B. 'Singular' means unique. A diverse pantheon of warriors serves in the armies, so Rin's path is by no means unique.

Why the other choices are wrong:

- A) 'Errorless' means free from mistakes, which does not fit the sentence, where the blank needs the sense of 'singular'.
- C) 'High-paying' means providing a large income, which does not fit the sentence, where the blank needs the sense of 'singular'.
- D) 'Laborious' means requiring hard effort, which does not fit the sentence, where the blank needs the sense of 'singular'.

5. Answer: A · TEXT STRUCTURE · HARD

Correct answer: A. The underlined sentence uses human metaphors ("human companions," "peculiar vitalities") to elaborate on the inherent physiological differences in sap-yield across the trees.

Why the other choices are wrong:

- B) The sentence says the trees differ in how readily they yield sap; it does not claim human intervention standardizes their responses.
- C) The loggers' exhaustion or alienation is never described; the sentence concerns the trees' varying qualities.
- D) The sentence does not portray the loggers as violent or the trees as delicate; it compares the pines to human companions with differing temperaments.

6. Answer: B · AUTHOR'S PURPOSE · HARD

Correct answer: B. The speaker questions the wall, highlighting the friction between his desire for order and the natural entropy/human meddling.

Why the other choices are wrong:

- A) The frozen ground-swell is one cause the poem mentions, but the poem is not a scientific explanation of geology.
- C) Hunters are mentioned as one source of damage, but the poem does not criticize them over property rights.
- D) The speaker questions the wall rather than arguing that walls are essential; nothing here praises them.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

7. Answer: D · CROSS-TEXT CONNECTIONS · HARD

Correct answer: D. Text 2 argues that "flexible helical coiling and active zooid orientation allowed colonies to exploit micro-turbulent boundary layers," directly replacing passive vertical drifting.

Why the other choices are wrong:

- A) Text 2 discusses micro-turbulent boundary layers and fluid dynamics, but it does not claim that ocean currents were nonexistent in Ordovician epicontinental seas.
- B) Text 1 establishes that the primary nema suspended the rhabdosome; Text 2 challenges the passive suspension model via hydrodynamic experiments without claiming the nema served zero structural purpose.
- C) Text 2 describes colonial adaptations like flexible helical coiling and active zooid orientation for planktonic/boundary-layer exploitation, not that diplograptids were benthic organisms.

8. Answer: D · TEXTUAL EVIDENCE · HARD

Correct answer: D. It is correct because recording higher maximal aerobic sprint capacity measurements among athletes who completed cold immersion directly weakens the claim that cold-water protocols do not change sprint capacity.

Why the other choices are wrong:

- A) Baseline training volume predicts general athletic performance rather than testing the specific impact of cold immersion.
- B) Lower capacity in passive recovery does not test the direct effect of completing cold-water immersion protocols.
- C) Muscle fatigue across both groups reflects general exertion levels rather than the specific impact of cold immersion.

9. Answer: B · TEXTUAL EVIDENCE · MEDIUM

Correct answer: B. Field expeditions producing audio recordings of traditional harvest songs and festival melodies directly support Gandarillas's objective to record indigenous folk songs.

Why the other choices are wrong:

- A) Analyzing symphonic works of German composers contradicts the recording of indigenous Andean folk songs.
- C) Focusing acquisitions on classical European piano sheet music contradicts the collection of traditional Andean songs.
- D) Publications concentrated on urban jazz club history are unrelated to remote Andean folk music.

10. Answer: B · QUANTITATIVE EVIDENCE · HARD

Correct answer: B. The glutamate-rich core pathway kept 11.8 mV at 200 ms and 11.5 mV at 500 ms. The belt zone fell to 6.2 mV and 2.1 mV, so the rich pathway stayed stable while the deficient one collapsed.

Why the other choices are wrong:

- A) The belt zone's drop from 11.9 to 6.2 to 2.1 mV is accurate, but this describes only the glutamate-deficient pathway. It never shows that receptor-rich pathways maintain signal clarity.
- C) The control does have the lowest amplitude at each timepoint, but it is a somatosensory pathway. This does not address whether glutamate-rich auditory pathways maintain clarity.
- D) No pathway reached zero: the lowest value at 500 ms is 1.2 mV, and the core pathway is still at 11.5 mV. The claim contradicts the table.

11. Answer: C · INFERENCES · EASY

Correct answer: C. Scrub regions with prescribed burning recovered more strongly, so carefully designed interventions can sometimes increase natural areas' ability to withstand disruption.

Why the other choices are wrong:

- A) Says fire is the primary cause of decline worldwide, contradicted by the better recovery after prescribed burns.
- B) Says fire-free landscapes always recover better, the opposite of the finding.
- D) Says fire has little connection to shrub regions, though treated and untreated regions differed.

12. Answer: D · INFERENCES · HARD

Correct answer: D. The lanthanum-stabilized washcoat had a lower surface area loss rate, so it can inhibit thermal sintering under consistent conditions.

Why the other choices are wrong:

- A) Claims the comparison materials improve stability to a greater extent, reversing the finding that the tested material degraded more slowly.
- B) Claims the comparison materials have equal potential, but the finding shows a measured difference between the tested material and the others.
- C) Says the favored material would withstand stress for a shorter time, reversing the finding: a lower rate means slower degradation and so greater durability.

13. Answer: D · CENTRAL IDEAS AND DETAILS · MEDIUM

Correct answer: D. The passage states that the software improved accuracy by directing doctors' attention to unusual areas they might otherwise overlook.

Why the other choices are wrong:

- A) The software only highlighted areas in images for diagnosis; it did not treat patients after diagnosis.
- B) The passage says the software did not replace physicians, and the final decision stayed with trained medical professionals.
- C) The software highlighted existing images and did not create higher-resolution ones, so image quality is not the stated reason.

14. Answer: B · CENTRAL IDEAS AND DETAILS · MEDIUM

Correct answer: B. Mangroves trapped sediment, allowing coastlines to grow outward.

Why the other choices are wrong:

- A) Volcanic activity is never mentioned.
- C) Rivers continued carrying sediment.
- D) Wave height is unrelated.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

15. Answer: B • BOUNDARIES • MEDIUM

Correct answer: B. The clause before the blank is complete, so a colon can introduce the explanation that follows; a comma sets off 'though' as an interrupter, and the colon then introduces the elaborating phrase about Einstein's visionary role.

Why the other choices are wrong:

- A) A period splits the sentence, but 'Though,' followed by a comma then opens a new sentence awkwardly, misusing the transitional word and cutting off the contrast the passage intends.
- C) Without a comma before 'though' and without punctuation after it, the sentence runs two clauses together, producing a run-on that never separates the introductory idea from the explanation.
- D) A comma after 'though' cannot link the complete preceding clause to the clause that follows, so this creates a comma splice joining two independent clauses.

16. Answer: B • BOUNDARIES • MEDIUM

Correct answer: B. The semicolon correctly joins the two independent clauses, and the transitional adverb that follows is set off with a comma.

Why the other choices are wrong:

- A) A comma before the transitional adverb and a semicolon after it reverses the correct pattern, creating a comma splice and cutting the adverb off from the clause it introduces.
- C) A comma alone cannot join two independent clauses, so this creates a comma splice before the transitional adverb.
- D) With no punctuation between the two independent clauses, this creates a run-on sentence.

17. Answer: A • BOUNDARIES • EASY

Correct answer: A. The sentence before the blank is a complete independent clause and 'Trehalose is produced' is another, so a period is needed to mark the sentence boundary and avoid a run-on.

Why the other choices are wrong:

- B) Placing two independent clauses side by side with no punctuation creates a fused sentence (run-on).
- C) A comma alone cannot join two independent clauses, so this creates a comma splice.
- D) Using 'and' leaves 'is produced' without a clear subject in the compound phrase, producing an ungrammatical construction.

18. Answer: B • BOUNDARIES • EASY

Correct answer: B. The series items already contain commas, so a semicolon is needed to end the first item after "England", and a comma after "Burger's Daughter" then sets off its description, matching the structure of the other items.

Why the other choices are wrong:

- A) A comma after "England" does not separate the items, so the first and second items run together as a comma splice in a series whose items contain commas.
- C) The comma after "England" fails to separate the items, and the colon after "Burger's Daughter" wrongly interrupts the sentence before its description.
- D) The semicolon is right, but omitting the comma after "Burger's Daughter" leaves its description attached without the punctuation that sets it off.

19. Answer: C • FORM, STRUCTURE, AND SENSE • MEDIUM

Correct answer: C. "obliging" is a present participle that begins a modifying phrase showing the result of the main clause, so the sentence is grammatical after the comma.

Why the other choices are wrong:

- A) "obliged" is a past participle with a passive sense, which does not fit an active modifier describing the result of the main clause.
- B) "obliges" is a finite verb; placed after a comma with no conjunction it joins two independent clauses and creates a comma splice.
- D) The infinitive "to oblige" cannot introduce this modifier and leaves the final phrase grammatically unconnected to the main clause.

20. Answer: A • FORM, STRUCTURE, AND SENSE • HARD

Correct answer: A. "analyzed" is simple past tense, which matches the past-tense narrative and presents a completed action that follows the earlier events.

Why the other choices are wrong:

- B) "analyzes" is present tense and breaks the consistent past-tense narrative of the passage.
- C) "had analyzed" is past perfect, which signals an action completed before another past action, but the action here follows the earlier events in sequence.
- D) "was analyzing" is past progressive, which suggests an ongoing background action, but the text describes a discrete, completed step.

21. Answer: D • FORM, STRUCTURE, AND SENSE • MEDIUM

Correct answer: D. The introductory participial phrase must be followed by the noun it describes. The possessive noun phrase names the work that was recognized and that led to the later result, so the modifier is not left dangling.

Why the other choices are wrong:

- A) Opening with the person's name makes the introductory participial phrase modify a person instead of the work that was recognized, creating a dangling modifier.
- B) The introductory participial phrase ends up modifying the person named right after the comma rather than the work that was recognized, leaving a dangling modifier.
- C) The introductory participial phrase attaches to the person named as the subject, so the work that was recognized is not what the phrase describes, creating a dangling modifier.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

22. Answer: B · TRANSITIONS · HARD

Correct answer: B. "As a result" is correct because the sentence states the result of the process described in the previous sentence.

Why the other choices are wrong:

- A) "Subsequently" signals a time order or a step in a sequence, but the sentence states the result of the process just described.
- C) "Conversely" signals a contrast or concession, but the sentence states the result of the process just described.
- D) "Thus" signals a result or consequence, but the sentence states the result of the process just described.

23. Answer: B · TRANSITIONS · MEDIUM

Correct answer: B. "Consequently" is correct because the sentence states what followed from the historical situation described in the first sentence.

Why the other choices are wrong:

- A) "Alternatively" signals a contrast or concession, but the sentence shows the consequence of the situation just described.
- C) "For example" introduces an example, but the sentence shows the consequence of the situation just described.
- D) "Specifically" narrows the point to a specific detail, but the sentence shows the consequence of the situation just described.

24. Answer: B · TRANSITIONS · HARD

Correct answer: B. "Consequently" is correct because the sentence states what followed from the historical situation described in the first sentence.

Why the other choices are wrong:

- A) "Conversely" signals a contrast or concession, but the sentence shows the consequence of the situation just described.
- C) "For example" introduces an example, but the sentence shows the consequence of the situation just described.
- D) "Likewise" signals a similarity or a parallel, but the sentence shows the consequence of the situation just described.

25. Answer: A · RHETORICAL SYNTHESIS · HARD

Correct answer: A. Choice A highlights the shared timeframe of establishment close to one another (1590 CE and 1635 CE).

Why the other choices are wrong:

- B) It contrasts their specific primary functions (flora cultivation vs. medicinal teaching).
- C) It states general background information on European botanical gardens.
- D) It contrasts collection expansion sources and behaviors.

26. Answer: B · RHETORICAL SYNTHESIS · MEDIUM

Correct answer: B. This choice introduces the Gutenberg Bible by explaining what it was, who produced it, when it was created, and why it was historically important.

Why the other choices are wrong:

- A) This choice introduces Gutenberg's technology rather than introducing the Gutenberg Bible itself.
- C) This choice provides a detail about surviving copies but does not introduce the book or its significance.
- D) This choice explains the impact of printing technology but does not introduce the Gutenberg Bible.

27. Answer: A · RHETORICAL SYNTHESIS · MEDIUM

Correct answer: A. Present the study (Kunieda, 2019, *R. varieornatus*) AND its findings (TDPs form a glass-like matrix that preserves cellular structures during drying). A fulfills both parts completely.

Why the other choices are wrong:

- B) It provides general facts about tardigrades without mentioning the 2019 study or its findings.
- C) It states the goal/topic of the study but omits the actual finding (TDP glassification).
- D) It presents background context/uncertainty rather than the study's scientific finding.

Math 1 Math, Module 1: Explanations**1. Answer: A · LINEAR EQUATIONS (1 VAR) · MEDIUM**

After w weeks the account holds $240 + 12w$ dollars. Setting $240 + 12w = 600$ and subtracting 240 from both sides gives $12w = 360$. Dividing by 12 gives $w = 30$ weeks.

2. Answer: A · LINEAR FUNCTIONS · MEDIUM

Find $f(g(a)) = 4(3a + 2) - 3 = 12a + 8 - 3 = 12a + 5$. Set $12a + 5 = 41$, so $12a = 36$ and $a = 3$.

3. Answer: B · LINEAR FUNCTIONS · HARD

Shifting 6 units to the left replaces x with $x + 6$, and the vertical stretch multiplies the output by 4. So $g(x) = 4f(x + 6) = 4((1/2)(x + 6) - 3) = 4((1/2)x) = 2x$.

4. Answer: 7/2 or 3.5 · LINEAR EQUATIONS (2 VAR) · MEDIUM

Subtract $3x$ from both sides: $6y = -3x + 24$. Divide by 6: $y = -(1/2)x + 4$. So $m = -1/2$ and $b = 4$, and $m + b = 7/2 = 3.5$.

Common mistakes:

- $9/2$ (4.5) comes from using the slope $+1/2$ instead of $-1/2$, a sign error when moving $3x$ to the right side.
- 2 comes from using the slope -2 , the reciprocal, because the coefficient of y was divided by the coefficient of x .

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

5. Answer: 14 · SYSTEMS OF EQUATIONS · MEDIUM

Multiply the first equation by 12 to get $4x + 3y = 48$. Multiply the second by 4 to get $2x - y = 4$. From the second, $y = 2x - 4$. Substitute: $4x + 3(2x - 4) = 48$, so $10x = 60$ and $x = 6$. Then $y = 8$, and $x + y = 14$.

Common mistakes:

- 6 is the value of x alone.
- 8 is the value of y alone.
- 2 is $y - x$, the difference rather than the sum.

6. Answer: B · SYSTEMS OF EQUATIONS · HARD

Distribute in each equation. The first becomes $3x + 6 - 2y + 2 = 6$, so $3x - 2y = -2$. The second becomes $2x - 2 + 4y + 12 = 30$, so $x + 2y = 10$ after dividing $2x + 4y = 20$ by 2. Adding $3x - 2y = -2$ and $x + 2y = 10$ gives $4x = 8$, so $x = 2$. Then $2y = 8$, so $y = 4$.

7. Answer: A · LINEAR INEQUALITIES · EASY

Test each pair in both inequalities. For (2, 3): $3 \geq 2 + 1 = 3$ is true, and $2 + 3 = 5 \leq 9$ is true. So (2, 3) satisfies both inequalities.

8. Answer: 3 or 3.0 · EQUIVALENT EXPRESSIONS · MEDIUM

Factor the radicand into perfect cubes: $54x^7 = 27 \cdot 2 \cdot x^6 \cdot x = 27x^6 \cdot 2x$. Then $\sqrt[3]{27x^6} = 3x^2$, so $\sqrt[3]{54x^7} = 3x^2\sqrt[3]{2x}$. Therefore $a = 3$.

Common mistakes:

- Dividing 54 by 9 gives $a = 6$, but $54 = 27 \cdot 2$ and only the perfect cube 27 comes out as 3.
- Leaving the 27 inside the root as 27 gives $a = 27$, but a cube root of 27 is 3.
- Using $54 \div 3 = 18$ treats the cube root as division by 3, which is not correct.

9. Answer: A · EQUIVALENT EXPRESSIONS · HARD

Apply the power to each factor: $32^{3/5} = (32^{1/5})^3 = 2^3 = 8$, and $(x^{10})^{3/5} = x^6$. Dividing by $x^{1/2}$ subtracts the exponent: $x^{6-1/2} = x^{11/2}$. The expression is $8x^{11/2}$.

10. Answer: 6 or 6.0 · NONLINEAR EQUATIONS · HARD

A quadratic has exactly one real solution when its discriminant is 0. Here the discriminant is $k^2 - 4(1)(k + 3) = k^2 - 4k - 12$. Setting it to 0 gives $(k - 6)(k + 2) = 0$, so $k = 6$ or $k = -2$. Since k is positive, $k = 6$.

Common mistakes:

- -2 also makes the discriminant 0, but the problem states that k is positive, so it is rejected.
- 12 is the constant term in $k^2 - 4k - 12$, not a root of it. At $k = 12$ the discriminant is 84, so there are two real solutions.
- 3 does not make the discriminant 0. At $k = 3$ the discriminant is $9 - 24 = -15$, so there are no real solutions.

11. Answer: 4/3 or 1.333 · NONLINEAR EQUATIONS · MEDIUM

Since $x^2 - 9 = (x - 3)(x + 3)$, multiply both sides by $(x - 3)(x + 3)$: $(x + 3) + 2(x - 3) = 1$. This gives $3x - 3 = 1$, so $x = 4/3$. This value does not make any denominator zero, so it is a valid solution.

Common mistakes:

- Answer 1/3 comes from writing $2(x - 3)$ as $2x - 3$, which gives $3x = 1$.
- Answer 3 and answer -3 make the denominators zero, so they cannot be solutions.
- Answer 4 comes from the slip $3x - 3 = 1$ solved as $3x = 12$.

12. Answer: B · NONLINEAR EQUATIONS · MEDIUM

Subtract $3x + 24$ from both sides to get $2x^2 + 2x - 24 = 0$. Dividing by 2 gives $x^2 + x - 12 = 0$, which factors as $(x + 4)(x - 3) = 0$. The solutions are $x = -4$ and $x = 3$, so the positive solution is 3.

13. Answer: B · NONLINEAR FUNCTIONS · EASY

When $x = 0$, $y = 2$, so the constant term is 2. Testing $x = 1$: $1 - 2 + 2 = 1$. Testing $x = -1$: $1 + 2 + 2 = 5$, and $x = 2$: $4 - 4 + 2 = 2$. All four pairs fit.

14. Answer: C · NONLINEAR FUNCTIONS · EASY

The value $f(c)$ equals 0 exactly where the graph meets the x -axis. The graph crosses the x -axis at $x = -4$, $x = -1$ and $x = 2$. Among the choices, only $x = -1$ is such a point, so $f(-1) = 0$.

15. Answer: C · RATIOS AND RATES · EASY

Water to sugar is 5 to 2, so $5/2 = x/8$. Multiplying both sides by 8 gives $x = 5 \times 8 \div 2 = 20$ cups of water.

16. Answer: B · PERCENTAGES · HARD

The sale price is 80% of the marked price, so the marked price is $72/0.80 = \$90$. The marked price is 150% of the cost, so $c = 90/1.50 = \$60$.

17. Answer: B · TWO-VARIABLE DATA · EASY

The points slope downward from left to right: the temperature is about 22°C at a depth of 1 meter and about 8°C at 12 meters. So temperature tends to decrease as depth increases.

18. Answer: A · SAMPLE STATISTICS · MEDIUM

The sample was selected at random from card holders who live in the East District, so the results can be generalized to that group and not to other card holders or to people without cards.

19. Answer: C · AREA AND VOLUME · EASY

A rectangular box has three pairs of identical faces. The areas are $8 \times 5 = 40$, $8 \times 3 = 24$, and $5 \times 3 = 15$ square feet. The total surface area is $2(40 + 24 + 15) = 2(79) = 158$ square feet.

20. Answer: C · AREA AND VOLUME · MEDIUM

Let the width be w , so the length is $w + 6$ and $w(w + 6) = 91$. Then $w^2 + 6w - 91 = 0$, which factors as $(w + 13)(w - 7) = 0$. The width must be positive, so $w = 7$ and the length is 13. The perimeter is $2(7 + 13) = 40$ feet.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

21. Answer: C · RIGHT TRIANGLES AND TRIG · EASY

The diagonal is the hypotenuse of a right triangle with legs 24 and 7. By the Pythagorean theorem, the diagonal is $\sqrt{(24^2 + 7^2)} = \sqrt{(576 + 49)} = \sqrt{625} = 25$ meters.

22. Answer: A · RIGHT TRIANGLES AND TRIG · MEDIUM

The angle of depression from the top of the tower equals the angle of elevation from the tent to the top, so the triangle has a 24° angle at the tent. The tower height 60 is opposite this angle and the distance d is adjacent, so $\tan 24^\circ = 60/d$ and $d = 60/\tan 24^\circ$.

Math 2 Math, Module 2: Explanations**1. Answer: C · LINEAR EQUATIONS (1 VAR) · EASY**

Substitute $x = 6$: $5(6) + k = 38$. This gives $30 + k = 38$, so $k = 8$. So the value of k is 8. Check: $5(6) + 8 = 38$.

2. Answer: B · LINEAR FUNCTIONS · HARD

Here $f(x + 3) = 2(x + 3) - 3 = 2x + 3$ and $3f(x) = 3(2x - 3) = 6x - 9$. Setting $2x + 3 = 6x - 9$ gives $12 = 4x$, so $x = 3$.

3. Answer: B · LINEAR FUNCTIONS · HARD

Since $f(2) = 5$, $f(f(2)) = f(5) = 11$. The slope is $(11 - 5)/(5 - 2) = 2$. Moving from $x = 2$ to $x = 0$ decreases the output by $2(2) = 4$, so $f(0) = 5 - 4 = 1$.

4. Answer: -8/5 or -1.6 · LINEAR EQUATIONS (2 VAR) · MEDIUM

Solve for y : $-5y = -4x + 10$, so $y = (4/5)x - 2$. Thus $m = 4/5$ and $b = -2$, and $m \times b = (4/5)(-2) = -8/5$, or -1.6 .

Common mistakes:

- $8/5$ comes from taking $b = 2$, a sign error when dividing 10 by -5 .
- $-5/2$ comes from dividing b by m instead of multiplying them.
- $-6/5$ comes from adding m and b instead of multiplying them.

5. Answer: 9 · SYSTEMS OF EQUATIONS · MEDIUM

Multiply the first equation by 10 to get $6x + 5y = 51$, and the second by 12 to get $3x - 4y = 6$. Doubling the second gives $6x - 8y = 12$. Subtracting from the first gives $13y = 39$, so $y = 3$ and $x = 6$. Therefore $x + y = 9$.

Common mistakes:

- 3 is the value of y alone, but the question asks for $x + y$.
- 6 is the value of x alone, but the question asks for $x + y$.
- 3 is also $x - y = 6 - 3$; the sum is 9.

6. Answer: C · SYSTEMS OF EQUATIONS · HARD

Multiply the first equation by 2: $10x + 6y = 140$. Subtract the second equation: $8x = 88$, so $x = 11$. Then $55 + 3y = 70$, so $y = 5$. A hardcover book costs \$11 and a paperback costs \$5, so a hardcover costs \$6 more.

7. Answer: B · LINEAR INEQUALITIES · HARD

April's output was 1.05(840) kWh. Being at most 12% less than April means the May output is at least $100\% - 12\% = 88\%$ of April's output. So $m \geq 0.88(1.05)(840)$, which is $m \geq 776.16$. The inequality must use $\geq$ and multiply the two percent factors.

8. Answer: 11/12 or .9166 · EQUIVALENT EXPRESSIONS · HARD

Rewrite each radical with a rational exponent: $\sqrt[4]{x^3} = x^{3/4}$, $\sqrt[3]{x^2} = x^{2/3}$, and $\sqrt{x} = x^{1/2}$. Multiplying adds exponents and dividing subtracts: $3/4 + 2/3 - 1/2 = 9/12 + 8/12 - 6/12 = 11/12$.

Common mistakes:

- $23/12$ comes from adding all three exponents, treating the division by $\sqrt{x}$ as multiplication.
- $17/12$ comes from adding $3/4$ and $2/3$ correctly but forgetting to subtract the $1/2$ for the square root in the denominator.
- $4/5$ comes from combining numerators and denominators separately, $(3 + 2 - 1)/(4 + 3 - 2)$, instead of finding the common denominator 12 first.

9. Answer: A · EQUIVALENT EXPRESSIONS · HARD

Subtract $1/R_2$ from both sides: $1/R_1 = 1/R - 1/R_2$. Combine over the common denominator $R \cdot R_2$: $1/R_1 = (R_2 - R)/(R \cdot R_2)$. Taking the reciprocal of both sides gives $R_1 = (R \cdot R_2)/(R_2 - R)$.

10. Answer: 6/5 or 1.2 · NONLINEAR EQUATIONS · HARD

Split into two cases. If $2x - 5 = 3x - 1$, then $x = -4$; but then $3x - 1 = -13$ is negative and cannot equal an absolute value, so reject it. If $2x - 5 = -(3x - 1)$, then $5x = 6$ and $x = 6/5$, where both sides equal 3.4. The only solution is $6/5$.

Common mistakes:

- $x = -4$ is an extraneous solution: it comes from the first case, but the right side $3x - 1$ would be -13 , which an absolute value cannot equal.
- Reporting both -4 and $6/5$ skips the check of each solution; the equation has only one valid solution.

11. Answer: 2 · NONLINEAR EQUATIONS · HARD

Since $9^x = (3^x)^2$ and $3^{x+1} = 3 \cdot 3^x$, let $u = 3^x$. Then $u^2 - 3u = 54$, so $u^2 - 3u - 54 = 0$ and $(u - 9)(u + 6) = 0$. Because 3^x is always positive, $u = 9$, so $3^x = 9$ and $x = 2$. Check: $9^2 - 3^3 = 81 - 27 = 54$.

Common mistakes:

- 9 is the value of 3^x , not of x ; solving $3^x = 9$ gives $x = 2$.
- -6 is the other root for u , which is rejected because 3^x cannot be negative.
- 3 comes from solving the equation $3^{x+1} = 54$ incorrectly after ignoring the 9^x term.

12. Answer: A · NONLINEAR EQUATIONS · HARD

The equation holds when $x + 3 = 2x - k$, giving $x = k + 3$, or when $x + 3 = -(2x - k)$, giving $x = (k - 3)/3$. It has exactly one solution when these two values are equal: $k + 3 = (k - 3)/3$, so $3k + 9 = k - 3$ and $k = -6$. Then the only solution is $x = -3$.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →

13. Answer: B · NONLINEAR FUNCTIONS · EASY

The flare hits the ground when $h(t) = 0$. Dividing $-16t^2 + 48t + 160 = 0$ by -16 gives $t^2 - 3t - 10 = 0$, which factors as $(t - 5)(t + 2) = 0$. The solutions are $t = 5$ and $t = -2$, and only $t = 5$ is a valid time.

14. Answer: C · NONLINEAR FUNCTIONS · MEDIUM

A rational function is undefined where its denominator equals 0. Factoring gives $x^2 - 2x - 15 = (x - 5)(x + 3)$, which is 0 when $x = 5$ or $x = -3$. Of the choices, only $x = 5$ makes the denominator 0.

15. Answer: D · RATIOS AND RATES · HARD

The press fills $36 \div 45 = 0.8$ bottles per second. Two hours is $2 \times 60 \times 60 = 7,200$ seconds, so it fills $0.8 \times 7,200 = 5,760$ bottles.

16. Answer: B · PERCENTAGES · MEDIUM

The acid in the mixture is $0.20(12) + 0.45(8) = 2.4 + 3.6 = 6$ liters. The total volume is $12 + 8 = 20$ liters. So the mixture is $6/20 = 0.30$, or 30% acid.

17. Answer: C · PROBABILITY · MEDIUM

Line X has $0.15 \times 2,400 = 360$ contaminated items and Line Y has $0.20 \times 1,600 = 320$. The total is 680 contaminated items out of 4,000, so the probability is $680/4,000 = 0.17$.

18. Answer: B · STATISTICAL CLAIMS · HARD

If employees with severe back pain avoided standing desks, the group with standing desks would have less back pain from the start. The difference could then reflect who requested the desks rather than the effect of the desks.

19. Answer: B · AREA AND VOLUME · MEDIUM

The diagonals split the rhombus into four congruent right triangles, so the area is $(1/2)(AC)(BD) = 120$, which gives $AC = 24$. Each side is the hypotenuse of a right triangle with legs 12 and 5, so it is 13. The perimeter is $4 \times 13 = 52$ inches.

20. Answer: B · LINES, ANGLES, TRIANGLES · MEDIUM

The triangles are similar because the sun's rays strike at the same angle, so height/shadow is the same for both: $6/9 = h/54$. Then $h = 54 \times 6/9 = 36$. The silo is 36 feet tall.

21. Answer: B · RIGHT TRIANGLES AND TRIG · MEDIUM

By the Pythagorean theorem, $JK = \sqrt{(29^2 - 20^2)} = \sqrt{441} = 21$. The cosine of J is the adjacent side JK divided by the hypotenuse JL, so $\cos J = 21/29$.

22. Answer: B · RIGHT TRIANGLES AND TRIG · HARD

For acute angles, $\sin \alpha = \cos (90^\circ - \alpha)$, so the two angles must add to 90° . Then $(7x + 2) + (2x + 7) = 90$, so $9x + 9 = 90$ and $x = 9$. Check: $\sin 65^\circ = \cos 25^\circ$.

Want a plan built around **your** student?

Book a free consultation. The short intake takes about a minute and opens right away.

BOOK NOW →